Long-Term Effect of the n-1 Criterion on Cascading Line Outages in an Evolving Power Transmission Grid

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***Abstract*—Cascading transmission line outages contribute to widespread blackouts. Engineers respond to the risk of cascading line outages by applying policies such as the n-1 criterion and upgrading lines involved in recent cascading outages. The transmission grid slowly evolves as these policies are applied to maintain reliability while the load grows. We suggest how to assess the long-term effect of these policies on the risk of cascading line outages by simulating both the cascading and the slow evolution of the transmission grid. The long-term effects of these policies on the probability distribution of outage size and the grid utilization are computed for the IEEE 118-bus test system. The results show complex system self-organization of an evolving transmission grid.**

***Index Terms*—Complex system, failure analysis, network reliability, power system security, power transmission reliability, risk analysis.**

I. Introduction

THERE are diverse and complicated phenomena involved in widespread blackouts, such as cascading overloads, transient stability, oscillations, voltage collapse, component failures, and the action of protection devices. We focus on cascading transmission line overloads and outages because they often play a role in large blackouts. For example, overloaded lines in Ohio cascaded in the August 2003 North American blackout [1] and overloaded lines cascaded from within Germany to Southern Europe in the November 2006 European blackout [2]. The impact of large blackouts on society justifies the analysis and simulation of blackout failure mechanisms such as cascading overloads and the quantification of the power system reliability with respect to these mechanisms. Although blackouts usually combine multiple failure mechanisms, it is pragmatic and traditional to study the various mechanisms one at a time, especially in the case of mechanisms that are hard to analyze such as cascading failures [3].

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In this paper we study transmission grid reliability with respect to cascading line overloads and outages. In particular, we quantify this reliability in a transmission grid evolving over time and taking into account load growth and policies such as the n-1 criterion or responding to blackouts. Reliability in an evolving grid is introduced in [4] and [5] and demonstrated with power system modeling and simulation in [6]–[8]. Improving the reliability of the evolving grid is considered in [9] and [10] and there is an explanatory overview in [11]. In the previous work, the power system evolution is largely driven by upgrades in response to recent cascading failures. This paper advances the previous work by driving the upgrades with the n-1 criterion and considering measures of grid utilization. Before reviewing the previous work in more detail and summarizing the goals of the paper, we explain concepts of the evolving grid and complex systems.

A. Long-Term Reliability That Accounts for an Evolving Grid

Electric power systems experience slow load growth and slowly evolve over time to be able to satisfy the increasing load with reliability and economy. The load growth by itself tends to reduce transmission reliability, but the transmission grid is also upgrading to maintain reliability according to reliability policies. Examples of reliability policies are the n-1 criterion, in which no single contingency causes further outages, or upgrading the grid in response to cascading failures (real or simulated) to prevent similar occurrences. The evolution of the grid is a slow, ongoing interaction between the load growth and reliability that is influenced by the reliability policy.

We are interested in how a reliability policy affects the reliability of a transmission grid with respect to cascading line overloads. Here are two ways to think about assessing this reliability.

1) Assume a fixed grid with a fixed load. The reliability analysis is done for this fixed grid with the reliability policy applied. The analysis can be done for several conditions, such as for different reliability policies or for different loadings, but the grid is fixed for each condition. This reliability analysis is short-term because it describes the reliability of the grid over a time period that is short enough that the grid does not evolve over the time period. This short-term reliability calculation is traditional and is a useful way to as-

sess reliability policies. Indeed, almost all the literature on power system reliability addresses short-term reliability.

2) Assume a grid with a slowly increasing load that is slowly evolving with the reliability policy applied. The load growth and the reliability will interact over time under the influence of the reliability policy. (As the grid upgrades, the patterns of power flow on the grid respond to the upgrades so that the evolution of the grid includes both changes in the physical grid and changes in how it is operated.) This reliability analysis is long-term because it describes the reliability of the grid taking into account the outcome of the slowly evolving interactions between loading and reliability. We illustrate this reliability analysis in this paper.

B. Complex Systems Aspects

The term "complex system" is sometimes applied rather loosely, but here we start from the general description by Arthur [12]:

"Common to all studies on complexity are systems with multiple elements adapting or reacting to the pattern these elements create. … the elements adapt to the world—the aggregate pattern—they co-create. … As the elements react, the aggregate changes; as the aggregate changes, elements react anew. … complex systems are systems in process that constantly evolve and unfold over time."

The grid transmission lines with their various maximum flow limits support patterns of power flow and experience patterns of cascading overloads. As the grid slowly upgrades in response to these patterns, the maximum flow limits of the lines are evolving in response to the patterns they are causing in accordance with Arthur's definition.

Part of the complex evolution of the power system is the changing patterns of power flow as the grid is upgraded. There are convincing examples, by both Kirschen [13] and Reppen [14], that an upgrade in power system equipment or improvement in operating procedures that is made for the purpose of reliability may soon be exploited to increase the economic rewards from power system transactions. Suppose that a single transmission line of a grid is upgraded so that it can transmit more power. The immediate effect of the upgrade, assuming that the power flows of the system remain fixed, is that the grid is more reliable with respect to line overloads because the upgraded line has more margin available. However power flows will eventually change to take advantage of the upgrade and the upgraded line can again become congested. Although there is economic benefit from the upgrade, the eventual outcome may not be an increase in reliability. That is, the upgrade has an immediate short-term improvement on reliability, but the system will eventually evolve to yield a different reliability in the long-term. This kind of complex systems effect, although not always addressed in engineering analysis, is common knowledge in modern life. For example, an analogous question in transportation asks whether widening a road will reduce traffic congestion. It is clear that widening a road will reduce congestion if traffic flows stay the same, but it is possible for traffic flows to eventually exploit the increased capacity and congest the road again.

Complex systems are subtle and complicated, but they are not impossible to analyze because some regularity can emerge from all the interactions. We are particularly interested in self-organization, which is the process by which the system settles down to a "complex systems steady state." This steady state is constantly evolving, but there are no average trends and the statistics of the steady state are stationary in time. If the evolving grid self-organizes, it will then have reliability statistics that are stationary in time.[1] These are long-term reliability statistics.

We emphasize that this paper investigates by simulation a model of an upgrading power grid that is based on well-known power system principles. The simulation results do show complex system self-organization, but this is an outcome of the power system modeling and is not based on an assumption that the power system is a complex system.

C. Review of Previous Work

1) Previous Work on Cascading Failure in Evolving Grids: The idea of modeling the evolving grid is first suggested in [4] and [5] and is implemented in the OPA[2] simulation of an evolving grid as fully described in [6] and summarized in Section II. Papers such as [9] and [10] use the OPA simulation to study the effects of upgrades and mitigation measures on the long-term cascading outage risk of test systems with 100–300 buses. For an overview of the evolving grid work, see [11].

Several papers extend the OPA simulation in various ways. Watts and Ren [8] rework the grid and generation upgrade in economic terms. Mei *et al.* [7] represent ac power flows and compute the reliability of a 30-bus power system. In this paper, we extend the OPA simulation to implement the n-1 criterion and introduce measures of grid utilization.

2) Previous Work Simulating Cascading Failure in Fixed Grids: The previous experience in simulation of cascading failure in fixed grids can obviously inform the more difficult problem of simulation of cascading in evolving grids. Cascading failure in a fixed grid remains a substantial challenge for simulation and modeling, but the practical constraints of simulation efficiency are much less severe so that for the fixed grid, much larger power systems may be simulated in considerably more detail.

A range of simulations analyze cascading failure in fixed power grids [15]–[22]. These simulations evaluate the effect of different reliability policies by showing, for example, that a particular cascading outage could have been avoided. This type of analysis is very useful in identifying weak points and can be used to guide the upgrade of the grid or improvements to procedures. From the point of view of this paper, these analyses compute the effect of the policy on the short-term reliability.

The simulations of cascading failure in fixed grids vary in the detail of their modeling. Several simulations of cascading overloads of transmission lines [15]–[17], [22] represent the power

[1]The complex systems steady state is also thought to have properties of criticality as discussed in [11].

[2]**OPA** stands for **O**ak Ridge National Laboratory, **P**ower Systems Engineering Research Center at the University of Wisconsin, University of **A**laska to indicate the institutions collaborating to devise the simulation.

grid at the level of detail of dc load flow and linear programming dispatch of generation. The simulation [16] also represents hidden failures of the protection system and [22] also represents the timing of cascading events, grid restoration and repair, and operator actions. The simulations [18]–[21] use an ac load flow to represent several types of cascading failure and approximately represent protection, operator actions and voltage collapse. The capabilities of all these simulations are summarized in [23, Appendix].

3) Previous Work Related to the n-1 Criterion: There are interesting discussions of the n-1 criterion and its probabilistic generalizations in the literature. Zima and Andersson [24] assume a loading-dependent probability of line trip and redispatch generation to minimize the risk of subsequent line trips for any contingency. They compare this policy with the n-1 criterion and find that it slightly reduces the probability of medium-size blackouts due to cascading line outages. Nippert [25] generalizes and alters the n-1 criterion to a probabilistic criterion that bounds the expected energy unserved based on observed failure statistics. He illustrates this approach in planning the maximum load of a 110/10 kV transformer station. Chen and McCalley [26] discuss the combinatorial difficulties of systematic treatment of higher order $\mathrm{n}-\mathrm{k}$ contingencies and select higher order contingencies based on their risk computed from their probability and from their impact assessed by evaluating dependencies caused by switching actions.

D. Goal of Paper

The goal of this paper is to suggest and illustrate assessing the reliability of a power transmission grid with respect to cascading line overloads in a long-term time scale. In particular, we simulate the long-term effects of two basic reliability policies. The simulation represents cascading line overloads and outages as well as upgrading the grid in response to a slow load increase and the reliability policy. The first reliability policy is a standard n-1 criterion. That is, upgrade of the transmission lines is done to satisfy the requirement that any single line outage in a contingency list does not overload any other line. The second reliability policy responds directly to cascading outages by upgrading the lines involved after each cascading outage that sheds load.

We assess and compare the long-term effect of these policies on the probability distribution of outage size and the grid utilization. The probability distribution of outage size describes the frequency of cascading outages as a function of outage size and can be combined with outage cost to yield estimates of the risk of various sizes of cascading outages. The grid utilization is related to the average line loading or the average line flow limits relative to the total power supplied. Higher grid utilization extracts more value from the grid investment. The long-term reliability statistics are obtained after running the simulation until the statistics become stationary in the complex system steady state.

II. MODELING CASCADING AND THE EVOLVING GRID

This section summarizes the OPA model of the evolving grid [15], [6] and describes its extension to represent the n-1 criterion. Refer to Fig. 1 for a flowchart. The parameter values used to produce the results in this paper are stated during the explanation of OPA. The strengths and weaknesses of modeling the evolution of the grid are discussed.

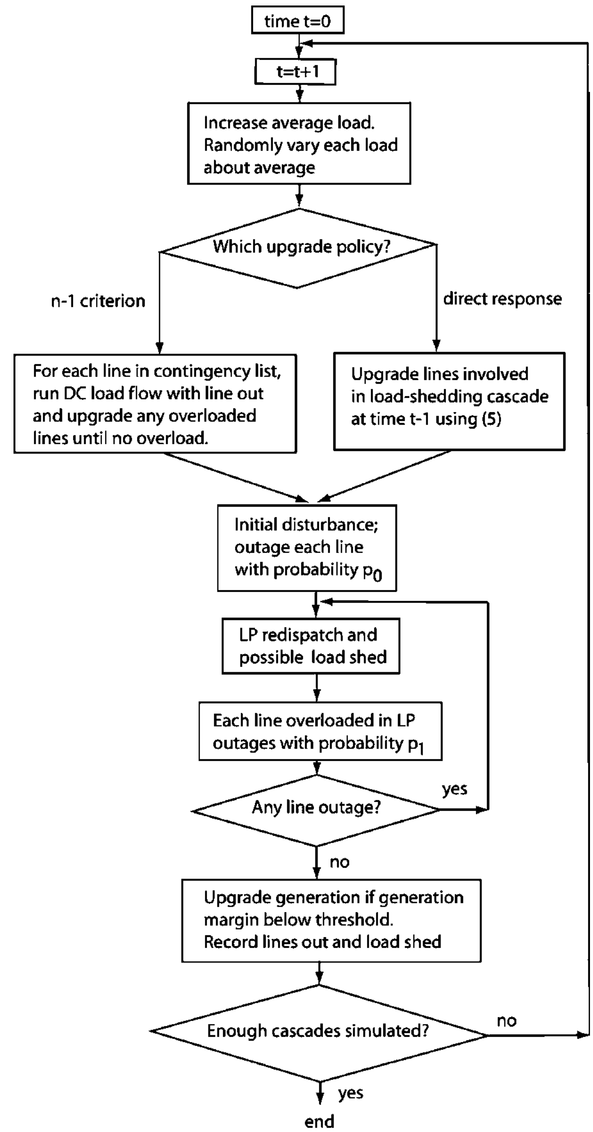

Fig. 1. Flowchart summarizing the OPA simulation.

A. Cascading Overloads

We describe how OPA simulates cascading line outages and overloads. Each simulated cascade, say, the cascade at time t, starts from a solution of a standard dc load flow

$$F = AP \tag{1}$$

where $F=(F_1(t),F_2(t),\ldots,F_{\mathrm{nline}}(t))^T$ is the vector of real power flows in the transmission lines, A is the matrix relating injections to flows, and $P=(P_1(t),P_2(t),\ldots,P_{\mathrm{nbus}-1}(t))^T$ is the vector of real power injections at all the buses except the slack bus. nline is the number of transmission lines and nbus is the number of buses. The initial disturbance is modeled by independently outaging each line with probability $p_0 = 0.001$.

That is, for each line, the simulation outages the line if a computer-generated random number uniformly distributed in the interval [0,1] is less than p_0. If any lines outage, A is recalculated and the power flows and injections are recomputed using (1) and standard linear programming of the generation redispatch [27]. The detail of the linear programming [27] is that the cost function

$$\text{cost} = \sum_{i \in \text{generators}} P_i(t) + 100 \sum_{i \in \text{loads}} P_i(t) \tag{2}$$

is minimized subject to the dc load flow (1) and the constraints of $0 \leq P_i(t) \leq P_i^{\max}(t)$ at generators, $P_i(t) \leq 0$ at loads, and $|F_j(t)| \leq F_j^{\max}(t)$ at lines, where $F_j^{\max}(t)$ is the maximum flow limit of line j at time t. The linear programming generation redispatch can shed load, but the weighting factor of 100 in the cost function ensures that load shedding is avoided where possible.

Lines that are overloaded during the calculation of the generation redispatch are assumed to be the lines vulnerable to further outage. The further outage of each of these vulnerable lines is modeled by independently outaging each vulnerable line with probability $p_1 = 0.15$. That is, for each vulnerable line, the simulation outages the line if a computer-generated random number uniformly distributed in the interval [0,1] is less than p_1. If none of these vulnerable lines outage, the simulated cascade stops. If some of these vulnerable lines outage, the load flow and redispatch are solved again and further vulnerable lines may outage. The cascade of line outages continues in this manner until no further lines outage. For each simulated cascade, the lines that have outaged (if any), the total amount of load shed, and diagnostic data are recorded. A simulated cascade with negligible load shed (less than 10^{-5} times the total load) is regarded as shedding zero load. This simplified modeling of cascading overloads produces a series of steady states that satisfy basic power systems constraints, but it does not represent the various mechanisms of overload and outage that may well produce another series of steady states in more detailed models. The simplified cascade modeling is used in [6]–[8], [15], and [16] and more sophisticated modeling is used in [18], [20], [22], and [28].

B. Grid Evolution

The grid evolves in time t by slowly upgrading system capacity to satisfy the gradual growth in load. The time t is discrete and is incremented by one for each simulated cascade. Therefore the time t also indicates the number of cascades that have been simulated. Note that not every simulated cascade produces significant cascading; a simulated cascade can have no lines outaged or no load shed or both. Since we are studying only the long-term steady state as time t goes to infinity, it does not matter how time is scaled.

The gradual growth in load is modeled by multiplying the average load by $\lambda = 1.00005$ before every simulated cascade. (If there were one simulated cascade per day, then $\lambda = 1.00005$ would correspond to an annual load growth of 1.8%.) To obtain diversity in the simulated cascades, the load at each load bus at the start of each simulated cascade is varied randomly about its average value by multiplying the load by a factor a uniformly distributed in $[2-\gamma, \gamma]$ with $\gamma = 1.67$. The overall effect is that if the average load on bus i at time $t-1$ is $\overline{P}_i(t-1)$, then the average load on bus i at time t is

$$\overline{P}_i(t) = \lambda \overline{P}_i(t-1) \tag{3}$$

and the load on bus i at time t is

$$P_i(t) = a\overline{P}_i(t). \tag{4}$$

The slow average load growth gradually makes the system more stressed and some reliability criterion finally can not be satisfied. Then system capacity has to be upgraded. The transmission lines are upgraded by increasing their maximum power flow limits $F_1^{\max}$, $F_2^{\max}, \ldots, F_{\text{nline}}^{\max}$. (In practice, there are number of ways of implementing an upgrade that have the effect of increasing a line maximum flow limit such as reconductoring, vegetation control, load voltage support, and upgrade elsewhere or improvement in operations that relaxes an operational limit on the line. Here we do not model changes in impedance, new substations, or new circuits.[3] Nor do we model the systematic long-term changes in load distribution patterns that would accompany and drive these network changes.) The choice of which transmission lines to upgrade and by how much is the upgrade policy described in the next subsection.

Of course, to satisfy adequacy, the generation also has to be upgraded. The generation upgrade is done as needed to maintain coordination with the transmission line upgrades. In particular, the generation is increased at randomly selected generators subject to coordination with the limits of nearby lines when the generator capacity margin falls below a threshold [6]. (The parameters that control the generator upgrade are fully explained in [6] and are normalized generator capacity margin threshold $\Delta P/P = 0.3$, fraction of total generation that is the discrete amount that generation is increased $\kappa = 0.04$, and generator upgrade delay $= 1$.)

C. Transmission Line Upgrade Policies

The transmission line upgrade policy determines how and when to increase the maximum line flow limits $F_1^{\max}, F_2^{\max}, \ldots, F_{\text{nline}}^{\max}$ of the transmission lines. We compare the following two upgrade methods:

1) $\text{n} - 1$ criterion. First a contingency list of the k most severe single line contingencies is defined as explained in the Appendix (choosing a larger k implements a stricter version of the n-1 criterion). Then the n-1 criterion requires no line to exceed its maximum flow limit for each of the contingencies in the contingency list.[4] At the beginning of each simulated cascade, given the initial pattern of loading, we test whether the system satisfies the n-1 criterion by solving the dc load flow equations for each contingency from the contingency list. Any line j that overloads in the test of the n-1 criterion is upgraded by increasing its maximum flow

[3]While it is straightforward to model the physics of new substations or circuits, since we are studying evolving engineered power grids as opposed to arbitrary power grids, it would be necessary to also model the effect or process of the engineering of these upgrades. In our simple model, retaining the topology of the IEEE 118-bus system retains an engineered grid.

[4]Generation is not redispatched during this process unless the contingency islands the power system.

limit $F_j^{\max}$ until it no longer overloads. No lines are upgraded if the $\mathrm{n}-1$ criterion is satisfied.

2) Direct response to load-shedding cascades. This policy upgrades a line when that line was outaged in the previous simulated cascade and there was nonzero load shed in the previous simulated cascade [6]. The line is upgraded by multiplying the maximum line flow limit by a factor of $\mu = 1.07$. That is, suppose that in simulated cascade at time $t-1$, load was shed and that the lines outaged were lines $j_1, j_2, \ldots, j_m$. Then the line flow limits that are changed at time t are given by

$$F_{j_k}^{\max}(t) = \mu F_{j_k}^{\max}(t-1), \quad k = 1, 2, \ldots, m. \tag{5}$$

The n-1 criterion and directly responding to load-shedding cascades are both used in practical power system design and operation. Here we are testing idealized forms of each method applied exclusively.

D. Discussion of the Modeling

The OPA model is "top-down" and represents the processes in greatly simplified forms, although the interactions between these processes still yield complex and complicated behaviors. The simple representation of the processes is desirable both to initially study only the main interactions governing the complex system and for pragmatic reasons of model tractability and simulation run time. (There is some tradeoff between modeling the upgrade process and how much detail can be included in modeling the cascading.) The modeling of the cascading overloads neglects the timing of events and does not consider the many other ways that disturbances can propagate in blackouts, such as protection system failures, dynamics, and human factors. However the cascading overloads are consistent with standard power system modeling at the level of dc load flow and linear programming generator dispatch. It is appropriate to use dc load flow in a simple model of cascading line overloads because dc load flow is a good first approximation of real power flows. The modeling of the grid evolution captures some simplified basic elements of the upgrade process and a reliability policy.

The modeling of the cascading overloads and the grid evolution is simple, but modeling both processes together is a significant innovation that allows the slow, complex dynamics of the interaction of the power system reliability and upgrade to be studied. One can think of the upgrade process as a feedback that adjusts the reliability of the power system. If the grid has too little capacity, there will be more cascading failures or security violations and the feedback will cause more upgrade. If the grid has excess capacity, there will be fewer cascading failures or security violations and the feedback will reduce the upgrades until load growth erodes the excess capacity. It is routine in control systems that system behavior is dominated by the feedback and is insensitive to the details of the "plant" being controlled. That is, a good model of the control system should represent the feedback and can use a simplified model of the plant. This analogy with control systems suggests that a basic model of power system reliability should represent the upgrade process and can use a simplified model of the failure mechanisms. Moreover, some robustness to parameters of the cascading process is expected [6]. In this regard, it is encouraging that the OPA simulation can approximately reproduce the form of the observed statistics of distribution of blackout sizes in North America [6].

There are many other mechanisms other than cascading line overloads involved in cascading blackouts, including voltage, transient, and small signal stability, hidden failures in protection systems and failures in planning, operations, communications and software. For an initial review of these challenges, see [3]. It would be interesting and useful to improve the modeling of the cascading to incorporate more of these mechanisms. For example, the role of voltage collapse could be studied using an ac load flow. Leaving aside issues of run time, this elaboration would not be trivial for an evolving power system model because it would require modeling of the upgrading of the reactive power capability of the grid with some assumed policy of reactive or voltage collapse margin requirements or reacting to voltage collapse blackouts. In complex systems terms, the upgrading of reactive capability would add an additional self-organizing feedback to the upgrading of line and generation capacity. Since many complex systems show robustness to modeling details, we would expect similar results from the improved modeling, but this robustness should be tested in future work.

Another, more traditional way to compute reliability is to consider a fixed power system that is not in an ongoing process of upgrading. Simulating such a power system with a different upgrade policy evaluates the short-term effect of the policy. This is definitely useful, but it does not account for the way in which the power system may evolve over the long term in response to the different upgrade policy. We suggest that our initial work with the OPA simulation shows how to evaluate the long-term effect of the policy and explores a complementary aspect of reliability.

More broadly, improving and augmenting the quantitative assessment of power system reliability is highly relevant for evaluating the benefits to society of new power system control devices and new operational strategies. We hope that the approach suggested in this paper can be developed and applied to quantify the impact of these devices and strategies on long-term reliability and that this will become a useful complement to more traditional approaches for quantifying their reliability.

III. Results

All results use OPA to simulate the IEEE 118-bus system with upgrade controlled either by the n-1 criterion or by responding directly to load-shedding cascades. The IEEE 118-bus system [29] represents a portion of a past American Electric Power Company transmission system. The standard base case [29] is used.

A key result of the power system modeling in the OPA simulation is that the evolving grid self-organizes and settles down to a complex system steady state after an initial transient. Each run simulates 50 000 cascades (some of these shed no load or outage no lines) and the last 30 000 of these simulated cascades are used to generate the steady state cascading failure statistics and grid utilization measures.

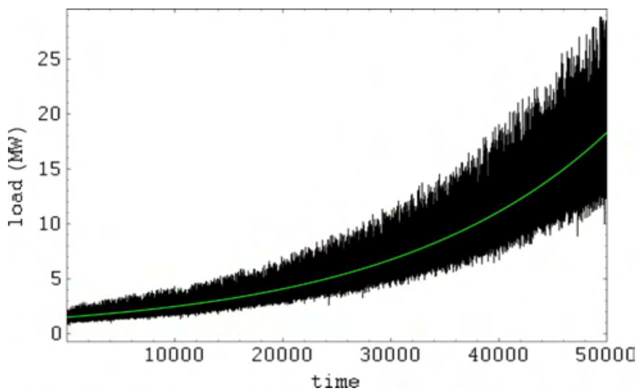

Fig. 2. Average load growth and the random variation about the average load. Time is discrete and increments by one for each simulated cascade. The average load growth is exponential because the average load is multiplied by 1.00005 before each simulated cascade.

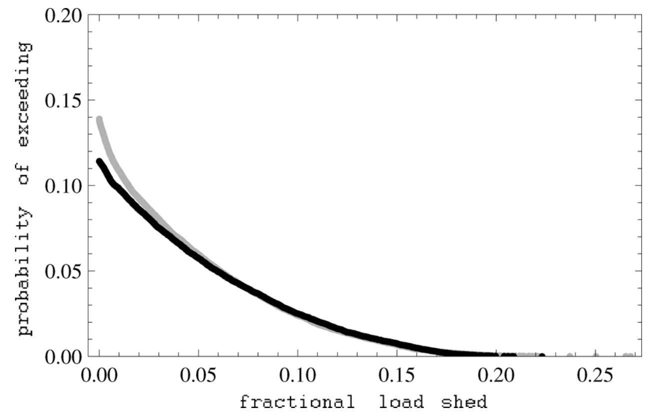

Fig. 4. Probability of fractional load power shed exceeding a given size. Black line is n-1 criterion with ten contingencies. Gray line is direct response to load-shedding cascades.

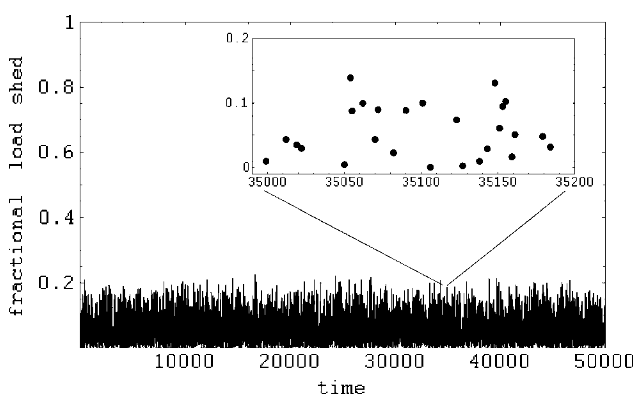

Fig. 3. Fractional load power shed. Inset shows detail for an example of 200 simulated cascades. Policy is n-1 criterion with ten contingencies.

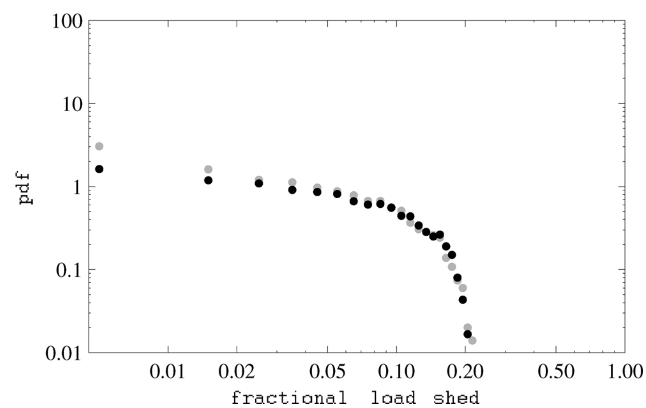

Fig. 5. Log–log plot of pdfs of fractional load power shed. Black dots are n-1 criterion with ten contingencies. Gray dots are direct response to load-shedding cascades.

A. Grid Evolving With n-1 Criterion With Ten Contingencies

We show the OPA results for the n-1 criterion reliability policy with a contingency list with ten contingencies. Fig. 2 shows the exponentially increasing average load growth and the random variability about that average load. The exponentially increasing average load causes the load power shed to have an exponentially increasing trend. We normalize the load power shed by defining the fractional load power shed at time t to be the load power shed divided by the load supplied at the end of the simulated cascade at time t. The statistics of the fractional load power shed become stationary as the grid evolves as shown in Fig. 3. The inset in Fig. 3 illustrates that in many cascades there is no load shed.

Fractional load power shed is a measure of cascade size. Figs. 4 and 5 show the probability distribution of the fractional load power shed in two ways. Fig. 4 plots the probability that a cascade has load shed exceeding a given amount. In particular, the black line in Fig. 4 shows the probability that a cascade has nonzero load shed is 0.114. That is, 11.4% of the cascades shed load. Fig. 5 shows the distribution of amount of load shed as a probability density function (pdf). The maximum likely cascade is about 20% of the supplied load. If we define risk as probability times cost and assume that cascade cost is proportional to load shed, then we obtain Fig. 6 which shows the distribution of risk of load shed.[5] Fig. 6 shows that the larger cascades have more than double the risk of the smaller cascades.

The fractional loading of line j is the line power flow $|F_j(t)|$ divided by the maximum flow limit $F_j^{\max}(t)$ at the beginning of simulated cascade at time t. One measure of grid utilization is the fractional line loading averaged over all the lines

$$\text{average fractional line loading at } t = \frac{1}{\text{nline}} \sum_{j=1}^{\text{nline}} \frac{|F_j(t)|}{F_j^{\max}(t)}. \tag{6}$$

Equation (6) indicates the average fraction of the grid capacity used at time t. Fig. 7 shows how the average fractional line loading at time t changes as the grid evolves from its initial condition. The statistics of (6) become stationary in the steady state. We define the average line loading as the average in time of (6) while the grid is in steady state. The average line loading is shown in Table I.

As the grid upgrades, the maximum flow limits of the lines increase to accommodate the exponential increase in average

[5]There are many uncertainties in determining direct and indirect cascading failure costs, particularly for large cascades, and the assumption of cascade cost proportional to load shed is crude. Despite this considerable uncertainty in determining costs, it is still worthwhile to illustrate a sample risk calculation with an assumption about costs.

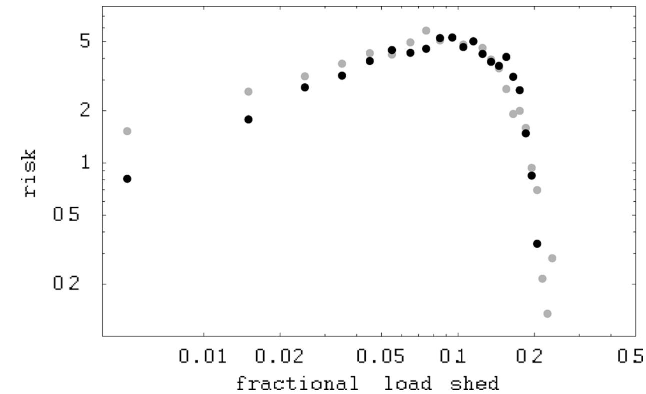

Fig. 6. Distribution of cascading failure risk. Black dots are n-1 criterion with ten contingencies. Gray dots are direct response to load-shedding cascades.

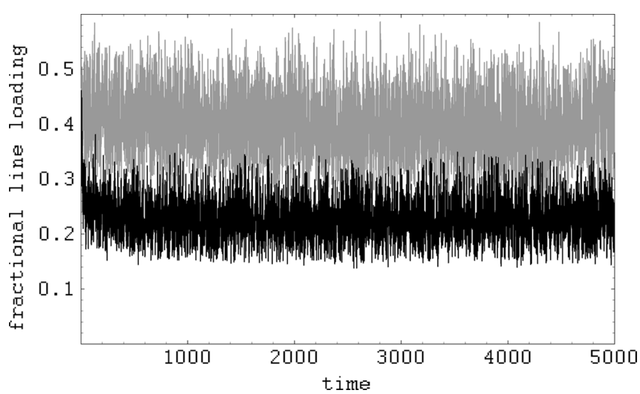

Fig. 7. Average fractional line loading for the first 5000 cascades. Black line is n-1 criterion with ten contingencies. Gray line is direct response to load-shedding cascades.

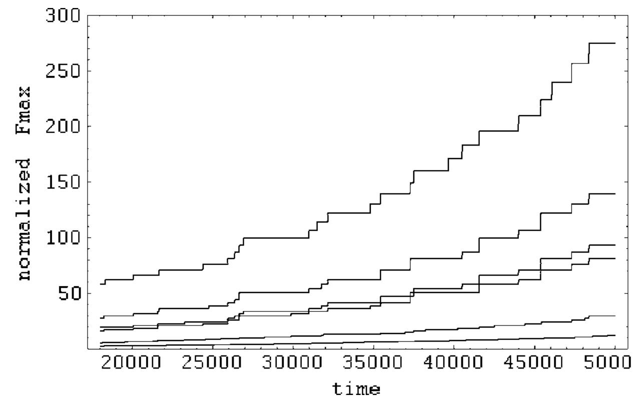

Fig. 8. Evolution of normalized line flow limits $F^{\max}(t)/F^{\max}(0)$ for six typical lines. Policy is n-1 criterion with ten contingencies.

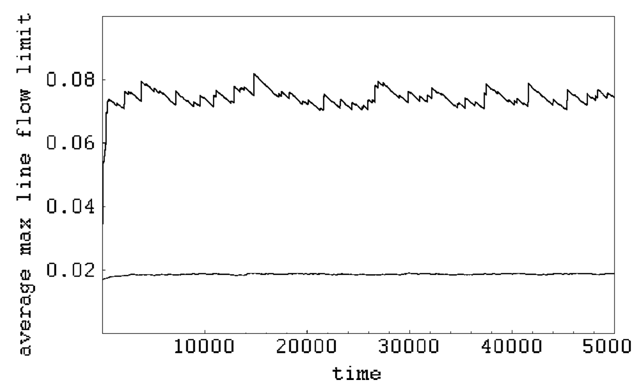

Fig. 9. Average maximum line flow limit per MW served. Black (upper) line is n-1 criterion with ten contingencies. Gray (lower) line is direct response to load-shedding cascades.

TABLE I
GRID UTILIZATION AND LOAD-SHEDDING CASCADE FREQUENCY

	direct response	n-1 list of 1	n-1 list of 10	n-1 list of 50
Average line loading	0.389	0.277	0.229	0.176
Average line flow limit per MW served	0.017	0.031	0.074	0.085
Cascade frequency	0.139	0.723	0.114	0.107

load as illustrated in Fig. 8. The average over the lines of the maximum flow limit at time t is

$$\text{average maximum line flow limit at } t = \frac{1}{\text{nline}} \sum_{j=1}^{\text{nline}} F_j^{\max}(t). \tag{7}$$

As the grid evolves, (7) has an exponentially increasing trend that follows the exponentially increasing system load. We normalize (7) by dividing it by the load supplied at the end of the cascade

$$\left.\begin{array}{r}\text{average line flow limit}\\ \text{per MW served at time } t\end{array}\right\} = \frac{\frac{1}{\text{nline}} \sum\limits_{j=1}^{\text{nline}} F_j^{\max}(t)}{\text{load supplied at } t}. \tag{8}$$

The statistics of (8) become stationary as the grid evolves as shown in Fig. 9. We define the average line flow limit per MW served as the average in time of (8) while the grid is in steady state. The average line flow limit per MW served is another measure of grid utilization and is shown in Table I. Since the grid investment is related to the maximum line flow limits and the societal benefit is related to the power served, the average line flow limit per MW served is one way to indicate the ratio of societal benefit to the grid investment.

B. Comparing Direct Response to Cascades With n-1 Criterion

The effect of changing the reliability policy from the n-1 criterion with ten contingencies to the direct response to the load-shedding cascades is examined. Figs. 4 and 5 compare the probability distributions of load shed. The two cases have almost the same probability distribution except that there are more small cascades with the direct response to load-shedding cascades. Fig. 6 compares the distributions of risk and Fig. 10 illustrates the line upgrading.

The grid utilization of the n-1 case and direct response to load-shedding cascades is compared in Table I. Also see Figs. 7 and 9. The average line loading for the $\mathrm{n}-1$ criterion with ten contingencies is about 41% smaller than with direct response to load-shedding cascades. It appears that the n-1 criterion gives a smaller average line loading because the network is upgraded unevenly, with line flows ranging from 10% of the flow limit to 95% of the flow limit. The average line flow limit per MW served for the n-1 criterion is about four times the line flow

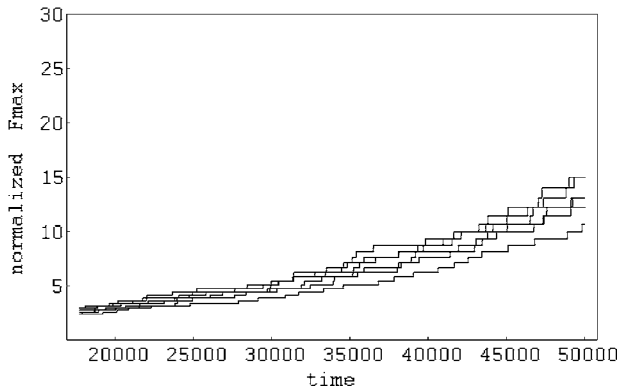

Fig. 10. Evolution of normalized line flow limits $F^{\max}(t)/F^{\max}(0)$ for six typical lines. Policy is direct response to load-shedding cascades.

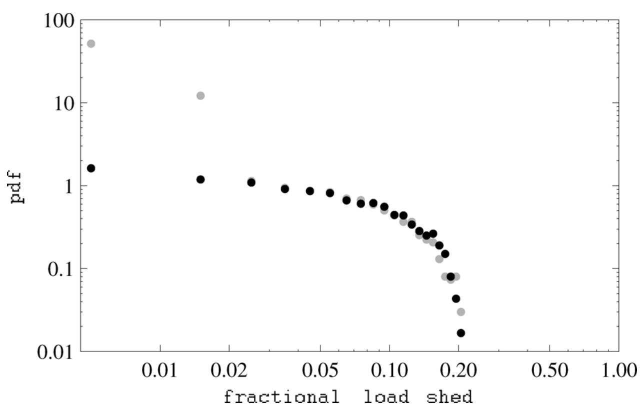

Fig. 11. Log–log plot of pdfs of fractional load power shed. Black dots are n-1 criterion with ten contingencies. Gray dots are n-1 criterion with one contingency.

limit per MW served for the direct response to load-shedding cascades.

C. Effect of Number of Contingencies

We examine the n-1 criterion with contingency lists with one, ten and fifty contingencies. Ten and fifty contingencies yield almost the same distribution of load shed, while one contingency has many more small cascades as shown in Fig. 11. The frequencies of load-shedding cascades with various numbers of contingencies are compared in Table I. The contingency lists with more contingencies have lower cascade frequencies and less grid utilization as shown by the lower average line loading and higher average line flow limit per MW served.

IV. Conclusion

We consider the n-1 criterion and another reliability policy that responds directly to load-shedding cascading outages by upgrading the lines that outaged in the cascade. We show how to assess the effect of these policies on the probability distribution that describes the long-term steady state frequency of various amounts of load shed due to cascading line outages. Transmission grids slowly upgrade and patterns of power flow change over time. This slow evolution of the grid is driven by a slow growth in load and the requirement to maintain reliability policies. Eventually this evolving grid self-organizes and settles down to a complex systems steady state in which, although there remains variability in the cascading failures, the cascading failure statistics and the grid utilization are stationary. We simulate this evolving grid together with the cascading line outages to compute these long-term cascading failure statistics and the grid utilization. Although each part of this complex system is represented simply, accounting for the joint evolution of the grid and the patterns of power flow gives a new type of reliability calculation that is complementary to reliability calculations that assess the short-term effect of policies on reliability by assuming a fixed grid.

To illustrate the approach, we compute the long-term effect of the n-1 criterion with ten contingencies on the reliability of the IEEE 118-bus test system. The long-term probability distribution of load power shed and measures of average grid utilization are computed. If it is assumed that cost is proportional to load power shed, then the risk of larger cascades exceeds the risk of smaller cascades.

To show how reliability policies can be compared, we study the effect of varying the number of contingencies in the contingency list and of changing the policy to a direct response to cascading outages that upgrades the lines involved after each cascade. Reducing the contingency list from ten contingencies to a single contingency greatly increases the frequency of small cascades and increases the grid utilization. Changing the n-1 criterion policy with ten contingencies to the direct response to cascades increases the frequency of small cascades somewhat and increases the grid utilization.

Although it is obvious that power grids are continually evolving to meet the demands of supplying an increasing load and maintaining reliability, it is exciting to suggest a way to describe the complex interactions between these processes and to take an initial step towards quantifying the long-term impact of reliability policies on reliability with respect to cascading overloads.

Appendix

The selection of a contingency list of k severe contingencies is described. Consider the outage of line m in the base case at time zero. The real power flow in line j after the outage of line m is approximated as

$$F_j^{(m)} = F_j(0) + LODF_{j,m}\, F_m(0)$$

where $LODF_{j,m}$ is the line outage distribution factor for line j from the outage of line m [30]. Following [31], the impact of the outage of line m on the system is measured by

$$I_m = \sum_{j=1}^{\text{nline}} \frac{1}{2} \left(\frac{F_j^{(m)}}{F_j^{\max}(0)} \right)^2 .$$

Then the contingency list is the lines with the k highest values of impact I_m.

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