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Multi-Area Generation Adequacy Planning Using Stochastic Programming

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***Abstract*—This paper proposes a mixed-integer stochastic programming approach to the solution of generation expansion planning problem. Generation and Transmission line expansion planning including system reliability considerations is considered a challenging problem. The optimal solution is expected to yield a favorable trade off between system reliability and cost. This helps guide the development of additional generation capacity that is optimal with respect to cost and reliability. The problem is stochastic due to random uncertainties in area generation, transmission lines, and area loads. The problem is formulated as a two-stage recourse model. Reliability index used in this problem is expected cost of load loss. The objective is to minimize the expansion cost in the first stage and the expected loss of load cost in the second stage. The problem is then solved by L-shaped algorithm. The method is illustrated by application to a three-area power system.**

***Index Terms*—Multi-area Power System, Power System Optimization, Reliability, Stochastic Programming, Two-stage Recourse Model.**

I. Nomenclature

A. Indices

I — $\{1,2,\ldots,n\}$ Set of network nodes
s — Source node
t — Sink node
i,j — Network nodes
ω — System state (scenario), $\omega \in \Omega$
Ω — State space (all possible scenarios)

B. Parameters

N^g — Maximum number of additional generation units
c_i^g — Cost of an additional generation unit at area *i* ($)
$c_i^l(\omega)$ — Cost of load loss in area *i* in state ω ($/MW)
M_i^g — Additional generation capacity in area *i* (MW)
$g_i(\omega)$ — Capacity of generation in area *i* in state ω (MW)
$t_{ij}(\omega)$ — Tie line capacity between area *i* and *j* in state ω (MW)
$l_i(\omega)$ — Load in area *i* in state ω (MW)

C. Decision variables

x_i^g — Number of additional generators in area *i*, integer
$y_{ij}(\omega)$ — Flow from arc *i* to *j* for system state ω

II. Introduction

A recent study of long term generation adequacy in a multi-area power system [2] uses an optimization procedure along with MARS to determine an excess or deficient amount of generation in each area. One of the contributions of this reference is to show the relationship between each area risk level and load changes. The drawback of this procedure [2] is that the method requires iterations between optimization and risk calculation which is obtained from several runs of MARS. In a single MARS run, the outage of each component in the system is simulated chronologically by Monte Carlo sampling which may demand long history to produce converged results.

Optimization methods have been previously applied to solve generation expansion planning [1], [2], [6], [7], [9] but these do not explicitly include system reliability considerations. The objective of this research is to solve the multi-area generation and transmission expansion problem with explicit considerations of reliability. Generation expansion problem was initially formulated as linear programming as the power flows are continuous variables. Mixed-integer programming and dynamic programming [9] were proposed to incorporate the discrete decision of additional capacity and to obtain the sequence of optimal decisions respectively. Depending upon application, power system networks are characterized by power flow equations (DC flow) or by capacity flow network. The common constraints are system capacity constraints and demand constraints. The common objective of all formulations is to minimize the expansion cost over a certain time period. Various optimization techniques; such as, Branch and Bound and Bender's decomposition have been proposed. Heuristic techniques such as Fuzzy logic, greedy adaptive search, genetic algorithm, and tabu search have been also used Finally meta-heuristic technique such as, simulated annealing have been also employed to find solution to the problem.

Attempts have been made to include the reliability aspect as a constraint on loss of load probability in [1]. However, the solution is obtained using heuristic techniques. In addition, unavailability cost resulting from unserved energy has been

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included in the objective function in [6] but the problem is formulated as linear programming and does not account for random uncertainties in generation, transmission lines and load. The formulation accounting random uncertainties in generation capacities and load has been shown in stochastic programming literature [11] but consideration of reliability indices is not included. In [11], the problem is formulated as two-stage recourse model where the first stage decision variables are the additional capacity units and the second stage decision variables are network flows. The objective is only to minimize the expansion cost in the first stage and operation cost in the second stage without consideration of reliability indices.

In this paper, the problem is formulated as two-stage recourse model. The first stage and second stage variables are the same as [11]; however, reliability is included in the second stage objective function. Unlike [11], the formulation does not require generation to meet demand at all time, it rather maximizes reliability within available resource, i.e. minimize expected loss of load cost subject to available expansion budget. It should be noted that this reliability index is also a stochastic variable and minimizing this index makes the problem more challenging than incorporating random uncertainties in system capacities and load. The overall objective is to minimize expansion cost in the first stage and at the same time to minimize expected loss of load cost in the second stage. L-shaped algorithm [8] is applied to solve the problem. The problem is implemented on a three-area power system.

III. Problem Formulation

Multi-area power systems are modeled as capacity flow network with area generation, area load, and tie-line connections between areas. This type of representation is considered adequate for multi-area configurations [3]. The problem is to determine the generation capacity requirement in each area with minimum cost and maximum reliability. In this analysis, it is assumed that tie-line equivalent parameters are given. The followings present detailed modeling of each unit namely area generation, area load, and tie lines.

A. Area Generation Model

The failure rate, mean repair time and capacity of each generating unit are assumed to be provided. Discrete probability distribution function for generation in each area is constructed based on unit parameters assuming two-stage Markov process shown in Fig. 1.

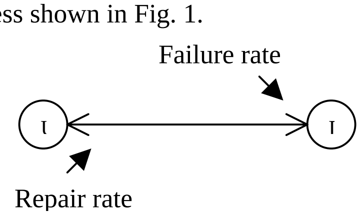

Fig. 1. Two-stage Markov Process

The distribution function is constructed utilizing sequential unit addition approach. The probability table contains levels of state capacity including zero and their corresponding probabilities.

$\vec{g}_i$ Generation capacity vector of area i

$\vec{p}_i^{\,g}$ Probability vector of generation capacity in area i such that $\Pr(\vec{g}_i) = \vec{p}_i^{\,g}$

For computational efficiency, the generation capacity is rounded off to a fixed increment so that only minimum capacity state and number of states in each area are stored. States with very small probability are ignored.

B. Area Load Model

Originally the hourly load data is available in the vector form shown in (1). To improve computational efficiency while still preserving correlation between area loads, they are grouped together utilizing clustering algorithm [12] to an appropriate number of states with corresponding probabilities.

$$\vec{l}^{\,h} = \left(l_1^h, l_2^h, \ldots, l_n^h\right) \tag{1}$$

where

$\vec{l}^{\,h}$ Load vector for the hour *h*

l_i^h Load for the hour *h* in area *i*

C. Tie Line Model

Tie-line parameters are its capacity, forced outage rate and repair rate. Discrete probability distribution of tie-line capacity between areas is constructed based on the given parameters assuming two-stage Markov process, up and down states. Like area generation model, the distribution function construction utilizes unit addition algorithm.

The Tie-line model is represented by (2), which contains the connection areas (from area, to area), its capacity and its corresponding probability.

$$\vec{t}_{ij} = \left(\vec{f}_{ij}, \vec{b}_{ij}\right) \tag{2}$$

where

$\vec{t}_{ij}$ Tie-line capacity vector from area *i* to area *j*

$\vec{f}_{ij}$ Tie-line capacity vector from area *i* to area *j* in forward direction

$\vec{b}_{ij}$ Tie line capacity vector from area *i* to area *j* in backward direction

$\vec{p}_{ij}^{\,t}$ Probability vector of tie line capacity from area *i* to area *j* such that $\vec{p}_{ij}^{\,t} = \Pr(\vec{t}_{ij})$

D. Stochastic Programming Model

Multi-area power system is formulated as a network flow problem where a node in the network represents an area. Source and sink nodes are artificially introduced to represent area generations and load as shown in Fig. 2. The overall objective is to minimize the expansion cost while also maximizing system reliability under uncertainty in area generation, load, and tie-lines. The capacity of every arc in the network is random variable with its discrete probability distributions.

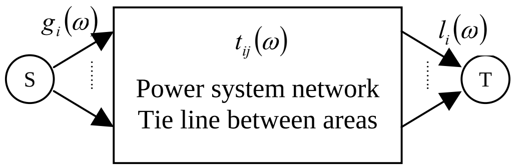

Fig. 2. Power System Network: Capacity Flow Model

An optimization approach based on mixed- integer stochastic programming is proposed for the solution of the generation expansion planning problem in multi-area power systems. Using expected system load loss as a reliability index, the problem is formulated as two-stage recourse model. The first stage decision variables are number of generators to be invested in each area which are determined before the realization of randomness in the problem. The second stage decision variables are the actual flows in the network. The failure probability of additional generators is taken into account by using their effective capacities [13]. The formulation is given in the following.

$$\text{Min} \quad z = \sum_{i \in I} c_i^g x_i^g + E_{\tilde{\omega}}\left[f(x, \tilde{\omega})\right] \tag{3}$$

$$\text{s.t.} \quad \sum_{i \in I} x_i^g = N^g \tag{4}$$

$$x_i^g \geq 0, \text{integer} \tag{5}$$

where the only constraint (4) in the first stage is a restriction on maximum number of additional generators in the system. Constraint (5) is an integer requirement for number of additional generators. The function in (3) is the second stage objective value of minimizing cost of load loss under a realization ω of Ω and is given as follows.

$$f(x, \omega) = \text{Min} \sum_{i \in I} c_i^l(\omega)\left(l_i(\omega) - y_{it}(\omega)\right) \tag{6}$$

$$\text{s.t.} \quad y_{si}(\omega) \leq g_i(\omega) + M_i^g x_i^g; \quad \forall i \in I \tag{7}$$

$$\left|y_{ji}(\omega) - y_{ij}(\omega)\right| \leq t_{ij}(\omega); \quad \forall i, j \in I, i \neq j \tag{8}$$

$$y_{it}(\omega) \leq l_i(\omega); \quad \forall i \in I \tag{9}$$

$$y_{si}(\omega) + \sum_{\substack{j \in I \\ j \neq i}} y_{ji}(\omega) = \sum_{\substack{j \in I \\ j \neq i}} y_{ij}(\omega) + y_{it}(\omega); \quad \forall i \in I \tag{10}$$

$$y_{ij}(\omega), y_{si}(\omega), y_{it}(\omega) \geq 0; \quad \forall i, j \in I \tag{11}$$

where, constraints (7), (8), and (9) are maximum capacity flow in the network under uncertainty in generation, tie line, and load arc respectively. Constraint (10) constitutes conservation of flow in network. Constraint (11) is non-negativity requirement for actual flow in the network.

It should be noted that the second stage objective function coefficient depends on system states. The calculation of this coefficient is performed separately and is shown in the following section.

E. Loss of Load Cost (LOLC) Coefficient Calculation

Loss of load cost depends on interruption duration as well as type of interrupted load. The most common approach to represent power interruption cost is through customer damage function (CDF) [4]. This function relates different types of load and interruption duration to cost per MW. In order to accurately calculate system expected LOLC, LOLC coefficient needs to be evaluated according to the mean duration time of each state (ω).

Mean duration of each stage can be assessed by taking a reciprocal of equivalent transition rate from that state to others [4]. State mean duration is presented in (12). Equivalent transition rate of all components can be calculated using the recursive formula in [10] when constructing probability distribution function. Equivalent transition rates of all components are shown in Appendix A.

$$D_\omega = \frac{24}{\sum_{i \in I} \lambda_{g_i}^{\omega+} + \sum_{i \in I} \lambda_{g_i}^{\omega-} + \sum_{\substack{i,j \in I \\ i \neq j}} \lambda_{t_{ij}}^{\omega+} + \sum_{\substack{i,j \in I \\ i \neq j}} \lambda_{t_{ij}}^{\omega-} + \sum_{k=1}^{m_l} \lambda_{l_k}^{\omega}} \tag{12}$$

where

D_ω — Mean duration of state ω (hours)

$\lambda_{g_i}^{\omega+}$ — Equivalent transition rate of generation in area *i* from a capacity of state ω to higher capacity (per day)

$\lambda_{g_i}^{\omega-}$ — Equivalent transition rate of generation in area *i* from a capacity of state ω to lower capacity (per day)

$\lambda_{t_{ij}}^{\omega+}$ — Equivalent transition rate of transmission line from area *i* to area *j* from a capacity of state ω to higher capacity (per day)

$\lambda_{t_{ij}}^{\omega-}$ — Equivalent transition rate of transmission line from area *i* to area *j* from a capacity of state ω to lower capacity (per day)

$\lambda_{l_k}^{\omega}$ — Equivalent transition rate of area load from state ω to other load states (per day)

m_l — Total number of area load states

Customer damage function used in this paper is taken from [5]. The function was estimated from electric utility cost survey in the US. For small-medium commercial and industrial load, interruption cost dollar per kW-h can be described, as a function of outage duration, by (13).

$$c^l(D_\omega) = e^{6.48005 + 0.38489 D_\omega - 0.02248 D_\omega^2} \tag{13}$$

IV. Solution Procedure

L-shaped algorithm [8] is the most common approach for stochastic programming procedure. At each iteration, the algorithm approximates the second stage objective function by generating piecewise linear function and appends it to the master problem. The algorithm is implemented with Xpress-IVE student edition.

Steps of L-shaped algorithm [8] for this problem are as follows.

Step 0. Initialization

- Find x^0 from solving master problem; discard the second stage objective function.

$$\text{Min} \quad \sum_{i \in I} c_i^g x_i^g$$

$$s.t. \quad \sum_{i \in I} x_i^g = N^g$$

$$x_i^g \geq 0, \text{integer}$$

- Set upper bound (UB) and lower bound (LB), i.e., $UB \leftarrow \infty$ and $LB \leftarrow -\infty$

Step 1. Solve subproblem at iteration k

- Reset the linear approximation function coefficients, $\beta_i^k \leftarrow 0; \forall i \in I$, its right-hand-side value $\alpha^k \leftarrow 0$, and the subproblem objective function value $f^k \leftarrow 0$.
- For all states ω = 1 to $|\Omega|$, solve sub problem k where each scenario has probability, p_ω

$$f_\omega^k = \text{Min}\sum_{i\in I} c_i^l(\omega)\left(l_i(\omega) - y_{it}(\omega)\right)$$

$$\begin{aligned} s.t. \quad & y_{si}(\omega) \le g_i(\omega) + M_i^g x_i^{g,k}; \quad \forall i \in I \\ & \left|y_{ji}(\omega) - y_{ij}(\omega)\right| \le t_{ij}(\omega); \quad \forall i, j \in I, i \ne j \\ & y_{it}(\omega) \le l_i(\omega); \quad \forall i \in I \\ & y_{si}(\omega) + \sum_{\substack{j\in I \\ j\ne i}} y_{ji}(\omega) = \sum_{\substack{j\in I \\ j\ne i}} y_{ij}(\omega) + y_{it}(\omega); \quad \forall i \in I \\ & y_{ij}(\omega), y_{isi}(\omega), y_{it}(\omega) \ge 0; \quad \forall i, j \in I \end{aligned}$$

- Obtain dual solution, $\vec{\pi}_\omega^k = \left(\pi_{\omega,i}^g, \pi_{\omega,ij}^t, \pi_{\omega,i}^l\right)$ associated with generation, transmission line capacities, and load constraints respectively.
- Update the generated cut from $\beta_i^k += p_\omega \pi_{\omega,i}^g M_i^g$, and

$$\alpha_k += p_\omega\left(\sum_{\forall i\in I}\pi_{\omega,i}^g g_i(\omega) + \sum_{\substack{\forall i,j\in I \\ i\ne j}}\pi_{\omega,ij}^t t_{ij}(\omega) + \sum_{\forall i\in I}\pi_{\omega,i}^l l_i(\omega)\right)$$

- Update subproblem objective value $f_k += p_\omega f_\omega^k$
- Update $UB = \min\left\{UB, \sum_{\forall i\in I} c_i^g x_i^{g,k} + f^k\right\}$, if changed, update the incumbent solution, $x^{incumbent} \leftarrow x^k$

Step 2. Solve master problem

- Append the following cut, $\eta \ge \alpha^k + \sum_{\forall i\in I}\beta_i^k x_i^g$
- Obtain solution $x_i^{g,k+1}, \eta^{k+1}$ from the following master problem

$$\begin{aligned} \text{Min} \quad & \sum_{\forall i\in I} c_i^g x_i^g + \eta \\ s.t. \quad & \sum_{\forall i\in I} x_i^g = N^g \\ & \eta \ge \alpha^q + \sum_{\forall i\in I}\beta_i^q x_i^g; q = 0,\ldots,k \\ & x_i^g \ge 0, \text{integer} \end{aligned}$$

- Update $LB = \max\left\{LB, \sum_{\forall i\in I} c_i^g x_i^{g,k+1} + \eta^{k+1}\right\}$

Step 3. Check convergence

- Compute percent gap from $\%gap = \dfrac{(UB - LB)}{UB}$
- If $\%gap \le \varepsilon$, stop and obtain optimal solution, $x^* \leftarrow x^{incumbent}$, and objective value from upper bound, else, $k \leftarrow k+1$, return to step 1.

V. Example System

A three area test system is shown in Fig. 3. There are 5, 6, and 5 generating units of 100 MW in area 1, 2, and 3 respectively. Each generator has failure rate of 0.1 per day and mean repair time of 24 hours. The system has three transmission lines each with 100 MW capacity, failure rate of 10 per year, and mean repair time of 8 hours. TABLE I, TABLE II and TABLE III show area generation, transmission line, and load probability distributions.

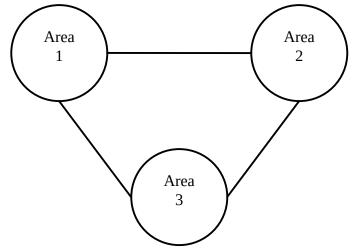

Fig. 3. Three Area Test System

TABLE I
THREE AREA GENERATION PROBABILITY DISTRIBUTIONS

State of Cap. arc	Area 1		Area 2		Area 3	
	Cap (MW)	Prob.	Cap (MW)	Prob.	Cap(MW)	Prob.
7			600	0.564474		
6	500	0.620921	500	0.338684	500	0.620921
5	400	0.310461	400	0.084671	400	0.310461
4	300	0.062092	300	0.011289	300	0.062092
3	200	0.006209	200	0.000847	200	0.006209
2	100	0.000310	100	0.000034	100	0.000310
1	0	0.000006	0	0.000001	0	0.000006

TABLE II
THREE AREA TIE-LINE PARAMETERS

State of Cap. arc	From Area - To Area					
	1-2		1-3		2-3	
	Cap (MW)	Prob.	Cap (MW)	Prob.	Cap (MW)	Prob.
2	100	0.990950	100	0.990950	100	0.990950
1	0	0.009050	0	0.009050	0	0.009050

TABLE III
THREE AREA LOAD PARAMETERS

Load State	Area 1 (MW)	Area 2 (MW)	Area 3 (MW)	Probability
1	500	600	500	0.028257
2	400	500	400	0.275288
3	300	400	300	0.436651
4	200	300	200	0.259803

Load cluster data is taken from [4]. It is assumed that the maximum number of additional units is 2 and the additional generators have capacity of 100 MW each. The cost of additional unit of the three-area system is 100 million dollars for all area.

The optimization procedure yields a solution to locate one generator in area 1 and one generator in area 2 which gives expected loss of load cost of 1.48 million dollar and expansion cost of 200 million dollar. The algorithm converges in 6 iterations. Upper bound and lower bound at each iteration are shown in Fig. 4. It should be noted that the expected loss of load cost depends on the customer damage function used. The objective of this paper is; however, to describe a new method of solving the multi area generation and transmission adequacy problem and the study presented here is only for illustration purposes.

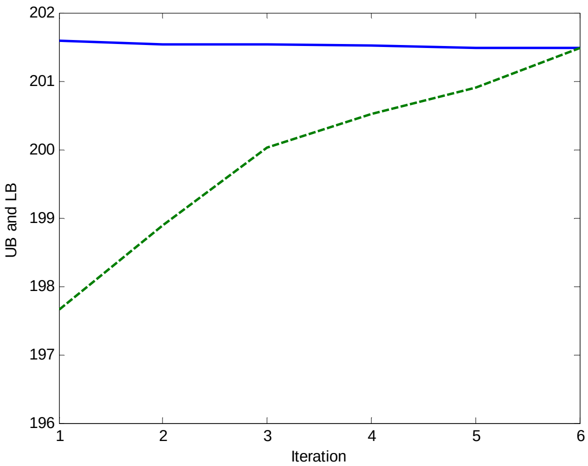

Fig. 4. Upper Bound and Lower Bound of Objective Function

VI. CONCLUSION

A solution procedure for multi-area generation adequacy planning problem is proposed. The problem is formulated as a two stage recourse model with the objective to minimize expansion cost and maximize reliability subject to total budget. L-shaped algorithm is implemented and applied to solve the problem. For larger systems with a very large number of scenarios, interior sampling by Monte-Carlo should be applied along with L-shaped algorithm.

If needed, a different reliability index can be used, for example, loss of load probability. To calculate loss of load probability, number of loss of load states has to be obtained. Therefore, minimizing loss of load probability is the same as minimizing number of loss of load states with the following second stage objective function in (13). The analysis should be made to verify that this function is convex on decision variables.

$$f(x,\omega) = \underset{i \in I}{\mathrm{Max}}\left(l_i(\omega) - y_{it}(\omega), 0\right) \tag{13}$$

Expected loss of load can also be used as a reliability index in the second stage objective function since it may be difficult to assess the loss of load cost coefficient for various systems. However, expected loss of load needs to be weighted in order to be compatible with the expansion cost in the first stage objective function. Instead of requiring maximum number of additional units, a budget constraint can be used to allow flexibility. Sensitivity analysis on the weight of load loss in each area can be conducted to provide the quantified information (expected loss of load reduction) of the next best generation location that improves system reliability subject to budget constraint.

If the budget constraint is relaxed or the loss of load weight in each area is different, the next best generation location will also change. Therefore, sensitivity analysis on these parameters should also be investigated in future studies. The problem can be formulated to minimize cost with subject to reliability constraint where reliability index can be obtained from different budget values. If reliability index (expected loss of load) is above the limit, budget can be increased. The algorithm has to be repeated until system reliability is below the limit.

APPENDIX

A. *Equivalent Transition Rate*

TABLE A.I, TABLE A.II and TABLE A.III show equivalent transition rate (per day) of area generation, transmission lines, and area load respectively.

TABLE A.I
EQUIVALENT TRANSITION RATES OF THREE AREA GENERATION

Area 1			Area 2			Area 3		
Cap (MW)	$\lambda^+_{g_1}$	$\lambda^-_{g_1}$	Cap (MW)	$\lambda^+_{g_2}$	$\lambda^-_{g_2}$	Cap (MW)	$\lambda^+_{g_3}$	$\lambda^-_{g_3}$
			600	0.6	0			
500	0.5	0	500	0.5	1	500	0.5	0
400	0.4	1	400	0.4	2	400	0.4	1
300	0.3	2	300	0.3	3	300	0.3	2
200	0.2	3	200	0.2	4	200	0.2	3
100	0.1	4	100	0.1	5	100	0.1	4
0	0	5	0	0	6	0	0	5

TABLE A.II
EQUIVALENT TRANSITION RATES OF THREE AREA TIE-LINE

From Area - To Area								
1-2			1-3			2-3		
Cap (MW)	$\lambda^+_{t_{12}}$	$\lambda^-_{t_{12}}$	Cap (MW)	$\lambda^+_{t_{13}}$	$\lambda^-_{t_{13}}$	Cap (MW)	$\lambda^+_{t_{23}}$	$\lambda^-_{t_{23}}$
100	0.0274	0	100	0.0274	0	100	0.0274	0
0	0	3	0	0	3	0	0	3

TABLE A.II
EQUIVALENT TRANSITION RATES OF THREE AREA LOAD

λ^{ij}_l	Load state, j			
Load state, i	1	2	3	4
1	0	1.3429	0.0206	0
2	0.3394	0	1.9753	0.0278
3	0.0085	1.3399	0	2.1036
4	0	0.0452	2.2370	0

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