Cournot Equilibria in Two-Settlement Electricity Markets with System Contingencies

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Abstract— **We study Nash equilibrium in two-settlement competitive electricity markets with horizontal market power, flow congestion, demand uncertainties and probabilistic system contingencies. The equilibrium is formulated as a stochastic Equilibrium Problem with Equilibrium Constraints (EPEC) in which each firm solves a stochastic Mathematical Program with Equilibrium Constraints (MPEC). We assume a no-arbitrage relationship between the forward prices and the spot prices. We find that, with two settlements, the generation firms have incentives to commit forward contracts, which increases social surplus and decreases spot energy prices. Furthermore, these effects are amplified when there are more firms in the markets.**

I. Introduction

The last decade has witnessed a fundamental transformation of the electric power industry around the world from one dominated by regulated vertically integrated monopolies to an industry where electricity is produced and traded as a commodity through competitive markets. In the US, this transformation was pioneered in the late 1990s by California and the northeastern power pools including Pennsylvania-New Jersey-Maryland (PJM) Interchange, New York and New England. A recent arrival is the ERCOT market in Texas.

While there are significant differences among the many implemented and proposed market designs that vary in terms of ownership structure, level of centralization and the authority of the system operator, the primary rationale for electricity restructuring in most markets has been to reap welfare gains by supplanting regulation with competition. Both theory and experience from other formerly regulated industries suggest that these gains will include increased short-run productive efficiency, enhanced allocation efficiency through pricing that more closely reflects physical and economic reality, as well as increased dynamic efficiency from improved incentives for investment.

A potentially significant obstacle to realizing these welfare gains is market power. Market power exercised by suppliers typically entails the withholding of output and an upward distortion in the market price. Market power is generally associated with various forms of economic inefficiency. Among the many proposed and implemented economic tools for mitigating market power is a multiple settlement approach wherein forward transactions, day ahead transactions, and real time balancing transactions are settled at different prices. The crisis in California in 2001 and the collapse of ENRON have drawn more attention to the role of forward markets in mitigating market power and in managing price risk in the electricity supply chain.

Theoretical analysis and empirical evidences suggest that forward contracting and multi-settlement systems reduce the incentives of sellers to manipulate spot market prices since under a multi-settlement approach, the volume of trading that can be affected by an increase in spot prices is reduced substantially. Thus, forward trading is viewed as an effective way of mitigating market power at real time. It is also argued that setting prices at commitment time provides incentives for accurate forecasting and provides ex-ante price discovery that facilitates trading. Accurate forecasting and advanced scheduling of generation and load also improves system operation and reliability while reducing the cost of reserves to handle unexpected deviations from schedule.

While intuitively the above arguments in favor of forward trading and multi-settlement systems are compelling, there is only limited theoretical analysis that supports these assertions and that analysis typically ignores network effects, flow congestion, generator outages, and other system contingencies. When flow congestion, system contingencies, and demand uncertainties are all present in the spot market, it is not clear to what extent producers are willing to engage in forward transactions, or how their incentives will be thus affected. Furthermore, it is not well understood whether forward trading may in fact help producers exercise market power in the spot market to lock in or even increase their Oligopoly rents. If indeed forward trading can be used to mitigate the exercise of market power but generators have little incentive to engage in such trading, then a natural public policy question is whether forward contracting should be imposed as a regulatory requirement and the market be designed to minimize spot transactions. Indeed, the current market rules in California and in Texas are designed to limit the scope of the real time balancing markets through penalties or added charges.

In this paper, we formulate the two-settlement competitive electricity markets as a two-period game, and its equilibrium as a subgame-perfect Nash equilibrium (see [8]) expressed in the format of an Equilibrium Problem with Equilibrium

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Constraints (EPEC), in which each firm faces a Mathematical Program with Equilibrium Constraints (MPEC) parametric on other firms' forward commitments. We apply the model to an IEEE 24-bus test network. With the specific data and simplifying assumptions of the example, it is shown that in equilibrium, firms commit certain quantities in forward transactions and adjust their positions in the spot market responding to contingencies and demand realization.

The rest of this paper is organized as follows. Related research on models with transmission constraints and forward markets are reviewed in the following section. Section III presents the model assumptions and the mathematical formulation. An example, numerical results and conclusions are discussed in sections IV and V.

II. Related Research

We review models of spot energy markets with transmission constraints as well as models that include forward markets. Most of the spot market models with transmission constraints assume either perfect competition or an Oligopoly based on the Cournot conjectural variation. Assuming that the agents act as price takers in the transmission market allows such models to be solved as complementarity problems or variational inequalities.

Wei and Smeers [20] consider a Cournot model with regulated transmission prices. They solve the variational inequalities to determine a unique long-run equilibrium. In subsequent work, Smeers and Wei [19] consider a separated energy and transmission market, where the system operator conducts a transmission capacity auction with power marketers purchasing transmission contracts to support bilateral transactions. They conclude that such a market converges to the optimal dispatch with a large number of marketers. Borenstein and Bushnell [3] use a grid search algorithm that iteratively converges to a Cournot model based on data from the California market.

Hobbs et al [12] calculate a Cournot equilibrium under the assumptions of linear demand and cost functions, which leads to a linear mixed complementarity problem. In a market without arbitrageurs, non-cost based price differences can arise because the bilateral nature of the transactions gives firms more degrees of freedom to discriminate between electricity demand at various nodes. This is equivalent to a separated market as in [19]. In the market with arbitrageurs, any non-cost differences are arbitraged by traders who buy and sell electricity at nodal prices. This equilibrium is shown to be equivalent to a Nash-Cournot equilibrium in a POOLCO-type market. In another paper [13], Hobbs presents an Oligopolistic market where each firm submits a linear supply function to the Independent System Operator (ISO). He assumes that firms can only manipulate the intercepts of the supply functions, but not the slopes, while power flows and pricing strategies are constrained by the ISO's linearized DC optimal power flow. Each firm in this model faces an MPEC problem with spatial price equilibrium as the inner problem.

Work on forward markets has focused on the welfare enhancing properties of forward markets and the commitment value of forward contracts. The basic model in Allaz [1] assumes that producers meet in a two-period market where there is some demand uncertainty in the second period. Allaz shows that generators have a strategic incentive to contract forward if other producers do not. This result can be understood using the concepts of strategic substitutes and complements of Bulow, Geneakoplos and Klemperer [4]. In these terms, the availability of the forward market makes a particular producer more aggressive in the spot market. Due to the strategic substitutes effect, this produces a negative effect on its competitors' production. The producer with access to the forward market can therefore use its forward commitment to improve its profitability to the detriment of its competitors. Allaz shows, however, that if all producers have access to the forward market, it leads to a prisoners' dilemma type of effect, reducing profits for all producers. Allaz and Vila [2] extend this result to the case where there is more than one time period where forward trading takes place. For a case without uncertainty, they establish that as the number of periods where forward trading takes place tends to infinity, producers lose their ability to raise market prices above marginal costs.

von der Fehr and Harbord [9] and Powell [18] study contracts and their impact on an imperfectly competitive electricity spot market: the UK pool. von der Fehr and Harbord [9] focus on price competition in the spot market with capacity constraints and multiple demand scenarios. They find that contracts tend to put downward pressure on spot prices. Although, this provides disincentive to generators to offer such contracts, there is a countervailing force in that selling a large number of contracts commits a firm to be more aggressive in the spot market, and ensures that it is dispatched into its full capacity in more demand scenarios. Powell [18] explicitly models recontracting by Regional Electricity Companies (RECs) after the maturation of the initial portfolio of contracts set up after deregulation. He adds risk aversion on the part of RECs to earlier models. Generators act as price setters in the contract market. He shows that the degree of coordination has an impact on the hedge cover demanded by the RECs, and points to a "free rider" problem which leads to a lower hedge cover chosen by the RECs.

Newbery [16] analyzes the role of contracts as a barrier to entry in the England and Wales electricity market. He extends earlier work by modeling supply function equilibrium (SFE) in the spot market. He further shows that if entrants can sign base load contracts and incumbents have enough capacity, the incumbent can sell enough contacts to drive down the spot price below the entry deterring level, resulting in more volatile spot prices if producers coordinate on the highest profit SFE. Capacity limits however may imply that incumbents cannot play a low enough SFE in the spot market and hence cannot deter entry. Green [11] extends Newbery's model showing that when generators compete in SFEs in the spot market, together with the assumption of Cournot conjectural variations in the forward market, imply that no contracting will take place unless buyers are risk averse and willing to provide a hedge premium in the forward market. He shows that forward sales can deter excess entry, and increase economic efficiency and long-run profits of a large incumbent firm faced with potential entrants.

Kamat and Oren [14] analyze the welfare and distributional properties of a two-settlement market, which consists of a nodal spot market over 2-node and 3-node networks with a single energy forward market. The system is subject to congestion with uncertain transmission capacities in the spot market, and to generators' market power. That work has been extended by Yao, Oren and Adler [21] where the formulation described in this paper was first introduced.

III. The model

A. Introduction and assumptions

We shall describe now our model for calculating the equilibrium quantities and prices of electricity over a given network with two settlements. We view the two settlements in the electricity market as a complete information game with two periods: a forward market (period 1), and a spot market (period 2). We model the equilibrium in this two-period game as a subgame-perfect Nash equilibrium.

In period one, firms enter forward commitments, by competing in a Cournot fashion anticipating the forward commitments of one another and the common knowledge of the expected spot market outcome in period two. In the spot market, the uncertain contingencies are realized and the generation firms act as Cournot competitors, choosing their spot production quantities for each generation unit, so as to maximize their total profit including the financial settlements of their forward commitments. In doing so they take as given the revealed forward commitments of all other generation firms, the conjectured spot production decisions of all other generators, and the redispatch decisions of the system operator (SO) specifying the import/export quantity at each node. Simultaneously with the generators' production decisions, the SO makes its redispatch decision determining imports and exports at each node so as to maximize total social welfare based on its conjectured spot production at each node, the transmission constraints, and the energy balance constraint.

Forward markets extending beyond one day ahead of real time typically involve zonal aggregation and trading hubs (e.g the Western Hub at PJM). Hence, our model permits different levels of locational granularity in the forward and spot markets. Specifically, we will assume that in the forward market nodes are clustered into zones and firms enter forward contracts which specify forward zonal quantity commitments at agreed upon zonal prices. Another key assumption underlying our formulation is that the forward market is sufficiently liquid so the forward price in each zone is uniform across all firms operating in the zone and the forward commitments are public knowledge in the spot market.

All forward contracts are settled financially in the spot market based on the difference between the forward zonal price and the spot zonal price, which is a weighted average of all spot nodal prices in the zone. The weights used in determining the spot zonal prices are constants that reflect historical load shares but are not endogenously determined based on actual load shares in the spot market. We also assume that risk neutral speculators take opposite positions to the generation firms and exploit any arbitrage opportunities so that the forward price in a zone equals to the corresponding expected spot zonal prices over all possible contingencies.

The available capacities of generation units and transmission lines in the spot market are unknown in period 1 and are subject to stochastic variations in period 2. We model the transmission network constraints in the spot market in terms of a lossless DC approximation of Kirchhoff's laws. Specifically, flows on lines can be calculated using power transfer distribution factors (PTDFs) which specify the proportion of flow on any particular line resulting from an injection of one unit at a particular node and a corresponding withdrawal at an arbitrary (but fixed) "slack bus" [6]. Uncertainty regarding the realized network topology in the spot market is characterized by different PTDF matrices with corresponding probabilities.

In order to avoid complications due to discontinuous payoffs in the spot market (see [5], [17]), we assume that agents do not game the transmission prices or consider the impact of their production decisions on congestion prices. For simplicity, we further assume that there is at most one generation facility at a node (this assumption can be easily relaxed).

B. Model notations:

Sets:

- N: The set of nodes (or buses).
- Z: The set of zones. Moreover, $z(i)$ represents the zone where node i resides.
- L: The set of transmission lines whose congestion in the spot market are under consideration. These lines are called flowgates.
- C: The finite set of states in the spot market.
- G: The set of generation firms. N_g denotes the set of nodes where generation facilities of firm g are located.

Parameters:

- $\underline{q}_i^c, \overline{q}_i^c$: The lower and upper capacity bounds of generation facility at node i in state c.
- $p_i^c(\cdot)$: The linear inverse demand function (IDF) at node i in state c:

  $$p_i^c(q) = \bar{p}^c - b_i q \quad i \in N, c \in C$$

  We assume that in each state c the price intercepts of the inverse demand curves are uniform across all nodes. We also assume that, for each node i, the nodal demand shifts inward and outward in different states, but the slope remains unchanged.
- $C_i(\cdot)$: The cost function at node i. In this model, the cost functions are assumed linear

  $$C_i(q) = d_i q \quad i \in N$$

  with given d_i.
- K_l^c : The flow capacity of line l in state c.
- $D_{l,i}^c$: The power transfer distribution factor in state c on line l with respect to node i.
- $Pr(c)$: The probability of state c of the spot market.
- δ_i: The weights used to settle the spot zonal prices ($\delta_i \geq 0, \sum_{i:z(i)=z} \delta_i = 1$).

Decision variables:

- $x_{g,z}$: Forward quantity committed by firm g to zone z.
- q_i^c: Generation level at node i in state c of the spot market.
- r_i^c : Import/export quantity at node i by the SO in state c of the spot market.

C. The Formulation

The zonal forward market ignores intra-zonal transmission congestion (although such congestion is implicitly accounted for through the rational expectation of the spot zonal price which is based on a weighted nodal price average). The spot market, on the other hand, is organized at a nodal level with all transmission constraints recognized in the SO redispatch.

The spot nodal price at each node i in each state c is given by the nodal inverse demand function $p_i^c(q_i^c+r_i^c)$ applied to the net local consumption that results from the local production decision by the generating firms and the redispatch decision by the SO.

The spot zonal (settlement) price u_z^c at a zone z in each state c is defined as the weighted average of the nodal prices in that zone with predetermined weights δ_i. Mathematically, the zonal spot settlement prices are given by:

$$u_z^c = \sum_{i:z(i)=z} \delta_i p_i^c(r_i^c + q_i^c), \quad z \in Z$$

The forward zonal prices h_z are the prices at which forward commitments are agreed upon in the respective zones. The no-arbitrage assumption implies that the forward zonal prices are equal to the expected spot zonal settlement prices:

$$\begin{aligned} h_z &= E^c[u_z^c] \\ &= \sum_{c \in C} Pr(c) u_z^c, \quad z \in Z \end{aligned} \tag{2}$$

In each state c of the spot market, the firms choose the production levels q_i^c. Each firm's profit in each state c is the sum of its forward commitment settlement (based on the difference between the zonal forward prices and the spot zonal settlement prices), and net profits from its production quantities that are paid at the spot nodal prices. So its profit is:

$$\begin{aligned} \pi_g^c = &\sum_{i \in N_g} p_i^c(r_i^c + q_i^c) q_i^c + \sum_{z \in Z} (h_z - u_z^c) x_{g,z} \\ &- \sum_{i \in N_g} C_i(q_i^c) \end{aligned}$$

Each firm g's objective in the spot market is to maximize its profit π_g^c. It solves the following profit maximization problem parametric on its forward commitments $x_{g,z}$ and the SO's redispatch quantities r_i^c:

$$\mathcal{G}_g^c : \quad \max_{q_i^c : i \in N_g} \quad \pi_g^c$$

subject to:

$$q_i^c \geq \underline{q}_i^c, \quad i \in N_g \tag{3}$$

$$q_i^c \leq \overline{q}_i^c, \quad i \in N_g. \tag{4}$$

In this program, constraints (3) and (4) ensure that the production levels q_i^c fall between the capacity bounds of the generation facilities in each state c.

The SO determines import/export quantities r_i^c at each node i. Its objective is to maximize the social surplus defined by the consumers' willing-to-pay minus the total generation cost. It solves a social welfare maximization problem:

$$\mathcal{S}^c : \max_{r_i^c} \quad \sum_{i \in N} [\int_0^{r_i^c + q_i^c} p_i^c(\tau_i) d\tau_i - C_i(q_i^c)]$$

subject to:

$$\sum_{i \in N} r_i^c = 0 \tag{5}$$

$$\sum_{i \in N} D_{l,i}^c r_i^c \geq -K_l^c, \quad l \in L \tag{6}$$

$$\sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{7}$$

Here constraint (5) represents energy balance (assuming no losses), whereas constraints (6) and (7) enforce the network feasibility, i.e. the power flows resulting from the SO redispatch must satisfy the thermal limits.

Since the nodal inverse demand functions as well as the cost functions are assumed linear, problems $\mathcal{G}_g^c$ and $\mathcal{S}^c$ are both strictly concave-maximization programs, which implies that their first order necessary conditions (the Karush-Kuhn-Tucker, KKT conditions) are also sufficient. The spot market outcomes can thus be characterized by the KKT conditions of the firms' problems and of the SO problem.

Let α^c, λ_{l-}^c and λ_{l+}^c be the Lagrange multipliers corresponding to constraints (5), (6) and (7), then the KKT conditions derived from problem $\mathcal{S}^c$ are:

$$\sum_{j \in N} r_j^c = 0 \tag{8}$$

$$\begin{aligned} &\bar{p}^c - (q_i^c + r_i^c) b_i - \alpha^c \\ &\quad + \sum_{t \in L} (\lambda_{t-}^c D_{t,i}^c - \lambda_{t+}^c D_{t,i}^c) = 0 \qquad i \in N \end{aligned} \tag{9}$$

$$\lambda_{l-}^c \geq 0 \qquad l \in L \tag{10}$$

$$\sum_{j \in N} D_{l,i}^c r_j^c + K_l^c \geq 0 \qquad l \in L \tag{11}$$

$$(\sum_{i \in N} D_{l,i}^c r_i^c + K_l^c) \lambda_{l-}^c = 0 \qquad l \in L \tag{12}$$

$$\lambda_{l+}^c \geq 0 \qquad l \in L \tag{13}$$

$$K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c \geq 0 \qquad l \in L \tag{14}$$

$$(K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c) \lambda_{l+}^c = 0 \qquad l \in L \tag{15}$$

Similarly, let ρ_{i-}^c and ρ_{i+}^c be the Lagrange multipliers corresponding to constraints (3) and (4), the KKT conditions for problem $\mathcal{G}_g^c$ are:

$$\begin{aligned} &\bar{p}^c - 2b_i q_i^c - b_i r_i^c - d_i + \\ &\quad \delta_i b_i x_{g,z(i)} + \rho_{i-}^c - \rho_{i+}^c = 0 && i \in N_g && (16)\\ &\rho_{i-}^c \geq 0 && i \in N_g && (17)\\ &q_i^c \geq \underline{q}_i^c && i \in N_g && (18)\\ &(q_i^c - \underline{q}_i^c)\rho_{i-}^c = 0 && i \in N_g && (19)\\ &\rho_{i+}^c \geq 0 && i \in N_g && (20)\\ &\overline{q}_i^c - q_i^c \geq 0 && i \in N_g && (21)\\ &(\overline{q}_i^c - q_i^c)\rho_{i+}^c = 0 && i \in N_g && (22) \end{aligned}$$

If we restrict

$$x_{g,z} = 0, \quad z \in Z, g \in G,$$

i.e. no firm commits to forward contracts, the solutions to the KKT conditions (8)-(22) characterize the outcomes of the single-settlement market, i.e. there is no forward market, and all firms act only in the spot market.

If there is no flow congestion in some state c of the spot market, the shadow prices corresponding to the transmission capacities are zeros, i.e.

$$\lambda_{l-}^c = \lambda_{l+}^c = 0, \quad l \in L$$

and the KKT conditions (9) reduce to

$$\bar{p}^c - (q_i^c + r_i^c)b_i - \alpha^c = 0, i \in N,$$

from which we solve for α^c:

$$\alpha^c = \bar{p}^c - \frac{\sum_{i\in N} q_i^c}{\sum_{i\in N} \frac{1}{b_i}}.$$

Therefore, if there is no flow congestion all the spot nodal prices are equal to α^c.

In the forward market, network feasibility is ignored and the forward contracts are settled financially. Each firm g conjectures the other firms' forward quantities and determines its own forward quantities. In general the firms' objectives are to maximize their respective expected utility function over total profit from spot productions and forward settlements. For simplicity, the firms are assumed here risk neutral so the firms' forward objectives are to maximize their expected spot profits subject to the "no-arbitrage" condition and the KKT conditions (8)-(22) which characterize the anticipated outcomes in the spot market. Each firm g solves the following stochastic MPEC program (see [15]) in the forward market:

$$\begin{aligned} &\max_{x_{g,z}} \quad E^c[\pi_g^c] = \sum_{c\in C} Pr(c)\pi_g^c \\ &\text{subject to:} \\ &h_z = \sum_{c\in C} Pr(c)u_z^c, \quad z \in Z \qquad (2)\\ &\text{and } (8)-(22), \text{ for all } c \in C \end{aligned}$$

Note that the forward settlement term in the objective function is cancelled due to constraint 2, so that the MPEC program for each firm g reduces to:

$$\begin{aligned} &\max_{x_{g,z}} \sum_{c\in C} Pr(c)[\sum_{i\in N_g} p_i^c(r_i^c + q_i^c)q_i^c - \sum_{i\in N_g} C_i(q_i^c)] \\ &\text{subject to:} \\ &(8)-(22), \text{ for all } c \in C \end{aligned}$$

The general structure of each firm's MPEC problem (after rearranging and relabelling the variables) is of the form:

$$\begin{aligned} &\min_{x_g,y,w} \quad f_g(x_g, x_{-g}, y, w) \\ &\text{subject to :} \\ &w = a + A^{-g}x_{-g} + A^g x_g + My \\ &w \geq 0, y \geq 0, w^T y = 0 \end{aligned}$$

In this program, x_g represents design variables controlled by firm g, x_{-g} are the corresponding design variables controlled by all other firms, whereas w and y are the shared state variables implied by these design variables through the constraints. Likewise a, A_g, A_{-g}, and M represent suitable vectors and matrices implied by the system's parameters. Due to the linearity of the demand functions and cost functions, the objective functions in these MPEC problems are quadratic and the variational inequality constraints (8)-(22) reduce to a Linear Complimentarity Problem (LCP) (see [7]). Combining all firms' MPEC programs, the equilibrium problem in the forward market is an EPEC which involves simultaneous solutions of the individual firms' MPECs. In the following numerical example we have employed a special purpose algorithm that we developed for such problems that exploits their special structure. The description of the algorithm is out of the scope of this paper and will be reported elsewhere.

IV. The 24-bus System

In this section, we apply our model to the IEEE 24-bus test network with different fictitious generator ownership structures, and observe the economic results of two settlements.

The 24-bus network is composed of 24 nodes and 38 lines (see figure 1). Eight lines are double lines connecting the same pairs of nodes, which, for computational purpose, are combined into single lines with adjusted thermal capacities and resistances. The simplified network contains 34 transmission lines. 10 nodes in this system have generation plants attached to.

Table I lists the nodal information, including inverse demand function slopes, marginal generation costs, full capacities of generation plants. We assume two zones in the system with nodes 1 through 13 in zone 1 and the rest nodes in zone 2. As to the thermal limits, we ignore the intra-zonal flows and focus only on the flowgates connecting the node pairs of [3,24], [11,14], [12,23], and [13,23].

We assume seven states in the spot market (see table II). In the first state, the demands are at peak, all generation plants operate at their full capacities, and all transmission lines are rated at their full thermal limits. The second state is the same as the first state except that it has shoulder demands. State 3 through 6 have also shoulder demands,

TABLE II

STATES OF THE SPOT MARKET

State	Prob.	IDF intercept ($/MWh)	Type and description
1	0.6	100	On-peak state: The demands are on the peak.
2	0.15	50	Shoulder state: The demands are at shoulder.
3	0.025	50	Shoulder demands with line breakdown: Line [3,24] goes down.
4	0.025	50	Shoulder demands with line breakdown: Line [11,14] goes down.
5	0.025	50	Shoulder demands with line breakdown: Line [12,23] goes down.
6	0.025	50	Shoulder demands with line breakdown: Line [13,23] goes down.
7	0.15	25	Off-peak state: The demands are off-peak.

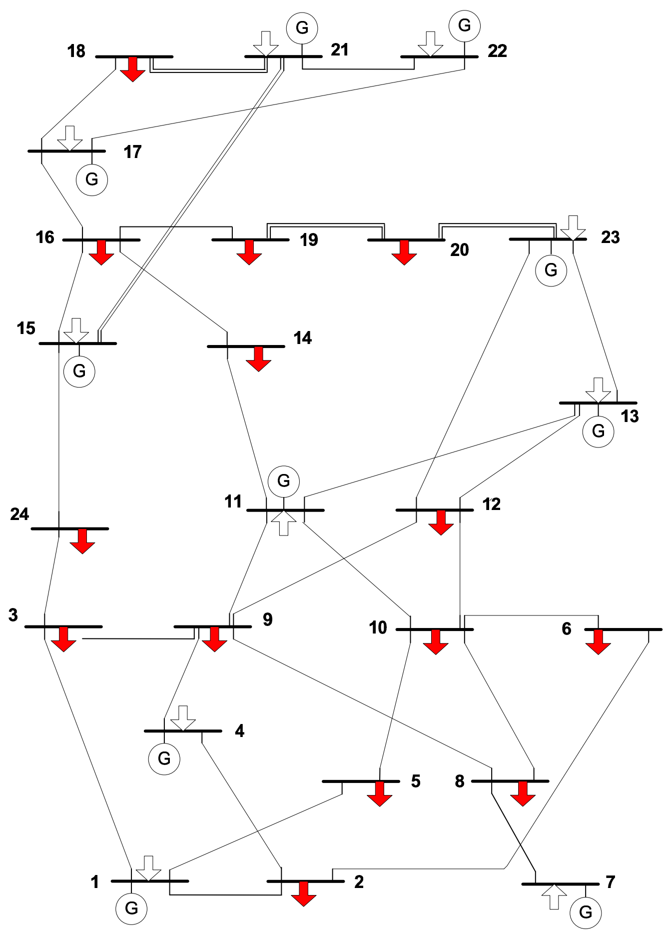

Fig. 1. The 24-bus network

but represent the contingencies of unavailability of the four flowgates respectively. Off-peak state 7 differs from state 1 and 2 with very low demand levels. Table II also illustrates the price intercepts of IDFs as well as the probabilities of the states.

We run tests on the system with single settlement and two settlements respectively, and observe the likelihood of congestion, generator output changes, social welfare changes and the behaviors of the spot nodal and zonal prices due to forward contracting. For the case of two settlements, we test different generator ownership structures with 2, 3, 4, 5 firms respectively. The details of the ownerships are listed in table III.

We observe that with two settlements, the firms have strategic incentives for committing to forward contracts. Further-

TABLE I

NODAL INFORMATION

Node	IDF slope	marg. cost ($/MWh)	capacity (MW)
1	1	30	70
2	.82	-	0
3	1.13	-	0
4	1.1	30	70
5	.93	-	0
6	.85	-	0
7	1	30	70
8	1	-	0
9	.88	-	0
10	.5	-	0
11	1	20	70
12	.73	-	0
13	1	30	70
14	.85	-	0
15	1	25	70
16	1.15	-	0
17	1	20	70
18	.79	-	0
19	.68	-	0
20	1.03	-	0
21	1	25	70
22	1.05	30	70
23	1	20	70
24	.73	-	0

more the incentive for forward contracting are strengthened by increased diversification in ownership. Figure 2 compares the total forward contracting quantities with different numbers of firms. It shows that the total forward contract quantity increases from 60MW with 2 firms to 640MW with 5 firms. Comparing further the results of two settlements to those of single settlement, we find that

- In all states of the spot market, the aggregated spot outputs are increased under two settlements; moreover, the more firms in the markets, the greater is such effect (see figure 3). Despite this phenomenon, some generators still decrease the outputs in some states. This is because, when facing intensive competition, some firms have to reduce productions of the generators located in the zones with lower spot prices, so as to sustain their profits by increasing their outputs from other plants. For example, when there are 5 firms in the markets, generators at nodes 15 and 21 only increase their production levels in the peak state, but reduce them in other six states (see table IV). Consequently, the expected spot outputs from these generators might be lower under two settlements than those under a single settlement. The expected generation

TABLE III
GENERATOR OWNERSHIP STRUCTURE

Node	number of firms			
	2	3	4	5
1	1	1	1	1
4	1	2	2	2
7	1	2	3	3
11	2	3	4	4
13	2	3	4	5
15	2	1	1	1
17	2	2	2	2
21	2	2	3	3
22	2	3	3	4
23	1	3	4	5

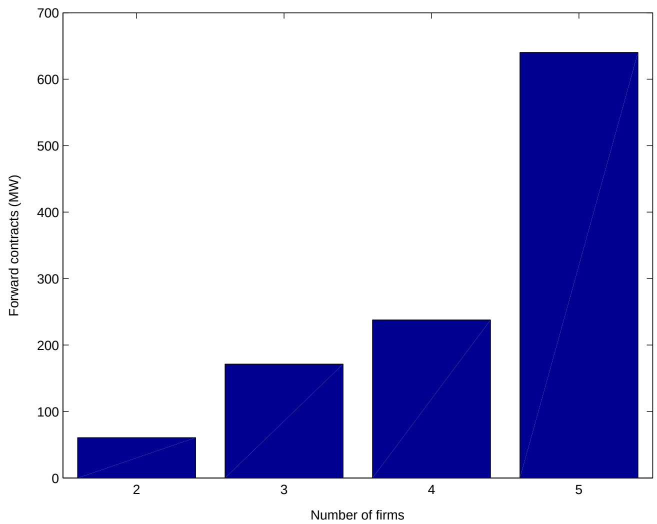

Fig. 2. Forward quantities

quantities are shown in figure 4 with the dark bars denoting the outputs for a single settlement, and the gray and white bars for two settlements with 2 firms and 5 firms respectively. It is shown that, under two settlements with 5 firms, the generators in both nodes 15 and 21 operate at expected levels that are lower than those under a single settlement.

- Spot nodal and zonal prices under two settlements decrease in all states. This follows directly from the fact that the aggregate output is increased in the spot market under two settlements. Figure 5 compares the expected spot nodal prices for a single settlement to those corresponding to two settlements with 2 and 5 firms respectively. The prices under a single settlement are drawn dark, while the prices under two settlements are in gray with 2 firms, and white with 5 firms. In table V, we report the spot zonal prices under a single settlement in columns 2 and 3, and the spot zonal prices under two settlements in column 4 through 7. The last row of this table lists the forward zonal prices. It is also shown in figure 5 and table V that the more firms compete in the two-settlement system, the lower are the spot nodal and zonal prices.
- Social surplus increases under two settlements. Moreover, the social welfare of the two-settlement system increases as the number of firms increases. The expected social welfare of a single settlement is \$7796/h which, under two settlements, is increased to \$8133/h with 2 firms and \$9383/h with 5 firms. Figure 6 shows that the same trend applies to the consumer and producer surpluses. These results are consistent with those of [2].
- Lines not congested in the single-settlement system might be congested in the spot market of the two-settlement system, or vice versa. This follows from the fact that the firms adjust their outputs, which alters the electricity flows on the transmission lines. For example, in state 3, the single-settlement market has only line [11,14] congested, however, the only congested line under two settlements with five firms is line [12,23] (see table VI).

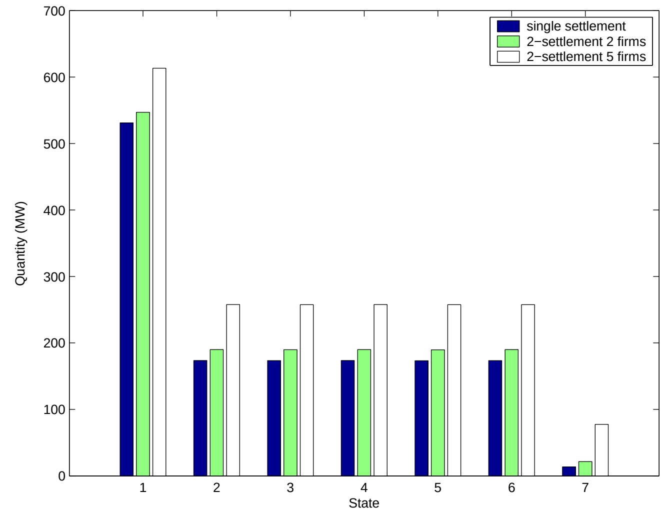

Fig. 3. Spot aggregated generation

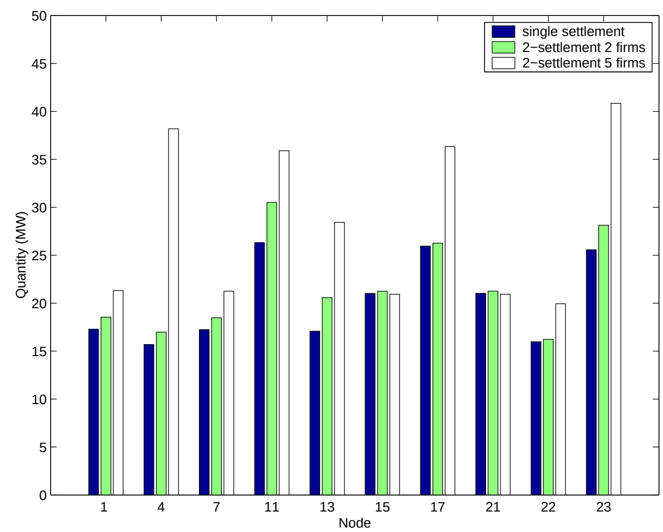

Fig. 4. Expected spot nodal generation

Finally, figures 7 and 8 illustrate the spot nodal prices and generator outputs for the seven states under two settlements with two firms. We note that the generators produce at levels between 40MW and 70MW in the peak state, and that only three generators operate in the off-peak state. Compared to their forward contracts, the firms are in fact net buyers in the

TABLE IV

OUTPUT LEVEL CHANGES FROM SINGLE SETTLEMENT TO TWO SETTLEMENTS WITH 5 FIRMS (MW)

state	node 1	node 4	node 7	node 11	node 13	node 15	node 17	node 21	node 22	node 23
1	4.6704	22.9899	4.4798	8.5305	11.2008	0.7479	11.1849	0.7684	5.5010	12.1438
2	4.7752	23.1898	4.7924	9.6474	12.2285	-0.3026	10.0909	-0.3045	4.4713	15.6964
3	4.7138	23.1340	4.7310	9.5054	12.1053	-0.1455	10.2480	-0.1474	4.6209	15.3486
4	4.7119	23.1146	4.6940	9.5360	12.0476	-0.2067	10.1942	-0.2048	4.5675	15.8469
5	4.7139	23.1192	4.7015	9.5613	12.0691	-0.2300	10.1697	-0.2287	4.5446	15.9265
6	4.8146	23.2347	4.8498	9.6961	12.2878	-0.3449	10.0448	-0.3487	4.4285	15.5619
7	0	18.8401	0	10.3492	7.4455	0	10.8999	0	0	16.4569

TABLE V

SPOT AND FORWARD ZONAL PRICES ($/MWH)

	single settlement		two settlements			
			2 firms		5 firms	
	zone 1	zone 2	zone 1	zone 2	zone 1	zone 2
spot: state 1	81.6964	77.7208	80.4398	77.2604	79.1841	75.2275
spot: state 2	43.8607	43.1124	42.7772	42.5626	41.4331	39.1074
spot: state 3	44.0055	42.9175	42.9319	42.3577	41.5875	38.9027
spot: state 4	43.9174	43.0761	42.7188	42.6434	41.4948	39.1408
spot: state 5	43.8697	43.1084	42.8144	42.5160	41.4314	39.2197
spot: state 6	44.0106	42.9529	42.8768	42.4506	41.5921	39.0052
spot: state 7	24.4973	24.4973	24.2061	24.2061	22.3382	21.8949
forward	-	-	45.6468	45.0068	44.2409	41.9395

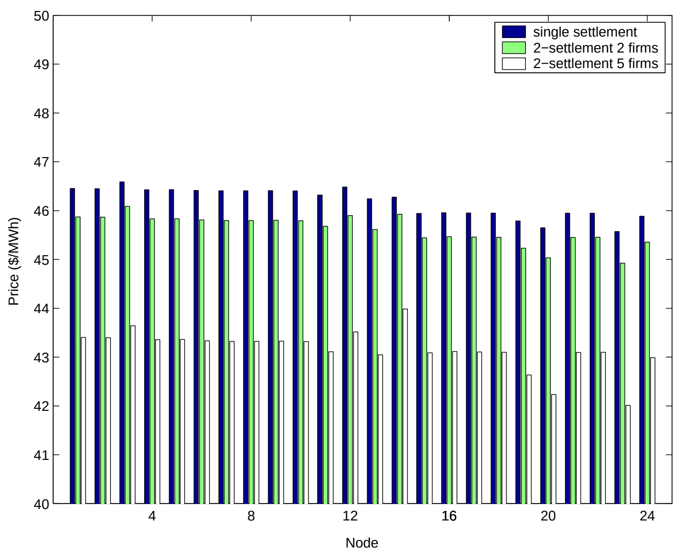

Fig. 5. Expected spot nodal prices

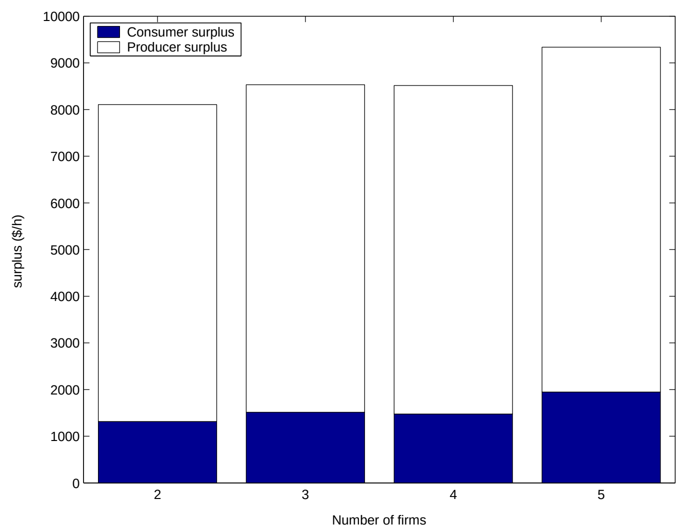

Fig. 6. Social surplus

off-peak state, and net supplies in the peak states.

V. CONCLUDING REMARKS

In this paper, we model the two-settlement electricity system as a two-period game with multiple states of the world in the second period. The Cournot equilibrium is a subgame perfect Nash equilibrium represented in the format of an EPEC. We assume linear demand functions and constant marginal generation costs, so the spot market equilibrium can be computed as a linear complementarity problem. In the forward market, firms solve MPECs subject to the "no-arbitrage" relationship between the forward prices and the expected spot zonal prices, and a linear complementarity problem defining the equilibrium outcomes of the spot market.

We apply our model to the 24-bus network, and observe from it the strategic incentives of the firms for forward contracting, the likelihood of congestion, increased generation quantities, increased social surplus and decreased spot prices with the introduction of a forward market. We also find that these effects are amplified when there are more firms in the network.

We plan to relax the "no-arbitrage" assumption between the forward and spot prices with a market-clearing condition that sets the forward prices based on the expected demands in the spot market. Such analysis will attempt to capture how lack of liquidity (or high risk aversion) on the buyers side might be reflected in a high risk premium embedded in the forward prices. We expect that such condition enhances firms' market

TABLE VI
FLOW CONGESTION COMPARISON

state	line	single settlement	two settlements
1	[3,24]	congested	congested
1	[11,14]	congested	congested
1	[12,23]	congested	congested
1	[13,23]	uncongested	uncongested
2	[3,24]	uncongested	uncongested
2	[11,14]	uncongested	uncongested
2	[12,23]	uncongested	uncongested
2	[13,23]	uncongested	uncongested
3	[3,24]	not available	not available
3	[11,14]	congested	uncongested
3	[12,23]	uncongested	congested
3	[13,23]	uncongested	uncongested
4	[3,24]	uncongested	uncongested
4	[11,14]	not available	not available
4	[12,23]	uncongested	uncongested
4	[13,23]	congested	uncongested
5	[3,24]	uncongested	uncongested
5	[11,14]	uncongested	uncongested
5	[12,23]	not available	not available
5	[13,23]	congested	congested
6	[3,24]	uncongested	uncongested
6	[11,14]	uncongested	uncongested
6	[13,23]	congested	congested
6	[12,23]	not available	not available
7	[3,24]	uncongested	uncongested
7	[11,14]	uncongested	congested
7	[12,23]	uncongested	uncongested
7	[13,23]	uncongested	uncongested

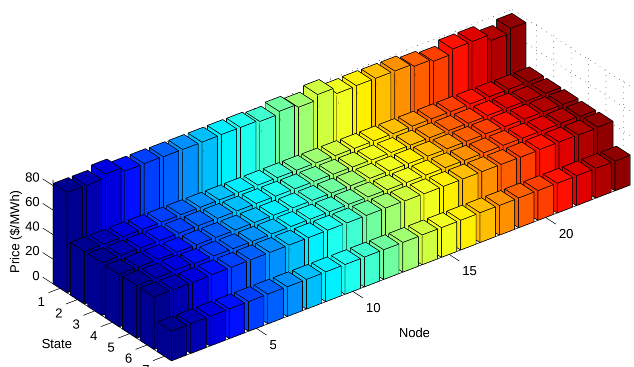

Fig. 7. Spot nodal prices with 2 firms

power and enables them to raise forward prices above the expected spot prices while increasing their profits.

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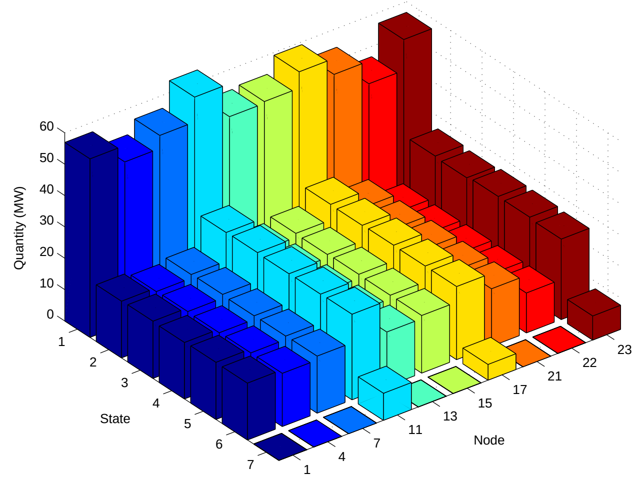

Fig. 8. Spot nodal prices with 2 firms

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