Non-Collocated Voltage and Current Measurements Used to Obtain Power

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***Abstract*— Voltage and current measurements are routinely used as inputs to transducers in order to obtain sensory information on active and reactive power. If the voltage and current measurements are not collocated, the power measurements will be incorrect. In this paper, a correction is calculated for non-collocated voltage and current measurements used to obtain power. The correction is obtained utilizing the non-collocated power and one voltage magnitude measurement. One application area is in state estimation sensory inputs.**

***Index Terms*— Measurements, instrumentation, state estimation, measurement error, complex power measurement.**

I. Introduction

POWER system measurements may possess error due to inherent sensor response, sensor interfaces, and instrument placement. One such power measurement error is non-collocated complex power measurements. Complex power is usually calculated from voltage and current measurements which must be taken at the same place. The usual instrumentation configuration is that phase-phase voltages and line currents are measured at the same location (i.e., the measurements are collocated), and these analog measurements are passed to a transducer which generates a digital signal corresponding to the active and reactive power. If the voltage and current measurements are not collocated, there will be an error in $P + jQ$. Non-collocated measurements may result from changes in original construction or physical difficulties in placement of current and potential transformers. It will be shown that an accurate power measurement can be calculated utilizing a non-collocated value plus a correction procedure. This correction calculation requires an accurate knowledge of the impedance between the points of instrumentation. One application area is in the utilization of active and reactive power measurements for use in state estimators [1] and for general energy management systems.

II. Non-collocated voltage and current measurements

Complex power is a function of the complex voltage and current, where $S=VI^*$. Power measuring transducers are a common part of power systems instrumentation. These power measurement devices sample the voltage, $v(t)$, and the current, $i(t)$, which are rendered to a digital signal by an analog / digital converter. The average power (P) is calculated from these signals, and the reactive power (Q) is calculated through the use of a phase shifted version of one of the signals. These values are provided to a remote system operator. When measuring or calculating complex power, the voltage and current must be taken at the same place in the system. Figure 1 shows the assumed single line equivalent configuration for the non-collocated case with a current transformer (CT) and potential transformer (PT) as the sensory elements, under the assumption of balanced conditions. In the case of a non-collocated measurement it is assumed that the instruments are not far apart and well within the range of typical short line modeling limits. Resistance in the reactance model has been assumed to be negligible.

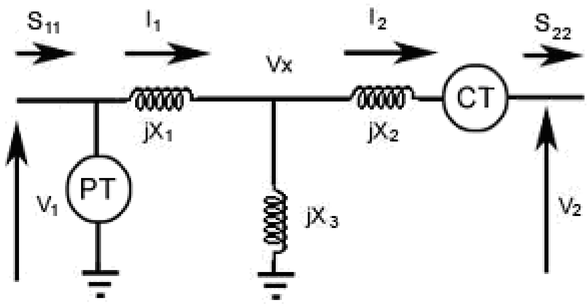

Fig. 1 Non-collocated power measurement sensory instrument placement. Note that $S_{11} = P_{11} + jQ_{11}$ and $S_{22} = P_{22} + jQ_{22}$.

The general concept of complex power may be generalized as follows: let V be a vector of complex voltage values at several buses (sinusoidal steady state, phasor notation), and let I be a vector of line currents. Let the dimensions of V and I be N_V and N_I respectively. Further let the N_V by N_I generalized complex power matrix S be defined as

$$S = VI^H,$$

where $(\cdot)^H$ denotes the hermitian operation (complex conjugate followed by transpose). Then elements of matrix S in positions like S_{aa} represent the familiar conventional complex power. However, elements like S_{ab}, $a \neq b$, represent non-collocated signals that are dimensionally like $P + jQ$, but do not represent conventional active and reactive power. The issue is the 'correction' of non-collocated terms like S_{ab} to obtain conventional active and reactive power like S_{aa}. Consider the case that $N_V = N_I = 2$, and the current vector I is written with polarity such that both currents are input to the two-port network. Then it is a simple matter to show that

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$$S = VI^H = ZII^H = VV^HY^H$$

where Z and Y are the bus impedance and admittance matrices of the two-port. The use of Z and Y imply that the bus current injection vector is $[I_1\ -I_2]^t$ using the notation of Fig. 1. Note that S is a complex, non-symmetric matrix which can easily be shown to be of deficient rank and hence $S_{11}S_{22} = S_{12}S_{21}$. There is a similar quantity s defined as $s = I^HV$ which is a scalar complex quantity that has the property $\mathrm{Re}\{s\} \geq 0$ for a passive two-port. In the field of circuit theory, a general n by n complex matrix Z is said to be a '*positive real matrix*' if for all complex n vectors I, the scalar valued function I^HZI has a non-negative real part [2]. A *positive real matrix* is characteristic of impedance and admittance matrices of passive circuits. Other properties of positive real matrices relate to causality, pole and zero location of the corresponding characteristic equation, and stability of the output / input transfer function. These and other properties and tests for 'positive realness' appear in classic electric circuits analysis texts, e.g., [4].

One method for correcting non-collocated measurements will be called the "Case A" method. For Case A, consider S as a two by two matrix and let the non-collocated measurements S_{12}, $|V_1|$, X_1, X_2, and X_3 be known or measured. S_{11} and S_{22} can be calculated in this case by obtaining a 'correction term'. This development also may be modified with a simple change of subscripts for the case that S_{21} and $|V_2|$ are known. Without loss of generality, the voltage V_1 can be made the reference voltage. Since $S_{12} = V_1 I_2^*$ and S_{12} and V_1 are known, I_2 can be calculated directly. To calculate S_{11} and S_{22}, V_2 and I_1 are needed. The following matrix equation is formed to solve for the remaining unknowns using the polarity notation in Fig. 1,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\begin{bmatrix} X_1 + X_3 & X_3 \\ X_3 & X_2 + X_3 \end{bmatrix}\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}. \qquad (1)$$

Equation (1) is a simple consequence of the bus impedance analysis of a linear AC circuit, namely $V_{bus} = Z_{bus} I_{bus}$, where Z_{bus} is the bus impedance matrix referred to ground [3]. Equation (1) can be multiplied out to give two equations that can be solved for the two unknowns, V_2 and I_1,

$$V_2 = \frac{jX_3V_1 + I_2(X_1X_2 + X_1X_3 + X_2X_3)}{j(X_1 + X_3)} \qquad I_1 = \frac{V_1 + jX_3I_2}{j(X_1 + X_3)}.$$

A second method denominated "Case B" applies when S_{21}, $|V_1|$ and the reactances X_1, X_2, and X_3 are known. To solve for S_{11} and S_{22}, the notation in Fig. 1 is used. Let

$$|V_1| = V_1\angle 0° = a.$$

Further, let

$$I_1 = b + jc \qquad V_2 = h + jk$$
$$I_2 = f + jg \qquad V_x = d + je.$$

Using basic power relationships, where $S=VI^*$, $P=Real\{S\}$, and $Q=Imag\{S\}$, the following relationships are evident,

$$P_{11} = ab \qquad Q_{22} = kf - hg$$
$$Q_{11} = -ac \qquad P_{21} = hb + kc$$
$$P_{22} = hf + kg \qquad Q_{21} = bk - hc$$

Parameters a, P_{21}, and Q_{21} are known values. To solve for the remaining unknowns, use the relationships between V_1 and V_X, V_X and V_2 and the Kirchhoff current law,

$$d + je = a - (b + jc)(jX_1)$$
$$h + jk = d + je - (f + jg)(jX_2)$$
$$b + jc = \frac{d + je}{jX_3} + f + jg$$

Using these equations and the power relationships, there are eight equations with eight unknown parameters. The eight equations are simplified and manipulated to obtain the real and imaginary parts of S_{11} and S_{22}

$$P_{11} = P_{21}\left(\frac{X_3}{X_3 + X_2}\right)$$
$$a = V_1 = 1\angle 0°$$
$$b = P_{11} = P_{21}\left(\frac{X_3}{X_3 + X_2}\right)$$
$$k = \frac{-P_{21}}{(X_2 + X_3)}(X_3X_1 + X_2X_3 + X_1X_2)$$

$$Q_{11} = -\left(\frac{P_{21} - \sqrt{P_{21}^2 - 4k^2b^2 + 4kbQ_{21}}}{2k}\right)$$
$$S_{11} = P_{11} + jQ_{11}$$
$$I_1 = \frac{S_{11}^*}{V_1^*}$$
$$V_2 = \frac{S_{21}}{I_1^*}$$
$$I_2 = I_1 - \frac{V_1 - jX_1I_1}{jX_3}$$

These equations are calculated in the order listed, i.e., the first column of equations followed by the second column of equations. While it is possible to eliminate the intermediate variables a, b, and k, the expression for S_{11} becomes unwieldy.

The Case B method can also be used when S_{12} and V_2 are known by a similar replacement of variables as explained in Case A.

III. Conclusions

Complex power can be calculated from a non-collocated measurement by two methods – denominated as Cases A and B in this paper. A local voltage measurement and a detailed model of the local impedances are required along with the non-collocated power measurements.

IV. References

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