Volumetric Hedging in Electricity Procurement

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***Abstract*— Load serving entities (LSE) providing electricity service at regulated prices in restructured electricity markets, face price and quantity risk. We address the hedging problem of such a risk averse LSE. Exploiting the correlation between consumption quantities and spot prices, we developed an optimal, zero-cost hedging function characterized by payoff as function of spot price. We then show how such a hedging strategy can be implemented through a portfolio of call and put options.**

I. Introduction

The introduction of competitive wholesale markets in the electricity industry has put high price risk on market participants, particularly on load serving entities (LSEs). The unique non-storable nature of electricity as a commodity eliminates the buffering effect associated with holding inventory, and makes the possibility of sudden large price changes more likely. Significant market risks that are faced by LSEs are not related to price alone. Volumetric risk (or quantity risk), caused by uncertainty in the electricity load is also an important exposure for LSEs since they are obligated to serve the varying demand of their customers at fixed regulated prices. Electricity volume directly affects the company's net earnings and more importantly the spot price itself. Hence, hedging strategies that only concern price risks for a fixed amount of volume cannot fully hedge market risks faced by LSEs. The price and volumetric risks are especially severe to LSEs because supply and demand conditions usually shift adversely together as demonstrated by the California electricity crisis in 2000 and 2001, which led three large LSEs in California to bankruptcy or near bankruptcy.

As a way of mitigating price risks in electricity markets, derivatives such as futures, forwards and options have been used. An electricity forward contracts obligates a party to buy and the other party to sell a specified quantity on a given date in the future at a predetermined fixed price. At the maturity date if the market price is higher than the contracted forward price, then the buyer would make a profit, conversely, if the market price is lower than the forward price then the buyer will suffer a loss. The profits and losses are paid when the delivery is completed. Put or call options are also used for different types of risk hedging: A call (put) option on electricity supply obligates the seller to reimburse the buyer for spot prices above (below) the strike price. LSEs would also use call options to avoid the risk of higher prices while still being able to forgo the contract and enjoy the benefit of lower spot prices.

While it is relatively simple to hedge price risks for a specific quantity, such hedging becomes difficult when the demand quantity is uncertain, i.e., volumetric risks are involved. When volumetric risks are involved a company should be hedged against fluctuations in total cost, i.e., quantity times price but unfortunately, there are no simple market instruments that would enable such hedging. Furthermore, the common approach of dealing with demand fluctuations for commodities by means of inventories is not possible in electricity markets where the underlying commodity is not storable.

The non-storability of electricity combined with the steeply rising supply function and long lead time for capacity expansion results in strong positive correlation between demand and price. When demand is high, for instance due to a heat wave, the spot prices will be high as well and vice versa. For example, the correlation coefficient between hourly price and load for two years from April 1998 in California[1] was 0.539. [2] also calculated the correlation coefficients between normalized average weekday price and load for 13 markets: for example, 0.70 for Spain, 0.58 for Britain, and 0.53 for Scandinavia. There are some markets where this price and load relationship is weak but in most markets load is the most important factor affecting price of electricity.

The correlation between load and price amplifies the exposure of an LSE having to serve the varying demand at fixed regulated prices and accentuates the need for volumetric risk hedging. An LSE purchasing a forward contract for a fixed quantity at a fixed price based on the forecasted demand quantity will find that when demand exceeds its forecast and it is underhedged the spot price will be high and most likely will exceed its regulated sale price, resulting in losses. Likewise, when demand is low below its forecast, the spot price at which the LSE will have to settle its surplus will be low and most likely below its purchase price, again resulting in losses.

Because of the strong causal relationship between electricity consumption and temperature, weather derivatives have been considered to be an effective means of hedging volumetric risks in the electricity market. Weather derivatives, whose

[1] During this period, all the regulated utilities in the California market procured electricity from the spot market at the Power Exchange (PX). They were deterred from entering into long term contracts through direct limitations on contract prices and disincentives due to ex post prudence requirements.

payoff is triggered by weather conditions, exploit the correlation between electricity demand and weather condition. For example, if the upcoming winter is milder than usual, the electricity demand would be low leaving an LSE with low revenue. The LSE can protect against such situation by buying a Heating Degree Days (HDD) put option, which gives a positive payoff if the winter was realized milder than the HDD strike value denotes and zero payoff otherwise. However, the speculative image of such instruments makes them undesirable for a regulated utility having to justify its risk management practices and the cost associated with such practices to a regulator.

In this paper we propose an alternative to weather derivatives which involves the use of standard forward electricity contracts and price based power derivatives. This new approach to volumetric hedging exploits the aforementioned correlation between load and price. Specifically, we address the problem of developing an optimal hedging portfolio consisting of forward and options contracts for a risk averse LSE when price and volumetric risks are present and correlated.

Electricity markets are generally incomplete markets in the sense that not every risk factor can be perfectly hedged by market traded instruments. In particular, the volumetric risks are not traded in the electricity markets. Thus, we cannot naively adopt the classical no-arbitrage approach for hedging volumetric risks. Our proposed methodology is based on the alternative approach offered by the economic literature for dealing with risks that are not priced in the market, by considering the utility of economic agents bearing such risks. Specifically, we maximize the expected utility over the LSE's profit in order to investigate the optimal hedging strategies for price and volumetric risks.

Hedging problems dealing with non-traded quantity risk has been analyzed in the agricultural literature. A pioneering article [1] shows that the correlation between price and quantity is a fundamental feature of this problem and calculated the variance-optimizing hedge ratio of futures contracts. [3] shows that quantity uncertainty provides a rationale for the use of options. They derived exact solutions for hedging decision on futures and options assuming a CARA utility function and multivariate normality for the distribution of price and quantity. However, they assumed that only one strike price of options is available. In the electricity market literature, [7] directly deals with an LSE's problem in a multi-period setting, but they don't consider options as their hedging instruments. Our result shows that an optimal hedging portfolio for the LSE includes options with various strike prices. The idea of volumetric hedging using a spectrum of options was also proposed in [6] from the perspective of a Public Utility Commissions who could impose such hedging on the LSE as a means of ensuring resource adequacy and market power mitigation.

Determining the optimal number of contracts from a set of available options requires the solution of a difficult optimization problem, even in a single-period setting since payoffs of options are non-linear. Instead, in this paper we tackle the problem by first determining a continuous optimal payoff function that represents payoff of a hedging portfolio as a function of spot price, and then developing a replicating strategy based on a portfolio of standard instruments. The idea of obtaining the optimal payoff function is adopted from [4] which derives and analyzes optimal payoff functions that should be acquired by a value-maximizing non-financial production firm facing multiplicative risk of price and quantity. Instead of assuming the existence of certain instruments, they derive the payoff function that the optimal portfolio will have. We extend their model to a more general setting and furthermore to the replication of the optimal payoff function using available forwards and options. We also derive an optimal payoff function in a closed form for an LSE considering a constant absolute risk aversion (CARA) utility function under a bivariate normal assumption on the distribution of quantity and logarithm of price.

The remainder of the paper is organized as follows. In section 2, we provide a mathematical model and obtain the optimal payoff function. In section 3, we explore a way of replicating the optimal payoff derived in section 2 using forwards and available call and put options. Section 3 shows an example and section 4 concludes the paper.

II. Obtaining the optimal payoff function

A. Mathematical Formulation

Consider an LSE who is obligated to serve an uncertain electricity demand q at the fixed price r.[2] Assume that the LSE procures electricity, that it needs in order to serve its customers, from the wholesale market at spot price p. We consider a single-period problem where hedging instruments are purchased at time 0 and all payoffs are received at time 1. Hedging portfolio has an overall payoff structure $x(p)$, which depends on the realization of the spot price p at time 1. Note that our hedging portfolio may include money market accounts, letting the LSE borrow money to finance hedging instruments.

Let $y(p, q)$ be the LSE's profit from serving the load at the fixed rate r at time 1. Then, the total profit $Y(p, q, x(p))$ after receiving payoffs from the contracts in the hedging portfolio is given by

$$Y(p, q, x(p)) = y(p, q) + x(p). \tag{1}$$

where

$$y(p, q) = (r - p)q.$$

The LSE's preference is characterized by a concave utility function U defined over the total profit $Y(\cdot)$ at time 1. LSE's beliefs on the realization of spot price p and load q are characterized by a joint probability function $f(p, q)$ for positive p and q, which is defined on the probability measure P. On the other hand, let Q be a risk-neutral probability measure by which the hedging instruments are priced, and $g(p)$ be the probability density function of p under Q. Note that this

[2] In fact, most of US states which opened their retail markets into competition have frozen their retail electricity prices.

probability measure may not be unique since the electricity market is incomplete, however, in this paper we ignore this issue.

We formulate the LSE's problem as follows:

$$\max_{x(p)} \; E\big[U[Y(p,q,x(p))]\big]$$
$$s.t. \; \frac{1}{B}E^Q[x(p)] = 0 \tag{2}$$

where $E[\cdot]$ and $E^Q[\cdot]$ denote expectations under the probability measure P and Q, respectively. B is the time-0 price of a risk-free asset paying \$1 at time 1. The constraint (2) means that the manufacturing cost of the portfolio is zero, because a contract is priced as the expected value of discounted payoff under the risk-neutral probability measure. This zero-cost constraint implies that purchasing derivative contracts may be financed from selling other derivative contracts or from the money market accounts. In other words, under the assumption that there is no limits on the possible amount of instruments to be purchased and money to be borrowed, our model finds a portfolio from which the LSE obtains the maximum expected utility over total profit.

B. Optimality Conditions

The Lagrangian function for the above constrained optimization problem is given by

$$\begin{aligned} L(x(p)) &= E\big[U(Y(p,q,x(p)))\big] - \lambda E^Q[x(p)] \\ &= \int_{-\infty}^{\infty} E\big[U(Y)|p\big] f_p(p)dp - \lambda \int_{-\infty}^{\infty} x(p)g(p)dp \end{aligned}$$

with a Lagrange multiplier λ and the marginal density function $f_p(p)$ of p under P. Differentiating $L(x(p))$ with respect to $x(\cdot)$ results in

$$\frac{\partial L}{\partial x(p)} = E\Big[\frac{\partial Y}{\partial x}U'(Y)\Big|p\Big] f_p(p) - \lambda g(p) \tag{3}$$

by the Euler equation. Setting (3) to zero and substituting $\frac{\partial Y}{\partial x} = 1$ from (1) yields the first order condition for the optimal solution $x^*(p)$ as follows:

$$E\big[U'(Y(p,q,x^*(p)))\big|p\big] = \lambda^* \frac{g(p)}{f_p(p)} \tag{4}$$

Here, the value of λ^* should be the one that satisfies the zero-cost constraint (2). If $g(p) = f_p(p)$ for all p, then (4) implies that the optimal payoff function makes an expected marginal utility from the variation in q to be the same for any p.

C. CARA utility

A CARA utility function has the form: $U(Y) = -\frac{1}{a}e^{-aY}$ where a is the coefficient of absolute risk aversion. We see from the special property $U'(Y) = -aU(Y)$ of a CARA utility function that the following condition holds:

$$E[U(Y^*)|p] = -\frac{\lambda^*}{a}\frac{g(p)}{f_p(p)},$$

which implies that the utility which is expected at any price level p is proportional to $\frac{g(p)}{f_p(p)}$.

It follows from $U'(Y) = e^{-aY}$ and (4) that the optimal condition is

$$E\big[e^{-a(y(p,q)+x^*(p))}\big|p\big] = \lambda^* \frac{g(p)}{f_p(p)}$$

for an LSE with a CARA utility function. Then,

$$\begin{aligned} x^*(p) &= \frac{1}{a}\ln\Big(\frac{1}{\lambda^*}\frac{f_p(p)}{g(p)}E\big[e^{-ay(p,q)}\big|p\big]\Big) \\ &= \frac{1}{a}\Big(-\ln\lambda^* + \ln\frac{f_p(p)}{g(p)} + \ln E\big[e^{-ay(p,q)}\big|p\big]\Big) \end{aligned} \tag{5}$$

The Lagrange multiplier λ^* in the equation should satisfy the zero-cost constraint (2), which is $\int_{-\infty}^{\infty} x^*(p)g(p)dp = 0$. That is,

$$\int_{-\infty}^{\infty}\Big(-\ln\lambda^* + \ln\frac{f_p(p)}{g(p)} + \ln E\big[e^{-ay(p,q)}\big|p\big]\Big)g(p)dp = 0 \tag{6}$$

Solving (6) for $\ln\lambda^*$ gives

$$\ln\lambda^* = \int_{-\infty}^{\infty}\Big(\ln\frac{f_p(p)}{g(p)} + \ln E\big[e^{-ay(p,q)}\big|p\big]\Big)g(p)dp$$

Substituting this into equation (5) gives the optimal solution:

$$\begin{aligned} x^*(p) =& \frac{1}{a}\Big(\ln\frac{f_p(p)}{g(p)} + \ln E\big[e^{-ay(p,q)}\big|p\big]\Big) \\ &- \frac{1}{a}\Big(E^Q\big[\ln\frac{f_p(p)}{g(p)}\big] + E^Q\big[\ln E\big[e^{-ay(p,q)}\big|p\big]\big]\Big) \end{aligned} \tag{7}$$

Note that if we can assume $P \equiv Q$ in the electricity market, then the optimal payoff function under CARA utility becomes

$$x^*(p) = \frac{1}{a}\Big(\ln E\big[e^{-ay(p,q)}\big|p\big] - E\big[\ln E\big[e^{-ay(p,q)}\big|p\big]\big]\Big) \tag{8}$$

and thus the utility expected at p after receiving the optimal payoff is

$$E[U(y + x^*(p))|p] = \exp(E[\ln E[U(y)|p]])$$

This implies that the optimal portfolio is such that the expected utility from the varying demand at given p is the same for all p.

Bivariate lognormal-normal distribution under $P \equiv Q$: Suppose for simplicity that the LSE assigns to each state the same probabilities as those given by the risk-neutral density function (i.e., $P \equiv Q$). We calculate the optimal payoff function (8) assuming the distribution of (p,q) to be bivariate lognormal-normal[3]: $(\log p, q) \sim N(u_p, \bar{q}, v_p^2, \sigma_q^2, \rho)$. We use the conditional distribution of q given p, which is

$$q|p \sim N\big(\bar{q} + \rho\frac{\sigma_q}{v_p}(\log p - u_p), \sigma_q^2(1-\rho^2)\big),$$

to obtain

$$\begin{aligned} \ln E\big[e^{-ay(p,q)}\big|p\big] =& \ln\big[e^{-a(r-p)q}\big|p\big] \\ =& -\bar{q}a(r-p) - \rho\frac{\sigma_q}{v_p}(\log p - u_p)a(r-p) \\ &+ \frac{1}{2}\sigma_q^2(1-\rho^2)a^2(r-p)^2. \end{aligned}$$

[3]price follows lognormal distribution and load follows normal distribution, but they are correlated each other.

Then, the optimal solution under $P \equiv Q$ from (8) becomes

$$\begin{aligned} x^*(p) =&\bar{q}(p - E[p]) \\ &+\rho\frac{\sigma_q}{v_p}\big(p\log p - E[p\log p] - u_p(p - E[p])\big) \\ &-\rho\frac{\sigma_q}{v_p}r\big(\log p - E[\log p]\big) \\ &+\frac{1}{2}\sigma_q^2(1-\rho^2)a\big(p^2 - E[p^2] - 2r(p - E[p])\big) \end{aligned}$$

For p, a lognormal random variable with parameter (u_p, v_p^2), we have $E[\log p] = u_p$, $E[p] = e^{u_p + \frac{1}{2}v_p^2}$, $E[p\log p] = (u_p + v_p^2)e^{u_p+\frac{1}{2}v_p^2}$, and $E[p^2] = e^{2u_p+2v_p^2}$. By substituting these, we obtain the following optimal payoff function:

$$\begin{aligned} x^*(p)=&(\bar{q} + \sigma_q^2(1-\rho^2)ar)(p - e^{u_p+\frac{1}{2}v_p^2}) \\ &+\rho\frac{\sigma_q}{v_p}(p - r)(\log p - u_p) - \rho\sigma_q v_p e^{u_p+\frac{1}{2}v_p^2} \\ &+\frac{1}{2}\sigma_q^2(1-\rho^2)a\big(p^2 - e^{2u_p+2v_p^2}\big) \end{aligned} \tag{9}$$

We note that scaling the quantity variable takes special care. Consider scaling the quantity so that $q' = cq$ instead of q. Then $q' \sim N(c\bar{q}, (c\sigma_q)^2)$. One might be led to think that the optimal payoff function would be just $x^*(p)$ obtained using (p, q'), multiplied by c; however, that is not true. The only case where scaling the quantity by c results in an optimal payoff function $cx^*(p)$, is when $\sigma_{q'} = \sqrt{c}\sigma_q$.

III. Replicating the Optimal Payoff Function

In the previous section, we've obtained the payoff function $x^*(p)$ that the optimal portfolio should have. In this section, we construct a portfolio that replicates the payoff $x(p)$.
In [5], Carr and Madan showed that any twice continuously differentiable function $x(p)$ can be written as in the following form:

$$\begin{aligned} x(p) &= [x(s) - x'(s)s] + x'(s)p \\ &+ \int_0^s x''(K)(K-p)^+ dK + \int_s^\infty x''(K)(p-K)^+ dK \end{aligned}$$

for an arbitrary positive s. This formula suggests a way of replicating the payoff $x(p)$. Let F be the forward price for a delivery at time 1. Evaluating the equation at $s = F$ and rearranging it gives

$$\begin{aligned} x(p) &= x(F)\cdot 1 + x'(F)(p - F) \\ &+ \int_0^F x''(K)(K-p)^+ dK + \int_F^\infty x''(K)(p-K)^+ dK. \end{aligned}$$

Note that $1, (p-F), (K-p)^+$ and $(p-K)^+$ at each term are payoffs at time 1 of a bond, forward contract, put option, and call option, respectively.

Therefore,
$x(F)$ units of bonds,
$x'(F)$ units of forward contracts,
$x''(K)dK$ units of put options with strike K for every $K < F$, and
$x''(K)dK$ units of call options with strike K for every $K > F$
gives the same payoff as $x(p)$.

The above implies that unless the optimal payoff function is linear, the optimal strategy involves purchasing (or selling short) a spectrum of both call and put options with continuum of strike prices. This result proves that LSEs should purchase a portfolio of options to hedge price and quantity risk together. Even if prices go up with increasing loads, more call options with higher strike prices are exercised, having an effect of putting price caps on each incremental load.

In practice, electricity derivatives markets, as any derivatives markets, are incomplete. Consequently, the market does not offer options for the full continuum of strike prices, but typically only a small number of strike prices are offered. Our purpose is to best-replicate the optimal payoff function using existing options only. Therefore, we need to decide what amount of options to purchase for each available strike price so that the total payoff from those options is equal or close to the payoff provided by the optimal payoff function. Let $K_1, \cdots, K_n$ be available strike prices for put options, $K'_1, \cdots, K'_m$ be available strike prices for call options where $0 < K_1 < \cdots < K_n < F < K'_1 < \cdots < K'_m$ and let $K_0 = 0$, $K_{n+1} = F = K'_0$, and $K'_{m+1} = \infty$.

Consider the following replicating strategy, which consists of
$x(F)$ units of bonds,
$x'(F)$ units of forward contracts,
$\frac{1}{2}((x'(K_{i+1}) - x'(K_{i-1}))$
units of put options with strike price K_i, $i = 1, \cdots, n$,
$\frac{1}{2}(x'(K'_{i+1}) - x'(K'_{i-1}))$
units of call options for a strike price K'_i, $i = 1, \cdots, m$.

This strategy replicates a payoff function $x(p)$ with an error $e(p)$ given below:
$e(p) = \frac{1}{2}\{(x'(p) - x'(K_j))(K_{j+1} - p) - (x'(F) - x'(K_n))(F - p)\}$, if $p \in (K_j, K_{j+1})$ for any $j = 1, \cdots, n-1$,
$e(p) = \frac{1}{2}\{(x'(p) - x'(K_n))(F - p) - (x'(F) - x'(p))(F - p)\}$ if $p \in (K_n, F)$,
$e(p) = \frac{1}{2}\{(x'(K'_1) - x'(p))(p - F) - (x'(p) - x'(F)\}(p - F)$ if $p \in (F, K'_1)$, and
$e(p) = \frac{1}{2}\{(x'(K'_j) - x'(p))(p - K'_{j-1}) - (x'(K'_1) - x'(F))(p - F)\}$ if $p \in (K'_{j-1}, K'_j)$ for any $j = 2, \cdots, m$.

We see that the error from the replicating strategy is very close to zero if there exist put and call options with strike price F (i.e., $K_n \simeq F \simeq K_1$) and if p is realized very close to one of the strike prices. The error will be smaller if strike prices are offered in smaller increments, especially

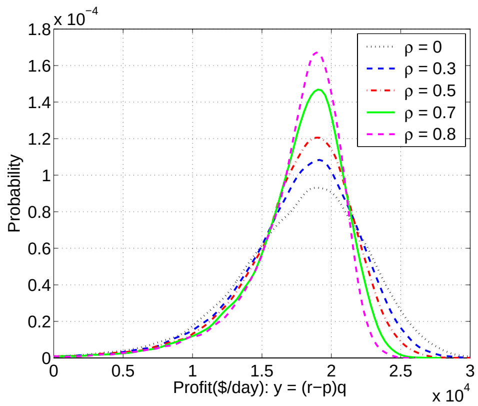


Fig. 1. Profit distribution for various correlation coefficients. Generated 50000 pairs of (p, q) from a bivariate normal distribution of $(\log p,\ q)$ with a various correlation ρ's, where $\log p \sim N(3.64, 0.35^2)$ and $q \sim N(300, 30^2)$, and plotted estimated probability density functions of the profit using normal kernel (assuming $r = \$100/\text{MWh}$).

for price intervals with high probabilities of containing p.[4]

IV. An Example

In this section, we illustrate the method that we derived in the previous sections. We consider the on-peak hours of a single summer day as time 1. Parameters were approximately based on the California Power Exchange data of daily day-ahead average on-peak prices and 1% of the total daily on-peak loads from July to September, 1999. Specific parameter values are imposed as follows:

- Price is distributed lognormally with parameters $u_p = 3.64$ and $v_p = 0.35$, in both the real-world and risk-neutral world: $\log p \sim N(3.64, 0.35^2)$ in P and Q. Note that the expect value of the price p under this distribution is $\$40.5/\text{MWh}$.
- Load that the LSE needs to serve is distributed normally with mean $\bar{q} = 300$ and variance $\sigma_q^2 = 30^2$.
- Correlation coefficient between $\log p$ and q is 0.7.
- The fixed rate r charged to the customers is 100.
- The LSE's risk preference is decided by CARA utility with the risk aversion $a = 1.5$.

We would like to point out a significant correlation-effect on profit distributions. Figure 1 shows that the profit distributions become quite different as the correlation between load and logarithm of price changes. Considering that the correlation coefficient of our data is 0.7, we observe that the correlation coefficient cannot be ignored in the analysis of profit.

The optimal payoff functions (9) are drawn in Figure 2 for various correlation coefficients between $\log p$ and q. Generally, low profit from high loads for very high spot prices and from low load for very low spot price is compensated with the cases where spot prices and loads are around the expected value. This can be seen from the graph where as the spot price goes away from r, positive payoff is received from the optimal portfolio while the payoff is negative around r. We also note that larger payoff can be received when the correlation is smaller. This is because the variance of profit is bigger when the correlation is smaller as we can see from Figure 1. Therefore, even when the correlation is zero, the optimal payoff function is nonlinear.

Figure 3 illustrates the optimal numbers of contracts to be purchased in order to obtain the payoff $x^*(p)$ for an LSE with a CARA utility function. It indicates large variations in the number of contracts purchased under the optimal portfoli as the correlation coefficient changes.

We see that the numbers of options contracts are very high relative to the mean volume. This is because we don't restrict the model with constraints such as credit limits. The zero-cost constraint (2) that we included in our model allows borrowing as much money as needed to finance any number of derivative contracts.

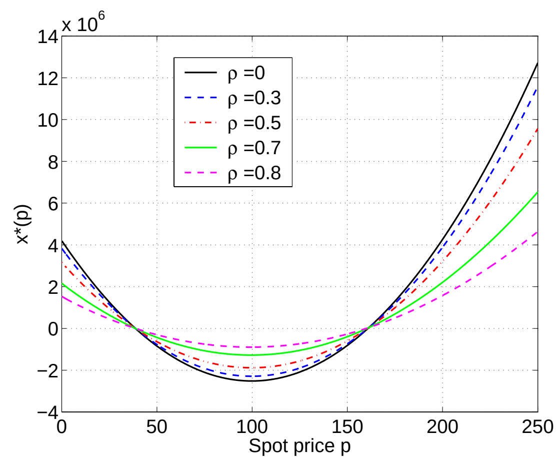


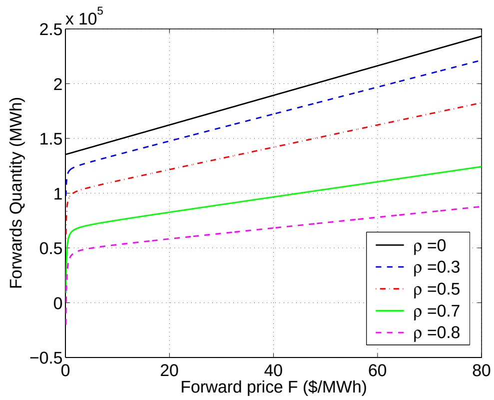
Fig. 2. The optimal payoff function under CARA utility when price and load follow bivariate lognormal-normal distribution

V. Conclusion

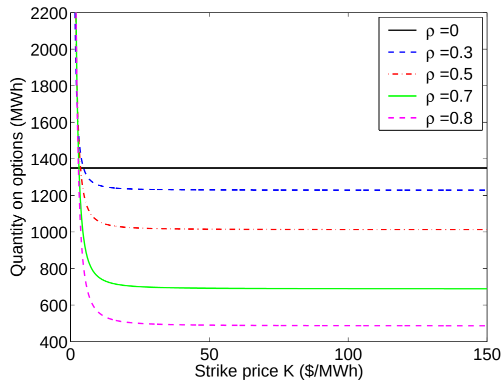
Price risk and its management in the electricity market have been studied by many researchers and it is well understood. However, price risk should bestudied in conjunction with volumetric risk (quantity risk), which is also significant. Volumetric risk has great impact on the profit of load-serving entities; therefore, there is a great need for methodology addressing volumetric risk management.

Weather derivatives are widely used to hedge volumetric risks since there is strong correlations between weather variables and power loads. In contrast, we propose an alternative approach that exploits the high correlation between spot prices and loads to construct a volumetric hedging strategy based on standard power contracts. In a one-period setting, we obtain

[4] In fact, the NYMEX offers the following strike prices for PJM electricity options: twenty strike prices in increments of $\$0.50$ per megawatt hour above and below the at-the-money strike price, and the next 10 strike prices in increments of $\$1.00$ above the highest and below the lowest existing strike prices for a total of at least 61 strike prices. The at-the-money strike price is nearest to the previous day's close of the underlying futures contract. Strike price boundaries are adjusted according to the futures price movements. (source: www.nymex.com)



(a) $x'(F)$



(b) $x''(K)$

Fig. 3. The graphs show numbers on forward and options contracts to be purchased to replicate the optimal payoff $x^*(p)$ that is obtained for the LSE with CARA utility. In this example, the optimal portfolio includes forward contracts for $x'(40.5)$ MWh, put options on $x''(K)dK$ MWh for $K < 40.5$ and call options on $x''(K)dK$ MWh for $K > 40.5$.

the optimal zero-cost portfolio consisting of bonds, forwards and options with a continuum of strike prices. Also the paper shows how to replicate the optimal payoff using available European put and call options. In a different paper we have obtained similar results for a mean-variance utility function and alternative joint probability distributions on quantity and price The model and methodology are applicable to other commodity markets and with different profit functions.

There are more extensions which can be made to the current model. First, the zero-cost assumption allows the LSE unlimited borrowing at time 0 to buy the options contracts. Imposing credit limits or Value-at-Risk limits on the hedging strategy would make the model more applicable. Second, the electricity market is incomplete, so the risk-neutral probability measure we choose would not be exactly the same as what the market uses for pricing. Therefore, a pricing error would exist, which can lead to inefficient hedging. A model that accounts for possible errors in choosing the risk-neutral probability measure would be a good extension for applications in the actual electricity markets.

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