

# **Coordination Mechanisms for Seamless Operation of Interconnected Power Systems**

Final Project Report

M-38

Power Systems Engineering Research Center Empowering Minds to Engineer the Future Electric Energy System

# Coordination Mechanisms for Seamless Operation of Interconnected Power Systems

**Final Project Report** 

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**PSERC** Publication 20-05

October 2020

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# Acknowledgements

We wish to thank our industry advisors whose help has been invaluable to the project. Our advisors include Jim Price (CAISO), Yohan Sutjandra (TEA), Anupam Thatte (MISO), Harvey Scribner (SPP), Jay Caspary (SPP), M.Gary Helm (PJM), Jianzhong Tong (PJM), Khosrow Moslehi (ABB), Mirrasoul Mousavi (ABB), Feng Zhao (ISO-NE), Tongxin Zheng (ISO-NE), Di Shi (GEIRI N. America), Qin Wang (EPRI), Erik Ela (EPRI), Nikita Singhal (EPRI), Evangelos Farantatos (EPRI), Miguel Ortega-Vasquez (EPRI).

We also thank Ye Guo (former, Cornell) and Kursat Mestav (Cornell) for their contributions on the project.

# **Executive Summary**

Parts of an interconnected power system are often managed by different independent system operators (ISOs). Each ISO operates wholesale electricity markets within its footprint (henceforth called an area). This project focuses on coordination mechanisms to increase the benefits of interchange among the players in wholesale markets of multiple areas that are connected via tie lines. Specifically, this final report analyzes the existing coordination mechanisms, offers algorithmic innovations to enable such mechanisms, and suggests systematic modifications to improve these mechanisms.

Why is coordination among areas important? Tie lines that interconnect different areas often have adequate transfer capabilities to cover a significant portion of the electricity demand in each area. However, tie lines are often underutilized or not used in an economically efficient manner. Power flows scheduled over tie lines often occur from an area with higher prices to an area with lower prices. Also, tie line schedules often fail to eliminate price differences between border buses of interconnected areas. Such schedules lead to large economic losses. For example, losses due to lack of coordination between ISO-NE and NYISO have been estimated to be \$784 million from 2006 to 2010, according to White and Pike in their 2011 report. Effective coordination among ISOs may lead to significant cost savings. Indeed, the final report of this project provides significant tools to allow ISOs to meet such goals.

Clearly, these inefficiencies in tie-line scheduling can be largely eliminated by the integration of market operations in different areas. However, such a fusion remains untenable in the near future, given the significant regulatory and legal structures of the wholesale markets that have separately developed in the different areas over more than two decades. Absent the possibility of an integrated market, the burden falls on coordination mechanisms to remove, or at least, minimize economic inefficiencies in scheduling power flows over tie lines. This project proposes schemes to improve coordination based on the insights obtained from the economic analysis of the main causes of inefficiencies and prescribes methods to efficiently overcome these limitations.

ISOs and market monitors have long recognized the need for comprehensive restructuring of tieline scheduling processes. Early research efforts sought algorithmic frameworks that allow ISOs of interconnected neighboring areas to compute a tie-line schedule that forms a part of a jointly optimal dispatch in a distributed fashion. The distributed nature of the algorithm seeks to obtain convergence to a jointly optimal dispatch in an iterative fashion without necessarily sharing all information from one area with the other. These efforts under the title of "Tie Optimization" (TO), however, were never realized in practice. Such a mechanism was perceived as one ISO taking a financial position in the other's markets, running counter to the financial neutrality of the ISOs. Such a mechanism also defined too far a departure from the earlier market-based, albeit inefficient, mechanism to determine tie-line flows. Multiple pairs of ISOs ultimately adopted a different market-based mechanism called "Coordinated Transaction Scheduling" (CTS) that even received approval by FERC. This project analyzes the main sources of inefficiencies in CTS, suggests both market innovations and algorithmic tools to tackle these inefficiencies. Specifically, the research identifies inefficiencies due to limited trading locations for CTS, market liquidity and strategic interaction of market participants, as well as those emanating from the sources of uncertainty in demand and supply conditions at the time of scheduling.

This report includes the results of efforts in three distinct thrusts:

A. Game-theoretic analysis of CTS markets—equilibrium and learning. The CTS mechanism involves market participants that compete to transport power from one area to the other. The obligation on the bidders is purely financial in that they are not responsible for the physical delivery of power, even though their bids define the tie-line schedule and they are paid based on the realized price spread between the proxy buses of neighboring areas. Naturally, the efficiency of the CTS market critically depends on the strategic interaction among the market participants. This work analyzes how the strategic incentives of market participants may be aligned with market efficiency. The analysis reveals several important insights. First, lack of liquidity has a strong impact on CTS market efficiency. Second, ISOs' forecasts of realized price spread play a key role in the CTS market outcomes. CTS bidders have some ability to correct systematic forecast errors by ISOs, but the quality of the forecasts largely dictates the outcomes. Third, transaction fees negatively impact CTS market outcome and can further hamper liquidity of the CTS market over the long run. The results from the game theoretic analysis are corroborated by simulation results obtained under the condition that bidders learn to bid in a realistic market environment.

*B. Algorithms for tie-line scheduling under uncertainty.* Tie-line schedules are always determined with a lead-time to power delivery, leading to uncertainty in demand and supply conditions during the scheduling process. This work develops a distributed solution architecture for robust tie-line scheduling. The technique draws on multi-parametric linear programming and defines an algorithmic framework to solve large distributed linear programs applicable to tie-line scheduling. The algorithm converges to an optimal tie-line schedule within a few iterations and does not require ISOs to reveal their sensitive information on the dispatch cost structures, the network constraints or the nature of their uncertainty sets to arrive at the optimal solution.

C. Generalized CTS market design to tackle limited trading locations. The CTS mechanism in practice allows bidders to trade power only between two "proxy" bus locations within neighboring areas. As a result, this mechanism does not accurately reflect the physics of the power flows over multiple tie-lines that may exist between two areas. A similar problem arises when three areas are interconnected via tie-lines. To overcome this so-called "loop flow" phenomenon in tie-line scheduling, an easy-to-implement modification of the CTS mechanism is proposed that allows CTS market participants to offer the delivery of power from any border bus in one area to any border bus in a neighboring interconnected area. When the CTS market has a sufficiently large number of bidders, the proposed mechanism is shown to completely correct the loop flow issue.

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- [2] Y. Ji, T. Zheng, and L. Tong, "Stochastic interchange scheduling in the real-time electricity market", IEEE Transactions on Power Systems. 32.3, pp. 2017-2027, 2017.
- [3] Y. Guo, L. Tong, W. Wu, B. Zhang, and H. Sun, "Coordinated multi-area economic dispatch via critical region projection", IEEE Transactions on Power Systems, 32.5, pp. 3736-3746, 2017.
- [4] Y. Guo, S. Bose, and L. Tong, "On robust tie-line scheduling in multi-area power systems," IEEE Transactions on Power Systems, 33.4, pp. 4144-4154, 2018.

- [5] Y. Guo, Y. Ji, and L. Tong, "Generalized Coordinated Transaction Scheduling: A Market Approach to Seamless Interfaces," IEEE Transactions on Power Systems, 33.5, pp. 4683-4693, 2018.
- [6] M. Ndrio, S. Bose, Y. Guo, and L. Tong, "Coordinated Transaction Scheduling in Multi-Area Power Systems with Strategic Participants," Proceedings of the IEEE PES General Meeting, 2019.
- [7] M. Ndrio, S. Bose, L. Tong, and Y. Guo, "Coordinated Transaction Scheduling in Multi-Area Power Systems: Equilibrium and Learning," In preparation, 2020.

# **Student Theses:**

- [1] Yuting Ji. "Operation under Uncertainty in Electric Grid: A Multiparametric Programming Approach". Ph.D. thesis. Cornell University. January 2017.
- [2] Ogün Yurdakul. "Analysis and Performance Evaluation of Coordinated Transaction Scheduling". M.S. thesis. University of Illinois at Urbana-Champaign. May 2018.
- [3] Mariola Ndrio. "Equilibrium and Learning in Electricity Markets: A Scalar-Parameterized Supply and Demand Function Approach". Ph.D. thesis. University of Illinois at Urbana-Champaign. January 2021.

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# 1. Introduction

The final report for the PSERC project M-38 collects our work on coordination mechanisms that exist among system operators (SOs) to schedule power flows over tie-lines. These tie-lines interconnect parts of the grids controlled by different SOs. We call the footprint of each SO an area in the sequel. The motivation behind this project lies in the gross inefficiencies that plagued coordination mechanisms for tie-line scheduling. The three following chapters tackle analysis of the current mechanism for tie-line scheduling, suggested modifications to how these markets are run, and novel algorithmic frameworks to compute tie-line schedules. The chapter descriptions are given below.

- Chapter 2 provides a game-theoretic analysis of Coordinated Transaction Scheduling (CTS) markets. The CTS mechanism involves profit-motivated market participants. In this work, we analyze if the strategic incentives of the CTS market participants are correctly aligned towards reducing the price spread between border buses in the two areas. We show that lack of liquidity, SOs' forecasts of realized price spread and transaction fees have a strong impact on the CTS market outcome and hence, its efficiency. The results from the game theoretic analysis coincide with simulations where bidders learn to bid in such a market environment.
- Chapter 3 develops novel algorithms for tie-line scheduling under uncertainty. The technique is inspired by interesting insights from multi-parametric linear programming and defines an algorithmic framework that is able to efficiently solve large distributed linear programs with applications to tie-line scheduling. The algorithm is shown to converge quite fast to an optimal tie-line schedule and does not require SOs to reveal critical information such as dispatch cost structures, network constraints, or natures of uncertainty sets to arrive at the optimal solution.
- Chapter 4 suggests a generalization to CTS market design to tackle an important limitation of the CTS mechanism-the bidders can only trade power between two "proxy" bus locations within neighboring areas, resulting in mismatch between scheduled power flows and the physics of power flows. A similar problem arises when three areas are interconnected via tie-lines. Our easy-to-implement modification is shown to circumvent this limitation when the number of traders in the CTS market is large enough.

# 2. Coordinated Transaction Scheduling: Equilibrium and Learning

#### 2.1 Introduction

Conceptually, power flows over tie-lines should be determined through a joint economic dispatch problem geared towards maximizing the efficiency of the interconnected power grid as a whole. However, historical and legal reasons render such an aggregation of market information from different areas at a central location untenable. Naturally, a considerable effort has been made to solve the joint dispatch problem in a distributed fashion, focusing on primal [2, 3] and dual decomposition methods [4, 5]. In such methods, SOs exchange information among themselves to compute the optimal tie-line schedule. This theoretical coordination mechanism, referred to as Tie Optimization (TO) in [6], proved challenging to implement in practice. It was perceived as requiring the SOs to trade directly with each other, violating their financial neutrality, in lieu of the earlier market-based, albeit inefficient, process for scheduling tie-line flows. Instead, many SOs adopted variants of Coordinated Transaction Scheduling (CTS), e.g., see [7,8], that sought to blend the earlier market-based tie-line scheduling with the theoretically optimal TO, after receiving approval from the Federal Energy Regulatory Commission (FERC). CTS is a market mechanism in which external market participants submit bids and offers to import or export from one area to the other. The schedule is computed using both participants' offers as well as the SOs' forecasts about price differences. CTS market design is predicated on the simple premise that arbitrage opportunity will attract more participants, whose profit motivation will ultimately shrink that opportunity, pushing the schedule closer to the theoretically optimum. CTS has certainly improved tie-line scheduling as per [9, 10], but significant inefficiencies remain. Motivated by these inefficiencies, we analyze the impacts of strategic interactions among CTS market participants on the performance of these markets through a game theoretic study. We provide palpable insights on the consequences of an illiquid market, errors in SOs' price forecasts and transaction fees on market efficiency. We remark that the use of proxy buses as trading locations results in the so-called 'loop flow' problem (see [11]) that negatively impacts CTS market performance. We refer the reader to [12] for mechanisms to tackle this problem, and instead focus on the repercussions of strategic interaction among market participants in this chapter.

We introduce CTS in Section 2.2. Then, we model CTS as a game among arbitrage bidders who compete through scalar-parameterized transport offers in Section 2.3. Our game formulation is inspired by supply function competition models considered in [13, 14]. We establish the existence of Nash equilibria for this game under mild assumptions and study the impact of various factors on the nature of said equilibria in Sections 2.4-2.6 to offer insights into the CTS market. First, we show that when transaction costs (levied on a per-megawatt hour basis on bidders) are absent, then a highly liquid CTS market is efficient. Market efficiency degrades with liquidity shortfall, exhibiting bounded efficiency loss for intermediate liquidity and unbounded losses in low liquidity regimes. Second, with transaction costs, CTS fails to eradicate the price spread between adjacent

markets even with a liquid market, implying that such costs undercut the vision behind the market design. Third, we show that SOs' estimate of the price spread plays a central role in the efficiency of CTS markets in that bidders have limited ability to correct the effects of SOs' forecast errors. Fourth, portfolios of financial transmission rights (FTR) held by CTS bidders can impact CTS market outcomes, revealing the dependency of the efficiency of these inter-area markets on other energy derivatives. Our equilibrium analysis reveals how the strategic incentives in CTS markets are oriented but does not illustrate if bidders can learn equilibrium behavior through repeated participation in these markets. We simulate repeated play using historical data from the NYISO–ISO-NE market and demonstrate that our conclusions from equilibrium analysis continue to hold in a statistical sense in our numerical experiments. Proofs are omitted for brevity and can be found in [15].

#### 2.2 The CTS mechanism

CTS is a market-based mechanism for tie-line scheduling that replaced an earlier market-based structure in an effort to streamline the bidding and scheduling process. Among the important changes, CTS unified the bid submission and clearing process among the neighboring SOs, reduced the tie-line schedule duration from one hour to 15-minute intervals, and decreased time delays among bidding, scheduling, and power delivery. To illustrate the economic rationale of the CTS mechanism we consider throughout a stylized two-area power system in Figure 2.1 connected via a single tie-line with the inter-area power flow denoted by Q. Each SO computes their supply stacks by solving an area-wise parametric economic dispatch by varying the amount of power Q flowing on the tie-line. An example of supply stack is shown in Figure 2.1. The stack of area a represents the incremental dispatch cost of delivering power at its side of the interface. Similarly, the stack of area b represents the decremental dispatch cost of reduced supply, shown in descending order. Since scheduling happens prior to power delivery, these stacks are based on SOs' forecasts. In this example, the optimal direction for the power flow is from area a to b since for zero scheduled flow, area b operates at higher dispatch costs than area a. At the level where dispatch costs at the border become equal or where the supply stacks intersect, is the TO schedule, denoted by  $Q_{TO}$ . This tie-line schedule minimizes the aggregate dispatch costs across the two areas and it serves as our theoretical benchmark to compare CTS performance. However, TO requires SOs to trade directly with each other on behalf of the market participants in their respective areas, which may be viewed as the SOs becoming active participants of trade rather than financially neutral market operators. Instead, the current practice of CTS relies on virtual traders whose offers/bids are utilized together with the supply stacks to arrive at the tie-line schedule.

CTS market participants in practice submit 'interface' bids that consist of three elements: the minimum price difference between the proxy buses in the two areas the bidder is willing to accept, the maximum quantity to be transacted and the direction of the trade, i.e., the source and the sink. A CTS market participant is a virtual bidder in that she can offer to transport power across areas without physically consuming or producing it. They only participate in the scheduling process, bearing no obligation for physical power delivery; the transaction is purely financial.



Figure 2.1: Illustration of the TO and CTS scheduling mechanisms.

Under CTS, one of the SOs pools the virtual bids at the proxy buses and the supply stacks from both operators to assemble the aggregate interface supply stack, shown in Figure 2.1. All the bids indicating the optimal direction are stacked from lowest to highest price to create their own "supply curve". The price spread curve is derived by subtracting the supply stack of area *a* from that of area *b*. The CTS schedule, denoted by  $Q_{CTS}$ , is set at the intersection of the interface supply stack and the price spread. An interface bid is accepted if its offer price is less than the price spread at the tie-line schedule. Therefore, all interface bids to the left of the CTS schedule are accepted; all bids to the right are not. All cleared interface bids are settled at the real-time proxy bus LMPs and there are not uplift credits or debits associated with tie-line schedules. In the next section, we extract a theoretical model for CTS and characterize its outcome under strategic interactions of interface bidders against the outcome of TO.

### 2.3 Modeling the CTS market as a game

We model the CTS market as a game among the virtual bidders who compete to transport power over a tie-line against an elastic inter-area price spread that varies with the power flow over the tie-line.<sup>1</sup> Recall that LMPs at the proxy bus in each area comes from the solution to an area-wise economic dispatch problem, parameterized by the tie-line power flow Q. For areas a and b, denote these LMPs by  $\mathcal{P}_a(Q)$  and  $\mathcal{P}_b(Q)$ , respectively. Without loss of generality, let area a export and

<sup>&</sup>lt;sup>1</sup> The study presented in [6] indicates that the primary interface between NYISO–ISO-NE typically operates at far less than its total transmission capacity (TTC). Specifically, the tie-line is congested 0.3% of the hours eastbound and 1.2% of the hours westbound in 2009. Hence, to avoid uneccessary complication of the analysis and facilitate exposition, we ignore the TTC of the tie-line in modeling the CTS game.

area b import power, and define

$$\mathscr{P}(Q) := \mathscr{P}_b(Q) - \mathscr{P}_a(Q) \tag{2.1}$$

as the price spread between the areas. Assume without loss of generality that  $\mathscr{P}$  is strictly decreasing, concave and differentiable in  $Q \ge 0$  with  $\mathscr{P}(0) > 0$ . Note that  $\mathscr{P}$  acts as the inverse demand function in a supply competition model with virtual bidders. Our framework adopts standard assumptions on the demand function that are employed in several supply function competition models to study electricity markets [16–19].

Consider N virtual bidders in the CTS market. Let the *i*-th bidder provide two parameters  $\theta_i$ ,  $B_i$  to the SOs with the understanding that she is willing to transport up to

$$x_i(p) := B_i - \frac{\theta_i}{p}, \quad \theta_i \ge 0$$
(2.2)

amount of power from area *a* to *b* at a price spread of *p*. Our transport offer is inspired by supply function competition models studied in [13, 14, 20]. Figure 2.2 reveals how the parameters  $\theta_i, B_i$  affect the shape of the transport offer. Bidder *i* is willing to transport a maximum quantity of  $B_i$ , but at a minimum price spread of  $\theta_i/B_i$ . The required price difference increases with the power transport and grows unbounded as the latter approaches  $B_i$ . In effect, transporting power above  $B_i$  requires an infinite price difference. The parameterized "hockey-stick" shaped transport offer in (2.2) is a smooth approximation to the one in practice where a player is willing to transport up to  $B_i$  at a specified price difference. The realized price spread is uncertain and a higher  $B_i$  exposes the player to a higher potential loss. Therefore, bidder *i* expresses her total budget for potential losses or her *liquidity* in  $B_i$ . Notice that since  $B_i$ 's express budget constraints for the bidders, we assume  $B_i$  does not vary strategically in day-to-day transactions.



Figure 2.2: Parameterized interface bid of CTS market participant.

The family of transport offers in (2.2) allows market participants to have one-dimensional action spaces and has been shown to posses a number of attractive properties including bounded price of anarchy and price markup at the Nash equilibrium [13, 14]. Moreover, they prohibit situations when market participants can bid above their means by explicitly incorporating maximum

budget/capacity in the offer structure, which is not straightforward to do with e.g. linear supply functions [17]. Other families of supply offers such as the (degenerate) pure price (Bertrand) or quantity (Cournot) competition are not suitable representations of the CTS interface bid. Although variations of the Bertrand model with capacity constraints may seem an attractive approach, however, in such settings pure Nash equilibria may not exist [21].

Given the liquidities  $B_1, \ldots, B_N$ , the choice of bids  $\theta_1, \ldots, \theta_N$  from the CTS bidders describe their willingness to transport power across the interface according to (2.2). Collect the liquidities and bids in  $\theta$  and B, respectively. The SOs allocate the aggregate tie-line schedule among N virtual bidders, given  $\theta, B$  as follows. They calculate  $x^* := (x_1^*, \ldots, x_N^*)$  as the allocations of the tie-line flow to the participants by solving

$$\boldsymbol{x}^{*}(\boldsymbol{\theta},\boldsymbol{B}) := \underset{\boldsymbol{x}\in\mathbb{R}^{N}}{\operatorname{argmax}} \quad \int_{0}^{\mathbb{1}^{\top}\boldsymbol{x}} \mathscr{P}(z)dz - \sum_{i=1}^{N} \int_{0}^{x_{i}} \frac{\boldsymbol{\theta}_{i}}{B_{i}-s}ds,$$
(2.3a)

subject to  $0 \le x_i \le B_i$ ,  $i = 1, \dots, N$ , (2.3b)

where,  $\mathbb{R}$  denotes the set of real numbers and  $\mathbb{1}$  is a vector of all ones. The above problem seeks a tie-line schedule where the offer stack for inter-area power transport from CTS market participants intersects the SOs' estimated price spread function. The tie-line schedule is then given by

$$Q_{\mathsf{CTS}}(\boldsymbol{\theta}, \boldsymbol{B}) := \mathbb{1}^{\mathsf{T}} \boldsymbol{x}^*(\boldsymbol{\theta}, \boldsymbol{B}).$$

The transport offer in (2.2) enters the SOs' problem through its implied cost of transport. This induced cost is calculated by equating the implied marginal cost curve to the transport offer. With this interpretation, the SOs' flow allocation problem in (2.3) seeks to maximize the social welfare of an economy that is composed of the wholesale markets in areas *a* and *b* together with the CTS bidders (see [12] for a similar interpretation of the CTS market objective). CTS identifies a single clearing price  $p(\theta, B)$  for its market as

$$p(\boldsymbol{\theta}, \boldsymbol{B}) = \mathscr{P}(Q_{\mathsf{CTS}}(\boldsymbol{\theta}, \boldsymbol{B})). \tag{2.4}$$

The market price as well as the CTS flow allocation to each participant depends on the liquidity of the market and how it compares to the maximum inter-area demand. When  $\mathbb{1}^T B < \mathscr{P}^{-1}(0)$ , the CTS flow allocation to bidder *i* and the resulting CTS tie-line schedule are respectively given by

$$x_i^*(\boldsymbol{\theta}, \boldsymbol{B}) = B_i - \frac{\theta_i}{p(\boldsymbol{\theta}, \boldsymbol{B})}, \qquad Q_{\mathsf{CTS}}(\boldsymbol{\theta}, \boldsymbol{B}) = \mathbb{1}^{\mathsf{T}} \boldsymbol{B} - \frac{\mathbb{1}^{\mathsf{T}} \boldsymbol{\theta}}{p(\boldsymbol{\theta}, \boldsymbol{B})}.$$
 (2.5)

On the other hand, when  $\mathbb{1}^{\mathsf{T}} \mathbf{B} \geq \mathscr{P}^{-1}(0)$ , we have  $p(\boldsymbol{\theta}, \mathbf{B}) > 0$  when  $\mathbb{1}^{\mathsf{T}} \boldsymbol{\theta} > 0$  and  $p(\boldsymbol{\theta}, \mathbf{B}) = 0$  for  $\mathbb{1}^{\mathsf{T}} \boldsymbol{\theta} = 0$ . To avoid difficulties due to a zero price, we adopt the convention

$$x_i(0, \boldsymbol{B}) = B_i$$
, for  $p(\boldsymbol{\theta}, \boldsymbol{B}) = 0$ .

In this case, liquidities of the players allow a tie-line schedule higher than  $\mathscr{P}^{-1}(0)$ . Current practice, however, rules out such a possibility. Hence, when  $\mathbb{1}^{\mathsf{T}}\theta = 0$  we define the proportionate flow allocation to bidder *i* 

$$x_i^*(\boldsymbol{\theta}, \boldsymbol{B}) = \frac{B_i}{\mathbb{1}^T \boldsymbol{B}} \mathscr{P}^{-1}(0).$$
(2.6)

This derivation allows a quick insight into when we expect CTS to emulate the SO-driven TO. Within our notational framework, TO determines the tie-line schedule as

$$Q_{\mathsf{TO}} = \operatorname*{argmax}_{Q \ge 0} \mathscr{W}(Q) := \int_0^Q \mathscr{P}(z) dz, \qquad (2.7)$$

that seeks to maximize  $\mathcal{W}$ , a measure of welfare for the wholesale markets in areas *a* and *b*.  $Q_{\text{TO}}$  is given by the schedule determined by the no-arbitrage condition, i.e., where the price spread vanishes. Notice that  $\mathcal{W}$  equals the CTS market objective in (2.3) with  $\theta = 0$ . Thus, we expect CTS to emulate TO only when all bidders bid zero  $\theta$ 's. We now proceed to formally define the CTS game and characterize its Nash equilibrium to understand what bidding behavior we expect, given the bidders' strategic incentives.

While virtual bidders do not incur any costs to physically transport power, many pairs of SOs levy transaction fees on a per-MWh basis, e.g., in CTS between NYISO and PJM, NYISO charges physical exports to PJM at a rate ranging from \$4-\$8 per MWh, while PJM charges physical imports and exports rates that average less than \$3 per MWh. See [10] for details. For a willingness to transport  $x_i$  MW of power from area *a* to *b*, assume that transaction cost equals  $c \cdot x_i$ , where *c* is measured in \$/MWh. Then, each bidder's payoff equals the total revenue garnered less the transaction costs, formally given in

$$\pi_i(\theta_i, \theta_{-i}) = \mathscr{P}(Q_{\mathsf{CTS}}(\theta, B)) x_i^*(\theta, B) - c x_i^*(\theta, B).$$
(2.8)

Formally, define  $\mathscr{G}(\boldsymbol{B},c)$  as the CTS game among *N* virtual bidders—henceforth referred to as players—who bid  $\boldsymbol{\theta} \geq 0$ , given *B*, and receive a payoff described by (2.8). Bidders selfishly seek to maximize their own payoffs, given their liquidities. A bid profile  $\boldsymbol{\theta}^{\mathsf{NE}}$  constitutes a *Nash equilibrium* of  $\mathscr{G}(\boldsymbol{B},c)$ , if

$$\pi_i\left(\theta_i^{\mathsf{NE}}, \theta_{-i}^{\mathsf{NE}}\right) \geq \pi_i\left(\theta_i, \theta_{-i}^{\mathsf{NE}}\right)$$

for all  $\theta_i \ge 0$ . That is, no player has an incentive for a unilateral deviation from the equilibrium offer. We establish the existence of such an equilibrium profile in our first result.

**Theorem 1** (Existence of Nash Equilibrium). *The CTS game*  $\mathscr{G}(B,c)$  *admits a Nash equilibrium if*  $\mathscr{P}$  *satisfies* 

$$\frac{\partial^2 \mathscr{P}(Q)}{\partial Q^2} (\mathbb{1}^{\mathsf{T}} B - Q) \ge 2 \frac{\partial \mathscr{P}(Q)}{\partial Q}$$
(2.9)

for  $0 \leq Q \leq \mathbb{1}^{\mathsf{T}}B$ .

The proof relies on Rosen's result in [22]. Uniqueness of the equilibrium remains challenging to prove. However, in the next sections, we establish uniqueness of the Nash equilibrium under specific price spread functions.

To explicitly characterize the Nash equilibrium we restrict our attention to affine price spread

$$\mathscr{P}(Q) := \alpha - \beta Q \tag{2.10}$$

with  $\alpha, \beta > 0$  to compute the equilibria and study its properties. It is straightforward to verify that  $\mathscr{P}$  as defined above satisfies (2.9) and hence, an equilibrium always exists for  $\mathscr{G}(B, c, \alpha, \beta)$ , according to Theorem 1. Indeed, the price spread can be shown to be affine in Q, when each area is represented as a copperplate power system, having a generator with quadratic generation cost and a fixed demand. This follows from properties of multiparametric quadratic programs in [23, Theorem 7.6]. To further justify our modeling choice, we perform a linear regression of New England's LMP at the proxy bus (Roseton) as  $\mathscr{P}_{NE} = w_1 \mathscr{P}_{NY} + w_2 Q + w_3$ , and obtain  $w_1 \approx 1.0$  with an adjusted  $R^2$  coefficient of 0.91, revealing an affine dependency of  $\mathscr{P}_{NY} - \mathscr{P}_{NE}$  in Q. Encouraged by this data analysis, we now proceed to analyze the CTS market with strategic participants for the affine price spread model.

#### 2.4 Impact of liquidity in CTS markets

Our first goal is to investigate the impacts of liquidity on the CTS scheduling efficiency. To isolate the effects of liquidity, neglect transaction fees and set  $c \approx 0$ . We define the efficiency of CTS as the ratio

$$\eta_{\mathsf{CTS}}(B) := \frac{\mathscr{W}\left(\mathcal{Q}_{\mathsf{CTS}}(\theta^{\mathsf{NE}}, B)\right)}{\mathscr{W}\left(\mathcal{Q}_{\mathsf{TO}}\right)},$$

where recall that  $\mathscr{W}$  measures the aggregate welfare of the wholesale markets in the two areas attained at a particular tie-line schedule. TO seeks to maximize this welfare with  $Q_{\text{TO}} = \alpha/\beta$ , while the outcome of CTS arises from the strategic interaction of the market participants. Our next result characterizes the equilibrium and provides key insights into the behavior of  $\eta_{\text{CTS}} \leq 1$  in different liquidity regimes.

**Proposition 1.** Consider the CTS game  $\mathscr{G}(\boldsymbol{B}, 0, \alpha, \beta)$ , where  $B_m$  is the unique maximal liquidity in  $\{B_1, \ldots, B_N\}$ . Then,  $\mathscr{G}(\boldsymbol{B}, 0, \alpha, \beta)$  admits a unique Nash equilibrium  $\boldsymbol{\theta}^{\mathsf{NE}}$  given by

$$\theta_m^{\mathsf{NE}} = \begin{cases} \frac{1}{4\beta} \left( \beta^2 B_m - \mathscr{P}^2(\mathbb{1}^\mathsf{T} B) \right), & \text{if } |\mathbb{1}^\mathsf{T} B - \alpha/\beta| < B_m, \\ 0, & \text{otherwise}, \end{cases}$$
(2.11)

and  $\theta_i^{NE} = 0$  for  $i \neq m$ . Furthermore, we have

$$\eta_{\mathsf{CTS}}(B) \begin{cases} = 1, & \text{if } \mathbb{1}^{\mathsf{T}} B - \alpha/\beta \ge B_m, \\ \ge \frac{3}{4}, & \text{if } |\mathbb{1}^{\mathsf{T}} B - \alpha/\beta| < B_m, , \\ = 2x - x^2, & \text{otherwise} \end{cases}$$
(2.12)

where  $x := \beta \mathbb{1}^{\mathsf{T}} B / \alpha$ .

Existence of the equilibrium follows from Theorem 1. The rest follows from analysis of the firstorder equilibrium conditions. The result highlights that allocation and the efficiency vary widely with liquidity and the player with the maximal liquidity plays a rather central role in determining the outcome of the CTS market. To offer more insights, distinguish three different liquidity regimes. Identify the liquidity as high when  $\mathbb{1}^{\mathsf{T}}B - \alpha/\beta \ge B_m$ , where the aggregate liquidity of all players but *m* is sufficient to cover the efficient schedule  $Q_{\mathsf{TO}} = \alpha/\beta$ . The intermediate liquidity occurs where the aggregate liquidity is different from  $Q_{\mathsf{TO}}$  by at most the liquidity of player *m*, i.e.,  $|\mathbb{1}^{\mathsf{T}}B - \alpha/\beta| < B_m$ . Finally, the low liquidity regime is where  $\mathbb{1}^{\mathsf{T}}B + B_m < Q_{\mathsf{TO}}$ . The outcome and the efficiency differ substantially across these regimes. Using the equilibrium profile, it is easy to see that the flow allocation is given by

$$x_m^*(\boldsymbol{\theta}^{\mathsf{NE}}, \boldsymbol{B}) = \begin{cases} \frac{1}{2}(\alpha/\beta - \mathbb{1}^{\mathsf{T}}\boldsymbol{B}_{-m}), & \text{if } |\mathbb{1}^{\mathsf{T}}\boldsymbol{B} - \alpha/\beta| < B_m, \\ B_m, & \text{otherwise}, \end{cases},\\ x_i^*(\boldsymbol{\theta}^{\mathsf{NE}}, \boldsymbol{B}) = B_i, \quad i \neq m, \end{cases}$$

where  $B_{-m}$  denotes the vector of liquidities of all players, except *m*. Thus, all but player *m* offer their maximum liquidity at equilibrium. These players benefit from being inframarginal, exploiting the bid of the marginal player *m*. This behavior is reminiscent of the so-called 'free-rider problem' (see [24]). When the liquidity is too high or too low, player *m* does not have enough market power and does not benefit from bidding nonzero  $\theta_m$ , implying that she does not withhold from her maximal budget  $B_m$  in her transport offer. In the intermediate liquidity case, player *m* enjoys market power and her flow allocation can be shown to be the Cournot best response to this residual price spread  $\mathscr{P}(Q - \mathbb{1}^T B_{-m})$ . See [25, 26] for details on Cournot competition.

The tie-line schedule at the equilibrium of  $\mathscr{G}(B, 0, \alpha, \beta)$  is given by

$$Q_{\mathsf{CTS}} = \begin{cases} Q_{\mathsf{TO}}, & \text{if } \mathbb{1}^{\mathsf{T}} \boldsymbol{B} - B_m \ge \alpha/\beta, \\ \frac{1}{2} (Q_{\mathsf{TO}} + \mathbb{1}^{\mathsf{T}} \boldsymbol{B}_{-m}), & \text{if } |\mathbb{1}^{\mathsf{T}} \boldsymbol{B} - \alpha/\beta| < B_m, \\ \mathbb{1}^{\mathsf{T}} \boldsymbol{B}, & \text{otherwise.} \end{cases}$$

When liquidity is high,  $Q_{CTS}$  coincides with  $Q_{TO}$ , implying that CTS yields the SOs' intended outcome. In other words, perfect competition arises as a result of strategic incentives. In the intermediate liquidity regime, CTS suffers welfare loss due to strategic interaction. The loss, however, is bounded; strategic behavior cannot cripple the welfare under perfect competition by more than 25%. When the liquidity is low, the lower bound on  $\eta_{CTS}$  can be small. However, in this case lack of efficiency is not due to strategic interactions but rather due to the very low market liquidity.

#### 2.4.1 Learning equilibria through repeated play

Nash equilibria characterize how the incentives of market participants are oriented. However, the power of said equilibria to predict market outcomes may appear limited in that players are endowed with intelligence over their opponents' payoff and the system conditions to compute such an equilibrium. In practice, players interact repeatedly in the market, facing a noisy reward. They either learn to predict the inter-area price spread or learn their optimal strategy through repeated interactions and exploration of the market environment. Motivated to investigate if players can learn equilibria through repeated play, we study the game dynamics where bidders adopt action-value

methods [27] to update their bids. More precisely, we implement an upper confidence bound (UCB) algorithm for each bidder. In such a setting, each player is agnostic to the presence of other players and the SOs' clearing process, i.e., they endogenize these as part of the environment that yields a random reward. UCB is a popular reinforcement learning algorithm that achieves logarithmic regret [28, 29] in static environments and balances between exploration and exploitation. In each round (an instance of a CTS market), each player selects the action that has the maximum observed payoff thus far plus some exploration bonus.

The game proceeds as follows. At each round, each bidder chooses  $\theta$  from a finite set of actions  $\Theta := \{\theta^1, \dots, \theta^M\}$ . Each bidder maintains a vector  $\mathbf{R} \in \mathbb{R}^M$  of average rewards from each action and the number of times  $\mathbf{T} \in \mathbb{N}^M$  each action is chosen, where  $\mathbb{N}$  denotes the set of naturals. Here, the reward equals the revenue less the transaction cost from the CTS market. Bidders initialize  $\mathbf{R}$  by selecting every action (possible bid from  $\Theta$ ) at least once. Upon bidding  $\theta^k \in \Theta$  at a certain round, say she receives the reward  $r^k$  from the CTS market. Then, the bidder updates  $T^k$  and  $R^k$  as

$$T^k \leftarrow T^k + 1, \quad R^k \leftarrow R^k + \frac{1}{T^k} \left( r^k - R^k \right).$$
 (2.13)

Then, the bidder bids the action  $\theta^k$ , where

$$k = \operatorname*{argmax}_{j \in \{1, \dots, M\}} \left\{ R^{j} + \rho \sqrt{\ln\left(\mathbb{1}^{\mathsf{T}} \boldsymbol{T}\right)/T^{j}} \right\},$$
(2.14)

The parameter  $\rho > 0$  controls the degree of exploration. The larger the  $\rho$ , the player is eager to explore actions that have not been tried often enough. The smaller the  $\rho$ , the player tends to choose an action largely based on the average reward seen thus far.

We utilize historical CTS data from the NYISO and ISONE markets to compute the affine price spread that yields  $Q_{TO} = 1493$  MW. We consider repeated play of the CTS game with five participants, first with B = (298, 223, 194, 149, 893) and then with B = (596, 522, 640, 373, 893). The first example corresponds to an intermediate liquidity regime with  $\theta^{NE} = (0, 0, 0, 0, 4882)$ . The second example belongs to the high liquidity category for which  $\theta^{NE} = (0, 0, 0, 0, 0, 0)$ . In our simulations, we use  $\rho = 2$  following [27, Chapter 2]. Each CTS bidder chooses from ten  $\theta$ 's in  $\Theta = [0, 6000]$  that includes the optimal actions. Figure 2.3 shows percentages of optimal actions selected by bidders in a total of 3000 games for the high and intermediate liquidity regimes.

In the intermediate regime, the pivotal and inframarginal players act in a rather 'greedy' fashion, exploiting their optimal action north of 99% of the games. This implies that the observed reward from playing the optimal action is large enough, even as the exploration bonus of other actions increases. Bidder 5 loses her role as the marginal player when liquidity is high. In this regime, players are slower to discover their optimal actions although selection percentages are north of 88% of the games. Our numerical experiments clearly demonstrate that even in a setting where players know little to nothing about the game setting, they are able to discover and play equilibrium actions (in majority of the games) through repeated play. This experiment lends credence to the



Figure 2.3: Plot of cumulative percentage of times the Nash action is chosen across 3000 games for bidders 1 (in blue) and 5 (in red). Bidder 5 is marginal for (a) and inframarginal for (b).



Figure 2.4: Comparison of tie-line schedules and price spreads for CTS markets with high liquidity (in blue) and intermediate liquidity (in red).

conclusions from our equilibrium analysis. Indeed,  $Q_{CTS}/Q_{TO}$  in Figure 2.4 remains close to unity and price spreads are below \$2/MWh in most games for a highly liquid CTS market. A liquidity reduction of around 40% has palpable effects on market performance, although in aggregate, the players have the capacity to meet  $Q_{TO}$ . In particular, the price spread for intermediate liquidity is more than \$6/MWh higher than the highly liquid case and  $Q_{CTS}/Q_{TO}$  remains well below 80%. This experiment highlights how rise of pivotal players exercising market power exploiting the lack of liquidity can impact market performance.

#### 2.5 Interactions with financial transmission rights (FTRs)

CTS performance may be influenced by potential uneconomic bidding that aims to benefit financial positions whose value is tied to CTS outcomes, such as FTRs. Price manipulation that involves uneconomic virtual transactions has emerged as a central policy concern for FERC, as shown by several high-profile enforcement cases that ended in multi-million dollar settlements [30]. Here, we investigate the CTS performance when any subgroup of market participants hold FTR positions. An FTR is a unidirectional financial instrument, defined in megawatts, from a source node to a sink

node. One unit of an FTR entitles its holder a payment equal to the difference between the LMPs at the sink and the source nodes [31, 32]. We focus on FTR positions that negatively impact CTS.

Denote by  $f_i^k$ , the FTR megawatt position of CTS bidder *i* from an internal node *k* inside area *b* to the CTS trading location. Let  $\mathscr{P}_b^k$  denote the LMP at node *k* in area *b*. Recall that we have assumed so far that  $\mathscr{P}_b - \mathscr{P}_a$  has an affine dependence on *Q*, the amount that flows from bus *a* to bus *b*. In general,  $\mathscr{P}_b - \mathscr{P}_b^k$  will also depend on *Q*. Assume a similar affine dependence

$$\mathscr{P}_b(Q) - \mathscr{P}_b^k(Q) = \alpha_{\rm in}^k - \beta_{\rm in}^k Q$$

for an internal node k. Albeit simplistic, this model is enough to reveal the impact of FTRs on CTS markets. The payoff of bidder i from her FTR positions then becomes  $\sum_k (\alpha_{in}^k - \beta_{in}^k Q) f_i^k$ , where the sum is taken with k ranging over buses within area b. To illustrate the coupling between FTR positions and CTS market, consider the joint payoff from them for bidder i in

$$\tilde{\pi}_{i}(\theta_{i},\theta_{-i}) = \underbrace{(\alpha - \beta Q)B_{i} - \theta_{i}}_{\text{from CTS}} + \underbrace{\sum_{k} (\alpha_{\text{in}}^{k} - \beta_{\text{in}}^{k} Q)f_{i}^{k}}_{\text{from FTR}},$$
(2.15)

where Q depends on CTS market clearing with bids  $\theta$  and liquidities B. Formally, call this game  $\mathscr{G}_{FTR}(B,c,\alpha,\beta,f,\alpha_{in},\beta_{in})$  with payoffs in (2.15). Here,  $\alpha_{in},\beta_{in},f$  collect the respective variables across all internal buses. Our next result characterizes the market outcome with FTR positions.

**Proposition 2.** The game  $\mathscr{G}_{FTR}(B, 0, \alpha, \beta, f, \alpha_{in}, \beta_{in})$  admits a unique Nash equilibrium if f is elementwise nonnegative, for which the tie-line schedule at the equilibrium is

$$Q_{\mathsf{CTS}} = \begin{cases} Q_{\mathsf{TO}}, & \text{if } \mathbb{1}^{\mathsf{T}} \boldsymbol{B} - \tilde{\boldsymbol{B}}_m \ge \alpha/\beta, \\ \frac{1}{2} \left( Q_{\mathsf{TO}} + \mathbb{1}^{\mathsf{T}} \boldsymbol{B} - \tilde{\boldsymbol{B}}_m \right), & \text{if } |\mathbb{1}^{\mathsf{T}} \boldsymbol{B} - \alpha/\beta| < \tilde{\boldsymbol{B}}_m, \\ \mathbb{1}^{\mathsf{T}} \boldsymbol{B}, & \text{otherwise}, \end{cases}$$

where  $\tilde{B}_i = B_i + \sum_k (\beta^k / \beta) f_i^k$  for i = 1, ..., N and m is the only player with maximal  $\tilde{B}_m$ .

Our proof again appeals to Rosen's result and the rest relies on analyzing the first-order conditions for equilibrium. The result reveals that the bidder with maximum combined CTS and FTR position emerges as the pivotal player in this market. Moreover,  $\tilde{B}_m \ge B_m$  dictates that less power is scheduled to flow in the tie-line when bidders have such FTR positions. This results from the incentives of the pivotal player who benefits from higher prices at the importing region *b*'s proxy bus as that yields a higher FTR payoff. In fact, the difference in the tie-line schedules with and without FTR grows with the difference between  $\tilde{B}_m$  and  $B_m$  that is directly proportional to the FTR positions. Opposite conclusions can be drawn if we consider players with FTR positions that source at area *b*'s proxy bus.

The following example illustrates the shift in market power and scheduling efficiency when participants hold FTRs. Consider the CTS market in Section 3.5 where the fifth bidder is pivotal

in the intermediate liquidity regime. At the equilibrium,  $Q_{CTS} = 1176$  MW. Assume that the first bidder holds an FTR  $f_1 = 800$  MW to an internal bus for which  $\alpha_{in} = 35.7$  and  $\beta_{in} = 0.02$ , while the rest of players do not have FTR portfolios. Then,  $\tilde{B} = [1018, 463, 193, 149, 893]$ . Notice that bidder one emerges as the new marginal bidder and has incentive to bid in a way that leads to less power being scheduled to flow into area *b*. Indeed, the new tie-line schedule is  $Q_{CTS} = 1113$  MW, 63 MW less than CTS without FTRs, falling even shorter of  $Q_{TO} = 1493$  MW.

#### 2.6 Impact of forecast errors and transaction costs

Our analysis of the CTS game so far has assumed that players and the SOs have perfect forecasts into the price spread function. In practice, tie-line scheduling takes place with a lead time to power delivery, meaning that there is an inherent uncertainty in the price spread when these markets are convened. To model this uncertainty, assume that the SOs conjecture an affine price spread function

$$\mathscr{P}_{\mathsf{SO}}(Q) = \alpha_{\mathsf{SO}} - \beta_{\mathsf{SO}}Q$$

with  $\alpha_{SO}$ ,  $\beta_{SO} > 0$ . The SOs use this spread to clear the CTS market as in (2.3). Let the realized price difference be

$$\mathscr{P}_{\star}(Q) = \alpha_{\star} - \beta_{\star}Q$$

with  $\alpha_{\star}, \beta_{\star} > 0$ . Then, the TO schedule and the optimal tie-line schedule, respectively, are given by

$$Q_{\mathsf{TO}} = lpha_{\mathsf{SO}} / eta_{\mathsf{SO}}$$
 and  $Q_{\star} = lpha_{\star} / eta_{\star}.$ 

Modeling the uncertainty explicitly at the time of scheduling reveals that  $Q_{TO}$  may not equal  $Q_*$ , the ex-post optimal tie-line schedule. Our interest lies in analyzing if strategic behavior of bidders in the CTS market can correct the errors in SOs' forecasts. Do bidders draw the outcome closer to  $Q_*$  than  $Q_{TO}$  or do they drive it further away as a result of their strategic interaction? We answer this question through a game-theoretic study. We also derive insights into how non-zero transaction fees (c > 0) affect these conclusions.

To isolate the impacts of uncertainty and transaction fees, we analyze the game under a simpler setting where the bidders are homogenous, each with liquidity B > 0 and conjectured price spread  $\mathscr{P}(Q) = \alpha - \beta Q$  with  $\alpha, \beta > 0$ . Notice that bidders' conjectured optimal schedule  $\alpha/\beta$  may be different from both  $Q_{\text{TO}}$  and  $Q_{\star}$ . We assume here that players share a common belief that the market operates at an intermediate liquidity where the aggregate liquidity NB is close to her conjectured optimal tie-line schedule  $\alpha/\beta$ , i.e.,

$$NB = \alpha/\beta + \mathcal{O}(1/N). \tag{2.16}$$

Under such an assumption, bidder *i* conjectures the market price from bidding  $\theta$  with liquidities  $B = B\mathbb{1}$  to be

$$p(\boldsymbol{\theta}, B\mathbb{1}) = \frac{1}{2} \left( \mathscr{P}(NB) + \sqrt{\mathscr{P}^2(NB) + 4\beta \mathbb{1}^{\mathsf{T}}\boldsymbol{\theta}} \right) = \sqrt{\beta \mathbb{1}^{\mathsf{T}}\boldsymbol{\theta}} + \mathscr{O}(1/N),$$

which yields the following perceived payoff for bidder *i*.

$$\pi_{i}(\theta_{i},\theta_{-i}) = p(\theta,B)B - \theta_{i} - c\left(B - \frac{\theta_{i}}{p(\theta,B\mathbb{1})}\right) \approx \sqrt{\beta\mathbb{1}^{\mathsf{T}}\theta}B - \theta_{i} - c\left(B - \frac{\theta_{i}}{\sqrt{\beta\mathbb{1}^{\mathsf{T}}\theta}}\right).$$
(2.17)

Call the CTS game with conjectured price spreads  $\mathscr{G}_{conj}(B, c, \alpha, \beta, \alpha_{SO}, \beta_{SO})$ , where  $\alpha, \beta$  satisfy (2.16) and the payoffs are given by (2.17). Assuming that all players offer based on an equilibrium profile for this game, the SOs then solve the CTS flow allocation problem in (2.3) with  $\mathscr{P}_{SO}$  to ultimately compute the tie-line schedule. Our next result characterizes both a (symmetric) equilibrium profile and the resulting tie-line schedule.

**Proposition 3.** The CTS game  $\mathscr{G}_{conj}(B,c,\alpha,\beta,\alpha_{SO},\beta_{SO})$  admits a unique symmetric Nash equilibrium given by  $\theta_i^{NE} = \frac{\gamma^2}{4N\beta}$  for i = 1, ..., N, for which the tie-line schedule at equilibrium is

$$Q_{\text{CTS}} = \frac{1}{2} \left[ Q_{\text{TO}} + NB - \sqrt{(Q_{\text{TO}} - NB)^2 + \frac{\gamma^2}{\beta \beta_{\text{SO}}}} \right],$$

*where*  $\gamma := c(2 - 1/N) + \beta B$ .

Notice that players bid solely based on their own conjectures. The tie-line schedule, however, depends on the conjectures of both the bidders and the SOs. This result allows us to study the effect of price spread forecasts and transaction costs on the scheduling efficiency in the sequel.

The lack of knowledge of  $Q_{\star}$  by the SOs and market participants prompts us to investigate whether CTS can yield a more efficient schedule than the pure SO-driven TO. Proposition 3 implies  $Q_{\text{CTS}} \leq Q_{\text{TO}}$ , meaning that CTS cannot yield a more efficient schedule than TO if  $Q_{\text{TO}} < Q_{\star}$ . Hence, CTS can only outperform TO when the SOs' forecast overestimates  $Q_{\text{TO}}$ . In this regime, Figure 2.1 yields that  $Q_{\text{CTS}}$  is always closer to  $Q_{\star}$  when  $Q_{\star} \leq Q_{\text{TO}}/2$ . Outside of this setting, the outcome of CTS depends on the liquidity and conjectures of players. Specifically, if  $NB \in \mathscr{A}_1 \cup \mathscr{A}_2$ , defined in Figure 2.5,  $Q_{\text{CTS}}$  is closer to  $Q_{\star}$  than  $Q_{\text{TO}}$ , if

$$\frac{\gamma^2}{\beta\beta_{\rm SO}} \le 8\left(Q_{\rm TO} - Q_\star\right)\left(Q_{\rm TO} - 2Q_\star + NB\right). \tag{2.18}$$

Such a premise appears to run counter to the intuition that TO is optimal. This situation can only arise under uncertainty where SOs make serious forecast errors in the expected price spread. Surprisingly, forecast errors are not that rare, according to [10], where the error in SOs' point forecast for the price spread between NYISO and ISO-NE averaged \$2.42/MWh. Notice how, in this liquidity regime, the presence of transaction fees makes it harder to satisfy (2.18). This is intuitively correct since transaction fees drive the tie-line schedule toward smaller values, as established in Proposition 3. When  $NB \in \mathcal{A}_3 \cup \mathcal{A}_4$ , liquidity is sufficiently high and the presence of costs might improve scheduling efficiency since players bid higher prices to counter costs. Overall,



Figure 2.5: Ability of market participants to correct SO's forecast error depends on liquidity and transactions costs.

players ability to correct SOs' forecast is somewhat limited and relies on many qualifications, indicating that the SOs forecasts and systematic bias plays a vital role in scheduling efficiency. Moving bid submittal and clearing timelines closer to power delivery should improve the efficiency of CTS.



Figure 2.6: The trajectory of CTS schedules cleared against SO's forecasted prices with 10% error with c = 0 and  $c = \frac{8}{MWh}$ .

Proposition 3 suggests that incentives of CTS bidders are aligned in a way that allows them to correct SOs' forecast errors in some settings. Can players learn such equilibria through repeated play. We employ the learning framework in Section 3.5, where players have their bids cleared against ( $\alpha_{SO}$ ,  $\beta_{SO}$ ) that are perturbed from ( $\alpha_{\star}$ ,  $\beta_{\star}$ ) learned from historical data. That is, in every round, bidders receive reward from the ex-post price spread described by  $\mathcal{P}_{\star}$ . The trajectory of tie-line schedules in Figure 2.6 with c = 0 reveals that bidding behavior of players results in CTS schedules consistently closer to the ex-post optimal than TO. Despite the SO's persistent forecast error, bidders 'correct' the tie-line schedule to an extent by seeking actions that maximize their observed reward.

The relation in (2.18) reveals that presence of nonzero transaction fees c make it more difficult for CTS market to drive the outcome closer to the ex-post optimal as  $\gamma$  increases with c. Bidders reacting to observed rewards with c = \$8/MWh in Figure 2.6 yield a CTS schedule farther from  $Q_{\star}$ , seeking actions that yield higher prices but smaller schedules. This result corroborates our theoretical finding that transaction fees impede bidders' ability to correct SOs' forecast errors.



Figure 2.7: Plot (a) depicts the time series of spread between NYISO and PJM proxy buses in 2018 (absolute mean = 8.92 \$/MWh, std. deviation = 22.11 \$/MWh). Plot (b) shows the same between NYISO and ISO-NE for the same year (mean = 0.44, absolute mean = 5.59 \$/MWh, std. dev. = 18.14 \$/MWh).

Notice that equilibrium bid grows with *c*, per Proposition 3. With c > 0, bidders are reluctant to offer their entire liquidity. A similar result can be shown under more general settings of Theorem 1. This may prevent the price spread from converging to zero, even if the market is liquid. And, transaction fees make it less attractive for CTS bidders overall, hurting long-term liquidity of the CTS market. Figure 2.7a indicates that the price spread in the CTS market between NYISO and PJM exhibits longer excursions from zero and higher volatility compared to that between NYISO and ISO-NE, depicted in Figure 2.7b. The average absolute spread between NYISO and PJM is approximately \$3.3/MWh higher than that between NYISO and ISO-NE. We surmise that transaction fees between NYISO and PJM and the lack thereof between NYISO and ISO-NE are largely responsible for this difference.

# 3. Robust Tie-Line Scheduling Via Critical Region Exploration

### 3.1 Introduction

For historic and technical reasons, different parts of an interconnected power system and their associated assets are dispatched by different system operators (SOs). We call the geographical footprint within an SO's jurisdiction an *area*, and transmission lines that interconnect two different areas as *tie-lines*. Power flows over such tie-lines are generally scheduled 15 - 75 minutes prior to power delivery. The report in [33] indicates that current scheduling techniques often lead to suboptimal tie-line power flows. The economic loss due to inefficient tie-line scheduling is estimated to the tune of \$73 million between the areas controlled by MISO and PJM alone in 2010. Tie-lines often have enough transfer capability to fulfill a significant portion of each area's power consumption [34]. Thus they form important assets of multi-area power systems.

SOs from multiple areas typically cannot aggregate their dispatch cost structures and detailed network constraints to solve a *joint* optimal power flow problem. Therefore, distributed algorithms have been proposed. Prominent examples include [1, 35, 36] that adopt the so-called *dual decomposition* approach. These methods are iterative, wherein each SO optimizes the grid assets within its area, given the Lagrange multipliers associated with inter-area constraints. Typically, a coordinator mediates among the SOs and iteratively updates the multipliers. Alternative *primal decomposition* approaches are also proposed in [37–39]. Therein, the primal variables of the optimization problem are iteratively updated, sometimes requiring the SO of one area to reveal part of its cost structure and constraints to the SO of another area or a coordinator.

Traditionally, solution techniques for the tie-line scheduling problem assume that the SOs and/or the coordinator has perfect knowledge of the future demand and supply conditions at the time of scheduling. Such assumptions are being increasingly challenged with the rapid adoption of distributed energy resources in the distribution grid and variable renewable generation like wind and solar energy in the bulk power systems. Said differently, one must explicitly account for the uncertainty in demand and supply in the tie-line scheduling problem. To that end, [40, 41] propose to minimize the expected aggregate dispatch cost and [42] propose to minimize the maximum of that cost. In this chapter, we adopt the latter paradigm – the robust approach.

# 3.1.1 Our contribution

With the system model in Section 3.2, we first formulate the deterministic tie-line scheduling problem in Section 3.3, where we propose an algorithm to solve this deterministic problem that draws from the theory of multiparametric programming [43]. The key feature of our algorithm is that a coordinator can produce the optimal tie-line schedule upon communicating only finitely many times with the SO in each area. In contrast to [39], our method does not require SOs to

reveal their cost structures nor their constraints to other SOs or to the coordinator. In Section 3.4, we formulate the robust counterpart of the tie-line scheduling problem. We then propose a technique that alternately uses the algorithm for the deterministic variant and a mixed-integer linear program to solve the robust problem. Again, our technique is proved to converge to the optimal robust tie-line schedule that requires the coordinator to communicate finitely many times with each SO. Also, SOs are not required to reveal the nature and range of the values the uncertain demand and available supply can take. Our proposed framework thus circumvents the substantial communication burden of the method proposed in [42] towards the same problem. We remark that [42] adopts the column-and-constraint generation technique described in [44] that requires SOs to reveal part of their network constraints, costs and ranges of demand and available renewable supply to the coordinator. We empirically demonstrate the performance of our algorithm in Section 3.5 and conclude in Section 3.6.

#### **3.2** System model

To formulate the tie-line scheduling problem, we begin by describing the model for multiarea power systems. Throughout, we restrict ourselves to a two-area power system, pictorially represented in Figure 4.5 for the ease of exposition. The model and the proposed methods can be generalized for tie-line scheduling among more than two areas.



Figure 3.1: An illustration of a two-area power system.

For the power network in each area, we distinguish between two types of buses: the *internal* buses and the *boundary* buses. The boundary ones in each area are connected to their counterparts in the other area via tie-lines. Internal buses do not share a connection to other areas. Assume that each internal bus has a dispatchable generator, a renewable generator, and a controllable load<sup>1</sup>. Boundary buses do not have any asset that can inject or extract power. Such assumptions are not limiting in that one can derive an equivalent power network in each area that adheres to these assumptions.

Let the power network in area *i* be comprised of  $n_i$  internal buses and  $\overline{n}_i$  boundary buses for each i = 1, 2. We adopt a linear DC power flow model in this chapter.<sup>2</sup> This approximate model sets all voltage magnitudes to their nominal values, ignores transmission line resistances and shunt

<sup>&</sup>lt;sup>1</sup> While we assume that all loads are controllable, uncontrollable load at any node can be easily modeled by letting the limits on the allowable power demand at that node to be equal.

<sup>&</sup>lt;sup>2</sup> See [45, 46], and the references therein for solution approaches for a multi-area ACOPF problem.

reactances, and deems differences among the voltage phase angles across each transmission line to be small. Consequently, the real power injections into the network is a linear map of voltage phase angles (expressed in radians) across the network. To arrive at a mathematical description, denote by  $g_i \in \mathbb{R}^{n_i}$ ,  $w_i \in \mathbb{R}^{n_i}$ , and  $d_i \in \mathbb{R}^{n_i}$  as the vectors of (real) power generations from dispatchable generators, renewable generators, and controllable loads, respectively. Let  $\theta_i \in \mathbb{R}^{n_i}$  and  $\overline{\theta}_i \in \mathbb{R}^{\overline{n}_i}$  be the vectors of voltage phase angles at internal and boundary buses, respectively. Then, the power flow equations are given by

$$\begin{pmatrix} B_{11} & B_{1\bar{1}} & & \\ B_{\bar{1}1} & B_{\bar{1}\bar{1}} & B_{\bar{1}\bar{2}} & \\ & B_{\bar{2}\bar{1}} & B_{\bar{2}\bar{2}} & B_{\bar{2}2} \\ & & & B_{2\bar{2}} & B_{22} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \overline{\theta}_1 \\ \overline{\theta}_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} g_1 + w_1 - d_1 \\ 0 \\ 0 \\ g_2 + w_2 - d_2 \end{pmatrix}.$$
(3.1)

Non-zero entries of the coefficient matrix depend on reciprocals of transmission line reactances, the unspecified blocks in that matrix are zeros. Throughout, assume that one of the boundary buses in area 1 is set as the slack bus for the two-area power system. That is, the voltage phase angle at said bus is assumed zero.

Power injections from the supply and demand assets at the internal buses of area *i* are constrained as

$$\underline{G}_i \le g_i \le \overline{G}_i, \ 0 \le w_i \le \overline{W}_i, \ \underline{D}_i \le d_i \le \overline{D}_i.$$
(3.2)

The inequalities are interpreted elementwise. The lower and upper limits on dispatchable generation  $\underline{G}_i, \overline{G}_i$  are assumed to be known at the time when tie-line flows are being scheduled. Our assumptions on the available renewable generation  $\overline{W}_i$  and the limits on the demands  $[\underline{D}_i, \overline{D}_i]$  will vary in the subsequent sections. In Section 3.3, we assume that these limits are known and provide a distributed algorithm to solve the deterministic tie-line scheduling problem. In Section 3.4, we formulate the robust counterpart, where these limits are deemed uncertain and vary over a known set. We then describe a distributed algorithm to solve the robust counterpart.

The power transfer capabilities of transmission lines within area *i* are succinctly represented as

$$\boldsymbol{H}_{i}\boldsymbol{\theta}_{i} + \overline{\boldsymbol{H}}_{i}\overline{\boldsymbol{\theta}}_{i} \leq f_{i} \tag{3.3}$$

for each i = 1, 2. Here,  $H_i$  and  $\overline{H}_i$  define the branch-bus admittance matrices, and  $f_i$  models the respective transmission line capacities. Similarly, the transfer capabilities of tie-lines joining the two areas assume the form

$$\overline{\boldsymbol{H}}_{12}\overline{\boldsymbol{\theta}}_1 + \overline{\boldsymbol{H}}_{21}\overline{\boldsymbol{\theta}}_2 \le f_{12}. \tag{3.4}$$

Again,  $\overline{H}_{12}$ ,  $\overline{H}_{21}$  denote the relevant branch-bus admittance matrices and  $f_{12}$  models the tie-line capacities.

Finally, we describe the cost model for our two-area power system. For respectively procuring  $g_i$  and  $w_i$  from dispatchable and renewable generators, and meeting a demand of  $d_i$  from controllable loads, let the dispatch cost in area *i* be given by

$$\left[P_{i}^{g}\right]^{\mathsf{T}}g_{i}+\left[P_{i}^{w}\right]^{\mathsf{T}}\left(\overline{W}_{i}-w_{i}\right)+\left[P_{i}^{d}\right]^{\mathsf{T}}\left(\overline{D}_{i}-d_{i}\right).$$
(3.5)

We use the notation  $v^{T}$  to denote the transpose of any vector or matrix v. The linear cost structure in the above equation is reminiscent of electricity market practices in many parts of the U.S. today. The second summand models any spillage costs associated with renewable generators. The third models the disutility of not satisfying all demands.

#### **3.3** The deterministic tie-line scheduling problem

Tie-line flows are typically scheduled ahead of the time of power delivery. The lead time makes the supply and demand conditions uncertain during the scheduling process. Within the framework of our model, the available capacity in renewable supply and lower and upper bounds on power demands, *i.e.*,  $\overline{W}_i$ ,  $\underline{D}_i$ ,  $\overline{D}_i$ , can be uncertain. In this section, we ignore such uncertainty and formulate the deterministic tie-line scheduling problem, wherein we assume perfect knowledge of  $\overline{W}_i$ ,  $\underline{D}_i$  and  $\overline{D}_i$  to decide the dispatch in each area and the tie-line flows. Our discussion of the deterministic version will serve as a prelude to its robust counterpart in Section 3.4.

To simplify exposition, consider the following notation.

$$x_i := (g_i, w_i, d_i, \theta_i)^{\mathsf{T}}, \ \xi_i := (\overline{W}_i, \underline{D}_i, \overline{D}_i)^{\mathsf{T}}, \ y := (\overline{\theta}_1, \overline{\theta}_2)^{\mathsf{T}}$$

for i = 1, 2. The above notation allows us to succinctly represent the constraints (4.1) - (3.3) as

$$\boldsymbol{A}_i^{\boldsymbol{x}} \boldsymbol{x}_i + \boldsymbol{A}_i^{\boldsymbol{\xi}} \boldsymbol{\xi}_i + \boldsymbol{A}_i^{\boldsymbol{y}} \boldsymbol{y} \leq \boldsymbol{b}_i$$

for each i = 1, 2 and suitably defined matrices  $A_i^x, A_i^{\xi}, A_i^y$  and vector  $b_i$ . Denote by  $m_i$  the number of inequality constraints in the above equation. Next, we describe transmission constraints on tie-line power flows in (3.4) as

$$y \in \mathscr{Y} \subset \mathbb{R}^Y$$
.

Without loss of generality, one can restrict  $\mathscr{Y}$  to be a polytope<sup>3</sup>. Finally, the cost of dispatch in area *i*, as described in (3.5), can be written as

$$c_i(x_i, \xi_i) := c_i^0 + [c_i^x]^\mathsf{T} x_i + [c_i^\xi]^\mathsf{T} \xi_i$$

for scalar  $c_i^0$  and vectors  $c_i^x$ ,  $c_i^{\xi}$ .

<sup>&</sup>lt;sup>3</sup> Assuming the power network to be connected, the modulus of the phase angle of any bus can be constrained to lie within the sum of admittance-weighted transmission line capacities connecting that bus to the slack bus.

Equipped with the above notation, we define the deterministic tie-line scheduling problem as follows.

#### 3.3.1 Distributed solution via critical region exploration

The structure of the optimization problem in (3.6) lends itself to a distributed solution architecture that we describe below. Our proposed technique is similar in spirit to the critical region projection method described in [37].<sup>4</sup> We assume that each area is managed by a system operator (SO), and a *coordinator* mediates between the SOs. Assume that the SO of area *i* (call it SO<sub>*i*</sub>) knows the dispatch cost  $c_i$  and the linear constraint involving  $x_i, \xi_i, y$  in (3.6) in area *i*, and that SOs and the coordinator all know  $\mathscr{Y}$ .

Our algorithm relies on the properties of (3.6) that we describe next. To that end, notice that (3.6) can be written as

$$\underset{y \in \mathscr{Y}}{\text{minimize } J^*(y, \xi_1, \xi_2)} := J_1^*(y, \xi_1) + J_2^*(y, \xi_2), \tag{3.7}$$

where

$$J_{i}^{*}(y,\xi_{i}) := \min_{x_{i}} c_{i}(x_{i},\xi_{i}),$$
  
subject to  $A_{i}^{x}x_{i} + A_{i}^{\xi}\xi_{i} + A_{i}^{y}y \leq b_{i}.$  (3.8)

Assume throughout that all optimization problems parameterized by y is feasible for each  $y \in \mathscr{Y}$ . Techniques from [42] can be leveraged to shrink  $\mathscr{Y}$  appropriately, otherwise. The optimization problem in (3.8) is a multi-parametric linear program, linearly parameterized in  $(y, \xi_i)$  on the right-hand side<sup>5</sup>. Such optimization problems are well-studied in the literature. For example, see [43]. Relevant to our algorithm is the structure of the parametric optimal cost  $J_i^*$ . Describing that structure requires an additional notation. We say that a finite collection of polytopes  $\{\mathscr{P}^1, \ldots, \mathscr{P}^\ell\}$  define a *polyhedral partition* of  $\mathscr{Y}$ , if no two polytopes intersect except at their boundaries, and their union equals  $\mathscr{Y}$ . With this notation, we now record the properties of  $J_i^*$  in the following lemma.

**Lemma 1.**  $J_i^*(y,\xi_i)$  is piecewise affine and convex in  $y \in \mathscr{Y}$ . Sets over which  $J_i^*(\cdot,\xi_i)$  is affine define a polyhedral partition of  $\mathscr{Y}$ .

<sup>&</sup>lt;sup>4</sup> The cost structure in [37] is quadratic; the linear cost case does not directly follow from [37].

<sup>&</sup>lt;sup>5</sup> The problem in (3.8) reformulated using the so-called epigraph form yields a multi-parametric program that is classically recognized as one linearly parameterized on the right-hand side.



Figure 3.2: A pictorial representation of the critical regions induced by the areawise parametric optimal costs  $J_1^*(\cdot, \xi_1), J_2^*(\cdot, \xi_2)$ , and the aggregate cost  $J^*(\cdot, \xi_1, \xi_2)$ . The trapezoids represent  $\mathscr{Y}$ . Differently shaded polytopes indicate different critical regions.

The proof is immediate from [43, Theorem 7.5]. Details are omitted for brevity. We refer to the polytopes in the polyhedral partition of  $\mathscr{Y}$  induced by  $J_i^*(\cdot, \xi_i)$  as *critical regions*. Recall that the feasible set of (3.8) is described by a collection of linear inequalities. Essentially, each critical region corresponds to the subset of  $\mathscr{Y}$  over which a specific set of these inequality constraints are active – *i.e.*, are met with equalities – at an optimal solution of (3.8).

A direct consequence of the above lemma is that the *aggregate cost*  $J^*(\cdot, \xi_1, \xi_2)$  is also piecewiseaffine and convex. Sets over which this cost is affine define a polyhedral partition of  $\mathscr{Y}$ . The polytopes of that partition – the critical regions – are precisely the non-empty intersections between the critical regions induced by  $J_1^*(\cdot, \xi_1)$  and those by  $J_2^*(\cdot, \xi_2)$ . The relationship between the critical regions induced by the various piecewise affine functions are illustrated in Figure 3.2.

In what follows, we develop an algorithm wherein the coordinator defines a sequence of points in  $\mathscr{Y}$  towards optimizing the aggregate cost. In each step, it relies on the SOs to identify their respective critical regions and the affine descriptions of their optimal costs at these iterates. That is, SO<sub>i</sub> can compute the critical region  $\mathscr{P}_i^y$  that contains  $y \in \mathscr{Y}$  and the affine description  $[\alpha_i^y]^T z + \beta_i^y$ of its optimal dispatch cost  $J_i^*(z, \xi_i)$  over  $z \in \mathscr{P}_i^y$  by parameterizing the linear program described in (3.8)<sup>6</sup>. The details of this step are omitted; see [47]. For any  $y \in \mathscr{Y}$ , we assume in the sequel that the coordinator can collect this information from the SOs to construct the critical region  $\mathscr{P}^y$ induced by the aggregate cost containing y and its affine description  $[\alpha_i^y]^T z + \beta_i^y$  for  $z \in \mathscr{P}^y$ , where

$$\mathscr{P}^{y} := \mathscr{P}^{y}_{1} \cap \mathscr{P}^{y}_{2}, \ \alpha^{y} := \alpha^{y}_{1} + \alpha^{y}_{2}, \ \beta^{y} := \beta^{y}_{1} + \beta^{y}_{2}.$$
(3.9)

In presenting the algorithm, we assume that the coordinator can identify the *lexicographically smallest* optimal solution of a linear program. A vector *a* is said to be lexicographically smaller than *b*, if at the first index where they differ, the entry in *a* is less than that in *b*. See [48] for details on

<sup>&</sup>lt;sup>6</sup> The critical region containing  $y \in \mathscr{Y}$  is unique, except when y lies at the boundary of critical regions. In that event, assume that the SO returns one of the critical regions containing y.

such linear programming solvers. When a linear program does not have a unique optimizer<sup>7</sup>, such a choice provides a tie-breaking rule. The final piece required to state and analyze the algorithm is an optimality condition that is both necessary and sufficient for a candidate minimizer of (3.7). Stated geometrically,  $y^* \in \mathscr{Y}$  is a minimizer of (3.7) if and only if

$$0 \in \partial J^*(y^*, \xi_1, \xi_2) + N_{\mathscr{Y}}(y^*).$$
(3.10)

The first set on the right-hand side of (3.10) is the sub-differential set of the aggregate cost  $J^*(\cdot, \xi_1, \xi_2)$  evaluated at  $y^{*8}$ . And, the second set denotes the normal cone to  $\mathscr{Y}$  at  $y^*$ . The addition stands for a set-sum.

Algorithm 1 delineates the steps for the coordinator to solve the deterministic tie-line scheduling problem. In our algorithm,  $||v^*||_2$  denotes the Euclidean norm of  $v^*$ . If  $\mathcal{D} := \{\alpha^1, \ldots, \alpha^{\ell_D}\}$  and  $N_{\mathcal{U}}(y^*) := \{z \mid \mathbf{K}^y z \ge 0\}$ , then computing the least-square solution  $v^*$  amounts to solving the following convex quadratic program.

minimize 
$$\frac{1}{2} \|v\|_2^2$$
,  
subject to  $v = \sum_{j=1}^{\ell_D} \eta_j \alpha^j + \zeta$ ,  $\mathbb{1}^{\mathsf{T}} \eta = 1$ ,  $\eta \ge 0$ ,  $\mathbf{K}^y \zeta \ge 0$  (3.11)

over the variables  $v \in \mathbb{R}^{\overline{n}_1 + \overline{n}_2}$ ,  $\eta \in \mathbb{R}^{\ell_D}$ , and  $\zeta \in \mathbb{R}^{\ell_N}$ , where 1 is a vector of all ones, and  $K^y \in \mathbb{R}^{(\overline{n}_1 + \overline{n}_2) \times \ell_N}$ .

#### 3.3.2 Analysis of the algorithm

The following result characterizes the convergence of Algorithm 1.

# **Theorem 2.** Algorithm 1 terminates after finitely many steps, and $y^*$ at termination optimally solves (3.7).

The above result fundamentally relies on the fact that each time the variable y is updated, it belongs to a critical region (induced by the aggregate cost) that the algorithm has not encountered so far. And, there are only finitely many such critical regions. That ensures termination in finitely many steps. Each time the algorithm ventures into a new critical region, we store the optimizer and the optimal cost over that critical region in the variables  $y^{opt}$  and  $J^{opt}$ . Forcing the linear program to choose the lexicographically smallest optimizer always picks a unique vertex of the critical region as  $y^{opt}$ . Unless  $J^{opt}$  improves upon the cost at  $y^*$ , we ignore the new point  $y^{opt}$ . However, the exploration of the new critical region provides a possibly new sub-gradient of the aggregate cost at

<sup>&</sup>lt;sup>7</sup> A linear program has non-unique optimizers when it is *dual degenerate*. See [48] for details.

<sup>&</sup>lt;sup>8</sup> We use the sub-differential characterization as opposed to the familiar gradient condition for optimality since  $J^*(\cdot, \xi_1, \xi_2)$  is piecewise affine and may not be differentiable everywhere in  $\mathscr{Y}$ .

Algorithm 1 Solving the deterministic tie-line scheduling problem.

```
1: Initialize:
               y \leftarrow any point in \mathscr{Y}, J^* \leftarrow \infty,
               \mathscr{D} \leftarrow \text{empty set}, \varepsilon \leftarrow \text{small positive number.}
 2: do
               Communicate with the SOs to obtain \mathscr{P}^{y} and \alpha^{y}, \beta^{y}.
  3:
               Minimize [\alpha^{y}]^{\mathsf{T}}z + [\beta^{y}] over \mathscr{P}^{y}.
  4:
               y^{\text{opt}} \leftarrow lexicographically smallest minimizer in step 4.
  5:
               J^{\text{opt}} \leftarrow \text{optimal cost in step 4.}
  6:
               if J^{\text{opt}} < J^*. then
  7:
                     y^* \leftarrow y^{\text{opt}}, J^* \leftarrow J^{\text{opt}}, \mathscr{D} \leftarrow \{\alpha^y\}.
  8:
 9:
               else
                      \mathscr{D} \leftarrow \mathscr{D} \cup \{\alpha^{\mathsf{y}}\}.
10:
               end if
11:
              v^* \leftarrow \operatorname{argmin}_{v \in \operatorname{conv}(\mathscr{D}) + N \mathscr{U}(v^*)} \|v\|_2^2.
12:
              y \leftarrow y^{\text{opt}} - \varepsilon v^*.
13:
14: while v^* \neq 0.
```

 $y^*$ . The sub-differential set at  $y^*$  is given by the convex hull of the sub-gradients of the aggregate cost over all critical regions that  $y^*$  is a part of. The set  $\mathscr{D}$  we maintain is such that  $conv(\mathscr{D})$  is a *partial* sub-differential set of the aggregate cost at  $y^*$ . Notice that

 $\operatorname{conv}(\mathscr{D}) \subseteq \partial J^*(y^*, \xi_1, \xi_2)$ 

throughout the algorithm. Therefore, any  $y^*$  that meets the termination criterion of the algorithm automatically satisfies (3.10). As a result, such a  $y^*$  is an optimizer of (3.7).

The proposed technique is attractive in that each SO only needs to communicate finitely many times with the coordinator for the latter to reach an optimal tie-line schedule. Further, each SO<sub>i</sub> can compute its optimal dispatch  $x_i^*$  by solving (3.8) with  $y^*$ . A closer look at the nature of the communication between the SOs and the coordinator reveals that an SO will not have to disclose the complete cost structure nor a complete description of the constraints within its area to the coordinator.

**Remark 1.** Algorithm 1 allows the coordinator to minimize

$$F(y) := F_1(y) + F_2(y)$$

in a distributed manner, where  $F_i : \mathscr{Y} \to \mathbb{R}$  satisfies two properties. First, it is piecewise affine and convex. Second, given any  $y \in \mathscr{Y}$ ,  $SO_i$  can compute an affine segment containing that y. While we do not explicitly characterize how fast the algorithm converges to its optimum, one can expect the number of steps to convergence to grow with the number of critical regions so induced. However, we do not expect our algorithm to explore all such critical regions on its convergence path.

#### **3.3.3** A pictorial illustration of the algorithm

To gain more insights into the mechanics of Algorithm 1, consider the example portrayed in Figure 3.3. The coordinator begins with  $y^A$  as the initial value of y. It communicates with SO<sub>i</sub> to obtain the critical region induced by  $J_i^*$  containing  $y^A$ , and the affine description of  $J_i^*$  over that critical region. Using the relation in (3.9), it then computes the critical region  $\mathscr{P}^A$  induced by the aggregate cost and the affine description of that cost  $[\alpha^A]^T z + \beta^A$  over that region. For convenience, we use

$$\mathscr{P}^A := \mathscr{P}^{y^A}, \quad \alpha^A := \alpha^{y^A}, \quad \beta^A := \beta^{y^A},$$

and extend the corresponding notation for  $y^B, \ldots, y^E$ .



Figure 3.3: An example to illustrate the iterative process of Algorithm 1.

The coordinator solves a linear program to minimize the affine aggregate cost  $[\alpha^A]^T z + \beta^A$  over  $z \in \mathscr{P}^A$ , and obtains the lexicographically smallest optimizer  $y^{\text{opt}}$ . Such an optimizer  $y^{\text{opt}}$  is always a vertex of  $\mathscr{P}^A$ . Identify  $y^B$  as that vertex in Figure 3.3. The optimal cost at  $y^B$  is indeed lower than the initial value of  $J^* = \infty$ , and hence, the coordinator sets  $y^* \leftarrow y^B$ . It also updates  $J^*$  to the aggregate cost at  $y^B$ , and the partial sub-differential set to  $\mathscr{D} \leftarrow \{\alpha^A\}$ .

Next, the coordinator solves the least square problem described in (3.11) to compute  $v^*$ . In so doing, it utilizes  $\mathscr{D} = \{\alpha^A\}$ , and  $\mathbf{K}^y = 0$  that describes the normal cone to  $\mathscr{Y}$  at  $y_B$ .<sup>9</sup> Suppose  $v^* \neq 0$ . The coordinator updates the value of y to  $y^C$ , obtained by moving a 'small' step of length  $\varepsilon$  from  $y^B$  along  $-v^*$ . Recall that  $y^C \notin \mathscr{P}^A$ . The coordinator again communicates with the SOs to obtain the new critical region  $\mathscr{P}^C$  induced by the aggregate cost that contains  $y^C$ . Again, it obtains the affine description of that cost and optimizes it over  $\mathscr{P}^C$  to obtain the new  $y^{\text{opt}}$ . In the figure, we depict the case when  $y^{\text{opt}}$  coincides with  $y^* = y^B$ .

Notice that the optimal cost  $J^{\text{opt}}$  at  $y^{\text{opt}}$  is equal to  $J^*$ , and hence, the coordinator only updates the partial sub-differential set  $\mathcal{D}$  to  $\{\alpha^A, \alpha^C\}$ . With the updated set of  $\mathcal{D}$ , the coordinator solves (3.11) to obtain  $v^*$ . In this example,  $v^*$  is again non-zero, and hence, the coordinator moves along a step

<sup>&</sup>lt;sup>9</sup> The normal cone to  $\mathscr{Y}$  at  $y^B$  is  $\{0\}$  because  $y^B$  lies in the interior of  $\mathscr{Y}$ .

of length  $\varepsilon$  along  $-v^*$  from  $y^B$  to land at  $y^D$ . Again,  $y^D \notin \{\mathscr{P}^A, \mathscr{P}^C\}$ . The coordinator repeats the same steps to optimize the aggregate cost over  $\mathscr{P}^D$  to obtain  $y^E$  as the new  $y^{\text{opt}}$ . Two cases can now arise, that we describe separately.

- If the optimal cost  $J^{\text{opt}}$  at  $y^{\text{opt}} = y^E$  does not improve upon the cost  $J^*$  at  $y^B$ , the coordinator ignores  $y^E$  and updates the set  $\mathcal{D}$  to  $\{\alpha^A, \alpha^C, \alpha^D\}$ . It computes  $v^*$  with the updated  $\mathcal{D}$ . Again, if  $v^* \neq 0$ , it traverses along  $-v^*$  to venture into a yet-unexplored critical region. The process continues till we get  $y^* = y^B$  as an optimizer (if  $v^* = 0$  at a future iterate), or we encounter the case we describe next.
- If  $J^{\text{opt}} < J^*$ , then the coordinator sets  $y^E$  as the new  $y^*$ . It retraces the same steps with this new  $y^*$ . In this example, since  $y^E$  is a vertex of  $\mathscr{Y}$ , one can show that (3.11) will yield  $v^* = 0$ , and hence,  $y^* = y^E$  will optimize the aggregate cost over  $\mathscr{Y}$ .

#### 3.4 The robust counterpart

The deterministic tie-line scheduling problem was formulated in the last section on the premise that available renewable supply and limits on power demands within each area are known at the time when tie-line schedules are decided. We now alter that assumption and allow these parameters to be uncertain. In particular, we let  $\xi_i = (\overline{W}_i, \underline{D}_i, \overline{D}_i)$  take values in a *box*, described by

$$\Xi_i := \{ \xi_i \in \mathbb{R}^{3n_i} \mid \xi_i^{\mathsf{L}} \le \xi_i \le \xi_i^{\mathsf{U}} \}$$
(3.12)

for i = 1, 2. The robust counterpart of the tie-line scheduling problem is then described by

$$\underset{y \in \mathscr{Y}}{\text{minimize}} \left( \max_{\xi_1 \in \Xi_1} J_1^*(y, \xi_1) + \max_{\xi_2 \in \Xi_2} J_2^*(y, \xi_2) \right).$$
(3.13)

We now develop an algorithm that solves (3.13) in a distributed fashion. Problem (3.13) has a *minimax* structure. Therefore, we employ a strategy in Algorithm 2 to alternately minimize the objective function over  $\mathscr{Y}$  and maximize it over  $\Xi_1 \times \Xi_2$ . Thanks to the following lemma, the maximization over  $\Xi_1 \times \Xi_2$  can be reformulated into a mixed-integer linear program.

**Lemma 2.** Fix  $y \in \mathscr{Y}$ . Then, there exists M > 0 for which maximizing  $J_i^*(y, \xi_i)$  over  $\xi_i \in \Xi_i$  is equivalent to the following mixed-integer linear program:

$$maximize_{w_{i},\rho,\lambda} \qquad c_{i}^{0} + [c_{i}^{\xi}]^{\mathsf{T}} \boldsymbol{\xi}_{i}^{\mathsf{L}} + (\boldsymbol{A}_{i}^{\xi} \boldsymbol{\xi}_{i}^{\mathsf{L}} + \boldsymbol{A}_{i}^{y} y - b_{i})^{\mathsf{T}} \boldsymbol{\lambda} + \mathbb{1}^{\mathsf{T}} \rho,$$
  

$$subject \ to \qquad c_{i}^{x} + [\boldsymbol{A}_{i}^{x}]^{\mathsf{T}} \boldsymbol{\lambda} = 0,$$
  

$$\rho \leq \mathsf{M} w_{i},$$
  

$$\rho \leq \mathsf{M} w_{i},$$
  

$$\rho \leq \mathsf{M}(\mathbb{1} - w_{i}) + \boldsymbol{\Delta}_{i}^{\xi} (c_{i}^{\xi} + [\boldsymbol{A}_{i}^{\xi}]^{\mathsf{T}} \boldsymbol{\lambda}),$$
  

$$w_{i} \in \{0, 1\}^{n_{i}}, \rho \in \mathbb{R}^{n_{i}}, \boldsymbol{\lambda} \in \mathbb{R}^{m_{i}}_{+}.$$

$$(3.14)$$

We use the notation  $\Delta_i^{\xi}$  to denote a diagonal matrix with  $\xi_i^{U} - \xi_i^{L}$  as the diagonal. The lemma builds on the fact that  $J_i^*(y, \xi_i)$  is convex in  $\xi_i$ , and hence, reaches its maximum at a vertex of  $\Xi_i$ .

The convexity is again a consequence of [43, Theorem 7.5]. Our proof in [47] leverages duality theory of linear programming and the so-called *big-M* method adopted in [49, Chapter 2.11] to reformulate the maximization of  $J_i^*(y, \cdot)$  over the vertices of  $\Xi_i$  into a mixed-integer linear program. An optimal  $\xi_i^{\text{opt}}$  can be recovered from  $w_i^*$  that is optimal in (3.14) using

$$\xi_i^{\text{opt}} := \xi_i^{\mathsf{L}} + \Delta_i^{\xi} w_i^*.$$

Next, we present our algorithm for solving the robust counterpart. In the algorithm, the SOs exclusively maintain and update certain variables; we distinguish these from the ones the coordinator maintains.

Algorithm 2 Solving the robust counterpart.

```
1: Initialize:
```

```
SO<sub>1</sub>: \mathscr{V}_1 \leftarrow \{ a \text{ vertex of } \Xi_1 \},\
SO<sub>2</sub>: \mathscr{V}_2 \leftarrow \{ a \text{ vertex of } \Xi_2 \}.
```

2: **do** 

3: Coordinator uses Algorithm 1 to solve

$$\underset{y \in \mathscr{Y}}{\text{minimize}} \left( \max_{\xi_1 \in \mathscr{V}_1} J_1^*(y, \xi_1) + \max_{\xi_2 \in \mathscr{V}_2} J_2^*(y, \xi_2) \right)$$

 $y^* \leftarrow$  optimizer in step 3. 4:  $J^* \leftarrow \text{optimal cost in step 3.}$ 5: For i = 1, 2, SO<sub>i</sub> performs: 6: Maximize  $J_i^*(y^*, \cdot)$  over  $\Xi_i$  using (3.14). 7:  $\xi_i^{\text{opt}} \leftarrow \text{optimizer in step 7.}$ 8:  $J_i^{\text{opt}} \leftarrow \text{optimal cost in step 7.}$ 9:  $\mathcal{V}_i \leftarrow \mathcal{V}_i \cup \{\xi_i^{\text{opt}}\}.$ 10: **return**  $J_i^{\text{opt}}$  to the coordinator. 11: 12: while  $J_1^{\text{opt}} + J_2^{\text{opt}} > J^*$ .

We summarize the main property of the above algorithm in the following result. See [47] for the proof, that is similar in spirit to [44, Preposition 2].

**Theorem 3.** Algorithm 2 terminates after finitely many steps, and  $y^*$  at termination optimally solves (3.13).

Our algorithm to solve the robust counterpart makes use of Algorithm 1 in step 3. The coordinator performs this step with necessary communication with the SOs. However, it remains agnostic to the uncertainty sets  $\Xi_1$  and  $\Xi_2$  throughout. Therefore, our algorithm is such that the SOs in general

will not be required to reveal their cost structures, network constraints, nor their uncertainty sets to the coordinator to optimally solve the robust tie-line scheduling problem. Further, Theorems 2 and 3 together guarantee that the coordinator can arrive at the required schedule by communicating with the SOs only finitely many times. These define some of the advantages of the proposed methodology. In the following, we discuss some limitations of our method.

The number of affine segments in the piecewise affine description of  $\max_{\xi_i \in \mathcal{V}_i} J_i^*(y, \xi_i)$  increases with the size of the set  $\mathcal{V}_i$ . The larger that number, the heavier can be the computational burden on Algorithm 1 in step 3. To partially circumvent this problem, we initialize the sets  $\mathcal{V}_i$  with that vertex of  $\Xi_i$  that encodes the least available renewable supply and the highest nominal demand. Such a choice captures the intuition that dispatch cost is likely the highest with the least free renewable supply and the highest demand. Our empirical results in the next section corroborate that intuition.

We make use of mixed-integer linear programs in step 7 of the algorithm. This optimization class encompasses well-known NP-hard problems. Solvers in practice, however, often demonstrate good empirical performance. Popular techniques for mixed-integer linear programming include branch-and-bound, cutting-plane methods, etc. See [49] for a survey. Providing polynomial-time convergence guarantees for (3.14) remains challenging, but our empirical results in the next section appear encouraging.

# 3.5 Numerical experiments

We report here the results of our implementation of Algorithm 2 on several power system examples. All optimization problems were solved in IBM ILOG CPLEX Optimization Studio V12.5.0 [50] on a PC with 2.0GHz Intel(R) Core(TM) i7-4510U microprocessor and 8GB RAM.

# 3.5.1 On a two-area 44-bus power system

Consider the two-area power system shown in Figure 3.4a, obtained by connecting the IEEE 14and 30-bus test systems [51]. The networks were augmented with wind generators at various buses. Transmission capacities of all lines were set to 100MW. The available capacity of each wind generator was varied between 15MW and 25MW. The lower limits on all power demands were set to zero, while the upper limits were varied between 98% and 102% of their nominal values. Our setup had 36 uncertain variables – 32 power demands and 4 available wind generation. Bus 5 in area 1 was set as the slack bus.

From the data in Matpower [52], we chose the linear coefficient in the nominal quadratic cost structure for each conventional generator to define  $P_i^g$  in (3.5). Further, we neglected wind spillage costs by letting  $P_i^w = 0$ , and defined  $P_i^d$  by assuming a constant marginal cost of \$100/MWh for not meeting the highest demands.

To run Algorithm 2, we initialized  $\mathcal{V}_i$  with the scenario that describes the highest power demands and the least available wind generation across all buses. To invoke Algorithm 1 in step 3, we initialized y with a vector of all zeros. When the algorithm encountered the same step in future



Figure 3.4: The two-area 44-bus system is portrayed on the left. It shows where the wind generators are added and the parameters for the tie-lines used in our experiments. The figure on the right plots the optimal aggregate costs from  $P_1$ ,  $P_2$  over 3000 samples of uncertain variables, and that of Algorithm 2 on this system.

Iteration	Step in Algorithm 2	Aggregate cost (\$/h)	Run-time (in ms)
1	Step 3 to compute $y^*$	9897.7	113.6
1	Step 7 to compute $\xi^{opt}$	9910.3	99.6
2	Step 3 to compute $y^*$	9899.3	93.4
2	Step 7 to compute $\xi^{opt}$	9899.3	121.5

Table 3.1: Evolution of aggregate cost of Algorithm 2 for the two-area power system in Figure 3.4a.

iterations, it was initialized with the optimal  $y^*$  from the last iteration to provide a *warm start*. Algorithm 2 converged in two iterations, *i.e.*, it ended when the cardinality of  $\mathcal{V}_1$  and  $\mathcal{V}_2$  were both two. The trajectory of the optimal cost and the run-times for each step are given in Table 3.1. In the first iteration, Algorithm 1 in step 3 with  $\varepsilon = 10^{-5}$  converged in four iterations<sup>10</sup> of its own and explored five critical regions induced by the aggregate cost. A naive search over  $\mathcal{Y}$  yielded that the aggregate cost induced at least 126 critical regions. Our simulation indicates that Algorithm 1 only explores a 'small' subset of all critical regions.

Step 7 of Algorithm 2 was then solved to obtain  $\xi_i^{\text{opt}}$ . As Table 3.1 suggests, the aggregate cost  $J_1^{\text{opt}} + J_2^{\text{opt}}$  exceeded  $J^*$  obtained earlier in step 3. Thus, the scenario of demand and supply captured in our initial sets  $\mathscr{V}_1$  and  $\mathscr{V}_2$  was *not* the one with maximum aggregate dispatch costs. To accomplish this step, two separate mixed-integer linear programs were solved – one with 13 binary variables (in area 1) and the other with 23 binary variables (in area 2). CPLEX returned the global optimal solutions in 15ms and 77ms, respectively. In the next iteration, step 3 was performed with  $\xi_i^{\text{opt}}$ 

<sup>&</sup>lt;sup>10</sup> The termination condition  $v^* = 0$  is replaced by checking that the Euclidean norm of a suitably normalized  $v^*$  is less than a threshold.

added to  $\mathscr{V}_i$ , where Algorithm 1 converged in five iterations, exploring only four critical regions. Finally, step 7 yielded  $J_1^{\text{opt}} + J_2^{\text{opt}} = J^*$ , implying that the obtained  $y^*$  defines an optimal robust tie-line schedule.

To further understand the efficacy of our solution technique, we uniformly sampled the set  $\Xi_1 \times \Xi_2$ 3000 times. With each sample ( $\xi_1, \xi_2$ ), we solved two optimization problems:

- $P_1$ : a deterministic tie-line scheduling problem solved with Algorithm 1,
- $P_2$ : the optimal power flow problem in each area with the optimal  $y^*$  obtained from Algorithm 2 for the robust counterpart.

The histograms of the optimal aggregate costs from  $P_1$  and  $P_2$  are plotted in Figure 3.4b. The same figure also depicts the optimal cost of the robust tie-line scheduling problem, which naturally equals the maximum among the costs from  $P_2$ . And for each sample, the gap between the optimal costs of  $P_1$  and  $P_2$  captures the cost due to lack of foresight. Figure 3.4b reveals that such costs can be significant. The median run-time of  $P_1$  was 48.5ms over all samples. The run-time for the robust problem was 458.2ms – roughly 10 times that median.

#### 3.5.2 On a three-area 187-bus system test

For this case study, we interconnected the IEEE 30-, 39-, and 118-bus test systems as shown in Figure 3.5. All transmission capacities were set to 100MW. Five wind generators were added to the 118-bus system (at buses 17, 38, 66, 88, and 111), three in the 39-bus system (at buses 3, 19, and 38), and two in the 30-bus system (at buses 11, and 23). Again, we adopted the same possible set of available wind power generations and power demands, as well as the cost structures as in Section 3.5.1. In total, our robust tie-line scheduling problem modeled 151 uncertain variables. For this multi-area power system, Algorithm 2 converged in the first iteration. The mixed integer programs in step 7 yielded the global optimal solution for each area, taking 62ms, 109ms, and 281ms, respectively. We again sampled the set  $\Xi_1 \times \Xi_2 \times \Xi_3$  3000 times, and solved  $P_1$ . The runtime of Algorithm 2 was 825.3ms, that is roughly 1.8 times the median run-time of  $P_1$ , given by 450.8ms.

We studied how our algorithm scales with the number of boundary buses by adding more tie-lines to the same system. The aggregate iteration count of Algorithm 1 is expected to grow with the number of induced critical regions, that in turn should grow with the boundary bus count. On the other hand, the iteration count of Algorithm 2 largely depends on the initial choice of the scenario encoded in the sets  $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3$ , and thus, varies to a lesser extent on the same count. Figure 3.6 validates these intuitions.

#### 3.5.3 Summary of results from other case-studies

We compared Algorithm 1 with a dual decomposition based approach proposed in [1]. That algorithm converges asymptotically, while our method converges in finitely many iterations. Table



Figure 3.5: A three-area 187-bus power system.



Figure 3.6: How our algorithms perform with variation in the number of tie-lines in the three-area 187-bus power system.

Items	Two-area 44-bus system	Three-area 187-bus system
# iterations in Algorithm 1	8	9
# iterations of [1]	23	78
Run-time of Algorithm 1 (ms)	458.2	825.3
Run-time of [1] (ms)	779.8	1227.5

Table 3.2: Comparison with the method in [1].

# areas	# buses	# uncertain variables	# boundary buses	# iter. in Algorithm 2	Run-time of Algorithm 2 (ms)	Run-time of joint problem (ms)
2	87	91	4	1	719.6	310.0
2	175	179	4	1	871.1	340.5
2	236	240	4	1	1732.6	391.5
2	418	42	10	1	1020.7	455.7
2	418	422	10	4	6124.5	461.4
3	354	360	12	3	4127.4	655.8
3	536	54	12	1	2557.6	699.7
3	536	546	12	3	18359.8	701.2

Table 3.3: Performance of Algorithm 2 on various multi-area power system examples in Figure 3.7.

3.2 summarizes the comparison.<sup>11</sup> Compared to that in [1], our algorithm clocked lesser number of iterations and lower run-times in our experiments.

Apart from the two systems considered so far, we ran Algorithm 2 on a collection of other multiarea power systems given in Figure 3.7. Tie-line capacities were set to 100MW and their reactances were set to 0.25*p.u.* Capacity limits on the transmission lines within each area were set to their respective nominal values in MATPOWER [52] wherever present, and to 100MW, otherwise. For all two-area tests, two wind generators were installed in the two areas at buses 6 and 14 in area 1 and buses 11 and 23 in area 2. For the three-area tests, we replicated the placements described in Section 3.5.2. Power demands and available wind generations were varied the same way as in Sections 3.5.1 and 3.5.2.

The results on these power systems are summarized in Table 3.3. Our experiments reveal that Algorithm 2 often converges within 1 - 4 iterations. The run-time of Algorithm 2 grows significantly with the number of uncertain parameters. The 418-bus and the 536-bus systems with 422 and 546 uncertain variables, respectively, corroborate that conclusion. Such growth in run-time is expected because the complexity of (3.14) grows with the number of binary decision variables that equals the number of uncertain parameters. Run-time of a joint multi-area optimal power flow problem with a sample scenario in the last column provides a reference to compare run-times for the robust one.

<sup>&</sup>lt;sup>11</sup> We say the method in [1] converges when the power flow over each tie-line as calculated by the areas at its end mismatches by < 0.01 p.u..



Figure 3.7: Additional power system examples considered for numerical experiments.

#### 3.6 Conclusion

This work presented an algorithmic framework to solve a tie-line scheduling problem in multi-area power systems. Our method requires a coordinator to communicate with the system operators in each area to arrive at an optimal tie-line schedule. In the deterministic setting, where the demand and supply conditions are assumed known during the scheduling process, our method (Algorithm 1) was proven to converge in finitely many steps. In the case with uncertainty, we proposed a method (Algorithm 2) to solve the robust variant of the tie-line scheduling problem. Again, our method was shown to converge in finitely many steps. Our proposed algorithms do not require the system operator to reveal the dispatch cost structure, network parameters or even the support set of uncertain demand and supply within each area to the coordinator. We empirically demonstrated the efficacy of our algorithms on various multi-area power system examples. Future directions include extending our framework to a multi-period setting, considering unit commitment decisions [53], reserve sharing decisions [54], and allowing for asynchronous updates from neighboring SOs [55].

We briefly review the state-of-the-art interchange mechanism CTS [34] in a deterministic setting. For a stochastic version of CTS, see [56].

For ease of presentation, we consider throughout this chapter a two-area power system illustrated in Figure 4.1(a). The proposed GCTS for more than two areas is straightforward and illustrated in the Appendix. We define a *boundary bus* as one to which a tie-line connecting two areas is attached. Other buses are called *internal buses*.

The system in Figure 4.1(a) is jointly operated by ISO 1 and ISO 2. In particular, each ISO controls the interior of its operating region defined by internal buses, and the two operators control jointly operating boundaries defined by boundary buses. The interchange problem is a two-stage process in which the neighboring ISOs jointly set the boundary state in a look-ahead scheduling, and each ISO optimizes internal states in real time subject to fixed interchange schedules.

In CTS, as shown in Figure 4.1(b), a "proxy bus" is selected among external boundary buses<sup>1</sup> in each area as a trading location of market participants who submit interface bids to the coordinator<sup>2</sup>. Each interface bid is a pair of buying and selling bids at proxy buses. They represent market participants' interest to arbitrage in a certain direction, which changes with the anticipated price gap. These bids are used to set the interchange defined as the net power transfer (rather than power flows on tie-lines) across boundaries.

Each interface bid has three attributes: an anticipated price difference  $\Delta \pi$  at proxy buses, a maximal quantity  $s_{max}$ , and an import/export direction. CTS bids are cleared 15-30 minutes prior to individual real-time markets by the coordinator. The clearing process is based on the minimizing of generation cost and the payoff to market participants. To this end, the coordinator collects demand/supply curves from system operators that are used in conjunction of bids from market participants to determine the interchange quantity. The demand/supply curve from each operator is obtained by computing the expected LMP at the proxy bus for its neighboring area for each interchange level<sup>3</sup>.

We use the graphical representation in Figure 4.2 from [34] to illustrate the clearing principle of

 $<sup>^{1}</sup>$  In case of internal buses of the neighboring area being placed as trading locations, we can preserve these trading buses in the equivalent network on the boundary in Figure 4.5(b). Thereby, similar method can be derived. For simplicity, we assume hereafter that all interface bids are at boundary buses

<sup>&</sup>lt;sup>2</sup> It is NYISO in the implementation between New York and New England

<sup>&</sup>lt;sup>3</sup> Injections and line capacities of the neighboring area are not used



Figure 4.1: The model of the interconnected power system and the proxy-bus model in CTS

CTS without tie-line congestion. Therein, curve  $\pi_i(q)$  represents the incremental cost of generation for Area *i*, and *Q* is the interface capacity. In this example, the direction of interchange is from Area 1 to Area 2<sup>4</sup>, so  $\pi_1(q)$  and  $\pi_2(q)$  serve as supply and demand curves, respectively. The third price curve  $\pi_2(q) - \Delta \pi(q)$  is the adjusted curve of  $\pi_2(q)$  by subtracting the aggregated interface bids  $\Delta \pi(q)$ . CTS interchange schedule  $q^{CTS}$  is set at the intersection of  $\pi_1(q)$  and  $\pi_2(q) - \Delta \pi(q)$ . All interface bids with prices lower than  $\Delta \pi(q^{CTS})$  are cleared.

Interface bids are separately settled in individual real-time markets where the proxy bus injection is set as  $q^{CTS}$ . The net interchange between the two areas will match with the scheduled  $q^{CTS}$ . The real-time LMP at proxy buses  $\pi_1^{RT}$  and  $\pi_2^{RT}$  are used to settle cleared interface bids. We note that there is a time latency between the clearing of interface bids and the physical power delivery. Such randomness may cause price deviations from the expected LMP difference at the time of interface bid clearing. Therefore, market participants with cleared bid offers are exposed to risks of losing money.

If there are tie-line congestions, *i.e.*, the intersection of  $\pi_1(q)$  and  $\pi_2(q) - \Delta \pi(q)$  is greater than Q, then the net interchange will be scheduled at  $q^{CTS} = Q$ . There is  $\pi_2(q^{CTS}) - \pi_1(q^{CTS}) > \Delta \pi(q^{CTS})$ . In CTS, such a price difference  $\rho$  is equally partitioned into congestion prices for the two areas. Specifically, interface bids are paid at  $(\pi_1^{RT} - \frac{\rho}{2})$  in Area 1 and charged at  $(\pi_2^{RT} + \frac{\rho}{2})$  in Area 2, respectively. Note that  $\rho$  is calculated in the look-ahead clearing process, whereas  $\pi_i^{RT}$  is determined in the real-time dispatch.

<sup>&</sup>lt;sup>4</sup> This is because  $\pi_1(0) < \pi_2(0)$ . If  $\pi_1(0) > \pi_2(0)$ , the direction would be opposite.



Figure 4.2: Illustration of CTS's clearing without tie-line congestion



Figure 4.3: Illustration of CTS's clearing with tie-line congestion

We review the process of CTS and the role of interface bids via Figure 4.4. Clearing interface bids will create imbalances of local supply and demand in each area. Physically, as in Figure 4.4(a), such local imbalances naturally compel power to flow across tie-lines in a interconnected power system. Financially, as shown in Figure 4.4(b), there is no direct cash flow between the two ISOs. The payment to excess power generations in Area 1 and the revenue from excess power consumptions in Area 2 are balanced by external market participants who buy from Area 1 and sell to Area 2. Note that, cleared interface bids are financial contracts and do not physically generate or consume. They simply provide financial compensations that allow each regional market to dispatch imbalanced generations and consumptions so that power can flow across their boundaries.

Although it is reported that the CTS approach has to some extent ameliorated the seams issue, inefficient scheduling still persists [57]. In particular, modeling the net interchange as the injection to the proxy bus may be highly inaccurate when there are multiple tie-lines. In what follows, we present a generalization of CTS by removing the proxy bus approximation.



(b) Independent Financial System

Figure 4.4: Physical power system versus the financial trading procedure.

#### 4.1 Introducing generalized CTS

# 4.1.1 Network model

Without loss of generality, we assume that no generator or load is on the boundary bus. This assumption is made for convenience of presentation. A boundary bus that has a generator can be split into a fictitious internal bus with a generator and a boundary bus without injection. We also assume that each internal bus has one generator and one load. Let  $g_i$  be the vector of power generations and  $d_i$  the vector of load in Area *i*.

We adopt the DC power flow model in this work. Specifically, nodal phase angles are state variables that are determined by active power injections. The state variables in Area *i* are partitioned into internal phase angles  $\theta_i$  and boundary phase angles  $\bar{\theta}_i$ . We also use  $\bar{\theta} = [\bar{\theta}_1, \bar{\theta}_2]^T$  to represent all boundary phase angles.

The DC power flow equation for the two-area power system in Figure 4.1(a) is

$$\begin{bmatrix} Y_{11} & Y_{1\bar{1}} & & \\ Y_{\bar{1}1} & Y_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} & \\ & Y_{\bar{2}\bar{1}} & Y_{\bar{2}\bar{2}} & Y_{\bar{2}2} \\ & & & Y_{2\bar{2}} & Y_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} g_1 - d_1 \\ 0 \\ 0 \\ g_2 - d_2 \end{bmatrix},$$
(4.1)

where  $Y_{1\bar{1}}$  is the nodal admittance sub-matrix<sup>5</sup> associated with the internal and boundary buses in

<sup>&</sup>lt;sup>5</sup> The matrix Y is composed of reciprocals of branch reactance and differs from the bus admittance matrix used in AC

Area 1, and  $Y_{\bar{1}\bar{2}}$  the sub-matrix associated with boundary buses in areas 1 and 2. Other terms in the coefficient matrix in (4.1) are similarly defined.



Figure 4.5: Boundary equivalent network model in GCTS

An equivalent network that captures completely the electrical properties at the boundary of the two networks can be derived as follows and is illustrated in Figure 4.5. Here boundary buses of Area *i* have equivalent self-admittance  $\tilde{Y}_{i\bar{i}}$ , mutual-admittance  $Y_{\bar{i}\bar{j}}$ , and generation  $\tilde{g}_{\bar{i}}$ . The power flow equation for the equivalent network for the interface is

$$\begin{bmatrix} \tilde{Y}_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} \end{bmatrix} \begin{bmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} \tilde{g}_{\bar{1}} \\ \tilde{g}_{\bar{2}} \end{bmatrix},$$
(4.2)

where

$$\tilde{Y}_{\bar{i}\bar{i}} = Y_{\bar{i}\bar{i}} - Y_{\bar{i}i}Y_{i\bar{i}}^{-1}Y_{i\bar{i}}, \tilde{g}_{\bar{i}} = -Y_{\bar{i}i}Y_{i\bar{i}}^{-1}(g_i - d_i).$$
(4.3)

The coefficient matrix in (4.2) does not change with nodal power injections. Throughout this chapter, we assume that the two-area system is on the same island, so the coefficient matrix in (4.2) is full rank after removing the reference bus. The equivalent power injection  $\tilde{g}_{\bar{i}}$  succinctly captures the external impact of internal power injections in Area *i*; it represents its power interchange schedule. Therefore, in what follows, those equivalent power injections are associated with interface bids from external market participants. Hereafter, we drop the word "external" if that does not cause any confusion.

#### 4.1.2 Definition of interface bids

GCTS uses the same format of bids as CTS. Namely, an interface bid *i* is defined by a triple

$$\mathscr{B} \triangleq \{ \langle B_{pm}, B_{qn} \rangle, \Delta \pi_i, s_{\max,i} \},\$$

where

1.  $\langle B_{pm}, B_{qn} \rangle$  is an ordered pair of boundary buses that specifies the bid as withdrawing at bus *m* in Area *p* and injecting the same amount at bus *n* in Area *q*. They need not be directly connected by a tie-line;

power flow model.

- 2.  $\Delta \pi_i$  is its price bidding on the anticipated price gap that the bid is settled in the two real-time markets<sup>6</sup>;
- 3.  $s_{\text{max,i}}$  is its maximum quantity.

The only difference between CTS and GCTS is that, in stead of using a single proxy bus in each area, GCTS allows bids to be submitted to all pairs of boundary buses across the boundary, as illustrated in Figure 4.6.



Figure 4.6: Network equivalence on the boundary. Dotted-line arrows represent three interface bids in the example below:  $s_1$  injects at  $B_{11}$  and withdraws at  $B_{21}$ ;  $s_2$  injects at  $B_{11}$  and withdraw at  $B_{22}$ ;  $s_3$  injects at  $B_{12}$  and withdraws at  $B_{22}$ .

We aggregate all interface bids with an incidence matrix  $M_i$  associated with boundary buses in Area *i*. Specifically, each row of  $M_i$  corresponds to a boundary bus of Area *i*, and each column of which corresponds to an interface bid. The entry  $M_i(m,k)$  is equal to one if interface bid *k* buys power at boundary bus  $B_{im}$  from Area *i*, minus one if it sells power at bus  $B_{im}$  to Area *i*, and zero otherwise. For example, if there are three bids as illustrated in Figure 4.6, matrices  $M_i(i = 1, 2)$  are

$$M_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} B_{11} \\ B_{12} \end{pmatrix}, M_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{pmatrix} B_{21} \\ B_{22} \end{pmatrix}.$$
 (4.4)

Consequently, let *s* be the vector whose *i*th entry  $s_i$  is the cleared quantity of bid *i*. Then  $M_is$  represents the aggregated equivalent power injection induced by cleared interface bids on boundary buses in Area *i*. By substituting the right-hand side in (4.2) by  $M_is$ , we have

$$\begin{bmatrix} \tilde{Y}_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} \end{bmatrix} \begin{bmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} M_1 s \\ M_2 s \end{bmatrix}.$$
(4.5)

<sup>&</sup>lt;sup>6</sup> This may not be equal to the LMP difference. See Subsection III-D and Remark 2 after Theorem 4 for mathematical and economical interpretations.

In (4.5), the interchange schedule is solely determined by the cleared interface bids from market participants. In the market clearing process of GCTS, as presented in the next subsection, Equation (4.5) will be incorporated as an equality constraint in the optimization model where the internal bids  $g_i$  and interface bids s are cleared together.

#### 4.1.3 Market clearing mechanism

0

GCTS preserves the architecture of CTS; it assumes the presence of a coordinator who collects interface bids and clears them via a look-ahead dispatch, and the interface bids are settled separately in the real-time markets. GCTS removes the proxy bus approximation, and its clearing of interface bids is based on a generalization of JED. The key idea is to clear interface bids by optimizing the boundary state as follows:

$$\underset{\{g_i, s, \bar{\theta}, \theta_i\}}{\text{minimize}} \quad c(g_1, g_2, s) = \sum_{i=1}^2 c_i(g_i) + \Delta \pi^T s,$$
(4.6a)

subject to 
$$\check{g}_i \le g_i \le \hat{g}_i, \qquad i = 1, 2,$$
 (4.6b)

$$\leq s \leq s_{\max},$$
 (4.6c)

$$H_i \theta_i + H_{\bar{i}} \bar{\theta}_i \le f_i, \quad i = 1, 2, \tag{4.6d}$$

$$\bar{H}_{\bar{1}}\bar{\theta}_1 + \bar{H}_{\bar{2}}\bar{\theta}_2 \le \bar{f},\tag{4.6e}$$

$$\begin{bmatrix} Y_{11} & Y_{1\bar{1}} & & \\ Y_{\bar{1}1} & Y_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} & \\ & Y_{\bar{2}\bar{1}} & Y_{\bar{2}\bar{2}} & Y_{\bar{2}2} \\ & & Y_{2\bar{2}} & Y_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \bar{\theta}_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} g_1 - d_1 \\ 0 \\ 0 \\ g_2 - d_2 \end{bmatrix},$$
(4.6f)
$$\begin{bmatrix} \tilde{Y}_{1\bar{1}} & Y_{\bar{1}\bar{2}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} \end{bmatrix} \begin{bmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} M_{1s} \\ M_{2s} \end{bmatrix},$$
(4.6g)

where decision variables are the cleared internal generation bids  $g_i$  with the quantity limit (4.6b), the cleared interface bids *s* with the quantity limit (4.6c), and the system states ( $\theta_i$ ,  $\bar{\theta}$ ) subject to internal and tie-line power limits (4.6d) and (4.6e). Any bid *i* with  $s_i = s_{\max,i}$  is fully cleared, any with  $s_i = 0$  is rejected, and any with  $0 < s_i < s_{\max,i}$  is partially cleared at amount  $s_i$ .

Note that, the term  $\Delta \pi^T s$  represents the market cost of clearing interface bids. Because the price difference in the real time is in general different from the look-ahead dispatch, market participants carry a certain amount of risk. Thus the bid  $\Delta \pi_i$  represents the willingness of the bidder *i* to take that risk. See [58] for details of the quantification for risks.

The market clearing model of GCTS (4.6a)-(4.6g) differs from JED in two aspects: (i) the market cost of clearing interface bids  $\Delta \pi^T s$  in the objective function and (ii) the additional equality constraint (4.6g) that determines the boundary state by clearing interface bids subject to their

quantity limits. The coordinator sets the interchange by clearing the interface bids to minimize the overall cost subject to operational constraints and constraints (4.6c) and (4.6g) imposed by the interface bids.

The clearing problem (4.6a)-(4.6g) of GCTS is a look-ahead economic dispatch where the load powers  $d_i$  are predicted values. It should be solved in a hierarchical or decentralized manner. Any effective multi-area economic dispatch method can be employed. See, *e.g.*, [39, 59] where (4.6a)-(4.6g) is solved with a finite number of iterations.

#### 4.1.4 Real-time dispatch and settlement

Interface bids are settled in the real-time market together with internal bids. There is no coordination required at this step. Specifically, ISO 1 solves its local economic dispatch with fixed boundary state  $\bar{\theta}$ :

$$\underset{\{g_1,\theta_1\}}{\text{minimize}} c_1(g_1), \tag{4.7a}$$

subject to  $H_1\theta_1 + H_{\bar{1}}\bar{\theta}_1 \le f_1$ ,  $(\eta_1^R)$  (4.7b)

$$\check{g}_1 \le g_1 \le \hat{g}_1, \ i = 1, 2,$$
  $(\bar{\xi}_1^R, \underline{\xi}_1^R)$  (4.7c)

$$\begin{bmatrix} Y_{11} & Y_{1\bar{1}} \\ Y_{\bar{1}1} & Y_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} g_1 - d_1^R \\ 0 \end{bmatrix}, \quad (\lambda_1^R)$$
(4.7d)

where  $d_1^R$  represents real-time internal loads, which may deviate from their predictions in the lookahead dispatch (4.6a)-(4.6g). The real-time internal dispatch in each area should be compliant with the pre-determined interchange schedule. To this end, boundary state  $\bar{\theta}$  is fixed at the solution to Equation (4.6g) with *s* cleared interface bids solved from (4.6a)-(4.6g). All multipliers are given to the right of corresponding constraints.

ISO 1 simultaneously settles internal and interface bids in the real-time market. Internal bids are settled at the LMP  $\lambda_1^R$ . To settle interface bids, we need to analyze the sensitivity of the local optimal cost in (4.7a) with respect to *s*. In the real-time dispatch (4.7a)-(4.7d), the impact of interface bids *s* is imposed via the fixed boundary state variables  $\bar{\theta}$ . The sensitivity of local optimal cost with respect to  $\bar{\theta}$  is

$$\nabla_{\bar{\theta}} c_1^* = \begin{bmatrix} Y_{\bar{1}1} & Y_{\bar{1}\bar{1}} \\ & Y_{\bar{2}\bar{1}} \end{bmatrix} \begin{bmatrix} \lambda_1^R \\ \bar{\lambda}_1^R \end{bmatrix} + \begin{bmatrix} H_{\bar{1}}^T \eta_1^R \\ 0 \end{bmatrix}.$$
(4.8)

The sensitivity of local optimal cost with respect to s is

$$\nabla_{s}c_{1}^{*} = [\nabla_{s}\bar{\theta}]^{T}\nabla_{\bar{\theta}}c_{1}^{*} = M^{T} \begin{bmatrix} \tilde{Y}_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} \end{bmatrix}^{-1} \nabla_{\bar{\theta}}c_{1}^{*} \triangleq \mu_{1}^{R}.$$
(4.9)

In the absence of tie-line congestion, interface bids pay prices  $\mu_1^R$  in Area 1 and  $\mu_2^R$  in Area 2 (they get paid if  $\mu_i < 0$ ). In general, interface bids are not settled at LMPs. This is because the change of the objective function (4.7a) with an increment of cleared *s* differs from that with an increment of load power.

If there are tie-lines congested, similar to CTS, we will compute congestion rents according to the look-ahead dispatch (4.6a)-(4.6g) and subtract them from the payment to interface bidders. Tie-line congestion prices associated with interface bids are calculated by

$$\rho = M^T \tilde{S}^T \bar{\eta}, \tilde{S} = [\bar{H}_{\bar{1}} \ \bar{H}_{\bar{2}}] \begin{bmatrix} \tilde{Y}_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} \end{bmatrix}^{-1},$$
(4.10)

where  $\bar{\eta}$  is the shadow price in (4.6e), and  $\tilde{S}$  is the shift factor of boundary buses with respect to tie-lines in Figure 4.6. Similar to CTS, we evenly split the congestion rent price  $\rho$  into two areas<sup>7</sup>. Namely, market participants pay  $\mu_i^R + \frac{\rho}{2}$  in Area i, i = 1, 2.

If  $d_i^R = d_i$ , one can prove that the real-time dispatch level and prices are consistent with the lookahead dispatch. Note that fixing some variables at their optimal values does not change optimal values of other primal and dual variables. If the real-time dispatch (4.7a)-(4.7d) is infeasible, ad hoc adjustments such as relaxations of flow limits can be employed in practice.

#### 4.2 **Properties of GCTS**

#### 4.2.1 Efficiency and price convergence of GCTS

By removing the proxy bus approximation and adopting a strict DC OPF model in (4.6a)-(4.6g), we are able to establish many important properties for GCTS. All proofs are omitted and can be found in [60].

We first show that GCTS asymptotically achieves seamless interfaces when more and more bidders participate in the competition at all possible pairs of trading locations. Intuitively, for GCTS to achieve the cost of JED, two conditions are necessary in general. First, there have to be enough bidders who try to capture the arbitrage profits across the interface so that they drive  $\Delta \pi \rightarrow 0$ . This follows the standard economic argument of perfect competition. Second, bids need to be diverse

<sup>&</sup>lt;sup>7</sup> When there are more than two areas, tie-line congestions may induce positive shadow prices  $\rho$  for interface bids over other interfaces. Nevertheless, the calculation of  $\rho$  is the same as in (4.10), and the shadow price should be evenly split by neighboring areas.

enough to make the matrix M full row rank so that the tie-line flows of the GCTS can match those of JED. It turns out that both conditions can be satisfied simultaneously by conditions below.

**Theorem 4.** (Asymptotic efficiency) Consider a market with N independent interface bidders. Assume that (i) both JED and GCTS are feasible and each has an unique optimum, (ii) the number of aggregated bids for each pair of source and sink buses grows unbounded with N, and (iii) bidding prices for all participants go to zero as  $N \to \infty$ , i.e.  $\lim_{N\to\infty} \Delta \pi = 0$ , then the scheduled tie-line power flows and generations in each area by GCTS converge to those of the JED as  $N \to \infty$ .

**Remark 1**: Recall that JED by a "super ISO" provides the lowest possible generation cost, thus achieving the overall market efficiency. In practice, the power system is artificially partitioned into multiple subareas that are operated by financially neutral ISOs, and interchange scheduling has to rely on bids from market participants. Such operational regulations will naturally create seams at interfaces. Theorem 4 shows that, however, GCTS asymptotically achieves seamless interfaces under mild conditions. This indicates that GCTS leads to the price convergence between regional electricity markets.

**Remark 2**: The price convergence implies that there is no arbitrage opportunity, and that the dispatch level of GCTS is the same as that of JED. Note that due to congestions, boundary buses may have different LMPs even under the administration of a "super-ISO". So the price convergence is in fact for shadow prices of *s*. This also explains why interface bids should be settled at  $\mu_i^R$  in (4.9) but not LMPs.

**Remark 3**: The assumptions that bidding locations are diverse enough and that  $\Delta \pi$  goes to zero as N increases come from the interpretation that, as the number of bidders increases, there are always enough bids that can be cleared to satisfy the desired interchange level. Thus individually, each bidder seeks trading locations with seams and reduces its bidding price so that it will have a better chance to be cleared.

# 4.2.2 Relation between GCTS and CTS

Next we establish connections between GCTS and CTS. Specifically, we show that the two mechanisms are equivalent in a particular simple setting.

**Theorem 5.** When there is a single tie-line between two areas, the clearing process of GCTS (4.6a)-(4.6g) provides the same interchange as that of CTS.

**Remark:** A natural corollary of Theorem 5 is that when there is a single tie-line between two areas and real-time load is the same as the load considered in the interchange scheduling, then CTS provides the optimal interchange schedule in the sense that the posterior real-time dispatch  $g_i^R$  minimizes the total cost of all internal and external market participants.

In practice, however, neither condition in these two theorems is likely to hold. In such cases, our

simulations show that GCTS generally has lower overall cost than CTS and its dispatch satisfies security constraints. CTS, may on the other hand, may violate security constraints due to the loop flow problem engendered by its proxy-bus approximation. See Section V for details.

#### 4.2.3 Revenue adequacy

In this subsection, we establish the revenue adequacy for the real-time market (4.7a)-(4.7d). Recall that, in the single-area economic dispatch, each area has a non-negative net revenue, which is equal to its congestion rent. We prove in the following theorem that each area achieves its revenue adequacy in the same fashion in GCTS in an interconnected power system.

**Theorem 6.** Assume that the real-time dispatch (4.7*a*)-(4.7*d*) is feasible and that the settlement process follows our description in Subsection III-D, then the net revenue of each area is non-negative and is equal to its congestion rent.

#### 4.2.4 Local performance

ISOs are mainly responsible of the efficiencies of their own regional markets, rather the overall efficiency. Therefore, an ISO may be reluctant to implement any interchange scheduling approach that worsens its local performance for the sake of the overall efficiency. We partly address this issue in this subsection.

In the conventional interchange scheduling before CTS, market participants split their bidding prices into  $\Delta \pi = \pi_1 + \pi_2$  and separately submit them to the two neighboring ISOs who clear these bids independently. Only bids cleared in both markets will be scheduled [61]. In essence, we take the minimum of the cleared quantities. In the following theorem, we prove that GCTS achieves higher local surpluses in all areas than the conventional approach under a simple setting:

**Theorem 7.** Assume that (i) there is a single tie-line between two neighboring areas, (ii) real-time load demands are the same as their look-ahead predictions, and (iii) each market clearing problem has an unique optimum, then there is

$$\tilde{LS}_i \ge \hat{LS}_i, \tag{4.11}$$

where  $L\tilde{S}_i$  is the local surplus of area *i* in its real-time market (4.7*a*)-(4.7*d*) with  $\bar{\theta}$  determined by the optimal  $\tilde{s}$  cleared in GCTS (4.6*a*)-(4.6*g*). Specifically, it is defined as

$$\tilde{LS}_i \triangleq (D_i - (\tilde{\lambda}_i^R)^T d_i) + ((\tilde{\lambda}_i^R)^T \tilde{g}_i - c_i(\tilde{g}_i)) + f_1^T \tilde{\eta}_1^R,$$
(4.12)

where  $D_i$  is the constant utility of consumers. Variables with tildes are solved with  $\tilde{s}$ . The local total surplus in Area i is the sum of its consumer surplus, supplier surplus, and the surplus of transmission owners. The local surplus  $\hat{LS}_i$  with  $\hat{s}$  the result of separate clearing is similarly defined.

We remove this result from our journal submission because this is more about CTS. In general, when there are multiple tie-lines, Theorem 7 may not hold for GCTS. Nevertheless, it is important

to look into performances in regional markets, especially for power systems that cover multiple regions or even countries. Investigating weaker conditions for Theorem 7 would be an interesting direction for future works.

### 4.3 Numerical tests

#### 4.3.1 Two-area 44-bus system

GCTS was tested on a two-area system composed of the IEEE 14-bus system (Area 1) and the 30-bus system [62] (Area 2). The system configuration and reactance and capacities of tie-lines are illustrated in Figure 4.7.



Figure 4.7: Configuration of the two-area power system

Two groups of simulations were conducted. First, we aimed to illustrate the market clearing process of GCTS. Second, we compared GCTS with JED and CTS and numerically demonstrated the asymptotic convergence of GCTS to JED as in Theorem 4.

#### Illustration of the market clearing process

Eight interface bids were considered in the first group of simulations. Their trading locations and prices  $\Delta \pi$  are listed in Table 4.1. Some market participants traded on boundary buses without direct connections, such as bids 2 and 5. The maximal quantities of all bids were set as 30MW.

From default prices in Area 1, we used a weighting factor w to generate scenarios with various degrees of price discrepancies. For all scenarios, cleared interface bids, tie-line power flows, marginal prices, and system costs are presented in Table 4.2:

The second block (second to ninth rows) in Table 4.2 lists cleared amounts for interface bids. With the increase of w, bids delivering power from Area 2 to Area 1 were cleared at greater quantities, see the fifth and eighth rows, while those delivering power in the opposite direction were cleared at smaller quantities, see the second row.

Indices	Sell to	Buy from	Price (\$/MWh)
1	Bus 15 (Area 2)	Bus 5 (Area 1)	1
2	Bus 28 (Area 2)	Bus 5 (Area 1)	2
3	Bus 5 (Area 1)	Bus 15 (Area 2)	1.5
4	Bus 5 (Area 1)	Bus 28 (Area 2)	0.5
5	Bus 15 (Area 2)	Bus 9 (Area 1)	1.0
6	Bus 28 (Area 2)	Bus 9 (Area 1)	2.0
7	Bus 9 (Area 1)	Bus 15 (Area 2)	1.5
8	Bus 9 (Area 1)	Bus 28 (Area 2)	0.5

Table 4.1: Profile of interface bids

The third block includes results on tie-line power flows. They were determined by the boundary power flow equation (4.5) and cleared amounts of bids in the second block. When w = 0.1, tie-line power flows were in both directions. When w was increased, which signified greater price discrepancies, tie-line power flows became unidirectional from the low-price area to the high-price area.

The fourth block are marginal prices for all boundary buses in the market clearing process, *i.e.*, multipliers associated with boundary equality constraint (4.6g). All bids whose prices were lower than marginal price gaps between their trading points were totally cleared, see the second, twelfth, and fourteenth rows when w = 0.1 and the second row in Table 4.1 as an example. All bids whose prices were higher were rejected, see the third, twelfth, and fifteenth rows when w = 0.1 and the third row in Table 4.1 as an example. For partially cleared interface bids, marginal price gaps between their trading points were equal to their bidding prices, see the eighth, thirteenth, and fourteenth rows when w = 0.1 and the eighth row in Table 4.1 as an example.

The last block are generation costs, costs of market participants, and total costs per hour in the proposed approach. GCTS considered the total market cost of internal and interface bidders.

#### Comparison with existing benchmarks

In the second group of simulations, we compared the proposed method with existing approaches on tie-line scheduling. Specifically, the following methods were compared:

i) JED that minimized the total generation cost;

ii) CTS wherein proxy buses were selected as bus 5 in Area 1 and bus 15 in Area 2;

iii) The proposed mechanism of GCTS.

Default generation prices were considered in this test. We used similar bids to those in Table 4.1

Weighting factor		0.1	0.15	0.2	1.0
	1	30	30	5.56	5.56
Classed	2	0	0	0	0
quantities	3	0	0	0	0
of	4	0	0	30	30
interface	5	0	0	0	0
(MW)	6	0	0	0	0
	7	10.40	30	30	30
	8	30	30	30	30
Tie-line	bus 15 to 5	-8.66	5.53	38.60	38.60
flow (MW)	bus 28 to 9	19.05	41.31	45.84	45.84
Manainal	bus 5	-0.10	0.02	0.82	15.62
nrices	bus 9	2.90	4.34	6.49	45.96
(\$/MWh)	bus 15	1.40	1.04	1.82	16.62
(4/112//11/	bus 28	0.0	0.0	0.0	0.0
Market	Internal	923.2	1148.2	1371.0	4525.4
costs	interface	58.1	90	80.56	80.56
(\$/h)	total	983.0	1238.2	1451.6	4605.9

Table 4.2: Results of interface bid clearing

for GCTS but their quantity limits and prices were uniformly set as  $s_{max} = 100MW$  and  $\Delta \pi =$ \$0.1/*MWh*. In CTS, all bids were placed at proxy buses with the same quantity limits and prices.

We compared market costs in the look-ahead interchange scheduling as well as those in the realtime local dispatch. For the latter, we generated 100 normally distributed realizations of realtime load consumptions whose mean values were their look-ahead predictions (default values in the system data) and standard deviations were 5% of their mean values. Comparisons on net interchange quantities, look-ahead generation costs and total costs, and real-time average total costs for all samples are recorded in Table 4.3:

Table 4.3:	Comparison	of JED,	CTS, a	and GCTS	for the	two-area	test
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	JED	CTS	GCTS
Net interchange amounts (MW)	87.0	80.3	87.0
Look-ahead generation costs (\$/h)	4039.8	4109.9	4039.8
Look-ahead total costs (\$/h)	_	4118.0	4048.5
Average real-time total costs (\$/h)	4096.2	4139.8	4115.7

From Table 4.3 we observed that GCTS achieved lower look-ahead and average real-time costs than CTS. Specifically, GCTS had lower real-time costs in 88 out of the 100 samples. In addition,

CTS suffered from the loop-flow problem in that branch power flows solved with the global power flow equation and real-time dispatch levels in both areas deviated from internal real-time schedules. In this test, average discrepancies on tie-line power flows were 18.25% for Area 1 and 16.38% for Area 2, respectively. As a result, CTS caused unpredicted overflows for transmission lines in all of the 100 scenarios, with 2.72 overflowed transmission lines in each scenario on average and the average ratio of overflows as 11.27%. In GCTS, however, such problems did not exist because it is based on the exact DC power flow model. Another takeaway of Table 4.3 is that, with sufficient bids and relatively low prices ( $\Delta \pi = \$0.1/MWh$ ), the interchange scheduled by GCTS was the same as that in JED in this test.

We illustrate the price convergence of GCTS with different values of w in Figure 4.8 by adjusting the uniform bidding price  $\Delta \pi$ . No bid was cleared when the bidding price  $\Delta \pi = \$100/MWh$ . When  $\Delta \pi$  decreased to small enough values (\$0.1/MWh in this test), generation costs of GCTS in all scenarios were equal to those of JED. In general, the more significant the price discrepancy was, the faster the price converged. This is consistent with our intuition that market participants could be cleared at higher prices when there is more room for arbitrations.



Figure 4.8: Price convergence of GCTS with different bidding prices

Note that such price convergence did not happen in CTS. For the test in Table 4.3, for example, if we set the bidding price of CTS as zero, the total generation cost would be \$4109.7 per hour, which was higher than that of JED.

#### 4.3.2 Three area 189-bus system test

The proposed method was also tested on a three-area system as shown in Figure 4.9. The system was composed of IEEE 14, 57, and 118-bus systems. Power flow limits on all lines were set as 100 MW. Eight interface bids were considered. For each tie-line, there were two interface bids who traded at their terminal buses but in opposite directions. The prices and maximum quantities for all interface bids were respectively set as \$0.5/(MW-h) and 100MW.



Figure 4.9: Configuration of the three-area power system

The results of market clearing are given in Figure 4.10, where internal parts of all areas are represented by their network equivalences. Cleared interface bids are denoted by power injections at boundary buses. Power flows through tie-lines are also shown, which were determined by the DC power flow equation for the network (4.5) in Figure 4.10.

The total cost of the three-area system was  $1.263 \times 10^5/h$ , in which the cost of market participants was 601.43/h and the rest was the generation cost. As a reference, if there is no interchange at all, the total generation cost would be  $1.394 \times 10^5/h$ . The reduction of generation cost largely exceeded the cost of market participants.



Figure 4.10: Clearing of interface bids and tie-line power flows (MW)

We did similar comparisons of JED, CTS, and GCTS for this three-area test as in Table 4.4. In CTS, interchange schedules were set in a pairwise manner, and proxy buses were always selected as ones with the smallest indices on their sides.

Our conclusions of comparisons were similar to those in the two-area test. GCTS had lower lookahead costs than CTS, which was close to JED. Although its real-time costs were similar to CTS,

	JED	CTS	GCTS
Look-ahead generation costs (\$/h)	$1.255 \times 10^{5}$	$1.261 \times 10^{5}$	$1.257 \times 10^{5}$
Look-ahead total costs (\$/h)	_	$1.262 \times 10^{5}$	$1.257 \times 10^{5}$
Average real-time total costs (\$/h)	$1.255 \times 10^{5}$	$1.263 \times 10^{5}$	$1.263 \times 10^{5}$

Table 4.4: Comparison of JED, CTS, and GCTS for the three-area test

GCTS removed the loop-flow problem in CTS. Namely, CTS suffered from overflow problems in 92 out of the 100 scenarios with randomly generated load powers.

#### 4.3.3 Cases with more than two areas

In this subsection, we generalize GCTS to cases with more than two areas. For each area, the network equivalence is illustrated in Figure 4.11. Therein, internal buses are eliminated, and the equivalent admittance matrix  $Y_{\bar{1}\bar{1}}$  and injection  $\tilde{g}_i$  are still calculated by (4.3). The calculation of  $Y_{\bar{1}\bar{1}}$  and  $\tilde{g}_i$  only requires local information.



(b) Equivalent system model

Figure 4.11: Network equivalence with more than two areas

Thereby, the equivalent model of the global power system, corresponding to the Figure 4.5, can be obtained by eliminating all internal buses. An example of a three-area system can be seen in Figure 4.10. In the clearing of GCTS with n areas, the constraint (4.6g) becomes

$$\begin{bmatrix} \tilde{Y}_{\bar{1}\bar{1}} & Y_{\bar{1}\bar{2}} & \dots & Y_{\bar{1}\bar{n}} \\ Y_{\bar{2}\bar{1}} & \tilde{Y}_{\bar{2}\bar{2}} & \dots & Y_{\bar{2}\bar{n}} \\ \dots & \dots & \dots & \dots \\ Y_{\bar{n}\bar{1}} & Y_{\bar{n}\bar{2}} & \dots & \tilde{Y}_{\bar{n}\bar{n}} \end{bmatrix} \begin{bmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \dots \\ \bar{\theta}_n \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ \dots \\ M_n \end{bmatrix} s.$$
(4.13)

The clearing problem can be solved in a distributed fashion via existing solutions like [59], which is capable to solve problems with more than two areas.

The real-time problem of Area i and the settlement process are similar to those of the two-area cases discussed above.

# 5. Conclusion

The aim of this work is to unify major approaches to interchange scheduling: JED that achieves the ultimate economic efficiency and CTS that is the state-of-the-art market solution. GCTS partially meets this goal by maintaining the same market structure as CTS while asymptomatically achieving the economic efficiency of JED under given assumptions. GCTS also ensures the revenue adequacy of each system operator.

Several important issues not considered here require further investigation. Among these are the impacts of strategic behavior of market participants, uncertainties in real-time operations, and the asynchronous mode of interchange scheduling among more than two areas.

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