An Energy Reference Bus Independent LMP Decomposition Algorithm

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***Abstract*—The volatility of the price of electricity in a locational marginal price (LMP) market makes it necessary to introduce financial price risk hedging instruments. The congestion-related and marginal-loss-related revenue surpluses collected by the regional transmission operator (RTO) are proposed to be redistributed to market players. It is important to be able to correctly decompose the LMP into its congestion and marginal-loss components, which are critical for the valuation and settlement of these financial instruments. A new energy reference bus independent LMP decomposition model using an ac optimal power flow (OPF) model is presented to overcome the reference bus dependency disadvantage of the conventional approach. The marginal effect of the generators' output variation with respect to load variation are used as the basis of this decomposition model. The theoretical derivation and a proof are given. The new model achieves a set of reference bus independent results. An example is presented comparing the new model with the conventional model.**

***Index Terms*—Congestion, decomposition, electricity markets, locational marginal price (LMP), losses, optimal power flow (OPF).**

NOMENCLATURE

$\mathbf{P}_{\mathcal{G}}$, $\mathbf{Q}_{\mathcal{G}}$	Generator real/reactive power output vectors.
$\mathbf{X}$	State variable vector.
$\mathrm{C}(\mathbf{P}_{\mathcal{G}})$	Total system cost function.
$\mathbf{f}_p(\mathbf{X})$	Bus real power balance functions.
$\mathbf{f}_q(\mathbf{X})$	Bus reactive power balance functions.
$\mathbf{P}_{\mathcal{D}}$, $\mathbf{Q}_{\mathcal{D}}$	Bus real/reactive power load vectors.
$\mathbf{S}(\mathbf{X})$, $\overline{\mathbf{S}}$	Branch power flows and MVA thermal limits.
$\mathbf{G}(\cdot)$	Variables bounds limits functions.
$L(\cdot)$	Lagrangian function.
$\boldsymbol{\lambda}_p$, $\boldsymbol{\lambda}_q$, $\boldsymbol{\mu}$, $\boldsymbol{\gamma}$	Lagrangian multipliers.
r	Energy reference bus index.
$-r$	Buses other than the reference bus r.
$\boldsymbol{\Psi}_r$	Transmission sensitivity matrix.
ς	Loss factor.
λ^e, $\boldsymbol{\lambda}_p^l$, $\boldsymbol{\lambda}_p^c$	Energy, loss, and congestion LMP components.
$w_{i,k}$	Marginal generator contribution factor.
$\partial\mathbf{x}/\partial\mathbf{y} \in \Re^{m\times n}$	If $\mathbf{x} \in \Re^{m\times 1}$ and $\mathbf{y} \in \Re^{n\times 1}$.

Manuscript received April 13, 2005; revised December 23, 2005. This work was supported in part by the National Science Foundation under Grant EEC 96-15792, in part by PSERC, and in part by the Grainger Foundation. Paper no. TPWRS-00204-2005.

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Digital Object Identifier 10.1109/TPWRS.2006.876703

I. INTRODUCTION

ELECTRICITY industry deregulation around the world has caused rapid changes in industry structure and policies. In order to efficiently use generation resources and the transmission grid in a competitive environment, nodal price theory, which was first formulated in [1], is increasingly being employed, particularly through the use of locational-based marginal pricing (LMP) [2]. The LMP at a location is defined as the marginal cost to supply an additional increment of load to the location without violating any system security limits. Usually the LMP varies throughout the system because of the effects of both transmission losses and transmission system congestion.

The locational prices in an LMP market are usually very volatile, which results in significant price risk for market participants. Financial price hedging instruments need to be used to reduce this price risk. Generally speaking, in an LMP electricity market that considers losses and congestion, the regional transmission operator (RTO) collects more payments from the loads than the amount paid to the generator. Financial instruments such as financial transmission rights (FTRs) [2] and loss hedging rights (LHRs) [3] have been introduced to help hedge these price risks by refunding the over-collected revenues to FTR and LHR owners. With the introduction of these financial instruments in a LMP-based market, there has been a desire to decompose the LMP value into three components: energy, losses, and congestion [8]. In particular, such a decomposition is desirable for the settlements of FTRs and LHRs. More specifically, FTRs, which are used to hedge the price risks caused by congestion by returning congestion-related over-collected revenue to the FTR owners using the marginal congestion components (MCCs) of the LMP, have been extensively studied. Similarly, the objective of LHRs is to hedge the price risks caused by marginal loss by returning marginal-loss-related over-collected revenue to the LHR owners using the marginal loss components (MLC) of the LMPs. Although there is still no widely accepted financial marginal loss price risk hedging instrument, LHRs, together with FTRs, present a general picture of full price risk hedging by using the LMP price components. A detailed mathematical formula of hedging can be found in [3, Appendix].

A model was proposed in [4] to breakdown the LMP into two components: an energy/loss component and a congestion component. A more general LMP decomposition method was presented in [5]. More recently, in [6], the New England independent system operator (ISO) has presented a marginal loss model based on a dc optimal power flow (OPF) [7] model with distributed reference buses. However, the selection of the optimal loss distribution factors is still a topic of research. Furthermore, the resultant LMP and dispatching results are dependent upon the loss distribution factors, which are preset. Also, an LMP components formulation based upon a distributed-slack ac power flow is proposed in [12]. More LMP decomposition methods involving various heuristic implementations are presented in [13]–[15], which produce arguable results because of the heuristic settings and slack assumptions used. Finally, the author in [8] argued that the LMP decomposition method should consider the actual marginal effect of congestion during the calculation of penalty factors, a key factor that had previously been neglected in the literature.

With respect to the underlying power flow model, the dc power flow approach has been widely used in LMP calculation in industry [6] due to its simplicity. However, it is shown in [11] that the OPF results (both dispatch and locational price) obtained from a dc model with a loss compensation technique applied differ from those of the more accurate ac OPF model. Thus, it is important to study the components of the LMP using an ac power flow model, an approach that is used exclusively in this paper.

In order to create a competitive energy market, it is important for the RTO to send out fair and accurate price signals to market participants. Currently, however, the LMP components determined by the conventional LMP decomposition methods are dependent upon the location of the energy reference bus since the conventional method determines the LMP components using the transmission constraint sensitivities and the loss sensitivity factors, which are reference bus dependent [9]. That is, in their calculation, the assumption is the compensating power (which must always be present) is picked up at a specified reference bus or a set of distributed slacks whose weights are preset. Before proceeding further, it is important to note that the energy reference bus (or buses in a distributed approach) need not be, and often are not, the actual power flow slack bus.

The impact of the reference bus selection can be illustrated through the use of the three-bus power system shown in Fig. 1. The Fig. 1 values represent the system's OPF solution, with a single binding transmission constraint—the flow on the line from bus 1 to bus 2 is constrained to its 100-MVA limit. While the LMP values themselves are reference bus independent, the decomposition of the LMP into its three components (Energy Component [EC], MLC, and MCC) depend on the assumed energy reference bus. For example, Table I shows the decomposition assuming bus 1 as the reference, while Table II shows it with bus 2 as the reference. Note, a characteristic of the reference bus is that its LMP has zero loss and congestion components.

Certainly the influence of the LMP decomposition, and how it would impact individual market participants, depends upon the specific market rules. However, utilizing LMP components that depend upon such an arbitrary value as which bus has been designated as the energy reference bus certainly could raise concerns about market fairness.

In this paper, we present a reference bus independent decomposition of the congestion and loss component of the LMP. Specifically, we present an algorithm in which both the MLC values and the differences in the MCC values are completely independent of the reference bus. Since only the differences in the MCC values are used in the FTR settlement, the proposed model can be considered to be independent of the reference bus. Of course, since by definition the MCC at the reference bus is zero, with our algorithm, changing the energy reference bus will still change both the EC and the MCC of all buses. However, with our algorithm, since the MLC and the MCC differences remain the same, a change in the energy reference bus only causes a constant shift in the EC and MCC values.

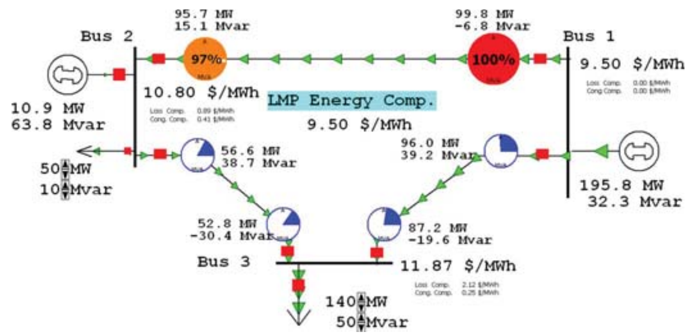

Fig. 1. Three-bus LMP example. (Color version available online at http://ieeexplore.ieee.org.)

TABLE I
LMP COMPONENTS WITH BUS 1 AS ENERGY REFERENCE BUS

Bus	LMP ($/MWh)	LMP Components ($/MWh)		
		EC	***MLC***	***MCC***
Bus 1	9.50	9.50	0.00	0.00
Bus 2	10.80	9.50	0.89	0.41
Bus 3	11.87	9.50	2.12	0.25

TABLE II
LMP COMPONENTS WITH BUS 2 AS ENERGY REFERENCE BUS

Bus	LMP ($/MWh)	LMP Components ($/MWh)		
		EC	***MLC***	***MCC***
Bus 1	9.50	10.80	-0.92	-0.38
Bus 2	10.80	10.80	0.00	0.00
Bus 3	11.87	10.80	1.28	-0.21

This paper is organized as follows. In Section II, we review the mathematical derivation of conventional LMP decomposition method and the useful concepts of loss sensitivity and congestion sensitivity. Our new decomposition method is presented in Section III. The effects of reference energy price of the new method are discussed in detail in Section IV, while Section V presents two examples.

II. CONVENTIONAL LMP DECOMPOSITION

In this section, we review the conventional model of breaking down LMP into energy, loss, and congestion components.

A. OPF Problem Formulation

The OPF problem is widely used to determine the optimal dispatch of generators by minimizing the total operation cost without violating various system security constraints. With the development of electricity market, bidding cost functions are used instead of the actual cost functions to represent generators' economic features. Mathematically, an ac OPF problem can be formulated as

$$\begin{aligned}
\min_{\mathbf{P}_{\mathcal{G}},\mathbf{Q}_{\mathcal{G}},\mathbf{X}} \quad & \mathrm{C}(\mathbf{P}_{\mathcal{G}}) & \\
\text{s.t.} \quad & -P_{\mathcal{G}_r}+P_{\mathcal{D}_r}-\mathrm{f}_{p_r}(\mathbf{X})=0 & \leftrightarrow \lambda_{p_r} \quad (1)\\
& -Q_{\mathcal{G}_r}+Q_{\mathcal{D}_r}-\mathrm{f}_{q_r}(\mathbf{X})=0 & \leftrightarrow \lambda_{q_r} \quad (2)\\
& -\mathbf{P}_{\mathcal{G}_{-r}}+\mathbf{P}_{\mathcal{D}_{-r}}-\mathbf{f}_{p_{-r}}(\mathbf{X})=\mathbf{0} & \leftrightarrow \boldsymbol{\lambda}_{p_{-r}} \quad (3)\\
& -\mathbf{Q}_{\mathcal{G}_{-r}}+\mathbf{Q}_{\mathcal{D}_{-r}}-\mathbf{f}_{q_{-r}}(\mathbf{X})=\mathbf{0} & \leftrightarrow \boldsymbol{\lambda}_{q_{-r}} \quad (4)\\
& \mathbf{S}(\mathbf{X})-\overline{\mathbf{S}}\le\mathbf{0} & \leftrightarrow \boldsymbol{\mu} \quad (5)\\
& \mathbf{G}(\mathbf{P}_{\mathcal{G}},\mathbf{Q}_{\mathcal{G}},\mathbf{X})\le\mathbf{0} & \leftrightarrow \boldsymbol{\gamma}. \quad (6)
\end{aligned}$$

Equations (1) and (2) are the power flow balance constraints at the energy reference bus r, which may be different from the voltage angle reference (assumed here to be bus 0). Equations (3) and (4) include the real and reactive power flow balance constraints. Equation (5) models the transmission system flow constraints (either transmission lines, transformers, or flowgates). Also, equation (6) models the upper and lower variable limits (e.g., bus voltage limits, generators' real and reactive power output limits, etc). The state variable vector $\mathbf{X}$ includes the voltage magnitudes and voltage angles of all buses, except for the actual system slack bus.

Next, the Lagrangian function for the OPF is

$$\begin{aligned}
&\mathcal{L}(\mathbf{P}_{\mathcal{G}},\mathbf{Q}_{\mathcal{G}},\mathbf{X},\boldsymbol{\lambda}_p,\boldsymbol{\lambda}_q,\boldsymbol{\mu})=\mathrm{C}(\mathbf{P}_{\mathcal{G}})\\
&\quad+\left[-P_{\mathcal{G}_r}+P_{\mathcal{D}_r}-\mathrm{f}_{p_r}(\mathbf{X})\right]^T\lambda_{p_r}\\
&\quad+\left[-Q_{\mathcal{G}_r}+Q_{\mathcal{D}_r}-\mathrm{f}_{q_r}(\mathbf{X})\right]^T\lambda_{q_r}\\
&\quad+\left[-\mathbf{P}_{\mathcal{G}_{-r}}+\mathbf{P}_{\mathcal{D}_{-r}}-\mathbf{f}_{p_{-r}}(\mathbf{X})\right]^T\boldsymbol{\lambda}_{p_{-r}}\\
&\quad+\left[-\mathbf{Q}_{\mathcal{G}_{-r}}+\mathbf{Q}_{\mathcal{D}_{-r}}-\mathbf{f}_{q_{-r}}(\mathbf{X})\right]^T\boldsymbol{\lambda}_{q_{-r}}\\
&\quad+\left[\mathbf{S}(\mathbf{X})-\overline{\mathbf{S}}\right]^T\boldsymbol{\mu}+\boldsymbol{\gamma}^T\mathbf{G}(\mathbf{P}_{\mathcal{G}},\mathbf{Q}_{\mathcal{G}},\mathbf{X}). \qquad (7)
\end{aligned}$$

Then the Kuhn–Tucker (KT) necessary conditions for optimality are given by

$$\left(\frac{\partial\mathcal{L}}{\partial\mathbf{P}_{\mathcal{G}}}\right)^T=\left(\frac{\partial\mathrm{C}}{\partial\mathbf{P}_{\mathcal{G}}}\right)^T-\boldsymbol{\lambda}_p+\left(\frac{\partial\mathbf{G}}{\partial\mathbf{P}_{\mathcal{G}}}\right)^T\boldsymbol{\gamma}=\mathbf{0} \qquad (8)$$

$$\left(\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_{\mathcal{G}}}\right)^T=-\boldsymbol{\lambda}_q+\left(\frac{\partial\mathbf{G}}{\partial\mathbf{Q}_{\mathcal{G}}}\right)^T\boldsymbol{\gamma}=\mathbf{0} \qquad (9)$$

$$\begin{aligned}
\left(\frac{\partial\mathcal{L}}{\partial\mathbf{X}}\right)^T=&-\left(\frac{\partial\mathrm{f}_{p_r}}{\partial\mathbf{X}}\right)^T\lambda_{p_r}-\left(\frac{\partial\mathrm{f}_{q_r}}{\partial\mathbf{X}}\right)^T\lambda_{q_r}\\
&-\left(\frac{\partial\mathbf{f}_{p_{-r}}}{\partial\mathbf{X}}\right)^T\boldsymbol{\lambda}_{p_{-r}}-\left(\frac{\partial\mathbf{f}_{q_{-r}}}{\partial\mathbf{X}}\right)^T\boldsymbol{\lambda}_{q_{-r}}\\
&+\left(\frac{\partial\mathbf{S}}{\partial\mathbf{X}}\right)^T\boldsymbol{\mu}+\left(\frac{\partial\mathbf{G}}{\partial\mathbf{X}}\right)^T\boldsymbol{\gamma}=\mathbf{0}. \qquad (10)
\end{aligned}$$

The LMP at a particular bus location is defined as the minimal cost for supplying an additional increment of load at the bus location without violating the system operation security constraints. Hence, the real power LMP and the reactive power LMP are the Lagrange multipliers, $\boldsymbol{\lambda}_p$ and $\boldsymbol{\lambda}_q$, associated with the corresponding real and reactive power balance equations. They are the by-products of the OPF problem solution and are often called the *shadow prices*.

B. Transmission Sensitivity Matrix

Define the Jacobian matrix

$$\mathbf{J}_r=\left[\left(\frac{\partial\mathbf{f}_{p_{-r}}}{\partial\mathbf{X}}\right)^T \quad \left(\frac{\partial\mathbf{f}_{q_{-r}}}{\partial\mathbf{X}}\right)^T\right] \qquad (11)$$

where r again refers to the energy reference bus. The Jacobian matrix is assumed to be non-singular; otherwise, the system is having a problem of voltage collapse or maximum loadability, which is discussed more from a steady-state stability stand point of view [16]. Then, differentiating (3) and (4) with respect to $\mathbf{P}_{\mathcal{D}_{-r}}$, we have

$$\begin{pmatrix}\frac{\partial\mathbf{f}_{p_{-r}}}{\partial\mathbf{X}}\\ \frac{\partial\mathbf{f}_{q_{-r}}}{\partial\mathbf{X}}\end{pmatrix}\frac{\partial\mathbf{X}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}=\mathbf{J}_r^T\frac{\partial\mathbf{X}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}=\begin{pmatrix}\mathbf{I}\\ \mathbf{0}\end{pmatrix}. \qquad (12)$$

Define the *transmission sensitivity matrix*

$$\boldsymbol{\Psi}_r=\frac{\partial\mathbf{S}}{\partial\mathbf{X}}\frac{\partial\mathbf{X}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}=\frac{\partial\mathbf{S}}{\partial\mathbf{X}}\left(\mathbf{J}_r^T\right)^{-1}\begin{pmatrix}\mathbf{I}\\ \mathbf{0}\end{pmatrix} \qquad (13)$$

where the transmission sensitivity matrix is used to measure the marginal variations of power flows on transmission lines with respect to small variation of real power load at the specified bus, assuming the generation at the energy reference bus r is responsible for the real power compensation. Hence, the transmission sensitivity matrix depends upon the energy reference bus.

C. Loss Sensitivity Vector

Define the loss sensitivity factor for bus k (assuming bus r as the reference bus) as

$$\varsigma_{r,k}=\frac{\partial P_{loss_{r,k}}}{\partial P_{\mathcal{D}_k}}=\frac{\partial P_{\mathcal{G}_r}-\partial P_{\mathcal{D}_k}}{\partial P_{\mathcal{D}_k}}=\frac{\partial P_{\mathcal{G}_r}}{\partial P_{\mathcal{D}_k}}-1. \qquad (14)$$

The loss sensitivity factor specifies how the real power losses change with respect to the load variation at bus k, again assuming the bus r generation is responsible for the load compensation.

Differentiating (1) with respect to $\mathbf{P}_{\mathcal{D}_{-r}}$, we have

$$\begin{aligned}
\left(\frac{\partial P_{\mathcal{G}_r}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}\right)^T&=-\left(\frac{\partial\mathbf{X}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}\right)^T\left(\frac{\partial\mathrm{f}_{p_r}}{\partial\mathbf{X}}\right)^T\\
&=-(\mathbf{I}\quad\mathbf{0})\mathbf{J}_r^{-1}\left(\frac{\partial\mathrm{f}_{p_r}}{\partial\mathbf{X}}\right)^T. \qquad (15)
\end{aligned}$$

Then define the loss sensitivity vector as

$$\begin{aligned}
\boldsymbol{\varsigma}_{r,-r}&=\left(\frac{\partial P_{loss}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}\right)^T=\left(\frac{\partial P_{\mathcal{G}_r}}{\partial\mathbf{P}_{\mathcal{D}_{-r}}}\right)^T-\mathbf{1}\\
&=-\left[(\mathbf{I}\quad\mathbf{0})\mathbf{J}_r^{-1}\left(\frac{\partial\mathrm{f}_{p_r}}{\partial\mathbf{X}}\right)^T+\mathbf{1}\right] \qquad (16)
\end{aligned}$$

where the values in the loss sensitivity vector again depend upon the energy reference bus selection. Of course, $\varsigma_{r,r}\equiv 0$.

D. LMP Decomposition Into Energy, Loss, and Congestion Components

Next, we present the standard LMP decomposition into its three components, making the reasonable assumption that the reactive power output at the reference bus does not hit its limits. With this assumption then from (9), it is clear that $\lambda_{q_r} = 0$. Then (10) can simplified as

$$\left[\left(\frac{\partial \mathbf{f}_{p_{-r}}}{\partial \mathbf{X}}\right)^T \quad \left(\frac{\partial \mathbf{f}_{q_{-r}}}{\partial \mathbf{X}}\right)^T\right] \begin{pmatrix} \boldsymbol{\lambda}_{p_{-r}} \\ \boldsymbol{\lambda}_{q_{-r}} \end{pmatrix} = -\left(\frac{\partial \mathrm{f}_{p_r}}{\partial \mathbf{X}}\right)^T \lambda_{p_r} + \left(\frac{\partial \mathbf{S}}{\partial \mathbf{X}}\right)^T \boldsymbol{\mu} + \left(\frac{\partial \mathbf{G}}{\partial \mathbf{X}}\right)^T \boldsymbol{\gamma}. \quad (17)$$

Thus, we have

$$\begin{pmatrix} \boldsymbol{\lambda}_{p_{-r}} \\ \boldsymbol{\lambda}_{q_{-r}} \end{pmatrix} = -\mathbf{J}_r^{-1} \left(\frac{\partial \mathrm{f}_{p_r}}{\partial \mathbf{X}}\right)^T \lambda_{p_r} + \mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{S}}{\partial \mathbf{X}}\right)^T \boldsymbol{\mu} + \mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{G}}{\partial \mathbf{X}}\right)^T \boldsymbol{\gamma}. \quad (18)$$

This allows (17) to be rewritten as

$$\begin{aligned} \boldsymbol{\lambda}_{p_{-r}} &= (\mathbf{I} \quad \mathbf{0}) \begin{pmatrix} \boldsymbol{\lambda}_{p_{-r}} \\ \boldsymbol{\lambda}_{q_{-r}} \end{pmatrix} \\ &= \mathbf{1}\lambda_{p_r} - \left[\mathbf{1} + (\mathbf{I} \quad \mathbf{0})\mathbf{J}_r^{-1} \left(\frac{\partial \mathrm{f}_{p_r}}{\partial \mathbf{X}}\right)^T\right] \lambda_{p_r} \\ &\quad + (\mathbf{I} \quad \mathbf{0}) \left[\mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{S}}{\partial \mathbf{X}}\right)^T \boldsymbol{\mu} + \mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{G}}{\partial \mathbf{X}}\right)^T \boldsymbol{\gamma}\right] \quad (19) \end{aligned}$$

which gives the desired breakdown of $\boldsymbol{\lambda}_{p_{-r}}$

$$\boldsymbol{\lambda}_p = \mathbf{1}\lambda^e + \boldsymbol{\lambda}_p^l + \boldsymbol{\lambda}_p^c \quad (20)$$

with the energy component given by (21), the loss component by (22), and the congestion component by (23)

$$\lambda^e = \lambda_{p_r}; \quad (21)$$

$$\begin{aligned} \boldsymbol{\lambda}_{p_{-r}}^l &= -\left[\mathbf{1} + (\mathbf{I} \quad \mathbf{0})\mathbf{J}_r^{-1} \left(\frac{\partial \mathrm{f}_{p_r}}{\partial \mathbf{X}}\right)^T\right] \\ \lambda_{p_r} &= \boldsymbol{\varsigma}_{r,-r}\lambda_{p_r}; \quad (22) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\lambda}_{p_{-r}}^c &= (\mathbf{I} \quad \mathbf{0}) \left[\mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{S}}{\partial \mathbf{X}}\right)^T \boldsymbol{\mu} + \mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{G}}{\partial \mathbf{X}}\right)^T \boldsymbol{\gamma}\right] \\ &= \boldsymbol{\Psi}_r^T \boldsymbol{\mu} + (\mathbf{I} \quad \mathbf{0})\mathbf{J}_r^{-1} \left(\frac{\partial \mathbf{G}}{\partial \mathbf{X}}\right)^T \boldsymbol{\gamma}. \quad (23) \end{aligned}$$

As discussed in Sections II-B and II-C above, both the loss sensitivity vector and the transmission sensitivity matrix depend on the selection of reference bus. Hence, the conventional decomposition of the LMP depends upon the reference bus. Another undesired feature of conventional LMP decomposition is that not only do the component values themselves depend upon the reference bus, but even the differences in these component values between any two buses depend upon the reference bus selection. The differences of LMP congestion/loss components of two buses represent the marginal congestion/loss costs of an additional transaction between these two buses. For example, in looking at the difference between the loss components at buses 3 and 2 in Table I, the value is $2.12 - 0.89 = 1.23$, while in Table II, it is 1.28. Such differences can certainly affect the perception of fairness in pricing losses and congestion and can possibly send out inaccurate price signals. In the next section, we present an alternative decomposition method.

III. New Decomposition Approach

A. Simplification of OPF Problem Formulation

To simplify the derivation, if one assumes a known OPF solution, then the following notational simplifications can be made. First, from the known OPF solution, we divide the generator buses into those that have marginal generators (i.e., not operating at a limit) and those that are non-marginal (i.e., operating at either an upper or lower limit). In the derivation that follows, the non-marginal generators will be thought of as having fixed real power outputs. Also, without loss of generality, one can assume there is only one marginal generator at each marginal generator bus. Conceptually, this could be done by replacing all the generator cost curves at a bus with an aggregate, single generator cost curve that assumes the bus's net generation is optimally allocated to the generators at the bus.

Second, the buses can be divided into those that have generators that are actively regulating their reactive power output and those that either have no generation or have generators operating at their reactive power limits. The reactive power equations can be deleted for the regulating buses since their corresponding Lagrange multipliers are zero.

Third, all the voltage variables operating at a limit can be removed from the state variable vector $\mathbf{X}$ by fixing them to the binding limits values. Last, one can remove all non-binding constraints from (6). The net result of these four simplifications is a new optimization problem that shares the same optimal solution with the original one. This new optimization can be written as

$$\begin{aligned} &\min_{\mathbf{u}} \quad \mathrm{C}(\mathbf{P}_{\mathcal{G}}) \\ &\text{s.t.} \; -\mathbf{P}_{\mathcal{G}} + \mathbf{P}_{\mathcal{D}_{\mathcal{G}}} - \mathbf{f}_{p_{\mathcal{G}}}(\mathbf{X}) = \mathbf{0} \quad \leftrightarrow \boldsymbol{\lambda}_{P_{\mathcal{G}}} \quad (24) \\ &\quad \mathbf{P}_{\mathcal{D}_{-\mathcal{G}}} - \mathbf{f}_{p_{-\mathcal{G}}}(\mathbf{X}) = \mathbf{0} \quad \leftrightarrow \boldsymbol{\lambda}_{p_{-\mathcal{G}}} \quad (25) \\ &\quad \mathbf{Q}_{\mathcal{D}} - \mathbf{f}_q(\mathbf{X}) = \mathbf{0} \quad \leftrightarrow \boldsymbol{\lambda}_q \quad (26) \\ &\quad \mathbf{S}(\mathbf{X}) - \overline{\mathbf{S}} = \mathbf{0} \quad \leftrightarrow \boldsymbol{\mu}. \quad (27) \end{aligned}$$

With the previous simplifications, $\mathbf{P}_{\mathcal{G}}$ now only contains the real power marginal generators, while the cost function $\mathrm{C}(\mathbf{P}_{\mathcal{G}})$ only includes the costs of the real power marginal generators. The corresponding values of $\mathbf{P}_{\mathcal{D}_{\mathcal{G}}}$, $\mathbf{P}_{\mathcal{D}_{-\mathcal{G}}}$, and $\mathbf{Q}_{\mathcal{D}}$ are also adjusted based upon the previous simplifications, while the functions $\mathbf{f}_q(\mathbf{X})$ and $\mathbf{S}(\mathbf{X})$ are also simplified.

B. Marginal Effect of Load Variation of OPF Problem

The Lagrangian for this new OPF problem is

$$\begin{aligned}\mathcal{L}(\mathbf{P}_{\mathcal{G}}, \mathbf{X}, \boldsymbol{\lambda}_{p_{\mathcal{G}}}, \boldsymbol{\lambda}_{p_{-\mathcal{G}}}, \boldsymbol{\lambda}_q, \boldsymbol{\mu}) &= \mathrm{C}(\mathbf{P}_{\mathcal{G}}) \\ &+ \left[-\mathbf{P}_{\mathcal{G}} + \mathbf{P}_{\mathcal{D}_{\mathcal{G}}} - \mathbf{f}_{p_{\mathcal{G}}}(\mathbf{X})\right]^T \boldsymbol{\lambda}_{p_{\mathcal{G}}} \\ &+ \left[\mathbf{P}_{\mathcal{D}_{-\mathcal{G}}} - \mathbf{f}_{p_{-\mathcal{G}}}(\mathbf{X})\right]^T \boldsymbol{\lambda}_{p_{-\mathcal{G}}} \\ &+ \left[\mathbf{Q}_{\mathcal{D}} - \mathbf{f}_q(\mathbf{X})\right]^T \boldsymbol{\lambda}_q + \left[\mathbf{S}(\mathbf{X}) - \overline{\mathbf{S}}\right]^T \boldsymbol{\mu}. \end{aligned} \tag{28}$$

The KT necessary condition for optimality is now

$$\left(\frac{\partial \mathcal{L}}{\partial \mathbf{P}_{\mathcal{G}}}\right)^T = \left(\frac{\partial \mathrm{C}}{\partial \mathbf{P}_{\mathcal{G}}}\right)^T - \boldsymbol{\lambda}_{P_{\mathcal{G}}} = \mathbf{0} \tag{29}$$

$$\begin{aligned}\left(\frac{\partial \mathcal{L}}{\partial \mathbf{X}}\right)^T &= \left(\frac{\partial \mathbf{F}_{p_{\mathcal{G}}}}{\partial \mathbf{X}}\right)^T \boldsymbol{\lambda}_{p_{\mathcal{G}}} + \left(\frac{\partial \mathbf{f}_{p_{-\mathcal{G}}}}{\partial \mathbf{X}}\right)^T \boldsymbol{\lambda}_{p_{-\mathcal{G}}} \\ &+ \left(\frac{\partial \mathbf{f}_q}{\partial \mathbf{X}}\right)^T \boldsymbol{\lambda}_q + \left(\frac{\partial \mathbf{S}}{\partial \mathbf{X}}\right)^T \boldsymbol{\mu} = \mathbf{0}. \end{aligned} \tag{30}$$

Note that according to the implicit function theorem, $\mathbf{P}_{\mathcal{G}}$, $\mathbf{X}$, $\boldsymbol{\lambda}_{p_{\mathcal{G}}}$, $\boldsymbol{\lambda}_{p_{-\mathcal{G}}}$, $\boldsymbol{\lambda}_q$, and $\boldsymbol{\mu}$ are all now functions of $\mathbf{P}_{\mathcal{D}}$.

Then, take the derivative of (24)–(27), (29), and (30) with respect to $P_{\mathcal{D}_k}$, writing the result in matrix form

$$\begin{pmatrix} \boldsymbol{\Lambda} & -\mathbf{I} & & & & \\ -\mathbf{I} & & & & & \mathbf{E} \\ & & & & & \mathbf{F}_1 \\ & & & & & \mathbf{F}_2 \\ & & & & & \mathbf{F}_3 \\ & \mathbf{E}^T & \mathbf{F}_1^T & \mathbf{F}_2^T & \mathbf{F}_3^T & \mathbf{D} \end{pmatrix} \begin{pmatrix} \partial \mathbf{P}_{\mathcal{G}}/\partial P_{\mathcal{D}_k} \\ \partial \boldsymbol{\lambda}_{p_{\mathcal{G}}}/\partial P_{\mathcal{D}_k} \\ \partial \boldsymbol{\lambda}_{p_{-\mathcal{G}}}/\partial P_{\mathcal{D}_k} \\ \partial \boldsymbol{\lambda}_q/\partial P_{\mathcal{D}_K} \\ \partial \boldsymbol{\mu}/\partial P_{\mathcal{D}_k} \\ \partial \mathbf{X}/\partial P_{\mathcal{D}_k} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ -\partial \mathbf{P}_{\mathcal{D}_{\mathcal{G}}}/\partial P_{\mathcal{D}_k} \\ -\partial \mathbf{P}_{\mathcal{D}_{-\mathcal{G}}}/\partial P_{\mathcal{D}_k} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{31}$$

where

$$\boldsymbol{\Lambda} = \frac{\partial^2 \mathrm{C}}{\partial \mathbf{P}_{\mathcal{G}}^2}; \quad \mathbf{E} = \frac{\partial \mathbf{f}_{p_{\mathcal{G}}}}{\partial \mathbf{X}};$$
$$\mathbf{F}_1 = \frac{\partial \mathbf{f}_{p_{-\mathcal{G}}}}{\partial \mathbf{X}}; \quad \mathbf{F}_2 = \frac{\partial \mathbf{f}_q}{\partial \mathbf{X}}; \quad \mathbf{F}_3 = \frac{\partial \mathbf{S}}{\partial \mathbf{X}}$$
$$\mathbf{D} = \sum_i \frac{\partial^2 \mathrm{f}_{p_i}}{\partial \mathbf{X}^2} \lambda_{p_i} + \sum_i \frac{\partial^2 \mathrm{f}_{q_i}}{\partial \mathbf{X}^2} \lambda_{q_i} + \sum_l \frac{\partial^2 S_l}{\partial \mathbf{X}^2} \mu_l. \tag{32}$$

Last, (31) can be solved using the result given in the Appendix to give the marginal variation of the generator outputs with respect to the bus real power load variation.

If bus k is a marginal generator bus, we have

$$\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial P_{\mathcal{D}_k}} = (\mathbf{I} - \mathbf{ETE}^T \boldsymbol{\Lambda}) \frac{\partial \mathbf{P}_{\mathcal{D}_{\mathcal{G}}}}{\partial P_{\mathcal{D}_{\mathcal{G}_k}}} = (\mathbf{I} - \mathbf{ETE}^T \boldsymbol{\Lambda}) \mathrm{e}_k \tag{33}$$

while if bus k is a non-marginal generator bus, we have

$$\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial P_{\mathcal{D}_k}} = \mathbf{EH}^{-1} \mathbf{F}^T \mathbf{U}^{-1} \frac{\partial \mathbf{P}_{\mathcal{D}_{-\mathcal{G}}}}{\partial P_{\mathcal{D}_{-\mathcal{G}_k}}} = \mathbf{EH}^{-1} \mathbf{F}^T \mathbf{U}^{-1} \begin{pmatrix} \mathbf{E}_k \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{34}$$

where matrices $\mathbf{T}$, $\mathbf{H}$, and $\mathbf{U}$ are as defined in the Appendix. Combining these results for every bus k and rewriting in the matrix form gives

$$\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial \mathbf{P}_{\mathcal{D}_{\mathcal{G}}}} = \mathbf{I} - \mathbf{ETE}^T \boldsymbol{\Lambda} \tag{35}$$

$$\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial \mathbf{P}_{\mathcal{D}_{-\mathcal{G}}}} = \mathbf{EH}^{-1} \mathbf{F}^T \mathbf{U}^{-1} (\mathbf{I} \quad \mathbf{0} \quad \mathbf{0})^T. \tag{36}$$

The net result is the $\partial \mathbf{P}_{\mathcal{G}}/\partial \mathbf{P}_{\mathcal{D}}$, a matrix that provides detailed information about how the generators would actually participate in optimally supplying additional power to each bus. This information then provides a reasonable basis for decomposing the LMP into its components. Such a decomposition is developed next.

C. LMP Decomposition With Piecewise Linear Cost Functions Model

Although theoretically one could adopt any valid generator cost functions for the OPF problem, linear cost functions are widely used to represent generators' economic features in industry. Therefore, the following derivation assumes the use of a piecewise linear generator cost function. With this assumption, we have

$$\boldsymbol{\Lambda} = \frac{\partial^2 \mathrm{C}}{\partial \mathbf{P}_{\mathcal{G}}^2} = \mathbf{0} \tag{37}$$

which simplifies (35) to

$$\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial \mathbf{P}_{\mathcal{D}_{\mathcal{G}}}} = \mathbf{I} \tag{38}$$

a result that states that additional real power variation at the marginal generator buses will be fully compensated by the generator(s) at the bus. Therefore, incremental load changes at these buses would have no impact of the system power flows and hence no impact on system losses. Thus, it is reasonable to claim that the LMP loss component at the marginal generator buses is zero, which implies that any LMP differences between these buses are totally caused by system congestion.

If non-piecewise linear cost functions are used, it is obvious that load variation at the marginal bus will be compensated by all the marginal generators instead of only by the local one. Thus, this will make marginal loss not equal to zero. However, nowadays in industry, piecewise linear cost functions are widely used instead of the quadratic ones (or other forms). It is believed that piecewise linear cost functions have already given us a good enough result for the industry standard.

If λ^e is chosen as the energy price throughout the system, then the LMP congestion component vector for the marginal generator buses is given by

$$\boldsymbol{\lambda}_{p_{\mathcal{G}}}^c = \boldsymbol{\lambda}_{p_{\mathcal{G}}} - \mathbf{1} \lambda^e. \tag{39}$$

This yields the desirable result that while different values for λ^e result in different LMP congestion components, the differences between these congestion components remains the same. The effect of the selection of reference energy price λ^e will be discussed in detail in Section IV.

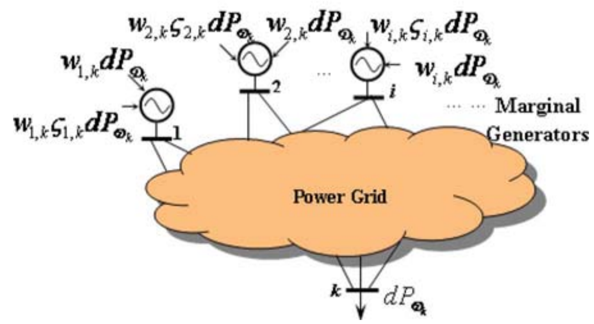

Fig. 2. Breakdown marginal generator variation into two parts. (Color version available online at http://ieeexplore.ieee.org.)

Next, we present the LMP decomposition for the other buses in the system, with the key assumption being that an incremental load change at bus k would be optimally supplied by the marginal generators. Hence, the variation in the marginal generation can be expressed as

$$d\mathbf{P}_{\mathcal{G}} = \frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial P_{\mathcal{D}_k}} dP_{\mathcal{D}_k}. \tag{40}$$

To proceed with the derivation, first divide $dP_{\mathcal{G}_i}$ into two parts: 1) $dP_{\mathcal{G}_i}/1+\varsigma_{i,k}$, which supplies the real power load variation at bus k, $dP_{\mathcal{D}_k}$, and 2) $(\varsigma_{i,k}/1+\varsigma_{i,k})dP_{\mathcal{G}_i}$, which supplies the real power loss variation. This division is illustrated in Fig. 2.

Next, define the *contribution weight* of generator i with respect to the load variation at bus k as

$$W_{i,k} = \frac{1}{1+\varsigma_{i,k}} \frac{\partial P_{\mathcal{G}_i}}{\partial P_{\mathcal{D}_k}} \tag{41}$$

where

$$\sum_i w_{i,k} = 1. \tag{42}$$

Of course, when bus k is a marginal generator bus, we have $w_{i,k} = 0\ i \neq k$, $w_{k,k} = 1$, and $\varsigma_{i,k} = 0$.

According to the relationship between the primal and dual variables, and also following the definition of the LMP, for every bus k (marginal or non-marginal generator bus), we have

$$\lambda_{p_k} = \boldsymbol{\lambda}_{p_{\mathcal{G}}}^T \frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial P_{\mathcal{D}_k}} \tag{43}$$

which can be expressed in matrix form as

$$\boldsymbol{\lambda}_p = \left(\frac{\partial \mathbf{P}_{\mathcal{G}}}{\partial \mathbf{P}_{\mathcal{D}}}\right)^T \boldsymbol{\lambda}_{p_{\mathcal{G}}}. \tag{44}$$

Substituting (41) into (43) gives

$$\begin{aligned}\lambda_{p_k} &= \sum_i \lambda_{p_{\mathcal{G}_i}} \frac{\partial P_{\mathcal{G}_i}}{\partial P_{\mathcal{D}_k}} = \sum_i \lambda_{p_{\mathcal{G}_i}} w_{i,k}(1+\varsigma_{i,k}) \\ &= \sum_i \lambda_{p_{\mathcal{G}_i}} w_{i,k} + \sum_i \lambda_{p_{\mathcal{G}_i}} w_{i,k}\varsigma_{i,k} \\ &= \sum_i \left(\lambda^e + \lambda^c_{p_{\mathcal{G}_i}}\right) w_{i,k} + \sum_i \lambda^1_{p_i} w_{i,k}\varsigma_{i,k} \\ &= \lambda^e + \sum_i \lambda_{p_{\mathcal{G}_i}} w_{i,k}\varsigma_{i,k} + \sum_i \lambda^C_{p_{\mathcal{G}_i}} w_{i,k}\end{aligned} \tag{45}$$

which provides the desired breakdown of the LMP by component

$$\lambda_{p_k} = \lambda^e + \lambda^l_{p_k} + \lambda^c_{p_k} \tag{46}$$

$$\lambda^l_{p_k} = \sum_i \lambda_{p_{\mathcal{G}_i}} w_{i,k}\varsigma_{i,k} \tag{47}$$

$$\lambda^c_{p_k} = \sum_i \lambda^c_{p_{\mathcal{G}_i}} w_{i,k}. \tag{48}$$

From the derivation above, it is clear that the proposed new decomposition method considers the marginal effect of load variation, and the marginal cost is broken down according to a reasonable explanation of each part's purpose.

IV. Effect of the Selection of Reference Energy Price

Using the derivation from the previous section and making use of (42), the difference between the congestion components for any two buses m and n is given by

$$\begin{aligned}&\lambda^c_{p_m} - \lambda^c_{p_n} \\ &\quad= \sum_i \left(\lambda_{p_{\mathcal{G}_i}} - \lambda^e\right) w_{i,m} - \sum_i \left(\lambda_{p_{\mathcal{G}_i}} - \lambda^e\right) w_{i,n} \\ &\quad= \sum_i \lambda_{p_{\mathcal{G}_i}} (w_{i,m} - w_{i,n}) - \sum_i \lambda^e w_{i,m} + \sum_i \lambda^e w_{i,m} \\ &\quad= \sum_i \lambda_{p_{\mathcal{G}_i}} (w_{i,m} - w_{i,n})\end{aligned} \tag{49}$$

a result that is clearly independent of λ^e. Furthermore, since $\lambda_{p_{\mathcal{G}_i}}$, $w_{i,k}$, and $\varsigma_{i,k}$ are all independent of reference energy price, the value of the loss component is also independent of the energy reference bus.

From the above discussion, we find several important features of the proposed decomposition scheme.

1) LMP at any bus is determined by the original OPF problem and is independent of the decomposition scheme and the reference energy price selection.
2) The values for the loss components are independent of the reference bus location, which obviously implies that the differences in these values between buses are also independent of the reference bus.
3) The reference bus location does change the value of the congestion component, but the differences in the congestion components between any two buses are not affected by the reference bus location.

The physical idea behind the decomposition process reveals the marginal impacts on loss and congestion. Also, the above properties satisfy the fairness, accuracy, and consistency requirements for an LMP decomposition scheme.

V. Numerical Example

A. Tutorial Three-Bus System Example

As an example, we again consider the three-bus system introduced in the introduction section, as shown in Fig. 1. After the OPF problem is solved, one can easily deduce that the generators at buses 1 and 2 are both marginal. From the previous discussion, this implies the LMP loss components for both buses are zero, i.e., $\lambda^l_{p_1} = 0\$/\text{MWh}$ and $\lambda^l_{p_2} = 0\ \$/\text{MWh}$. Because

TABLE III
THREE-BUS SYSTEM LMP COMPONENTS BY PROPOSED DECOMPOSITION SCHEME

Bus	LMP ($/MWh)	LMP Components ($/MWh)		
		Energy	*Loss*	*Congestion*
Bus 1	9.50	9.50	0.00	0.00
Bus 2	10.80	9.50	0.00	1.30
Bus 3	11.87	9.50	1.66	0.71

the selection of the energy reference bus does not affect the difference between buses of the loss components or the congestion components, we can arbitrarily select the energy reference bus, set to bus 1 in this example. Thus, $\lambda^e = \lambda_{p_1} = 9.50$ \$/MWh, which gives $\lambda^c_{p_1} = 0$ \$/MWh and $\lambda^c_{p_2} = 1.30$ \$/MWh. Next, to break λ_{p_3} into components, do the following.

1) Calculate the loss sensitivity factors at the optimal solution point, $\varsigma_{1,3} = 0.2232$, $\varsigma_{2,3} = 0.1188$.
2) Solve for marginal variations using (34) or (36), $(\partial P_{\mathcal{G}_1}/\partial P_{\mathcal{D}_3}\ \partial P_{\mathcal{G}_2}/\partial P_{\mathcal{D}_3}) = (0.5561\ 0.6102)$.
3) Determine the contribution factors using (41), $w_{1,3} = 0.4546$ and $w_{2,3} = 0.5454$.
4) Calculate the loss and congestion components of λ_{p_3} using (47) and (48)

$$\lambda^l_{p_3} = \lambda_{p_{\mathcal{G}_1}} w_{1,3}\varsigma_{1,3} + \lambda_{p_{\mathcal{G}_2}} w_{2,3}\varsigma_{2,3} = 1.66$$
$$\lambda^c_{p_3} = \lambda^c_{p_{\mathcal{G}_1}} w_{1,3} + \lambda^c_{p_{\mathcal{G}_2}} w_{2,3} = 0.71.$$

The final results are presented in Table III.

To further illustrate, suppose there is an incremental increase in the real power load at bus 3, say, 1 MW. The marginal variations indicate that to optimally supply this additional 1 MW, 0.5561 MW would come from the bus 1 generator, 0.6102 MW would come from bus 2, and the system losses would increase by 0.163 MW. Hence the bus 3 LMP is easily verified by multiplying these changes in generation by the corresponding LMPs for the generators' buses

$$0.5561 \times 9.50 + 0.6102 \times 10.80 = 11.87\ \$/\text{MWh}. \quad (50)$$

With the assumption that the change in bus 3's load is allocated optimally to the marginal generators, the change in system losses and hence the loss sensitivity at bus 3 (or any other bus) is independent of the reference bus.

B. *Thirty-Seven-Bus System Example*

The method is next demonstrated on the 37-bus design1 system from [10], with the modification that two of the three transmission lines between UIUC69 and BLT69 have been opened to create congestion. The system's OPF solution is shown in Fig. 3. At this solution, there is a single binding transmission constraint—the flow on the line from bus UIUC69 to bus BLT69 is constrained to its 100 MVA limit. At the optimal solution, the two marginal generators are located at LAUF69 and BLT69. The bus LMPs, their decomposition into the loss and congestion components (assuming LAUF69 as the reference energy bus), the associated variation in the two marginal generators, and their loss are shown in Table IV. Again, using the table data, the LMP values can be easily verified. For example, if the load is increased by 1 MW at TIM345, 0.757 MW would come from the generator at LAUF69, while 0.239 MW would come from the generator at BLT69. Using the same approach as (50), the result is

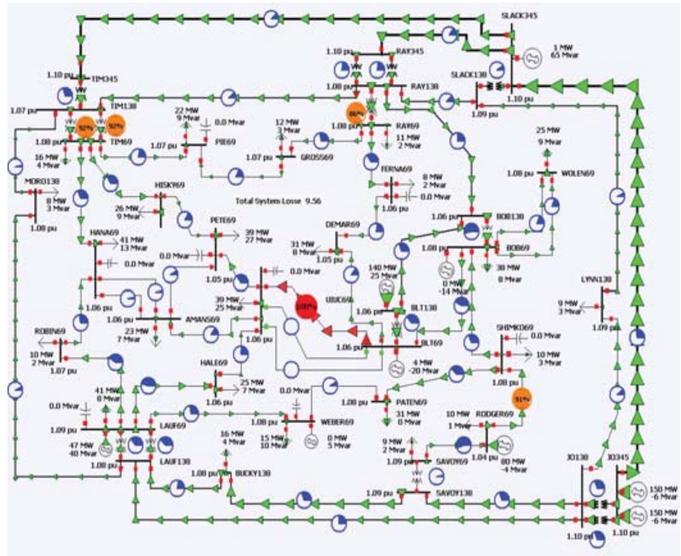

Fig. 3. Thirty-seven-bus example. (Color version available online at http://ieeexplore.ieee.org.)

$$0.757 \times 28.90 + 0.239 \times 22.53 = 27.26\ \$/\text{MWh}. \quad (51)$$

In Table V, we compare the MLC and MCC obtained from the proposed model and the distributed-slack model (two marginal generators are equally weighted). It is obvious that the selection of distributed-slack buses and their weights has big impact on LMP decomposition, while in the proposed new model in this paper, the slack buses are not predefined and the slack used in decomposition is actually the physical slack buses at the optimal solution point with weights optimally calculated according to optimal dispatch at the solution point.

The new model gives a more reasonable decomposition, which is also consistently independent of slack selection.

VI. CONCLUSION

This paper proposes a new LMP decomposition model using the ac OPF model. The LMP components from the conventional decomposition model usually depend on the selection of reference bus. By analyzing the actual marginal effect of load variation, the proposed model overcomes the disadvantage of conventional model and achieves reference energy price independent results. The loss components reflect the actual cost of marginal consumption of loss. The congestion components differences do not change with the selection of energy reference price. The proposed model is proved to be accurate, consistent, and fair. In this paper, we only discuss the ac OPF model using piecewise linear cost functions. Although piecewise linear cost functions are widely accepted by industry, theoretically, more general results using other valid cost functions [the marginal effects of load variations are obviously more complex according

TABLE IV
THIRTY-SEVEN-BUS SYSTEM LMP COMPONENTS ($/MWh) BY PROPOSED DECOMPOSITION SCHEME WITH $\lambda^e = 28.90$ \$/MWh

Bus Name	LMP	LMP Components		Marg. Variation	
		MLC	MCC	LAUF69	BLT69
TIM345	27.26	-0.109	-1.536	0.757	0.239
MORO138	28.30	0.047	-0.650	0.900	0.102
ROBIN69	30.29	0.487	0.904	1.160	-0.143
RAY69	26.31	-0.116	-2.477	0.610	0.386
TIM69	29.13	0.190	0.041	1.013	-0.006
FERNA69	25.48	0.269	-3.693	0.426	0.584
WEBER69	27.71	0.287	-1.477	0.777	0.233
UIUC69	33.05	0.748	3.399	1.565	-0.541
PETE69	32.16	0.783	2.481	1.421	-0.396
PIE69	28.50	0.367	-0.764	0.892	0.121
HANA69	31.44	0.795	1.740	1.304	-0.278
GROSS69	27.87	0.249	-1.278	0.807	0.201
SHIMKO69	25.26	-0.041	-3.601	0.435	0.563
WOLEN69	23.99	0.106	-5.015	0.215	0.789
HALE69	32.17	0.684	2.588	1.434	-0.411
HISKY69	31.73	0.716	2.116	1.361	-0.337
JO345	26.89	-0.419	-1.594	0.739	0.245
JO138	27.00	-0.451	-1.449	0.760	0.223
BUCKY138	27.54	-0.241	-1.121	0.817	0.174
SLACK345	26.95	-0.184	-1.771	0.718	0.275
SAVOY138	27.32	-0.312	-1.273	0.792	0.197
SAVOY69	26.52	-0.533	-1.847	0.697	0.283
PATEN69	26.61	0.264	-2.557	0.606	0.404
SLACK138	26.77	-0.220	-1.914	0.695	0.297
AMANS69	31.59	0.791	1.897	1.329	-0.303
RAY345	26.80	-0.147	-1.951	0.691	0.304
RAY138	26.41	-0.112	-2.374	0.626	0.370
TIM138	28.32	0.084	-0.665	0.899	0.104
LAUF138	28.22	-0.058	-0.623	0.901	0.097
LAUF69	28.90	0.000	0.000	1.000	0.000
BOB138	24.31	-0.002	-4.590	0.281	0.719
BOB69	23.90	0.010	-5.012	0.214	0.786
RODGER69	25.09	-0.991	-2.815	0.537	0.424
BLT138	24.06	-0.031	-4.813	0.245	0.753
BLT69	22.53	0.000	-6.370	0.000	1.000
DEMAR69	24.33	0.395	-4.963	0.226	0.791
LYNN138	26.92	-0.340	-1.640	0.734	0.253

TABLE V
LMP COMPONENTS OBTAINED USING DISTRIBUTED-SLACK MODEL AND NEW PROPOSED MODEL

Bus Name	LMP	New Model (Physical Slacks)		Distributed Slack (LAUF69:50% BLT69:50%)	
		MLC	MCC	MLC	MCC
TIM345	27.26	-0.109	-1.536	1.690	-0.145
MORO138	28.30	0.047	-0.650	2.620	-0.015
ROBIN69	30.29	0.487	0.904	4.310	0.350
RAY69	26.31	-0.116	-2.477	0.710	-0.125
TIM69	29.13	0.190	0.041	3.355	0.125
FERNA69	25.48	0.269	-3.693	-0.550	0.340
WEBER69	27.71	0.287	-1.477	1.730	0.260
UIUC69	33.05	0.748	3.399	6.925	0.515
PETE69	32.16	0.783	2.481	5.965	0.585
PIE69	28.50	0.367	-0.764	2.525	0.325
HANA69	31.44	0.795	1.740	5.225	0.605
GROSS69	27.87	0.249	-1.278	1.975	0.235
SHIMKO69	25.26	-0.041	-3.601	-0.500	-0.005
WOLEN69	23.99	0.106	-5.015	-1.950	0.215
HALE69	32.17	0.684	2.588	6.090	0.475
HISKY69	31.73	0.716	2.116	5.575	0.540
JO345	26.89	-0.419	-1.594	1.605	-0.480
JO138	27.00	-0.451	-1.449	1.750	-0.515
BUCKY138	27.54	-0.241	-1.121	2.090	-0.300
SLACK345	26.95	-0.184	-1.771	1.440	-0.215
SAVOY138	27.32	-0.312	-1.273	1.925	-0.370
SAVOY69	26.52	-0.533	-1.847	1.305	-0.595
PATEN69	26.61	0.264	-2.557	0.565	0.290
SLACK138	26.77	-0.220	-1.914	1.285	-0.250
AMANS69	31.59	0.791	1.897	5.355	0.625
RAY345	26.80	-0.147	-1.951	1.255	-0.170
RAY138	26.41	-0.112	-2.374	0.815	-0.125
TIM138	28.32	0.084	-0.665	2.610	0.035
LAUF138	28.22	-0.058	-0.623	2.635	-0.140
LAUF69	28.90	0.000	0.000	3.295	-0.110
BOB138	24.31	-0.002	-4.590	-1.490	0.075
BOB69	23.90	0.010	-5.012	-1.940	0.100
RODGER69	25.09	-0.991	-2.815	0.285	-1.045
BLT138	24.06	-0.031	-4.813	-1.720	0.055
BLT69	22.53	0.000	-6.370	-3.350	0.165
DEMAR69	24.33	0.395	-4.963	-1.920	0.555
LYNN138	26.92	-0.340	-1.640	1.565	-0.390

to (56)] can be obtained easily following the scheme and physical meaning proposed in this paper. Also, more research is needed studying FTRs and loss hedging rights using an ac power flow model.

APPENDIX

This Appendix derives the solution (31) under the assumption that the left-hand matrix of (31) is nonsingular. First, to present this matrix more compactly, define

$$\mathbf{F}^T = \begin{pmatrix} \mathbf{F}_1^T & \mathbf{F}_2^T & \mathbf{F}_3^T \end{pmatrix} \tag{52}$$

$$\mathbf{H} = \mathbf{D} + \mathbf{E}\mathbf{\Lambda}\mathbf{E}^T \tag{53}$$

$$\mathbf{U} = \mathbf{F}^T\mathbf{H}^{-1}\mathbf{F} \tag{54}$$

$$\mathbf{T} = \mathbf{H}^{-1} - \mathbf{H}^{-1}\mathbf{F}\mathbf{U}^{-1}\mathbf{F}^T\mathbf{H}^{-1}. \tag{55}$$

Its inverse can then be derived using elementary row operations. In the interest of brevity, (56) only lists the portion of the inverse germane to the solution of (31)

$$\begin{pmatrix} \mathbf{\Lambda} & -\mathbf{I} & & \\ -\mathbf{I} & & & \mathbf{E} \\ & & & \mathbf{F} \\ & \mathbf{E}^T & \mathbf{F}^T & \mathbf{D} \end{pmatrix}^{-1} = \begin{pmatrix} \cdots & \mathbf{E}\mathbf{T}\mathbf{E}^T\mathbf{\Lambda} - \mathbf{I} & \mathbf{E}\mathbf{H}^{-1}\mathbf{F}^T\mathbf{U}^{-1} & \cdots \\ \cdots & \mathbf{\Lambda}\mathbf{E}\mathbf{T}\mathbf{E}^T\mathbf{\Lambda} - \mathbf{\Lambda} & \mathbf{\Lambda}\mathbf{E}\mathbf{H}^{-1}\mathbf{F}^T\mathbf{U}^{-1} & \cdots \\ \cdots & \mathbf{U}^{-1}\mathbf{F}\mathbf{H}^{-1}\mathbf{E}^T\mathbf{\Lambda} & -\mathbf{U}^{-1} & \cdots \\ \cdots & \mathbf{T}\mathbf{E}^T\mathbf{\Lambda} & \mathbf{H}^{-1}\mathbf{F}^T\mathbf{U}^{-1} & \cdots \end{pmatrix}. \tag{56}$$

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