Stochastic Co-optimization for Hydro-Electric Power Generation

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***Abstract*—This paper proposes a stochastic programming framework for solving the optimal scheduling problem faced by a hydro-electric power producer that simultaneously participates in multiple markets. Specifically, the hydro-generator participates in both the electricity spot market and the ancillary services market as a price taker. It seeks to maximize its profit by jointly optimizing its energy/capacity sales and scheduling into all markets subject to market uncertainties and operational constraints. The impact of market uncertainties on the co-optimization problem over a pre-specified time horizon is analyzed through the stochastic programming formulation incorporating both ancillary service and market price uncertainties. Numerical case studies on the advantages of the proposed stochastic co-optimization strategy for a hydro-generator to hedge market uncertainties are carried out with a set of realistic parameters. The proposed model can also be adapted for determining the optimal scheduling and bidding strategy for a power producer facing additional types of market and operational uncertainties.**

***Index Terms*—Ancillary service, electricity energy market, co-optimization, stochastic programming,**

I. Nomenclature

H_A	Set of planning time horizon hours, $H_A = \{h = 1,2,\cdots,H\}$.
H_P	Set of on-peak hours $H_p = \{h \in H_A \mid 13 \le \mathrm{mod}(h,24) \le 20\}$.
H_n	Set of hours at stage n, $H_n = \{h = I_{n-1}+1, I_{n-1}+2,\cdots, I_n\}$, where I_{n-1} denotes the index of the last hour at stage (*n-1*), and $I_0 = 0$, $I_N = H$.
j	Scenario of market price, $j = 1,2,\cdots,J$.
k	Scenario of ancillary service, $k = 1,2,\cdots,K$.
L_h	Indicator of market price scenario in hour h.
n	Stage index, $n = 1,2,\cdots,N$.
$\mathbf{p}_h$	Vector of market settlement prices in hour h.
$\mathbf{q}_h$	Vector of $[q_h^E, q_h^U, q_h^D, q_h^S, q_h^N]^T$ with the components denoting generation scheduled (GS) in energy market, regulation-up, regulation-down, spin reserve, and non-spin reserve services in hour h, respectively.
q_h^R	Total GS quantity in reserve services in hour h.
q_h^C	Total GS capacity in hour h.
$\mathbf{R}_h$	Vector of $[R_h^E, R_h^U, R_h^D, R_h^S, R_h^N]$ with the components denoting the expected revenue from energy market, regulation-up, regulation-down, spin reserve, and non-spin reserve services in hour h, respectively.
S_n	Indicator of ancillary service scenario at stage n.
T_n^l / T_n^u	Lower/upper MW-equivalent water inflow target at stage n, $T_N^l = T_N^u = T_N$.
$\tilde{y}_h / y_h$	Random number/expected value of MW generated in hour h.
Y_n	Accumulated MW generated at stage n.
$\mathbf{\Lambda}_h$	Vector of $[p_h^E, p_h^U, p_h^D, p_h^S, p_h^N, p_h^B]^T$ with the components denoting the prices of energy market, regulation-up, regulation-down, spin reserve, non-spin reserve service markets and balance market in hour h, respectively
$\mathbf{\Pi}$	State transition matrix of a Markovian market price process.
$\mathbf{\Phi}$	Limiting probability vector of the Markovian price process.
$\boldsymbol{\kappa}_h$	A constant coefficient vector, $\boldsymbol{\kappa}_h = [1,1,0,1,1]$.
$\boldsymbol{\gamma}_h$	A constant coefficient vector, $\boldsymbol{\gamma}_h = [0,1,0,1,1]$.
$\mathbf{\Psi}$	A constant coefficient vector, $\mathbf{\Psi} = [1, \rho^U, -\rho^D, 0, 0]$
$\mathbf{\Gamma}$	A coefficient matrix converting vector $\mathbf{\Lambda}_h$ to $\mathbf{p}_h$.
$\alpha^{(k)} / \beta^{(k)}$	Ancillary service scenario k -dependent coefficients.
$\rho^{\{U,D,S,N\}}$	Call probabilities of {regulation-up regulation-down, spin reserve, non-spin reserve} services.
$\tilde{\rho}_h^{\{U,D,S,N\}}$	Bernoulli random variables. with expected values being $\rho^{\{U,D,S,N\}}$.
φ_n^k	Probability of $S_n = k$ at stage n.
ϕ_h^j	Probability of $L_h = j$ in hour h.
τ_{j_1,j_2}	One step state transition probability from market price level scenario j_1 to j_2.
ξ_n^S	Ex-post information of ancillary service at stage n.
ξ_h^L	Ex-post information of market prices in hour h.
$\pi(\cdot)$	Revenue function over the whole planning time horizon (or in stage n if augmented with subscript n).

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Note that when augmented with superscript $(k_1, k_2, \cdots, k_{n-1})$, (j_h), or both, the above variables represent their values given scenario $S_i = k_i, (i = 1,2,\cdots, n-1)$, $L_h = j$, or both. When augmented with subscript of a set of hours, the above variables represent a vector of the variables indexed with each h in the set.

II. Introduction

In electricity market designs such as the one implemented in California, there often exist multiple markets for selling and buying electric power. These markets typically include forward energy markets, ancillary services (AS) markets, and real-time energy markets. While the overall transaction volume of the ancillary services markets is smaller than that of the energy markets, the revenue from selling ancillary services can yield significant profit potential or cost reduction. A mature pool-based electricity market offers participants the choice (and certain obligation) to participate in the ancillary services markets besides the energy markets. To determine the best portfolio strategy for selling electricity into these markets, market participants need to jointly optimize (or, co-optimize) the allocation of electricity generation capacity dedicated to each market incorporating both price and operational uncertainties in the energy and ancillary services markets. There has been a large amount of research on the co-optimization problem of selling electricity into multiple markets given deterministic price forecasts. However, much less literature is available on such a problem subject to stochastic prices as well as random service requests on the committed ancillary services capacity. This paper attempts to address this disparity by providing a stochastic co-optimization framework for optimizing the operations of a hydro-electric generator. The framework can be extended to other applications such as optimal bidding of generation capacity in multiple electricity markets facing price and operational uncertainties.

While the electricity contracts, ancillary services and real-time markets are similar in the sense that their transactions are all completed through auctions, they differ in the types of products offered for trading. Unlike the forward and real-time (instantaneous delivery) energy markets which are for firm energy delivery, the ancillary services market is a forward market for capacity with obligation to deliver energy only when called in real-time. The need for ancillary services as a form of reserve capacity comes from the fact that it is impossible to forecast system demand exactly and then purchase electricity for all the customers ahead of time. Although the real-time energy market is also available for load/generation balancing due to errors in forecasts (or strategic bidding), it is very difficult to predict how much capacity would show up in real-time since there are no forward obligations by the participants. In order to “lock-in” some reserves ahead of time and thus ensure that at least some additional capacity is available in real-time to balance potential discrepancy between load and generation, the independent system operator (ISO) pays a reservation price to all the participants who clear the AS market to keep their capacity idle and available for generation increase (or load decrease) if needed in real-time. In addition to the capacity reservation price, participants in the AS markets also receive payments for energy generated in real-time at the real-time market clearing price, which is determined by the aggregate energy bids in the real-time market.

Normally, market clearing prices in the energy and AS markets are formed from their respective bids received. There are four major ancillary services (namely, regulation-up, regulation-down, spinning reserve, and non-spinning reserve), which are differentiated by the flexibility and response-time of service offered. Regulation services are devoted to the continuous balancing of generation resource and load to assist in maintaining normal system frequency. They are accomplished by online synchronized generation capacity ready to respond to automatic generation control signals. Regulation-up and regulation-down services are procured separately. Spin and non-spin reserves (namely, operating reserves) are prepared for purposes such as peak load shaving and security maintenance in case of plant or transmission outages. Regulation up, spin and non-spin all come from generation units operating at less than full capacity. Regulation-up is replaced gradually by operating reserves if the system balancing requirements persist.

When simultaneously participating in the energy and AS markets, participants strive to get the best strategy for allocating their capacity between energy and AS markets so as to maximize their profits or minimize their overall costs. In order to solve this problem, the market participants need to be able to forecasting prices and their volatility level in all of the energy and ancillary services markets. While there has been significant research done in the area of price forecasting, the task of getting accurate energy price forecasts is still daunting at best. This suggests that the incorporation of a rigorous yet practical stochastic price model into a co-optimization framework is quite challenging.

The ancillary service uncertainty is another important type of uncertainty borne by a market participant. AS capacity is procured as an insurance against unpredictable real-time imbalance between demand and supply. The regulation capacity actually called in real-time is highly variable. However, we are not aware of any research addressing the ancillary service uncertainty explicitly.

This paper tackles the problem of obtaining a profit-maximizing capacity allocation policy in the presence of energy and AS market uncertainties. Specifically, we consider a profit-pursuing hydro-power producer participating in energy and AS markets as a price taker. The co-optimization model of capacity allocation in multiple markets for this hydro producer is formulated and solved utilizing stochastic programming, which gives rise to an optimal scenario-dependent scheduling strategy instead of just a fixed trajectory as is the case of deterministic optimization. We employ stochastic programming techniques for hedging potential revenue loss against market uncertainties by explicitly taking into account the energy price volatility and the AS request variability in the form of regulation-up and down service call probabilities. The advantages of the stochastic programming solutions over their deterministic counterparts have been demonstrated over a wide spectrum of applications in [2].

The remainder of the paper is organized as follows. Section III provides a literature review. A deterministic multi-market hydro-generation scheduling model is presented in Section IV. Sections V and VI discuss the modeling of ancillary service and market price uncertainties, respectively. In Section VII, a complete stochastic programming model for the co-optimization problem incorporating both types of uncertainties is presented. A case study with realistic data is carried out to illustrate the advantage of the proposed stochastic formulation in Section VIII with output graphs and tables. Section IX concludes and outlines some future research directions.

III. Literature Review

Literature on stochastic co-optimization is scant. Most of the early works on the co-optimization problem adopt a deterministic formulation and employ point estimates for the random variables.

A deterministic framework of scheduling in an integrated energy-reserve market is proposed in [4]. [9] introduces a security-constrained unit commitment model to achieve simultaneous optimization of energy and ancillary services markets given market demand information. In the spot market, power producers take on the responsibility of estimating the tradeoff between participating in alternative markets and determine the optimal capacity allocation to each market in response to time-varying market conditions over a given time horizon. [1] addresses the optimal response of a thermal unit to energy and spin reserve spot markets assuming accurate market price forecasts. However, the applicability of the results hinges on the accuracy of the price estimates. Profit opportunities can be lost if the estimates deviate from the realized values significantly.

An overview of the application of stochastic programming techniques in energy markets is provided in [15]. The increased awareness of market uncertainties calls for rigorous risk management of exposures to both ancillary service and market price uncertainties. For a power producer, risk exposures are controlled by putting proper financial instruments, such as futures contracts and flexible-load contracts, into its portfolio. [10] presents a risk management approach that integrates futures contract hedging and hydro generation scheduling in the energy markets. The risk level can be controlled by setting prudent value-at-risk targets. Moreover, the operation scheduling strategy can be expanded via more involved mathematical programming techniques for hedging against uncertainties. Early applications of stochastic programming techniques to unit commitment problems can be found in [3], [8], and [14]. Specifically, [3] models the uncertainty of demand and unit failures; [8] introduces PG&E's optimal scheduling system SOCRATES which coordinates hydro-generation with other energy sources with respect to stream-flow forecasting models and other hydrological information; [14] reports a multi-stage stochastic programming model for unit commitment considering demand uncertainty. An optimal intelligent control of powerhouse-flows and spill-flows in a hydro power system with a network of river basins is presented in [6], where the uncertainties in exogenous water availability and electricity demand are considered. [12] addresses the problem of scheduling the turbines in a chain of stations down a river valley subject to uncertain demand. [11] uses chance constrained programming to ensure a high probability of satisfying load. [7] presents a dynamic programming two-stage algorithm approach to the long-term hydrothermal scheduling of multi-reservoir systems where stochastic behaviors of inflows are treated by considering a large finite sample of hydrological sequences. Only in recent years has price uncertainty come to attract the research attention. For example, [13] studies optimal bidding strategies for the day-ahead energy market and automatic generation control (AGC) markets in the presence of price uncertainties. Although the literature on thermal unit commitment is plentiful, the literature on hydro-electric units is relatively limited. In [5], the self-scheduling of cascaded hydro generating plants along a river basin is studied in the context of the plants selling energy in a day-ahead market. However, the ancillary service uncertainty in the ancillary services markets, namely, the real-time service call randomness, is ignored in all these works.

IV. Deterministic Model

In this section, we present a deterministic model. Similar versions of the model are widely used in industry. We consider a hydro-electric power producer participating in an electricity energy market and four ancillary services markets, namely, regulation-up, regulation-down, spin reserve, and non-spin reserve. The energy market and the ancillary markets clear simultaneously. The producer forecasts the hourly prices of each market and determines the hourly capacity allocations correspondingly. Assume we have an estimation $\mathbf{\Lambda}$ of the hourly price process $\tilde{\mathbf{\Lambda}}$ for the time horizon of interest, where

$$\mathbf{\Lambda}_h = \left[p_h^E, p_h^U, p_h^D, p_h^S, p_h^N, p_h^B\right]^T, h \in H .$$

The revenue from each market is the product of the corresponding price and committed capacity. Assuming equal incremental and decremental market prices, $\mathbf{R}_h$ can be determined as follows.

$$\begin{aligned}
R_h^E &= p_h^E \cdot q_h^E \\
R_h^U &= \left(p_h^U + \rho^U p_h^B\right) \cdot q_h^U \\
R_h^D &= \left(p_h^D - \rho^D p_h^B\right) \cdot q_h^D \\
R_h^S &= \left(p_h^S + \rho^S p_h^B\right) \cdot q_h^S \approx p_h^S \cdot q_h^S \\
R_h^N &= \left(p_h^N + \rho^N p_h^B\right) \cdot q_h^S \approx p_h^S \cdot q_h^S
\end{aligned}$$

where ρ^S, ρ^N, ρ^U and ρ^D denote the transpired values of real-time call probabilities of spin reserve, non-spin reserve, regulation-up, and regulation-down services, respectively. Note that the revenues from ancillary services consist of both capacity payment and real-time balancing energy payment. However, due to the fact that ρ^S and ρ^N are typically very small (less than 1% on average), the energy payment terms in spin and non-spin reserve revenues are ignored in this paper.

The objective of the optimization problem is to maximize the expected total revenues defined in the following compact form.

$$\pi(\mathbf{q}_H)=\sum_{h\in H}\mathbf{p}_h{}^T\mathbf{q}_h \qquad (1)$$

where, $\mathbf{p}_h=\mathbf{\Gamma\Lambda}_h$ (2)

$$\mathbf{\Gamma}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&\rho^U\\0&0&1&0&0&-\rho^D\\0&0&0&1&0&0\\0&0&0&0&1&0\end{bmatrix}. \qquad (3)$$

The components of the vector $\mathbf{p}_h$ correspond to settlement prices of the five aforementioned markets. The operation and maintenance costs are ignored in the objective function since they are relatively fixed and low compared with market revenues.

For scheduling of hydro units, the water supply is a major concern. We simplify the problem by limiting the modeling to hydroelectric plants that are not hydraulically coupled. The total MW generation potential within a time period depends on the elevation difference between headwater and tailwater, reservoir inflows and outflows, and/or contracted water flow limits. In the operational model of a hydro-electric producer, we can envisage specification of weekly or monthly target values for water flows, which are normally determined by some long-term hydro planning model. Such water flow constraints are termed water target constraints herein. Because of small ρ^S and ρ^N values, we can neglect q_h^S and q_h^N in the expected MW generated in hour h and define

$\tilde{y}_h=q_h^E+\tilde{\rho}^U q_h^U-\tilde{\rho}^D q_h^D$, and

$$y_h=E[x_h]=q_h^E+\rho^U q_h^U-\rho^D q_h^D=\mathbf{\Psi q}_h,\ h\in H\ . \qquad (4)$$

For each stage n, the water target constraint is imposed as

$$T_n^l\le\sum_{h\in\{H_1,\cdots,H_n\}}y_h\le T_n^u. \qquad (5)$$

It ensures that certain reservoir levels or contract specifications are maintained during the corresponding time period. Note that the Lagrange multiplier of the target constraint actually reveals the marginal value (namely, shadow price) of the corresponding water resource. Indeed, this Lagrange multiplier is one of the most important inputs for setting optimal price bids.

In light of the market rule against gaming behavior of double booking capacity, we impose a cap of operating generation capacity on the regulation-down capacity as follows.

$$q_h^D\le q_h^E,\ h\in H\ . \qquad (6)$$

The total committed capacity in hour h is

$$q_h^C=q_h^E+q_h^U+q_h^S+q_h^N=\mathbf{\kappa}_h\mathbf{q}_h$$

out of which the capacity committed to reserve is

$$q_h^R=q_h^U+q_h^S+q_h^N=\mathbf{\gamma}_h\mathbf{q}_h\ .$$

Note that q_h^D denotes downward commitment of capacity, it is not included in q_h^C nor q_h^R above.

We also impose bound constraints on q_h^R, q_h^C, and $\mathbf{q}_h$ to reflect operational characteristics and leave capacity margin for both energy and operating reserve markets as follows.

$$\begin{aligned}&q_h^R\le\overline{q_h^R}\\&q_h^C\le\overline{q_h^C}\\&0\le\mathbf{q}_h\le\overline{\mathbf{q}_h}\qquad h\in H\ .\end{aligned} \qquad (7)$$

Without meaningful bounds, a generator may tend to dedicate most capacity to one of the markets while leaving others out of consideration. Such an allocation would not be favorably considered as it could be viewed as manipulative from the market monitoring point of view. A long-term market participant would let go potential market opportunities in respecting these implied regulatory constraints. Note that $\overline{\mathbf{q}_h}$ can be quite different for $h\in H^P$ and $h\in H/H^P$ due to different market conditions. For example, during off-peak hours, most generation units are operated at low output levels, the upper bound of q_h^D is set low since little or no regulation-down service capacity can be committed.

As in most other scheduling problems, uncertainty is a key ingredient of the generation co-optimization problem. We do not have perfect information on all the parameters such as those corresponding to ancillary services $(\tilde{\rho}^U,\tilde{\rho}^U)$ and market prices $(\tilde{\mathbf{\Lambda}})$. The decisions based on (1) are questionable if the range of these random variables is too large. A generalized stochastic formulation of (1) is proposed as follows.

$$\max_{\mathbf{q}_H\in\chi}E[\pi(\mathbf{q}_H,\mathbf{\Omega})] \qquad (8)$$

where $\chi\subset R^N$ denotes the set of all feasible decisions which is defined by the constraints (5-7), and $\mathbf{\Omega}$ denotes the set of possible realization of $[\tilde{\mathbf{\Lambda}},\tilde{\rho}^U,\tilde{\rho}^D]$.

The deterministic formulation (1-3) is actually an expected value scenario of (8) defined by

$\max_{\mathbf{q}_H\in\chi}\pi(\mathbf{q}_H,\overline{\mathbf{\Omega}})$, where $\overline{\mathbf{\Omega}}=[E[\tilde{\mathbf{\Lambda}}],E[\tilde{\rho}^U],E[\tilde{\rho}^D]]$.

Usually, deterministic optimization leads to an inferior solution since inherent randomness is disregarded altogether. According to Jensen's Inequality [2], for any χ and $\mathbf{\Omega}$-concave function $G(\mathbf{q}_H,\mathbf{\Omega})$, we have $G(\mathbf{q}_H,\overline{\mathbf{\Omega}})\ge E[G(\mathbf{q}_H,\mathbf{\Omega})]$, which leads to $\max_{\mathbf{q}_H\in\chi}\pi(\mathbf{q}_H,\overline{\mathbf{\Omega}})\ge\max_{\mathbf{q}_H\in\chi}E[\pi(\mathbf{q}_H,\mathbf{\Omega})]$. Thus, the optimal value of the deterministic optimization is biased upward relative to the optimal value of the stochastic optimization, due to the fact that the variability of $\mathbf{\Omega}$ is ignored.

The generation scheduling is a discrete-time control problem with fixed decisions occurring at different points in time. The essence of resorting to stochastic programming is to incorporate market risks into decision making. The scheduling problem is essentially to strike a careful balance between using water now and using it at a future point. While deterministic multi-period optimization yields decisions for all periods, a stochastic approach yields policies and strategies. In stochastic programming with recourse, recourse actions are taken after some uncertainty has been resolved. The set of decisions is divided into periods before and after the realization of uncertainty. In our problem, the sequence of events and decisions, when considering ancillary service uncertainty at stage n only, is $\mathbf{q}_{H_n}\to\xi_n^S\to\mathbf{q}_{\{H_{n+1},\cdots H_N\}}$. The optimization problem corresponding to stage n is

$$\text{Max}\ \ \mathbf{p}_{H_n}{}^T\mathbf{q}_{H_n}+E_{\mathbf{\Omega}}\left[Max\left[\mathbf{p}_{\{H_{n+1},\cdots H_N\}}(\mathbf{\Omega})^T\mathbf{q}_{\{H_{n+1},\cdots H_N\}}(\mathbf{\Omega})\right]\right] \qquad (9)$$

s.t. $\mathbf{AQ}_{Hn} = \mathbf{b}$

$\mathbf{T}(\mathbf{\Omega})\mathbf{q}_{H_n} + \mathbf{Wq}_{\{H_{n+1},\cdots H_N\}}(\mathbf{\Omega}) = \mathbf{h}(\mathbf{\Omega})$

$\mathbf{q}_{H_n} \geq 0$, $\mathbf{q}_{H_{n+1}}(\mathbf{\Omega}) \geq 0$

where $E_{\mathbf{\Omega}}$ denotes mathematical expectation with respect to $\mathbf{\Omega}$, $\mathbf{A}$, $\mathbf{b}$, $\mathbf{h}$, $\mathbf{T}$, and $\mathbf{W}$ are (scenario-dependent) matrices or vectors. Specifically, $\mathbf{W}$ is called the recourse matrix [2].

To solve (9) numerically, assuming a discrete distribution of random data with a finite number (say, K) of possible realizations $\mathbf{\Omega}^{(k)}$ with probabilities $p^{(k)}$, the stochastic program has the following extensive form which describes the decision variables explicitly in each scenario.

$$\text{Max} \quad \mathbf{p}_{H_n}{}^T \mathbf{q}_{H_n} + \sum_{k=1}^{K} p^{(k)} \mathbf{p}^{(k)}_{\{H_{n+1},\cdots H_N\}}{}^T \mathbf{q}^{(k)}_{\{H_{n+1},\cdots H_N\}} \tag{10}$$

s.t. $\mathbf{Aq}_{H_n} = \mathbf{b}$

$\mathbf{T}^{(k)}\mathbf{q}_{H_n} + \mathbf{Wq}^{(k)}_{\{H_{n+1},\cdots H_N\}} = \mathbf{h}^{(k)}$

$\mathbf{q}_{H_n} \geq 0$, $\mathbf{q}^{(k)}_{\{H_{n+1},\cdots H_N\}} \geq 0$, $k = 1,2,\cdots,K$.

The recourse model transforms the randomness contained in a stochastic program into some specific parameters according to some random distributions. Modeling details such as possible set of outcomes or scenarios and the coarseness of the period structure must be specified with great care so as to achieve an optimal trade-off between realism of the optimization model, which affects the usefulness and quality of the obtained decisions, and tractability of the problem, which is necessary for practical implementation.

The approach proposed in this paper is based on probabilistic modeling of uncertainties in ancillary service and price dynamics. The objective function and constraints of the corresponding mathematical programming models are defined based on market scenarios of interest.

V. Ancillary Service Uncertainty

In hour h, the regulation services can be either called or not. According to (4), $\tilde{y}_h$ has the following possible values depending on which regulation service is called.

$$\tilde{y}_h = \begin{cases} q_h^E + q_h^U + q_h^D & if \quad \tilde{\rho}_h^U = 1, \tilde{\rho}_h^D = 1 \\ q_h^E + q_h^U & if \quad \tilde{\rho}_h^U = 1, \tilde{\rho}_h^D = 0 \\ q_h^E - q_h^D & if \quad \tilde{\rho}_h^U = 0, \tilde{\rho}_h^D = 1 \\ q_h^E & if \quad \tilde{\rho}_h^U = 0, \tilde{\rho}_h^D = 0 \end{cases} . \tag{11}$$

Note that even though regulation-up and regulation-down services can be called in the same hour, such situation does not occur often. Therefore, we exclude this scenario here. This mild assumption can be easily relaxed, albeit at the cost of higher dimensions of the search space. By discretizing $\tilde{y}_h$ according to (11) and following formulation (9), we get the recourse actions corresponding to various ancillary service scenarios in every hour. However, if we model all possible scenarios for each hour h, the number of total scenarios throughout the whole planning horizon would be 4^{H-1}, which is astronomical even for a moderate value of H and, thus, impractical to handle. An approximation is in order.

Define independent Bernoulli random variables ω_h^U and ω_h^D with $\text{Prob}(\omega_h^U = 1) = \rho^U$ and $\text{Prob}(\omega_h^D = 1) = \rho^D$.

Corresponding to each stage n, we set

$$Y_n \equiv \sum_{h \in H_n} \tilde{y}_h = \sum_{h \in H_n} \left(q_h^E + \tilde{\rho}_h^U q_h^U - \tilde{\rho}_h^D q_h^D\right). \tag{12}$$

Assume that $\tilde{\rho}_h^U$'s and $\tilde{\rho}_h^D$'s are independent. For a realization of $[q_h^E, q_h^U, q_h^D], h \in H_n$, given the large number of hours within stage n, we have, approximately,

$Y_n \sim Norm(\mu, \sigma^2)$

where,

$$\mu = E[Y_n] = \sum_{h \in H_n}\left(q_h^E + \rho^U q_h^U - \rho^D q_h^D\right) = \sum_{h \in H_n} \mathbf{\Psi q}_h$$

$$\sigma^2 = Var(Y_n) = \rho^U\left(1-\rho^U\right)\sum_{h \in H_n}\left(q_h^U\right)^2 + \rho^D\left(1-\rho^D\right)\sum_{h \in H_n}\left(q_h^D\right)^2.$$

Discretize Y_n to $K = 3$ realizations which represent low, medium and high scenarios of accumulated participation, respectively. Specifically,

$Y_n^{(k)} = \mu + \sigma\alpha^{(k)}$

where $\alpha^{(k)} = k - 2$ is set to match the first two moments of Y_n given $\varphi_n^k = \text{Prob}\left(Y_n = Y_n^{(k)}\right) = 1/K$. We assume that $\text{Prob}\left(S_n = k \middle| \xi_{n-1}^S\right) = \text{Prob}\left(S_n = k \middle| \xi_1^S, \xi_2^S, \cdots, \xi_{n-1}^S\right) = \varphi_n^k$. According to this scheme, we have K^n possible realizations of total capacity commitment scenarios by the end of any intermediate stage n. Given the discretization of Y_n, in order to represent the expected regulation service revenues at stage n in scenario k, which corresponds to discrete value $Y_n = Y_n^{(k)}$, we make ex-post expected values of $\tilde{\rho}_h^U$ and $\tilde{\rho}_h^D$ for $h \in H_n$ to be

$E\left[\tilde{\rho}_h^U\right] = \rho^{U(k)} = \rho^U \beta^{(k)}$, and $E\left[\tilde{\rho}_h^D\right] = \rho^{D(k)} = \rho^D \beta^{(k)}$

where $\beta^{(k)} = Y_n^{(k)}/\mu$.

The ex-post expected value of market price vector $\mathbf{p}_h^{(k)}$ corresponding to the realization of scenario $S_n = k$ becomes

$$\mathbf{p}_h^{(k)} = \mathbf{\Gamma}^{(k)}\mathbf{\Lambda}_h \tag{13}$$

where

$$\mathbf{\Gamma}^{(k)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \rho^{U(k)} \\ 0 & 0 & 1 & 0 & 0 & -\rho^{D(k)} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The scheduling strategy can be easily determined since the deterministic equivalent formulation of the stochastic programming model is amenable to any standard optimization solver. The overall formulation is given below.

$$\text{Max} \quad E_{\xi_1^S,\xi_2^S,\xi_3^S,\xi_4^S}\left[\pi\left(\mathbf{q}_{H_1}, \mathbf{q}_{H_2}^{(k_1)}, \cdots, \mathbf{q}_{H_N}^{(k_1,k_2,\cdots,k_{N-1})}\right)\right] \tag{14}$$

$$= \sum_{k_1=1}^{K} \varphi_1^{k_1} \left\{ \begin{array}{l} \mathbf{p}_{H_1}^{(k_1)} \mathbf{q}_{H_1} \\ + \sum_{k_2=1}^{K} \varphi_2^{k_2} \left[\begin{array}{l} \mathbf{p}_{H_2}^{(k_2)} \mathbf{q}_{H_2}^{(k_1)} \\ + \left(\begin{array}{l} \cdots \\ + \sum_{k_4=1}^{K} \varphi_N^{k_N} \mathbf{p}_{H_N}^{(k_N)} \mathbf{q}_{H_N}^{(k_1,\cdots k_{N-1})} \end{array} \right) \end{array} \right] \end{array} \right\}$$

s.t. $\forall\left(h, n, k_1, k_2, \cdots, k_{n-1}\right)$,

$$T_n^l \leq \sum_{i=1}^{n-1} Y_{i-1}^{(k_{i-1})} + \sum_{h \in H_n} y_h^{(k_1,k_2,\cdots,k_{n-1})} \leq T_n^u$$

$$q_h^{RD(k_1,k_2,\cdots,k_{n-1})} \leq q_h^{E(k_1,k_2,\cdots,k_{n-1})}$$

$$q_h^{C(k_1,k_2,\cdots,k_{n-1})} = \mathbf{\kappa}_h \mathbf{q}_h^{(k_1,k_2,\cdots,k_{i-1})} \leq \overline{q_h^C}$$

$$q_h^{R(k_1,k_2,\cdots,k_{n-1})}=\boldsymbol{\gamma}_h\mathbf{q}_h^{(k_1,k_2,\cdots,k_{i-1})}\le\overline{q_h^R}$$
$$0\le\mathbf{q}_h^{(k_1,k_2,\cdots,k_{n-1})}\le\overline{\mathbf{q}}_h$$

where $E_{\xi_n^S}$ denotes mathematical expectation with respect to realization of ξ_n^S, $Y_0=0$, and $y_h^{(k_1,k_2,\cdots,k_{n-1})}=\boldsymbol{\Psi}\mathbf{q}_h^{(k_1,k_2,\cdots,k_{n-1})}$.

Objective function (14) is readily extendible to the $E[\pi(\cdot)]-\eta\cdot Var[\pi(\cdot)]$ form to incorporate the mean-variance tradeoff, where $\eta>0$ is the risk-aversion coefficient. We do not examine the mean-variance model in this paper. It is reserved for future research.

VI. Market Price Uncertainty

Significant price volatilities have been observed since the market de-regulation. In this section, we focus on hedging against fluctuating market prices. We assume the producer has a projected hourly market price process $\boldsymbol{\Lambda}$ and, according to (2), the market price process vector $\mathbf{p}$ in place. For example, in Southern California Edison, a team of researchers is dedicated to price forecasting. A variety of commercial price forecasting services are employed for validation purposes. Instead of developing a full-fledged stochastic model for the price evolving processes, we model the price uncertainty by considering a set of discrete price levels which are expressed in proportions of the forecast and match the means and projected volatilities. The uncertainty is compensated for by stochastic programming with recourse. For each hour $h\in H$, we assume J market settlement price scenarios with

$$\mathbf{p}_h^{(j)}=\boldsymbol{\eta}^{(j)}\mathbf{p}_h \qquad (15)$$

and corresponding probability ϕ_h^j with $\sum_{j=1}^{J}\phi_h^j=1$. Also we assume that $\mathrm{Prob}(L_h=j|\xi_{h-1}^L)=\mathrm{Prob}(L_h=j|\xi_1^L,\xi_2^L,\cdots,\xi_{h-1}^L)$. The decision strategy is that for each realization of energy price level scenario $L_{h-1}=j$ of hour $h-1$, decision $\mathbf{Q}_h^{(j)}$ is selected for hour h. A Markovian process is employed for modeling the evolution of price process. While this Markovian assumption simplifies the complex energy price behavior, we concentrate our efforts on dealing with price fluctuations. To construct the price model, we choose $\boldsymbol{\eta}^{(j)}$ in a way to keep $\sum_{j=1}^{J}\phi^j\mathbf{p}_h^{(j)}=\sum_{j=1}^{J}\phi^j\boldsymbol{\eta}^{(j)}\mathbf{p}_h=\mathbf{p}_h$ and keep the correlation matrix of market settlement prices $\boldsymbol{\Xi}$ intact. To simplify the formulation, we assume that the decision strategy applies from $h=2$ onward, meaning that the solution for the first hour is not scenario-dependent. The objective function (1) of the optimization problem becomes

$$\sum_{j_1=1}^{J}\phi^{j_1}\left\{\begin{array}{l}\mathbf{p}_1^{(j_1)}\mathbf{q}_1\\ +\sum_{j_2=1}^{J}P(L_2=j_2|L_1=j_1)\left[\begin{array}{l}\mathbf{p}_2^{(j_2)}\mathbf{q}_2^{(j_1)}\\ +\left(\begin{array}{l}\cdots\\ +\sum_{j_H=1}^{J}P(L_H=j_H|L_{H-1}=j_{H-1})\left(\mathbf{p}_H^{(j_H)}\mathbf{q}_H^{(j_{H-1})}\right)\end{array}\right)\end{array}\right]\end{array}\right\}. \qquad (16)$$

Assuming the Markovian state transition matrix of the price level process being

$$\boldsymbol{\Pi}=\left[\tau_{j_1,j_2}=\mathrm{Prob}(L_2=j_2|L_1=j_1)\right],\ j_{\{1,2\}}=1,2,\cdots,J,$$

with limiting probability vector

$$\boldsymbol{\Phi}=\left[\phi^j\right],\ \phi^j=\sum_{j'=1}^{J}\phi^{j'}\tau_{j',j},$$

the objective function (16) can be rewritten as follows.

$$\sum_{j_1=1}^{J}\phi^{j_1}\left\{\begin{array}{l}\mathbf{p}_1^{(j_1)}\mathbf{q}_1\\ +\sum_{j_2=1}^{J}\tau_{j_1,j_2}\left[\begin{array}{l}\mathbf{p}_2^{(j_2)}\mathbf{q}_2^{(j_1)}\\ +\left(\begin{array}{l}\cdots\\ +\sum_{j_H=1}^{J}\tau_{j_{H-1},j_H}\mathbf{p}_H^{(j_H)}\mathbf{q}_H^{(j_{H-1})}\end{array}\right)\end{array}\right]\end{array}\right\} \qquad (17)$$

or, equivalently,

$$\mathbf{q}_1E[\mathbf{p}_1]+\sum_{h=2}^{H}\left[\sum_{j_{h-1}=1}^{J}\left(\phi^{j_{h-1}}\mathbf{q}_h^{(j_{h-1})}\sum_{j_h=1}^{J}\tau_{j_{h-1},j_h}\mathbf{p}_h^{(j_h)}\right)\right]. \qquad (18)$$

Corresponding to the water target constraint (5), we impose

$$z_h^{Min}\le\psi\mathbf{q}_h^{(j)}\le z_h^{Max},\forall j$$
$$\sum_{h\in\{H_1,\cdots,H_n\}}z_h^{Min}\ge T_n^l,\quad\sum_{h\in\{H_1,\cdots,H_n\}}z_h^{Max}\le T_n^u,\ \forall n$$

where the introduction of z_h^{Min} and z_h^{Max} is to ensure the water target constraints are satisfied for each scenario. The following constraints corresponding to (6-7) also apply $\forall j$:

$$q_h^{RD(j)}\le q_h^{E(j)}$$
$$q_h^{C(j)}=\boldsymbol{\kappa}_h\mathbf{q}_h^{(j)}\le\overline{q_h^C}$$
$$q_h^{R(j)}=\boldsymbol{\gamma}_h\mathbf{q}_h^{(j)}\le\overline{q_h^R}$$
$$0\le\mathbf{q}_h^{(j)}\le\overline{\mathbf{q}}_h.$$

VII. A Complete Model

A complete stochastic model taking into account both ancillary service and market price uncertainties is presented in this section. Corresponding to realizations of S_n and L_h, we define market price vector $\mathbf{p}_h^{(j)(k)}=\boldsymbol{\eta}^{(j)}\boldsymbol{\Gamma}^{(k)}\boldsymbol{\Lambda}_h$ by combining (13) and (15).

Corresponding to the finite discretization of ancillary service and market prices uncertainties, the probability assigned to a scenario node $(S_n=k,L_h=j)$ is the product of probabilities φ_n^k and ϕ_h^j.

The following Fig illustrates the decision tree for a 4-stage complete model considering both ancillary service and market price uncertainties with discretization parameters $K=3$ and $J=3$.

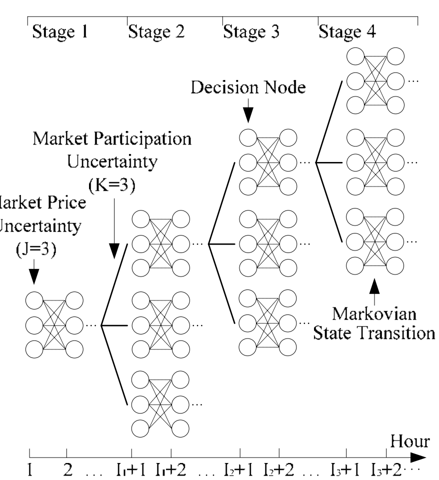

Fig. 1. Scenario tree with both uncertainties

The model aims to maximize

$$E_{\xi_h^L,\xi_h^S}\left[\pi\left(\mathbf{q}_{H_1}^{(j_{h-1})},\mathbf{q}_{H_2}^{(j_{h-1})(k_1)},\cdots,\mathbf{q}_{H_N}^{(j_{h-1})(k_1,k_2,\cdots,k_N)}\right)\right] \quad (19)$$

$$=\pi_1\left(\mathbf{q}_{H_1}^{(j_{h-1})}\right)+\sum_{k_1=1}^{K}\varphi_1^{k_1}\left\{\begin{array}{l}\pi_2\left(\mathbf{q}_{H_2}^{(j_{h-1})(k_1)}\right)\\+\left[\begin{array}{l}\cdots\\+\sum_{k_N=1}^{K}\varphi_{N-1}^{k_{N-1}}\pi_N\left(\mathbf{q}_{H_N}^{(j_{h-1})(k_1,k_2,\cdots,k_{N-1})}\right)\end{array}\right]\end{array}\right\}$$

where,

$$\pi_1\left(\mathbf{q}_{h\in H_1}^{(j_{h-1})}\right)=\mathbf{q}_1E[\mathbf{p}_1]+\sum_{h\in H_1}\left[\sum_{j_{h-1}=1}^{J}\left(\phi^{j_{h-1}}\mathbf{q}_h^{(j_{h-1})}\sum_{j_h=1}^{J}\tau_{j_{h-1},j_h}\mathbf{q}_h^{(j_h)}\right)\right]$$

$$\pi_n\left(\mathbf{q}_{h\in H_n}^{(j_{h-1})(k_1,k_2,\cdots,k_n)}\right)=\sum_{h\in H_n}\left[\sum_{j_{h-1}=1}^{J}\left(\phi^{j_{h-1}}\mathbf{q}_h^{(j_{h-1})(k_1,k_2,\cdots,k_n)}\sum_{j_h=1}^{J}\tau_{j_{h-1},j_h}\mathbf{p}_h^{(j_h)}\right)\right]$$

for $n>1$.

s.t. $\forall(h,n,j,k_1,k_2,\cdots,k_{n-1})$,

$$z_{h\in H_n}^{Min\ (k_1,k_2,\cdots,k_{n-1})}\le\mathbf{\Psi}\mathbf{q}_{h\in H_n}^{(j_{h-1})(k_1,k_2,\cdots,k_{n-1})}\le z_{h\in H_n}^{Max\ (k_1,k_2,\cdots,k_{n-1})}$$

$$\sum_{i=1}^{n-1}Y_{i-1}^{(k_{i-1})}+\sum_{h\in H_n}z_h^{Min(k_1,k_2,\cdots,k_{n-1})}\ge T_n^l$$

$$\sum_{i=1}^{n-1}Y_{i-1}^{(k_{i-1})}+\sum_{h\in H_n}z_h^{Max(k_1,k_2,\cdots,k_{n-1})}\le T_n^u$$

$$q_h^{RD(j)(k_1,k_2,\cdots,k_{n-1})}\le q_h^{E(j)(k_1,k_2,\cdots,k_{n-1})}$$

$$q_h^{C(j)(k_1,k_2,\cdots,k_{n-1})}=\mathbf{\kappa}_h\mathbf{q}_h^{(j)(k_1,k_2,\cdots,k_{n-1})}\le\overline{q_h^C}$$

$$q_h^{R(j)(k_1,k_2,\cdots,k_{n-1})}=\mathbf{\gamma}_h\mathbf{q}_h^{(j)(k_1,k_2,\cdots,k_{n-1})}\le\overline{q_h^R}$$

$$0\le\mathbf{q}_h^{(j)(k_1,k_2,\cdots,k_{n-1})}\le\overline{\mathbf{q}}_h\ .$$

VIII. CASE STUDY

We consider a hydro-electric power producer controlling capacity up to 900MWh doing hourly multi-market generation scheduling for 1 month (30 days, 24 hours per day). Corresponding to weekly and monthly targets, the whole time horizon is divided into $N=4$ stages with the first 3 weeks being the first 3 stages and the rest being the 4th stage. Therefore, we have $I_1=168, I_2=336, I_3=504, I_4=H=720$. The MW-equivalent water inflow target for the month is $T_N=200000$, and for targets of intermediate stage n, we set

$$\left[T_n^d,T_n^u\right]=\left[1-\varepsilon_n,1+\varepsilon_n\right]\frac{I_n}{I_N}T_N$$

where $\varepsilon_1=10\%$, $\varepsilon_2=5\%$, $\varepsilon_3=2\%$, and $\varepsilon_4=0$.

The rationale for the decreasing tolerance levels for target deviations is that the monthly target is normally a hard target and there is more leeway for adjustment in the beginning of the month than toward the end of the month. Actually, in reality, small deviation (1% to 2%, and up to 5% in emergency cases) is allowed for monthly targets. However, for scheduling purposes, monthly targets are normally treated as hard targets, which explains why we have $\varepsilon_4=0$.

Using Southern California Edison's historical (June - August 2005) market prices data, we calibrate the joint distribution of market prices as follows. The price vector $\mathbf{p}_h$ has the following correlation matrix $\mathbf{\Xi}$.

$$\mathbf{\Xi}=\begin{bmatrix}1.0000 & 0.4397 & -0.7768 & 0.6451 & 0.6099 & 0.6413\\0.4397 & 1.0000 & -0.2641 & 0.5133 & 0.4888 & 0.2261\\-0.7768 & -0.2641 & 1.0000 & -0.3905 & -0.3619 & -0.5341\\0.6451 & 0.5133 & -0.3905 & 1.0000 & 0.7250 & 0.4311\\0.6099 & 0.4888 & -0.3619 & 0.7250 & 1.0000 & 0.4052\\0.6413 & 0.2261 & -0.5341 & 0.4311 & 0.4052 & 1.0000\end{bmatrix}$$

with triangular marginal distributions $Tri(\mu_T,\varpi_T)$ where μ_T and ϖ_T denote the mean and width of the support, respectively. The parameters are listed in Table I.

TABLE I
PARAMETERS OF PRICE DISTRIBUTIONS (IN \$/MWH)

	On-Peak $(h\in H^P)$		Off-Peak $(h\in H/H^P)$	
	μ_T	ϖ_T	μ_T	ϖ_T
p_h^E	78.42	28.29	44.99	25.65
p_h^U	25.30	13.71	17.80	9.62
p_h^D	7.67	4.52	23.09	16.26
p_h^S	29.63	19.96	14.79	7.49
p_h^N	24.72	15.92	7.81	2.22
p_h^B	57.73	26.95	39.48	24.99

The expected call probabilities of regulation services are assumed to be $\rho^U=\rho^D=0.3$, and the ad hoc capacity constraints and the regulatory constraints are:

$$\overline{q_{H^P}^R}=200,\ \overline{q_{H/H^P}^R}=450,\ \overline{q_{H^P}^U}=150,\ \overline{q_{H/H^P}^U}=300,\ \overline{q_{H^V}^D}=50,\ \overline{q_{H/H^V}^D}=200.$$

To address the uncertainty of regulation service calls, we set $K=3$ and discretize Y_n as described in section V. To capture the uncertainty of market prices, we set $J=7$ and consider a set of discrete price levels of p_h^E:

$$\mathbf{L}^E=\begin{bmatrix}0.85 & 0.90 & 0.95 & 1.00 & 1.05 & 1.10 & 1.15\end{bmatrix}^T.$$

Considering the co-movement of ancillary service prices, the following discrete price level matrix is calculated with each row corresponds to each component of $\mathbf{p}_h$

$$\mathbf{L}=\begin{bmatrix}\mathbf{L}^E\\\mathbf{L}^U\\\mathbf{L}^D\\\mathbf{L}^S\\\mathbf{L}^N\\\mathbf{L}^B\end{bmatrix}=\begin{bmatrix}0.8500 & 0.9000 & 0.9500 & 1.0000 & 1.0500 & 1.1000 & 1.1500\\0.8147 & 0.9707 & 1.0732 & 1.0109 & 1.1691 & 0.9793 & 0.9822\\1.1196 & 1.0296 & 1.1121 & 0.9391 & 0.9652 & 1.0259 & 0.8085\\0.8158 & 0.9524 & 1.1318 & 0.9823 & 0.9634 & 1.0393 & 1.1151\\0.7861 & 1.0751 & 1.0233 & 0.9537 & 0.9935 & 1.1132 & 1.0552\\0.9446 & 0.9735 & 0.9748 & 0.8365 & 1.0710 & 1.0165 & 1.1831\end{bmatrix}.$$

Correspondingly, diagonal matrix $\mathbf{\eta}^{(j)}$ can be determined with its diagonal components being the j^{th} column of $\mathbf{L}$.

We assume the Markovian price level process to be mean reverting and set the state transition matrix to be

$$\mathbf{\Pi}=\left[\tau_{j_1,j_2}\right],\quad \tau_{j_1,j_2}=\begin{cases}\frac{1}{14} & (j_1-3)(j_2-3)>0\\\frac{1}{7} & (j_1-3)(j_2-3)=0\\\frac{3}{14} & (j_1-3)(j_2-3)<0\end{cases}$$

with the limiting probability vector being

$$\mathbf{\Phi}=\left[\phi^j\right],\quad \phi^j=\frac{1}{J},\quad j=1,2,\cdots,J.$$

In the presence of uncertainties, it is impossible to find a solution that is ideal under all circumstances. Using the above parameters and probability measures, we model and solve the stochastic optimization problems defined in sections V, VI, and VII (referred to as models S1, S2, and S3 hereinafter). These models are compared to the deterministic model given in section IV (referred to as model D).

With the price process parameters given, we simulate a projected price process $\tilde{\mathbf{\Lambda}}_h$ for $h \in H$. GAMS is used to solve the models D, S1, S2, and S3 and obtain the solutions.

An immediate comparison between the models D and S1 is done through plotting their respective expected revenues with respect to ρ^U and ρ^D. Since model S1 accommodates the uncertainty of regulation service calls, its expected revenue derived is less sensitive to the regulation call probabilities than that of model D.

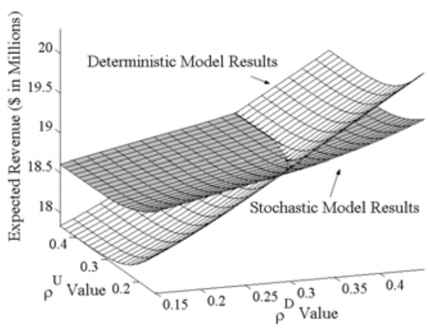

Fig. 2. Sensitivity of expected revenues with respect to ρ^U and ρ^D

The advantages of the stochastic programming formulation are demonstrated via Monte Carlo simulation. By generating Bernoulli random variables ω_h^U and ω_h^D for each $h \in H$, we simulate the regulation service calls. The realized revenue resulting from the generation scheduling solutions of models D and S1 can then be calculated. Repeating the simulation for 200 times, the realized revenues are plotted in Fig. 3. Note that the S1 solutions lead to higher revenues under most circumstances.

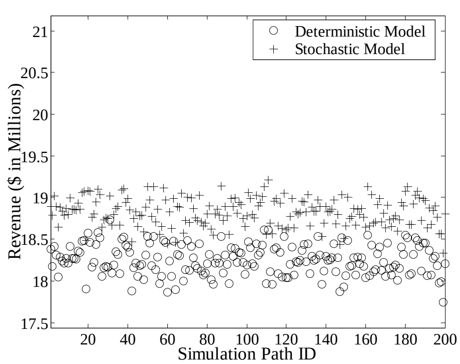

Fig. 3. Comparison of simulated revenues with regulation service call uncertainty

Similarly, by simulating the market price transition process with random starting level at $h = 1$, the realized revenue corresponding to the respective solutions of models D and S2 are compared and shown in Fig. 4. It is clear that the model S2 solution consistently outperforms the model D solution.

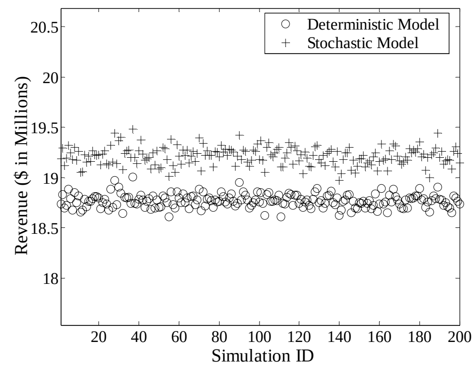

Fig. 4. Comparison of simulated revenues with market price uncertainty

To mimic the uncertainties arising from regulation service calls and market prices, we simulate the regulation service call process and the market price dynamics simultaneously and plot the realized revenues corresponding to the respective solutions of model D and S3 in Fig. 5 below.

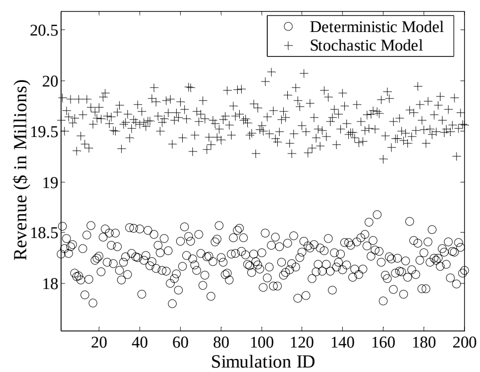

Fig. 5. Comparison of simulated revenues with regulation service call and market price uncertainties

The advantage of our stochastic programming approach is clearly demonstrated by the comparison of the revenue distributions resulting from the stochastic programming approach and the deterministic optimization approach as shown in Fig. 6.

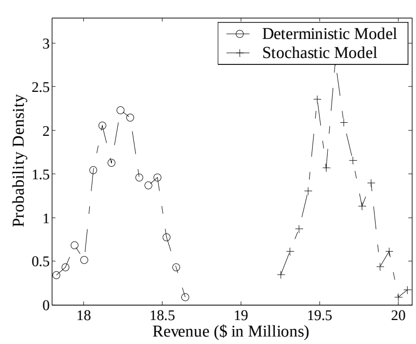

Fig. 6. Comparison of revenue PDF of deterministic and stochastic solutions

The minimum variance unbiased estimates of the parameters of the normal distribution $N(\mu, \sigma)$ and the 95% Confidence Intervals (CI) of the simulated revenue by the deterministic and the stochastic models are displayed in Table II.

TABLE II
ESTIMATES OF NORMAL DISTRIBUTION PARAMETERS (IN MILLION $)

Model	μ	95% CI of μ	σ	95% CI of σ
Deterministic	18.24	[18.22, 18.27]	0.18	[0.17, 0.20]
Stochastic	19.61	[19.59, 19.63]	0.17	[0.15, 0.19]

The comparison reveals a more than 7% improvement in the realized revenue on average. To a real market participant, it translates into a significant revenue gain.

IX. CONCLUSIONS

This paper proposes a stochastic programming framework which explicitly takes into consideration the market uncertainties for solving the co-optimization problem faced by hydro-electric power generators. Compared with the deterministic modeling approach, our approach yields a stochastic scheduling strategy which can effectively hedge the potential revenue loss due to uncertainties associated with the energy price and the realized ancillary service request. The proposed methodology can be adapted for determining optimal strategies that accommodate other sources of uncertainties such as unplanned unit maintenance and water inflow.

Under the stochastic programming framework, the generation scheduling problem can be tackled in different ways. For instance, instead of resorting to the multi-stage formulation described in the paper, a two-stage formulation can be used. As market uncertainties get revealed periodically, the solutions are updated in a rolling horizon fashion, every time by solving an updated two-stage problem.

Several potential extensions of this model deserve further investigation. We can explicitly take into consideration the call probabilities of spin and non-spin reserve services, say, by modeling the spin and non-spin calls as Poisson processes. Impacts of a producer's decisions on the market prices, especially on the ancillary service prices, can be modeled to make the model more realistic. Another fruitful direction is to extend the model to address the co-optimization problem for a power producer who owns a portfolio of hydro, thermal and other types of generation units.

X. ACKNOWLEDGMENT

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XII. BIOGRAPHIES

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