A Sensitivity Approach to Detection of Local Market Power Potential

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***Abstract*—Market power gives certain market participants the ability to manipulate the market to their advantage when their product is not substitutable by competitors. Identification of generators which have the potential for market power either individually or within a small group is performed using sensitivity information from the linear programming optimal power flow (LP OPF). The impact of network constraints on admissible price perturbations are used to group generators that have the potential to exhibit local market power. Specific price perturbation vectors are found that highlight a constraint-induced locational advantage for these suppliers. In practice, this is most commonly observed in "load pockets," for which ISO policies mitigate market power.**

***Index Terms*—Eigenvectors, electricity markets, linear sensitivity analysis, market power potential.**

I. Introduction

MARKET power is referred to as the ability of a market participant to profitably maintain prices above a competitive level for a significant period of time [1]. Numerous types of market power can distort competition in electric markets including vertical ownership, generation withholding, and locational market power [2]. The source of this ability to manipulate a market necessarily arises from some advantage a supplier has over would-be competitors such that these other participants cannot adequately replace, or "substitute", for supply. An obvious example is a "pivotal" supplier without whose resources the demand cannot be met. More generally, combinations of two or more dominant suppliers may share the property that their combined resources may not be substitutable by others and potentially they can exercise joint market power.

In an electricity market, the ability of one supplier to substitute for another depends on the structure and capacity of the electric power grid. When the market is competitive, a generator that unilaterally raises its price above marginal cost of production would simply lose its customers to competitors [3]. When the transmission network congests, potential for "local" market power arises.

Local market power is a major concern to ISOs. It has been recognized by PJM, since the local market power rule proposed in 1997, that local markets created by transmission constraints are generally not structurally competitive. As stated by the FERC, "In markets with very little demand elasticity, a pivotal supplier could extract significant monopoly rents during peak periods because customers have few, if any alternatives." A monopolist generating unit facing perfectly inelastic demand will not need to rely on competitive market mechanisms to set its price paid; thus, a regulated price must be set for its services. Local market power mitigation is enforced to limit the amount of market power exercised by such a pivotal unit and to prevent it from leveraging prices earned by other units in its portfolio [4]. Local market power mitigation is performed in PJM, California, and New York. In PJM, the three-pivotal supplier test is performed for every binding transmission constraint. It determines whether excess supply is available to a subregion and if the market is adequately competitive. When a generating unit is found pivotal to relieving the transmission constraint, it is deemed to possess local, structural market power and is assessed for local market power mitigation [5].

In the PJM Interconnection, a group of state offices called the Joint Consumer Advocates (JCA) was created by statute to represent the interest of electricity consumers. The primary interest of consumers is reliable service at just and reasonable rates, and the JCA has been actively involved in compensation discussions for units located in load pockets. The JCA states many load pockets have sufficient generation capacity to serve the load but are subjected to the exercise of market power by dominant suppliers, and under no circumstances is it reasonable to pay the incumbent in excess of full cost for reliability must-run services [6].

The idea of nonsubstitutability is implicitly and explicitly applied in various metrics to evaluate supplier potential for market power. The FERC applies a market share and a pivotal supplier screen for supplier applications for market-based rates [Order 697] [7]. A supplier with greater than 20% market share or that is a pivotal supplier at peak load is deemed to have market power. The pivotal supplier test is an explicit test of supplier nonsubstitutability—the peak demand cannot be met without a portion of the supplier's capacity. The market share test is an implicit test of this property; a supply with greater than 20% market share is presumed to have market power (FERC rules allows the suppliers to rebut this presumption of market power). ISOs also use these and other metrics to evaluate market competitiveness and market power. In [5], PJM reports the highest market share and

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HHI indices. The latter is a scalar metric combining information about the market share of all suppliers. Higher numbers indicate higher concentration of supply and presumably less substitutability. In real time, PJM applies a three-pivotal supplier test for market power related to transmission constraints. This expansion of a pivotal supplier explicitly checks whether three or fewer suppliers are needed to relieve a transmission constraint. The combined resources of these few suppliers are not substitutable for relief of the constraint.

In this paper, we present a method for identifying suppliers with market power potential. Specifically, we examine network constraints that allow for possible price perturbations to be isolated to a small number of suppliers. The most common example of this of concern to ISOs is the prices in load pockets. Because the supply within the load pocket is not substitutable by outside competitors, the suppliers within the pocket possess the ability to raise prices.

Importantly, the pattern of possible prices is entirely governed by the network characteristics and constraints. Each binding network constraint introduces a degree of freedom in possible prices with a particular pattern. The possible, or "admissible," patterns of prices do not depend on generator capacities, dispatch, offers, or other characteristics. The realized prices, among the network-only dependent possible set of prices, do depend on the generator characteristics. An excellent and thorough treatment of this is found in [8]. Here we focus on the possibility for isolated price disparities that offer the potential for local market power. It is intended that this approach will serve as an additional screening tool for local market power for market monitors. Subsequent, post-screening analysis of supplier characteristics and behavior is needed to assess the exercise of market power. Such analysis is not the focus of this work.

The approach we pursue here builds on prior work [3], [9], [10] in which the sensitivity of dispatch and revenues to price are analyzed, accounting for the impact of transmission congestion. This work is also related to the market sensitivity analyses presented in [8] that aim to capture the role of transmission system constraints on market power potential. Locational marginal prices (LMPs) which satisfy the necessary conditions for optimality are "admissible" LMPs and constraints confine admissible LMPs to a particular subspace. A need is identified to efficiently determine cases where zonally differentiated admissible LMPs exist and clustering is used to group similar admissible LMPs.

Here we note that the pattern of admissible LMPs is spanned by vectors associated with constrained line sensitivities. This allows for the efficient calculation of the basis for the price patterns. It also enables a comparison to methods that rely on the constrained line sensitivities, such as the PJM three-pivotal supplier test.

The three-pivotal supplier test used by PJM is the closest screen for market power in practice to the approach presented in this paper. The three-pivotal supplier test uses system constraint sensitivity analysis to identify suppliers with potential market power [11]. Specifically, the analysis determines whether three suppliers are pivotal for relieving a constraint. In practice, this is applied to each constraint separately. In this paper, we examine the simultaneous effect of multiple constraints which may give advantage to one or more suppliers in a manner that would not be apparent in the analysis of a single constraint.

In this paper, we present algorithms to identify potential pockets of price disparities made possible by network constraints. The suppliers associated with these pockets have market power potential, if subsequent analysis confirms that their capabilities allow them to achieve these prices.[1]

II. Linear Algebra for Identification of Local Market Power Potential

The local market power potential we consider is identified through the examination of the space of possible price perturbations for instances of price vectors with a relatively small number of concentrated entries. Associated with these entries, a small number of suppliers may jointly share the ability to manipulate prices and could potentially discover this ability through price perturbations. In economic experiments such as those done at Cornell University [10], pairs of suppliers with joint market power potential always discover this ability without direct collusion. However, buyers and sellers who are less favored in terms of market power cannot overcome this disadvantage through learning [12].

In theory, all suppliers acting together could jointly adjust prices. However, it is expected that in practice, there will be too many suppliers to exercise this ability without overt coordination and concentration measures are useful for assessing competitiveness. At times, network constraints can separate the market such that a smaller number of suppliers have market power potential as a result of reduced supplier substitutability in those areas. In such cases, the prices in the local area can be manipulated by the suppliers, presumably higher and to their advantage. The pattern of locally higher prices must appear in the space of all possible prices. In this paper, we examine the space of possible prices to identify possible price patterns.

The identification of local market power potential in this work relies on a basis of vectors that describe patterns of network-admissible prices. In prior work, a basis for these vectors was extracted from the null space of a calculated price dispatch sensitivity matrix, or from properties of the first-order conditions for optimality [5], [8], [13], [14]. We note that the same information can be obtained from transmission constraint sensitivities. Specifically, the basis for these vectors is the same as that of the augmented constrained line dispatch sensitivities (generation shift factors). Thus, constrained line sensitivities efficiently determine a basis for admissible price perturbation vectors. The main purpose of this paper is to present an algorithm that uses this sensitivity information to highlight certain vectors which show concentrated price variations in the fewest entries. Suppliers associated with these entries may warrant detailed scrutiny for local market power.

The constrained line sensitivities are explained in Section II-A and the local market power potential discrimination algorithm is outlined in Section II-B. To illustrate these concepts and the algorithm, we introduce the seven-bus

[1]Indeed, in the special case in which the price perturbations identified in this work are used as offers, the prices could be achieved with no change in generation dispatch.

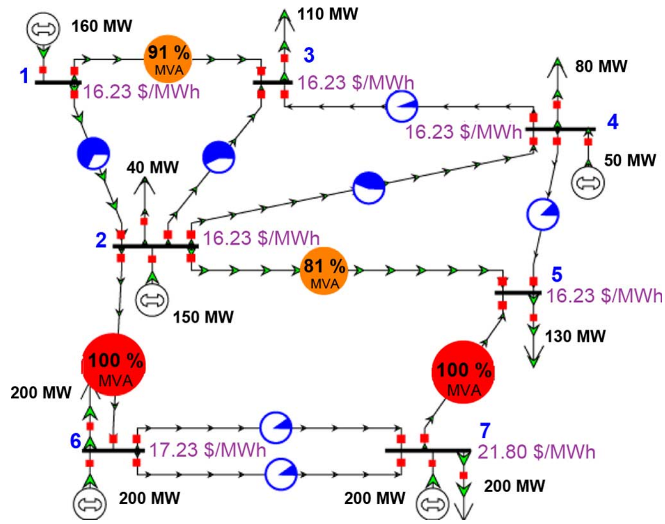

Fig. 1. Seven-bus system—Initial LMPs and generator dispatches are shown.

example system in Fig. 1. The market power potential we study depends on the network constraints. In this example, the lines connecting buses (2, 6) and buses (5, 7) are constrained, resulting in higher LMPs at buses 6 and 7.

Before leaving this section, we clarify an important point concerning the price perturbations discussed in this paper. While we focus on the price perturbations at the supply, these should not be confused with supplier offers into a market. The price perturbations should be interpreted as actual clearing price perturbations and not necessarily offers. The perturbed prices could be achieved by matching the offers to these prices, or through some other set of generator offers.

A. Constrained Line Sensitivities

The step to determine basis vectors for admissible LMPs, denoted $\mathbf{B}$ here, is computationally identical to determining the tableau for a linear programming optimal power flow (LP OPF). The LP OPF formulation is a well-known method for solving optimal redispatch problems [15]–[17]. The objective function and the constraints are linearized. The initial basis or tableau is $\mathbf{L}^o = [\mathbf{B}^o]\mathbf{\Delta P}^o$ where $\mathbf{L}^o$ represents the incremental flow on lines with active limits and rows of $\mathbf{B}^o$ represent the sensitivity of the line flow to incremental generator change. Thus, constrained line sensitivities are easily determined as a by-product of the LP OPF.

Currently, the only OPF controls considered are generator MW outputs. The extension of this model is a lengthy topic and is thus left as a subject of future work. The current model is sufficient to explain the ideas presented in this paper. For this purpose, we present a DC OPF model for which the constrained lines are specified:

$$\min_{P_g} \sum_i C_i(P_{gi}, w_i) \quad \text{such that} \tag{1}$$

$$\mathbf{A}\mathbf{diag}(\mathbf{b})\mathbf{A}^{\mathbf{T}}\boldsymbol{\theta} - \begin{bmatrix} \mathbf{P_g} \\ \mathbf{P_d} \end{bmatrix} = \mathbf{0} \tag{2}$$

$$\mathbf{diag}(\mathbf{b_f})\mathbf{A_f^T}\boldsymbol{\theta} - \mathbf{P_{flow}} = \mathbf{0} \tag{3}$$

where $C_i(P_{gi})$ is the generator cost function that depends on the dispatch P_{gi}. Equation (2) represents the relation of power injections to bus angle $\boldsymbol{\theta}$ and (3) imposes the line power flow $\mathbf{P_{flow}}$ constraints. Branch susceptances are specified in $\mathbf{b}$. Subscript f denotes quantities associated with the constrained lines. Unconstrained lines are below their limits and are not shown.

Matrix $\mathbf{A}$ is a branch-node incidence matrix that describes the topology of the network. Each column, corresponding to a branch, is zero except for a single $+1$ element and a single -1 element at rows corresponding to the terminal buses. Importantly, each column of $\mathbf{A}$ is orthogonal to a vector of all ones. This property will be exploited below. Finally, the formulation above is independent of slack bus choice, as we have specified neither a slack bus nor an angle reference. In fact, the above formulation depends only on line statuses and admittances of the transmission system elements.

We start by deriving a form for the constrained-line sensitivities that relate changes in power injections to changes in line power flow. We assume here that the demand remains constant and we only consider changes in generator injections. This allows us to consider the changes in Pg and Pd just by considering $\mathbf{\Delta P_{g0}}$. Then (2) may be solved for $\mathbf{\Delta}\boldsymbol{\theta}$ in terms of $\mathbf{\Delta P_{g0}}$, where the subscript $\mathbf{0}$ indicates that the sum of its elements must equal zero:

$$\mathbf{\Delta}\boldsymbol{\theta} = \left(\mathbf{A}\mathbf{diag}(\mathbf{b})\mathbf{A}^{\mathbf{T}}\right)^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{\Delta P_{g0}} + a.c.v. \tag{4}$$

where $a.c.v.$ denotes an *arbitrary constant vector*.[2] Combining (4) with (3), we obtain the sensitivities for the lines of interest:

$$\mathbf{\Delta P_{flow}} = \left(\mathbf{diag}(\mathbf{b_f})\mathbf{A_f^T}\left(\mathbf{A}\mathbf{diag}(\mathbf{b})\mathbf{A}^{\mathbf{T}}\right)^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\right)\mathbf{\Delta P_{g0}} \\ = \mathbf{K}\mathbf{\Delta P_{g0}}. \tag{5}$$

The constrained line sensitivities are described by $\mathbf{K}$. This solution is exact; the $a.c.v.$ in (4) is eliminated because it is orthogonal to $\mathbf{A_f^T}$. It is important to emphasize that the sensitivity matrix $\mathbf{K}$ only depends on network parameters, topology, and binding network constraints. It is independent of generator characteristics.

To satisfy power balance, the elements in $\mathbf{\Delta P_{g0}}$ must sum to zero, as mentioned above. This requirement is typically satisfied by choosing a slack bus whose incremental injection is the opposite of the sum of the other generator's incremental injections. Formally, this may be represented by

$$\mathbf{\Delta P_{g0}} = \mathbf{R}\mathbf{\Delta P_g}. \tag{6}$$

The requirement that $\mathbf{\Delta P_{g0}}$ must sum to zero holds if each column of $\mathbf{R}$ sums to zero. For example, if the first generator is chosen as the slack, $\mathbf{R}$ is the following:

$$\mathbf{R} = \begin{bmatrix} 0_{1x1} & \mathbf{-1}_{1x(n-1)} \\ \mathbf{0}_{(n-1)x1} & \mathbf{I}_{(n-1)x(n-1)} \end{bmatrix}. \tag{7}$$

[2]We purposefully abuse notation slightly by indicating a matrix inverse when the matrix enclosed is singular. When we restrict $\mathbf{\Delta P_{g0}}$ to sum to zero, then the vector of incremental power injections is orthogonal to the null space of $\mathbf{A}\mathbf{diag}(\mathbf{b})\mathbf{A}^{\mathbf{T}}$ and the solution in (4) exactly satisfies (2). Alternatively, the use of pseudoinverse notation, for example, might lead the reader to suppose that the solution in (4) is some sort of best fit solution, when in fact it is exact.

Then

$$\mathbf{\Delta P_{flow}} = \mathbf{KR\Delta P_g} = \mathbf{K_R \Delta P_g}. \quad (8)$$

Different slack buses can be chosen and consequently sensitivity matrix $\mathbf{K_R}$ is slack bus dependent. The elements in $\mathbf{K_R}$ are often referred to as generation shift factors. The choice of slack only adds a constant row vector to each row; the corresponding rows of $\mathbf{K}$ and $\mathbf{K_R}$ differ only by a constant.

We use the matrix $\mathbf{S}$ to represent the transpose of $\mathbf{K}$ augmented by a column of all ones:

$$\mathbf{S} = \begin{bmatrix} & 1 \\ \mathbf{K^T} & \vdots \\ & 1 \end{bmatrix}. \quad (9)$$

The basis of the column space of $\mathbf{S}$, $\mathrm{Col}(\mathbf{S})$, spans the same vector space as $\mathbf{B}$, the basis for admissible LMPs. To show this, recall that LMPs can be decomposed in terms of a uniform component and a congestion component (neglecting losses). Using the DC OPF model here, the LMPs can be expressed as

$$\boldsymbol{\lambda} = [1]\boldsymbol{\lambda_0} - \left[\left(\mathbf{A diag(b) A^T}\right)^{-1} \mathbf{A_f diag(b_f)}\right] \boldsymbol{\mu} \quad (10)$$

where λ is the vector of LMPs, λ_0 is the uniform component of the LMP, and μ is a vector of multipliers associated with the network constraints. A discussion of this LMP decomposition can be found in [8] and [18]. Extracting the LMPs associated with generators gives

$$\boldsymbol{\lambda_g} = [\mathbf{I} \quad \mathbf{0}]\boldsymbol{\lambda} = -\mathbf{K}^T\boldsymbol{\mu} + a.c.v. \quad (11)$$

where $[\mathbf{I}\ \mathbf{0}]$ selects generator entries that without loss of generality are ordered as the initial entries in the vector and a.c.v. is the arbitrary constant value associated with the uniform component of the LMP. The space of network admissible LMPs and LMP perturbations (since this is a linear model) is spanned by

$$\mathbf{B} = [\mathbf{K}^T \quad \mathbf{1}] \quad (12)$$

which is identical to $\mathbf{S}$.

The relation between the shift factors and LMPs suggests a rationale for the algorithm to follow. Large entries in $\mathbf{S}$ are associated with generators that most efficiently resolve network constraints, allowing for higher clearing prices at these locations. Combinations of constraints in which groups of generators appear increase the potential for high clearing prices. The actual impact on prices will depend on the capabilities of these generators. The following algorithm examines the space of potential impacts as a screen and subsequent detailed analysis would be needed to examine the capabilities.

B. Outline of the Two-Stage Algorithm

In the previous subsections, we discussed a basis for possible price perturbation vectors. The remaining challenge is to develop a method to determine particular vectors, among the infinite possibilities, that highlight concentrated entries that show opportunity for local market power. These vectors of interest have a particular structure: the entries are almost entirely zero, or near zero, except for a few relatively large entries. We consider this a nontrivial problem.

For the seven-bus Fig. 1 system, $\mathbf{S}$ is given by (13):

$$\mathbf{S} = \begin{matrix} Gen\ 1 \\ Gen\ 2 \\ Gen\ 4 \\ Gen\ 6 \\ Gen\ 7 \end{matrix} \begin{bmatrix} 0.23 & -0.06 & 1.00 \\ 0.24 & -0.05 & 1.00 \\ 0.17 & -0.11 & 1.00 \\ -0.58 & 0.14 & 1.00 \\ -0.21 & 0.50 & 1.00 \end{bmatrix}. \quad (13)$$

We seek linear combinations of these vectors that result in price perturbation vectors of the desired structure. Already we see suggestive patterns in the initial vectors. For example, the entries in the first three rows and last two rows have opposite signs in the first two columns. This suggests a possible separation between the first three generators (buses 1, 2, and 4) and the remaining two (buses 6 and 7). Not obvious is another separation between the last two.

We can gain insight from examination of the network topology and constraints (Fig. 1). A load pocket situation exists for bus 6 and bus 7. Incrementally, the load at these buses can only be supplied by the generators at these buses due to the constrained line between buses 2 and 6. Also, both Gen. 6 and Gen. 7 can independently exercise market power. To see why this is true, look at the one-line diagram and suppose Gen. 6 increases its price by \$1/MWh. The load will accept this price in order to satisfy power balance and in order not to further overload the constrained lines. The same situation is true for Gen. 7. Thus, we expect that Gen. 6 alone, Gen. 7 alone and Gen. 1, 2, and 4 combined will have some amount of market power potential.

The two-stage algorithm is detailed in the next sections. We start with the augmented sensitivities $\mathbf{S}$ from the LP OPF and compute an orthonormal basis $\mathbf{B}$ using singular value decomposition (SVD). Briefly, SVD expresses a matrix as $\mathbf{S} = \mathrm{U}\Sigma\mathrm{V}*$, where Σ is the matrix of singular values of $\mathbf{S}$. If r is the rank of $\mathbf{S}$, the first r left singular vectors or columns of $\mathbf{U}\{\mathbf{u}_1, \ldots, \mathbf{u}_r\}$ are an orthonormal basis $\mathbf{B}$ for $\mathrm{Col}(\mathbf{S})$. According to [19], computation is of order $\mathrm{O}(nm^2)$ and m is small because the number of constraints is small. The first stage of the algorithm clusters the rows of $\mathbf{B}$ to form groups of similar generators. In the second stage of the algorithm, for each cluster, we use eigen-analysis to maximize the entries associated with the generators in a cluster while minimizing other entries. Additional refinement may be performed.

III. Generator Grouping Algorithm

The generator grouping algorithm comprises two steps: an initial generator clustering step followed by a price perturbation refinement step. The clustering step forms sets of binary groupings, each of which contains a candidate group of generators. The refinement step mathematically seeks a price perturbation vector in the space of network-admissible vectors that maximizes the price perturbations for the candidate generators while minimizing the price perturbations for the other generators.

In theory, the initial clustering step would be unnecessary if we could simply evaluate all possible combinations of candidate generators. This is a combinatorial calculation in terms of the number of generators. Furthermore, such a brute-force approach appears unnecessary. Work is currently in progress to investigate different clustering algorithms. To date, almost any sensible clustering algorithms appears to work well and a Quality Threshold (QT) clustering algorithm and a K-Means clustering algorithm have been implemented. An overview of these two algorithms is provided below. The clustering is performed on the rows of matrix $\mathbf{B}$, as we are grouping generators.

A. Quality Threshold Algorithm

The QT algorithm can be used to cluster the rows of $\mathbf{B}$. The original algorithm was developed to cluster genes [20]. The criterion which motivated its design is the need to form an unknown number of potentially large clusters which satisfy a "quality guarantee," meaning here that the cluster diameters should not exceed a certain threshold.

A distance matrix $\mathbf{D}$ with elements D_{ij} for rows i and j gives the distances between all rows of $\mathbf{B}$. Any measure of distance may be used; here we use Euclidean distance. A threshold and optionally a maximum cluster size are specified initially. For each row of $\mathbf{B}$, we build a candidate cluster that contains all other rows of $\mathbf{B}$ which are closer in distance than the threshold. If a row of $\mathbf{B}$ is not within the threshold of any other rows, it forms its own cluster. The candidate cluster with the most elements becomes a true cluster and all points already in a cluster are removed from further consideration. The process then iterates until all points belong to a cluster. The QT algorithm is computationally intensive, $\mathrm{O}(n^2)$ [21], as it requires a distance metric to be computed between all points.

B. K-Means Algorithm

The K-means clustering technique is one of the oldest and most widely used algorithms and is described in detail in [22]. The algorithm is simple and fast (polylogarithmic) in practice [23], although in theory, it has polynomial smoothed running time [24]. K-means is a prototype-based, partitional clustering technique where clusters are represented by their centroids. In prototype-based clusters, the objects in a cluster are closer to the prototype of that cluster than to the prototype of any other cluster. The centroid used as the prototype in the K-means algorithm is typically the mean of the points in the cluster, hence the name K-means. Partitional clustering is a division of the objects into non-overlapping subsets, which implies that any object may only be in one cluster.

The K-means algorithm proceeds as follows: First, k points are chosen as the initial cluster centroids. Then, each point is assigned to the closest centroid and each collection of points is a cluster. The centroids for each cluster are recomputed. The process continues until points stop changing clusters which is equivalent to when the centroids stop changing. A drawback is that the K-means algorithm requires the user to specify *a priori* the number of clusters, k. Another difficulty is specifying the initial choice of clusters.

IV. Price Perturbation Algorithm

The generator grouping algorithm provides us with the following block matrix structure:

$$\begin{bmatrix} \mathbf{B_1} \\ \mathbf{B_2} \\ \cdot \\ \mathbf{B_k} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \Delta\mathbf{y_1} \\ \Delta\mathbf{y_2} \\ \cdot \\ \Delta\mathbf{y_k} \end{bmatrix} \tag{14}$$

where k is the number of clusters, $\mathbf{x}$ is a weighting vector (to be determined) that produces a particular network-admissible price perturbation vector, $\mathbf{\Delta y}$. In (14), the rows of the original $\mathbf{B}$ are simply grouped together with other generators in the same cluster.

Consider only one generator cluster at a time. For each cluster i, we partition $\mathbf{B}$ into a matrix $\mathbf{B_i}$ consisting of the rows of $\mathbf{B}$ which are in i and a matrix $\mathbf{B_{-i}}$ consisting of all other rows. The subscript $-i$ is used to denote generators which are not in group i. This gives us k problems of the following form:

$$\begin{bmatrix} \mathbf{B_i} \\ \mathbf{B_{-i}} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{\Delta y_i} \\ \mathbf{\Delta y_{-i}} \end{bmatrix} \tag{15}$$

where the elements in $\mathbf{\Delta y}_i$ should be much larger than those of $\mathbf{\Delta y_{-i}}$. That is, for each cluster i, we want to achieve an $\mathbf{x}$ so that price perturbation vector $\mathbf{\Delta y}$ is of the desired form. This problem may be written as follows:

$$\begin{aligned} Max \quad & \mathbf{\Delta y_i^T \Delta y_i - \Delta y_{-i}^T \Delta y_{-i}} \\ st \quad & \|\mathbf{x}\| = 1. \end{aligned} \tag{16}$$

The requirement that the norm of the vector $\mathbf{x}$ is equal to one is imposed since otherwise, any scaling of the optimal value of $\mathbf{x}$ would maximize the objective function. Substituting (15) into (16), the objective function of (16) may be rewritten as

$$\begin{aligned} Max \quad & \mathbf{x}^{\mathrm{T}} (\mathbf{B_i'}) \mathbf{x} \\ st \quad & \|\mathbf{x}\| = 1 \end{aligned} \tag{17}$$

where

$$\mathbf{B_i' = B_i^T B_i - B_{-i}^T B_{-i}}. \tag{18}$$

This is an eigenvalue problem and the maximum is obtained by choosing $\mathbf{x}$ to be the eigenvector corresponding to the largest eigenvalue $\lambda_{\max}$ of $\mathbf{B_i'}$ for each cluster i. The proof follows from the definition of eigenvalues and eigenvectors and is given in the Appendix.

By maximizing (17), the price perturbation scenario highlighting the largest price increase for generators in cluster i and smallest price change for the remaining generators is found. This automated process helps identify candidate clusters for local market power.

With $\mathbf{B}$ divided into k clusters, the objective function (17) is executed k times, giving k price perturbation vectors. However, it is useful to shift key rows to and from $\mathbf{B_i}$ and $\mathbf{B_{-i}}$ in order to refine the clusters, recalculate (17), and observe additional potentially improved price perturbation vectors. If the computational power were available, we could execute (17) for all combinations of $\mathbf{B_i}/\mathbf{B_{-i}}$ and examine all 2^n:

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n \tag{19}$$

resulting price perturbation vectors; n being the number of generators in $\mathbf{B}$. This number of computations is unfeasible; however, a complete enumeration on combinations of clusters may be possible. Therefore, (17) can reasonably be calculated 2^k times where $k < n$ and $2^k \ll 2^n$. Instead of executing (17) just k times by allowing only one shift and one price perturbation vector per cluster, we perform the shift and calculation for all possible 2^k combinations of the k number of clusters. Afterwards, the best set of vectors that highlight the potential for local market power, of the 2^k price perturbation vectors are selected for final results.

V. Numerical Examples With Test Systems

The first example presented uses the seven-bus test case. The algorithm steps are explained in detail for this example in order to provide insight for the interpretation of larger test systems' results. Other examples are given for the IEEE 118-bus system [25].

A. Seven-Bus Example

The seven-bus example follows Fig. 1. The augmented constrained-line sensitivities are given by $\mathbf{S}$ in (13). SVD is applied to (13) and an orthonormal basis $\mathbf{B}$ is obtained:

$$\mathbf{B} = \begin{matrix} Gen\ 1 \\ Gen\ 2 \\ Gen\ 4 \\ Gen\ 6 \\ Gen\ 7 \end{matrix} \begin{bmatrix} -0.065 & 0.026 & -0.576 \\ -0.062 & -0.013 & -0.582 \\ -0.083 & 0.234 & -0.544 \\ 0.640 & 0.756 & 0.030 \\ 0.759 & -0.611 & -0.182 \end{bmatrix}. \tag{20}$$

Then, we use the QT clustering algorithm with a threshold of 0.25 and identify clusters 1,2, and 3 containing buses $\{7\}$, $\{6\}$, and $\{1,2,4\}$, respectively.

For each cluster, a price perturbation vector of the desired form is found. For example, consider cluster 3 which has the following matrices:

$$\mathbf{B_3} = \begin{bmatrix} -0.065 & 0.026 & -0.576 \\ -0.062 & -0.013 & -0.582 \\ -0.083 & 0.234 & -0.544 \end{bmatrix},$$

$$\mathbf{B_{-3}} = \begin{bmatrix} 0.640 & 0.756 & 0.030 \\ 0.759 & -0.611 & -0.182 \end{bmatrix}$$

$$\begin{aligned} \mathbf{B}_3' &= \mathbf{B_3^T B_3} - \mathbf{B_{-3}^T B_{-3}} \\ &= \begin{bmatrix} -0.970 & -0.041 & 0.238 \\ -0.041 & -0.889 & -0.268 \\ 0.238 & -0.268 & 0.932 \end{bmatrix}. \end{aligned} \tag{21}$$

The largest eigenvalue of (21) is 0.9635 and the corresponding right eigenvector is $[0.124, -0.145, 1]^{\mathrm{T}}$. The price perturbation vector for cluster 3 is found from (15) to be $[-0.577, -0.577, -0.577, 0.00, 0.00]^{\mathrm{T}}$ by multiplying $\mathbf{B}$ in (20) by $\mathbf{x} = [0.124, -0.145, 1]^{\mathrm{T}}$. To ease comparison, normalization is done so that the largest component has a value of one. Price perturbation vectors for the clusters (CL) are shown in the following:

$$[\Delta\mathbf{y}_{\mathbf{CL\#1}}, \Delta\mathbf{y}_{\mathbf{CL\#2}}, \Delta\mathbf{y}_{\mathbf{CL\#3}}]$$

$$= \begin{matrix} \\ Gen\ 1 \\ Gen\ 2 \\ Gen\ 4 \\ Gen\ 6 \\ Gen\ 7 \end{matrix} \begin{matrix} \begin{matrix} \mathrm{CL\#1} & \mathrm{CL\#2} & \mathrm{CL\#3} \end{matrix} \\ \begin{pmatrix} 0.04 & -0.04 & 1.00 \\ 0.07 & -0.07 & 1.00 \\ -0.11 & 0.11 & 1.00 \\ 0.02 & 1.00 & 0.00 \\ 1.00 & 0.02 & 0.00 \end{pmatrix} \end{matrix}. \tag{22}$$

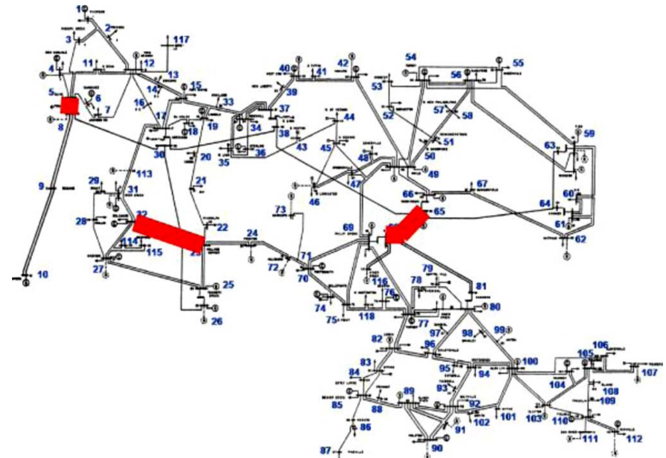

Fig. 2. IEEE 118-Bus system, Ex. 1 constrained lines.

B. 118-Bus Examples

In the IEEE 118-bus case [25], we consider 19 online generators. An operating point is obtained where the constraints on the following three lines are binding: (5,8), (23,32), and (65,68). This scenario is shown in Fig. 2.

Five clusters are found *a priori* by the K-means clustering algorithm, and the objective function is executed five times to compute the price perturbation vectors shown in Table I.

This first pass provides informative results, but it can be refined to better isolate a small number of suppliers. Note that $\Delta\mathbf{y}_{\mathbf{CL\#1}}$ has a large entry at Bus 12 which is not in cluster 1, $\Delta\mathbf{y}_{\mathbf{CL\#2}}$ has a large entry at Bus 10 which is not in cluster 2, and the other vectors could be refined as well. A complete combinatorial check of the five clusters improves results, at a modest increase in computation. The objective function (17) is then executed $2^k = 2^5 = 32$ times. These 32 results are finally clustered into six column clusters to remove similar or redundant vectors and the best performing vector is selected from each column cluster. The refined results are shown in Table II.

Price perturbation vectors $\Delta\mathbf{y}_{\mathbf{CL\#1}}$ and $\Delta\mathbf{y}_{\mathbf{CL\#5}}$ remain unchanged. The other four vectors are combinations of two clusters which improve the distinction between the many small entries and few large entries.

TABLE I
118-Bus, Ex. 2, 4 Clusters, Price Perturbation Vectors

Bus #	Gen MW	Cluster #	$\Delta y_{CL\#1}$	$\Delta y_{CL\#2}$	$\Delta y_{CL\#3}$	$\Delta y_{CL\#4}$	$\Delta y_{CL\#5}$
10	191.68	1	1	-0.42	0.4	0.46	0.05
12	136.88	2	-0.75	1	0.24	0.28	0.03
25	190.25	4	0.01	0.02	-0.17	0.79	0.23
26	400	4	0.16	0.01	-0.02	0.68	0.17
31	0	3	0.72	0.24	1	-0.27	-0.01
46	0	4	0.14	0.05	0	0.7	0.11
49	400	4	0.14	0.05	-0.01	0.76	0.05
54	0	4	0.14	0.05	-0.02	0.83	-0.02
59	0	4	0.15	0.05	-0.03	0.9	-0.09
61	0	4	0.15	0.05	-0.04	0.93	-0.12
65	400	4	0.15	0.05	-0.05	1	-0.19
66	400	4	0.14	0.05	-0.03	0.88	-0.06
69	400	5	0.01	0.01	-0.02	0.08	0.9
80	400	5	0.02	0.01	0.01	-0.03	1
87	0	5	0.01	0.01	0	-0.01	0.99
89	400	5	0.02	0.01	0	-0.01	0.99
100	358.19	5	0.02	0.01	0	-0.02	0.99
103	0	5	0.02	0.01	0	-0.02	0.99
111	0	5	0.02	0.01	0	-0.02	0.99

TABLE II
118-Bus, Ex. 2, 5 Clusters, Price Perturbation Vectors

Bus #	Cluster #	$\Delta y_{CL\#1}$	$\Delta y_{CL\#1,2}$	$\Delta y_{CL\#1,3}$	$\Delta y_{CL\#1,4}$	$\Delta y_{CL\#2,3}$	$\Delta y_{CL\#5}$
10	1	1	-0.6	0.59	0.79	-0.02	0.05
12	2	-0.75	1	0	0	1	0.03
25	4	0.01	0.01	-0.13	0.75	-0.12	0.23
26	4	0.16	-0.04	0.02	0.7	-0.01	0.17
31	3	0.72	0	1	0	0.99	-0.01
46	4	0.14	0	0.04	0.71	0.04	0.11
49	4	0.14	0	0.03	0.77	0.03	0.05
54	4	0.14	0	0.02	0.84	0.02	-0.02
59	4	0.15	0	0.01	0.9	0.01	-0.09
61	4	0.15	0	0.01	0.93	0.01	-0.12
65	4	0.15	0	0	1	0	-0.19
66	4	0.14	0	0.01	0.88	0.02	-0.06
69	5	0.01	0	-0.01	0.08	-0.01	0.9
80	5	0.02	0	0.01	-0.02	0.01	1
87	5	0.01	0	0	-0.01	0	0.99
89	5	0.02	0	0	-0.01	0	0.99
100	5	0.02	0	0.01	-0.01	0.01	0.99
103	5	0.02	0	0.01	-0.01	0.01	0.99
111	5	0.02	0	0.01	-0.01	0.01	0.99

Consider the same 118-bus case, except suppose that now, the three lines with binding constraints are (5,8), (8,9), and (38,65), as shown in Fig. 3.

This example illustrates the tendency of line constraints to separate both generator supply and generator prices. Fig. 3 illustrates the potential local market power clusters determined by the algorithm. The dotted lines illustrate the division of the system caused by these binding constraints. There are three groups with a small number of generators (CL#1-CL#3) and one group with many generators (CL#4). The price perturbation vectors are given in Table III.

VI. Conclusion

The method presented in this paper allows groups of generators with the potential for local market power to be quickly identified for a given network model and set of binding constraints.

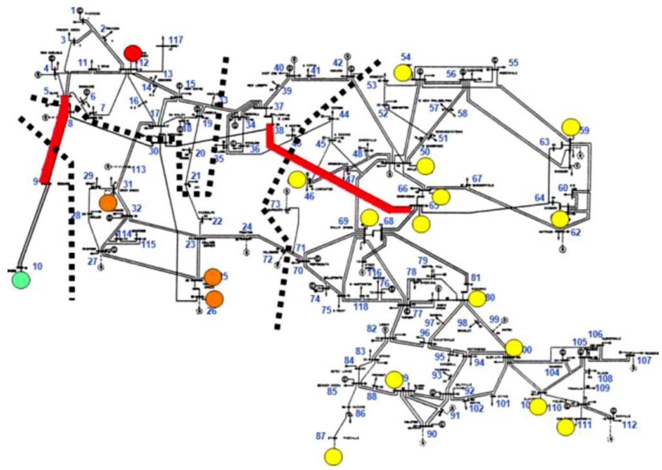

Fig. 3. 118-Bus system, division due to constraints.

TABLE III
118-Bus, Ex. 3, Price Perturbation Vectors

Bus #	Gen MW	Cluster #	$\Delta y_{CL\#1}$	$\Delta y_{CL\#2}$	$\Delta y_{CL\#3}$	$\Delta y_{CL\#4}$
10	450	3	0	0	1	0
12	85	2	0	1	0	0
25	220	1	0.92	-0.02	0	0.03
26	314	1	1	-0.05	0	-0.02
31	7	1	0.87	0.08	0	-0.01
46	19	4	0.15	0	0	0.75
49	204	4	0.09	0	0	0.81
54	48	4	0.03	0	0	0.87
59	155	4	-0.02	0	0	0.92
61	160	4	-0.05	0	0	0.94
65	391	4	-0.11	0	0	1
66	392	4	0	0	0	0.9
69	390	4	0.02	0	0	0.88
80	477	4	-0.01	0	0	0.91
87	4	4	-0.01	0	0	0.9
89	607	4	-0.01	0	0	0.9
100	252	4	-0.01	0	0	0.9
103	40	4	-0.01	0	0	0.9
111	36	4	-0.01	0	0	0.9

We show that the set of base vectors for admissible price perturbations are easily computed for the LP Tableau. Our approach searches over the space of possible prices to identify small clusters of price disparities that show opportunity for local market power.

This analysis does not necessarily conclude that certain generators are exercising market power. We also have not calculated the range over which this potential may be exercised. The key point is that the ideas and results from this analysis should serve as a screening tool. In situations where analysis suggests that participants are likely to be able to exercise market power, further analysis needs to be done to investigate under what conditions such suppliers would prefer to behave competitively.

The approach presented in this paper focuses on supply and presently ignores elasticity in demand. The mathematics can be extended to include demand participation and will be part of our future work. Presently the supplier focus of the screening tool is consistent with the application of other market power metrics discussed in the introduction, that also rely on supplier concentrations and supplier ability to relieve constraints.

The algorithm presented in this paper relies on sensitivity information related to certain distribution factors relating constrained lines and supplier dispatch. Such distribution factors are also employed in the PJM three pivotal supplier test [11]. In the three pivotal supplier test, each constraint is examined individually to determine whether three suppliers are jointly pivotal to relieve a constraint. PJM asserts that this approach is more accurate than concentration and market share measures [11]. The underlying mathematical sensitivities in our work are identical to those used in the three-pivotal supplier test. Our algorithm differs in that it predominantly focuses on the simultaneous effect of multiple constraints, whereas the three-pivotal supplier test examines each constraint separately. This is a significant addition, as the presence of multiple constraints could accentuate a supplier's potential for market power, or offer patterns of market power potential not evident in the analysis of a single constraint.

The next stage of our work will focus more on the computational issues associated with finding the orthonormal basis $\mathbf{B}$ and with the grouping of generators as discussed in Section III. Ensuring that the algorithm is as efficient as possible is important for the practicality of its real-time use on large systems, as utilities will be the ultimate users.

Appendix

Proof of the Price Perturbation Algorithm: If $\mathbf{x}$ is an eigenvector of $\mathbf{B_i'}$, then $\mathbf{B_i'x} = \lambda\mathbf{x}$ and (17) becomes

$$\mathbf{x}^{\mathrm{T}}\mathbf{B_i'x} = \mathbf{x}^{\mathrm{T}}\lambda\mathbf{x} = \lambda\left\|\mathbf{x}\right\|^2 = \lambda \tag{23}$$

where we used the fact that λ is a scalar and the norm of $\mathbf{x}$ equals 1. Thus, from (23), it is clear that if $\mathbf{x}$ is an eigenvector of $\mathbf{B_i'}$, the maximum of (17) is obtained when $\mathbf{x}$ is the eigenvector corresponding to $\lambda = \lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{B_i'}$.

All that is left is to show that the optimal $\mathbf{x}$ is in fact an eigenvector of $\mathbf{B_i'}$. To do this, we consider $\mathbf{x}$ which is not an eigenvector of $\mathbf{B_i'}$. Then, we show that any such $\mathbf{x}^{\mathrm{T}}\mathbf{B_i}\mathbf{x}$ is less than the $\mathbf{x}^{\mathrm{T}}\mathbf{B_i}\mathbf{x}$ given by $\mathbf{x} = \text{eigenvector corresponding to } \lambda_{\max}$.

If $\mathbf{x}$ is not an eigenvector of $\mathbf{B_i'}$, then it is some linear combination of the eigenvectors of $\mathbf{B_i'}$

$$\mathbf{x} = c_1\mathbf{v_1} + \cdots + c_n\mathbf{v_n}, \quad \sum_{i=1}^{n} c_i^2 = 1 \tag{24}$$

where $\mathbf{v_1}, \ldots, \mathbf{v_n}$ are normalized eigenvectors of $\mathbf{B_i'}$. Since $\mathbf{B_i'}$ is real-valued and symmetric (by construction), the eigenvalues are real and the eigenvectors are orthogonal. The squared coefficients must sum to 1. This necessarily implies that the magnitudes of the individual coefficients are less than or equal to 1. Then

$$\mathbf{B_i'x} = \mathbf{B_i'}\left(c_1\mathbf{v_1} + \cdots + c_n\mathbf{v_n}\right) \tag{25}$$

so pre-multiplying with $\mathbf{x}$ gives the following:

$$\mathbf{xB_i'x} = \mathbf{xB_i'}\left(c_1\mathbf{v_1} + \cdots + c_n\mathbf{v_n}\right) \tag{26}$$

$$= \left(c_1\mathbf{v_1} + \cdots + c_n\mathbf{v_n}\right)\mathbf{B_i'}\left(c_1\mathbf{v_1} + \cdots + c_n\mathbf{v_n}\right) \tag{27}$$

$$= c_1^2\lambda_1\left\|\mathbf{v_1}\right\|^2 + \cdots + c_n^2\lambda_n\left\|\mathbf{v_n}\right\|^2, \quad \left\|\mathbf{v_i}\right\| = 1 \quad \forall i \tag{28}$$

$$= \sum_{i=1}^{n} c_i^2\lambda_i, \quad \sum_{i=1}^{n} c_i^2 = 1 \tag{29}$$

$$\leq \lambda_{\max}. \tag{30}$$

The last line (30) above can be understood from the fact that we are essentially adding fractions of eigenvalues in (29). If we add a fraction of $\lambda_{\max}$ to a fraction of $\lambda(\neq \lambda_{\max})$, this will always be less than adding only fractions of $\lambda_{\max}$. If $c_i = 1$ corresponds to $\lambda_i = \lambda_{\max}$ and all other c_i are zero, then the last line in (30) is equality. Thus, to achieve the maximum, $\mathbf{x}$ must be the eigenvector corresponding to $\lambda_{\max}$.

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