Non-Collocated Power Measurements in a Power System State Estimator

B. Mann, G. Heydt, G. Strickler

Abstract— **This paper addresses a problem in state estimators for power systems. The issue of non-collocated measurements is studied. The measurement of *P+jQ* in a line or at a bus is usually accomplished by measuring voltage and current at an appropriate point. A transducer converts the voltage and current measurement to active and reactive power which are transmitted to the state estimator. If the voltage and current measurements are not at the same point in the circuit, that is the measurements are non-collocated, error is introduced. The paper reports a way to correct for non-collocation of measurements.**

Index Terms—**State estimation, measurements, measurement error, complex power measurement.**

I. Introduction

STATE estimation is a widely used tool in power system energy management systems. The essence of state estimation is that measurements are taken of active and reactive power, and system voltage magnitudes and phase angles (i.e., the 'states') are estimated. The process usually uses minimum least squares methods.

Power system measurements sometimes possess a degree of error, due to anything from instruments installed with reversed polarities to information loss through analog to digital conversion. State estimation methods can flag and smooth out bad measurements, but state estimation has better accuracy overall when there is less error. In a deregulated power market it is increasingly important to find cost effective ways to improve system visibility and account for measurement errors. If the error is due to an incorrectly installed instrument, then a software solution would likely save more money than instrument reinstallation.

One such power measurement error is non-collocated complex power measurements. Complex power is usually calculated from voltage and current measurements, which must be taken at the same place to have any meaning. In the non-collocated case the current transformer (CT) and the potential transformer (PT) of a power measurement instrument is separated by an amount of impedance.

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It will be shown that a more accurate power measurement can be calculated utilizing a non-collocated value plus a correction procedure. This correction calculation requires an accurate knowledge of the impedance between the instruments, the instruments location relative to the impedance and access to a PT reading. Given these values, a non-collocated power measurement can be corrected through calculation rather than a costly and interruptive reinstallation of hardware.

This paper gives detail on the nature of non-collocated measurements and presents a method to calculate more accurate power values. An 11 bus test bed has been created to demonstrate the effect non-collocated power measurement correction has on improving state estimation results over a large system.

The general topic of power system state estimation is well documented in the literature. References [1-3] are a small sampling of the available literature. Concerning error in state estimation, the following are offered as samples of relevant resources:

- Reference [4] describes network topology errors and how to identify and correct these.
- Reference [5] relates to parameter errors.
- Reference [6] describes error in circuit breaker status.
- Reference [7] deals with bad data measurements.

II. Non collocated power measurements

Complex power is a function of the complex voltage and current, where $S=VI^*$. Power measuring wattmeters are a very common part of power systems instrumentation. These power measurement devices work by sampling the voltage, *v(t)*, and the current, *i(t)*, which are turned into a digital signal by an analog / digital converter and the power is calculated by a transducer. This value is provided to a remote system operator and sometimes the individual voltage or current measurement as well. When measuring or calculating complex power, the voltage and current must be taken from the same place in the system. In the case of a non-collocated power measurement, this is not the case. The impedance between the CT and PT can be represented as the circuit shown in Fig. 1. In the case of a non-collocated measurement it is assumed that the instruments are not far apart and well within the range of typical short line modeling limits. Because the CT and PT are often not separated by even a kilometer, resistance in the impedance model has been assumed to be negligible. An example of a non-collocated instrument placement is shown in Fig. 2.

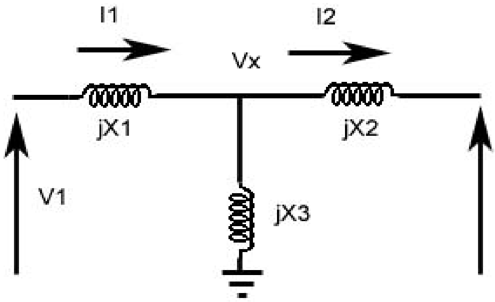

Figure 1 A model for the reactance between CT and PT in a non-collocated measurement

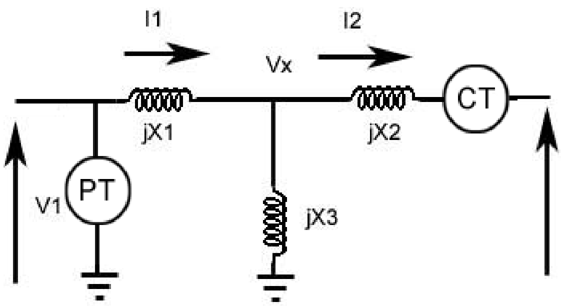

Figure 2 An example of a non-collocated power measurement instrument placing

When discussing power values in this paper, the notation S_{ab} will be used. The subscript a will be the same subscript as the voltage used to calculate S_{ab} and the subscript b will be the subscript of the current used to calculate S_{ab}. For instance, meaningful power values from the system in Fig. 1 would be S_{11} and S_{22}, which are calculated as follows,

$$S_{11} = V_1 I_1^*$$
$$S_{22} = V_2 I_2^* .$$

Examples of non-collocated power measurements from the system in Fig. 1 would be S_{12} (shown with instrument placing in Fig. 2) or S_{21}. Each of these is calculated as follows,

$$S_{12} = V_1 I_2^*$$
$$S_{21} = V_2 I_1^* .$$

It is possible to calculate all of the power, voltage, and current values of the circuit in Fig. 1 (including S_{11} and S_{22}) given only the systems reactance, X_1, X_2, and X_3, a voltage magnitude, $|V_1|$ or $|V_2|$, and a non-collocated complex power measurement, S_{12} or S_{21}, which are values from the circuit equation shown in Fig. 1. This can be done using one of two methods: Case A and Case B, which will be detailed in this section. Case A is for instances when the voltage of the non-collocated power is known and Case B is used when a voltage is know that isn't part of the non-collocated power value.

One method for correcting non-collocated measurements will be called the "Case A" method. For Case A, the non-collocated measurement S_{12} is known, as well as $|V_1|$, X_1, X_2, and X_3. S_{11} and S_{22} can be calculated. This method can also be changed to work with S_{21} and $|V_2|$ being known values, which will be explained later. First, the voltage magnitude V_1 can be made the reference voltage, so

$$V_1 = 1\angle 0° .$$

Since $S_{12} = V_1 I_2^*$ and S_{12} and V_1 is known, I_2 can be calculated immediately.

$$I_2 = \frac{S_{12}^*}{V_1^*}$$

To calculate S_{11} and S_{22}, V_2 and I_1 are needed. The following matrix equation can be formed to solve for the remaining unknowns,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\begin{bmatrix} X_1 + X_2 & X_2 \\ X_2 & X_2 + X_3 \end{bmatrix}\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}. \tag{1}$$

Equations (1) are a simple consequence of the bus impedance analysis of a linear AC circuit, namely $V_{bus} = Z_{bus} I_{bus}$, where Z_{bus} is the bus impedance matrix referred to ground [8].

Equation (1) can be multiplied out to give two equations that can be solved for the two unknowns, V_2 and I_1. Solving for V_2 and I_1 yields the following equations

$$\frac{1}{j(X_1 + X_3)}\begin{bmatrix} jX_3 & X_1X_2 + X_1X_3 + X_2X_3 \\ 1 & jX_3 \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}.$$

The matrix relationship simplifies into the following

$$V_2 = \frac{-jX_3V_1 - I_2(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)}{-j(X_1 - X_3)}$$

$$I_1 = \frac{-V_1 - jX_3I_2}{-j(X_1 - X_3)}$$

This Case A method can be changed to work when S_{21} and $|V_2|$ are known. The diagram in Fig. 3 shows how the polarities of these values are changed using Case A with these numbers. The preceding method can be used, but V_2 is used in place of V_1, V_1 is used in place as V_2, I_2 is replaced by $-I_1$, I_1 is replaced by $-I_2$, X_2 is used for X_1 and X_1 is used for X_2. For instance, in the first step of Case A, it is $|V_2|$ that becomes the reference voltage instead of $|V_1|$. V_2 and S_{21} next solve for $-I_1$, and so on. A detailed guide for parameter replacement is shown in Table 1.

Table 1 A parameter replacement guide to use Case A with S_{21} and V_2 known or Case B with S_{12} and V_2

Parameter	Replace With
V_1	V_2
V_2	V_1
I_1	$-I_2$
I_2	$-I_1$
X_1	X_2
X_2	X_1
S_{21}	S_{12}
S_{12}	S_{21}

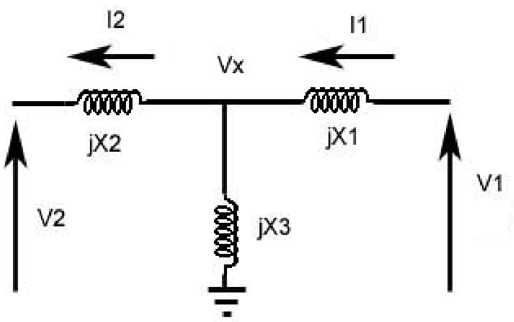

Figure 3 Circuit diagram for using Case A to calculate power given V_2 and S_{21}

The next method is called the "Case B" method. This is for when S_{21} and $|V_1|$ are known, along with the reactance X_1, X_2, and X_3. It will later be shown how Case B can solve for the instance where S_{12} and $|V_2|$ are known.

Since one of the current cannot be immediately calculated with the given voltage and non-collocated power, Case B is a bit more difficult. To solve for S_{11} and S_{22}, a new circuit model is needed, shown in Fig. 4.

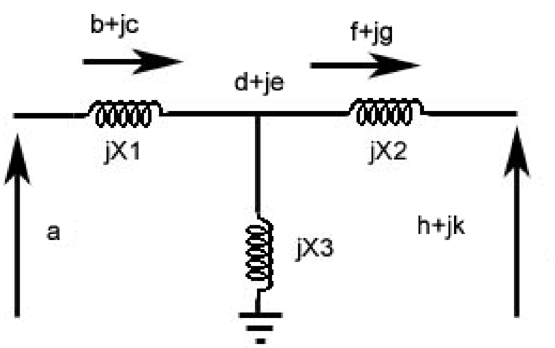

Figure 4 A new notation for the non-collocated impedance model for use in Case B

The given voltage is again the reference voltage, so using the new notation given in Fig. 4

$$|V_1| = V_1\angle 0° = a\,.$$

Using basic power relationships, where $S=VI^*$, $P=Real\{S\}$, and $Q=Imag\{S\}$, the following relationships can be made using the new notation

$$P_{11} = ab$$
$$Q_{11} = -ac$$
$$P_{22} = hf + kg\ .$$
$$Q_{22} = kf - hg$$
$$P_{21} = hb + kc$$
$$Q_{21} = bk - hc$$

Currently a, P_{21}, and Q_{21} are known values. To help solve for the rest, more equations can be made using the relationship between V_1 and V_X, V_X and V_2 and Kirchhoff's current law,

$$d + je = a - (b + jc)(jX_1)$$
$$h + jk = d + je - (f + jg)(jX_2)$$
$$b + jc = \frac{d + je}{jX_3} + f + jg$$

Here X_1, X_2, and X_3 are known values, along with a. Using these equations and the power relationships, there are eight equations with eight unknown parameters. The eight equations can be simplified even more to the following

$$d = a + cX_1$$
$$e = -bX_1$$
$$h = d + gX_2$$
$$k = e - fX_2$$
$$b = f + \frac{e}{X_3}$$
$$c = g - \frac{d}{X_3}$$
$$P_{21} = hb + kc$$
$$Q_{21} = bk + hc\ .$$

Solving for the unknown parameters is a simple but time consuming process and left as an exercise for the reader. Once all of the parameters in Fig. 4 are solved for, the real and unreal parts of S_{11} and S_{22} can be calculated with the following equations

$$P_{11} = P_{21}\left(\frac{X_3}{X_3 + X_2}\right)$$
$$a = V_1 = 1\angle 0°$$
$$b = P_{11} = P_{21}\left(\frac{X_3}{X_3 + X_2}\right)$$
$$k = \frac{-P_{21}}{(X_2 + X_3)}(X_3X_1 + X_2X_3 + X_1X_2)$$
$$Q_{11} = -\left(\frac{P_{21} - \sqrt{{P_{21}}^2 - 4k^2b^2 + 4kbQ_{21}}}{2k}\right)$$
$$S_{11} = P_{11} + Q_{11}$$
$$I_1 = \frac{{S_{11}}^*}{{V_1}^*}$$
$$V_2 = \frac{S_{21}}{{I_1}^*}$$
$$I_2 = I_1 - \frac{V_1 - jX_1I_1}{jX_3}$$

The Case B method equations shown directly above can also be used when S_{12} and V_2 are known. The diagram in Fig. 3 shows how the polarities of these values are changed using Case B with these numbers. The Case B method can be used, but V_2 is used in place of V_1, V_1 is used in place as V_2, I_2 is replaced by $-I_1$, I_1 is replaced by $-I_2$, X_2 is used for X_1 and X_1 is used for X_2. For instance, in the first step of Case A, it is $|V_2|$ that becomes the reference voltage instead of $|V_1|$. V_2 and

S_{21} next solve for $-I_1$, and so on. A detailed guide for parameter replacement is shown in Table 1, which works for Case A and Case B.

Case A and Case B show that complex power can be calculated from a non-collocated measurement. A local voltage measurement and a detailed model of the local impedance are required along with the non-collocated power measurement.

III. ILLUSTRATIVE EXAMPLE

To illustrate the effect of non-collocated power measurements, an 11 bus test bed has been created. The test bed has been loosely based on the 500 kV transmission line power system in the United States southwest. The reactance and network configurations have been "invented" to obtain a convenient test bed. The black diamonds in Fig. 5 represent wattmeters. Line parameters were calculated from the actual line configurations and approximate line length. These parameters are shown in Table 2 in per unit on a 100 MVA, 500 kV base. Each bus was assigned a per unit voltage value. Knowing the bus voltage and line parameters allowed calculation of the individual line currents and complex power flow. The 11 bus system and state estimation calculations were performed using MATLAB mathematical software. The goal in this test bed is to introduce random error into the measurements and also a non-collocated measurement to one of the buses to study the effect on state estimation. The no-error case is used as a basis of comparison.

Table 2 Line reactances for test bed, on a 100 MVA, 500 kV base

Line Parameter	Reactance (p.u.)
Xa	0.017
Xb	0.004
Xc	0.0335
Xd	0.0015
Xe	0.0062
Xf	0.0483
Xg	0.0064
Xh	0.0206
Xj	0.0039
Xk	0.0099
Xl	0.0156

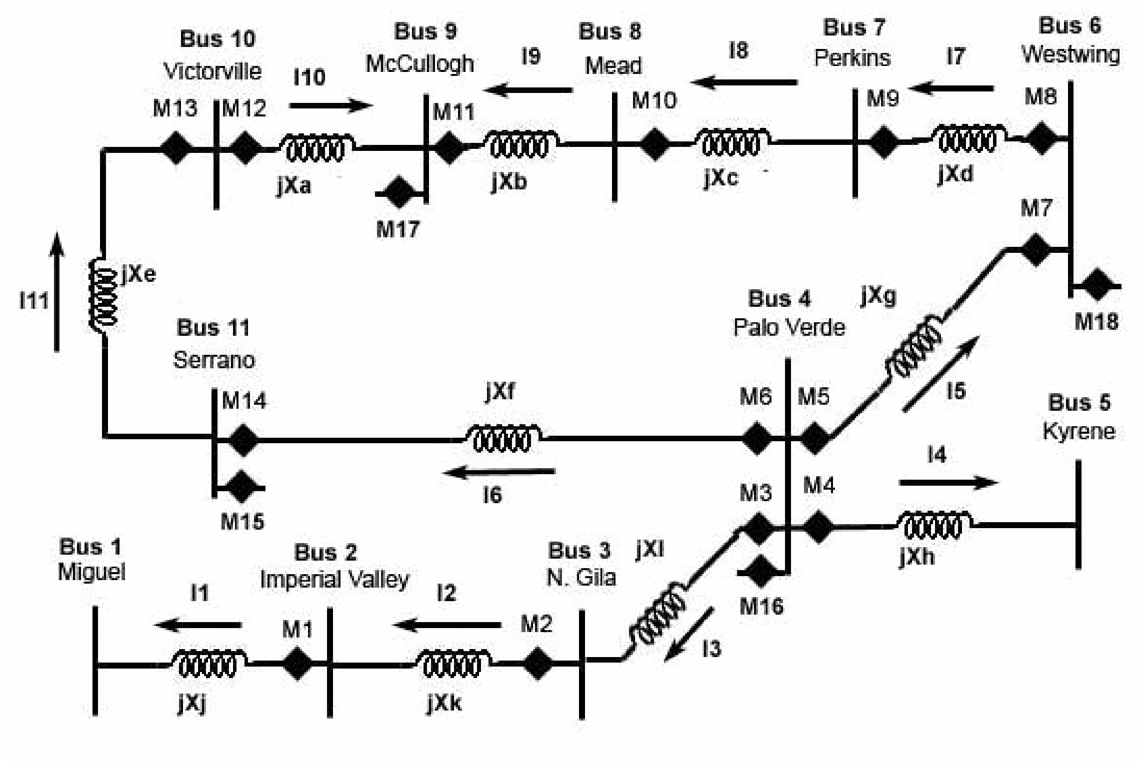

Figure 5 An 11 bus test bed inspired by the 500 kV lines of the US southwest

The test bed uses 18 power instruments to estimate the voltage angle and magnitude at each bus. Least squares state estimation is the most common form of state estimation used by power utilities and will be used in this example. Least squares state estimation multiplies a vector of measurements [z] to the pseudo-inverse of a matrix [H], which gives a state estimate vector [x]. The matrix [H] is a matrix of coefficients that is N_m (the number of measurements) by N_s (the number of states). Least squares state estimation is improved by a large number of measurements and in power engineering the case is always overdetermined, where $N_m>N_s$. The final unweighted, overdetermined case is shown here

$$[x] = \left[[H]^T[H]\right]^{-1}[H]^T[z].$$

For calculating the bus angles with real power measurements, a simplified version of the following power flow equation is used.

$$P = \frac{|V_1||V_2|}{X}\sin(\delta_1 - \delta_2)\cdot$$

Since the voltages are all very close to 1 per unit during the steady state and the angles very close to zero, the general power flow equation can be simplified. The sine of small angles is near the angle itself. The following power equation is used.

$$P = \frac{1}{X}(\delta_1 - \delta_2)\cdot$$

In the context of least squares state estimation, the matrix [z] is made of real power measurements and the matrix [H] is made from the inverse of line reactance. The state matrix [x] is composed of the bus reference angles to be estimated.

Now the state estimation equation for the imaginary power is created. Again voltages are assumed to be very close to 1 per unit value and the reference angles close to zero. The equation can be simplified as follows.

$$V_1 = 1 + \Delta V_1$$

$$V_2 = 1 + \Delta V_2$$

$$Q = \frac{|V_1|^2 - |V_1||V_2|\cos(\Delta\delta)}{X} \approx \frac{1}{X}\left[\Delta V_1 - \Delta V_2\right]$$

Whether calculating voltage magnitude or angle, the [H] matrix can be made from the inverse of each lines reactance. Here the measurement matrix [z] is the reactive power measurements and the state vector [x] is made of bus voltage magnitudes.

The test bed will have introduced a non-collocated measurement at Bus 2 for the non-collocation case studies. The impedance used in all instances is shown in Table 3, relative to the general non-collocated impedance circuit shown in Fig. 1. The X_1 and X_2 impedance values could represent large series capacitors used to decrease the change in voltage angle over these lines that somehow got installed between the wattmeter CT and PT. The non-collocated power instrument will calculate power from the V_2 and I_1 positions, relative to the diagram in Fig. 1. The power at instrument *M1* should read 5.14 per unit, but in this non-collocated instance it reads 1.05 per unit.

Table 3 Test bed non-collocated reactance values, on a 100 MVA and 500 kV base

	Value in per unit
X_1	-0.01022
X_2	-0.00995
X_3	1/1.14

The test will be conducted in five cases, called Cases 0, 1, 2, 3, 4, and 5. For each case, the complex power measurements are used to calculate the state variables δ and $|V|$. The state variables will have the no-error, no non-collocation Case 0 results subtracted from them, giving δ-$\delta_{no\text{-}error}$ and $|V|$-$|V|_{no\text{-}error}$ for each Case. The δ-$\delta_{no\text{-}error}$ and $|V|$-$|V|_{no\text{-}error}$ values will be calculated 1000 times and the average value will be analyzed by observing the 2-norm residual, mean, and variance. The Case 0 will have no power measurements error and no non-collocated instruments, so the δ-$\delta_{no\text{-}error}$ and $|V|$-$|V|_{no\text{-}error}$ values are expected to be close to zero. Case 1 will have power measurements with a random amount of error, with a maximum error of 10%, and no non-collocated power instruments. Case 2 will have power measurements with a random amount of error, with a maximum error of 30%, and have no non-collocated power measurements. Case 3 will have power measurements with a random amount of error, with a maximum error of 10%, and one non-collocated power instrument at Bus 2. Case 4 will have power measurements with a random amount of error, with a maximum error of 30%, and one non-collocated power instrument at Bus 2. The percent error represents typical random error in power systems measurements. It is hoped that by comparing Cases 0, 1, 2, 3 and 4 that the amount of error due to random noise (the percent error in cases 1, 2, 3, and 4) and the non-collocated instrument (Cases 3 and 4 only) can be discerned. The results of this test are shown in Tables 4, 5, 6, and 7.

Table 4 Test bed average δ after 1000 runs compared to actual δ, for all cases

		State δ-$\delta_{\text{no-error}}$		
Case	Noise	$\|\|r\|\|_2$ of δ	Mean	Variance
0	none	0.0945948	0.028521	1.62E-32
1	10%	0.0945958	0.028521	5.13E-09
2	30%	0.094598	0.028521	5.26E-08
3	10%, one non-collocated measurement	0.095822	0.028521	2.39E-05
4	30%, one non-collocated measurement	0.09586	0.028521	2.43E-05

Table 5 Test bed average $|V|$ after 1000 runs compared to actual $|V|$, for all cases

		State $\lvert V\rvert$-$\lvert V\rvert_{\text{no-error}}$		
Case	Noise	$\lVert r\rVert_2$ of $\lvert V\rvert$	Mean	Variance
0	none	0.081075	-0.022879	8.18E-05
1	10%	0.081079	-0.022879	8.20E-05
2	30%	0.081119	-0.022879	8.11E-05
3	10%, one non-collocated measurement	0.081558	-0.022879	8.88E-05
4	30%, one non-collocated measurement	0.081511	-0.022879	8.92E-05

Table 6 Test bed average P measurement after 1000 runs compared to actual P, for all cases

		P-$P_{\text{no-error}}$		
Case	Noise	$\lVert r\rVert_2$ of P	Mean	Variance
0	none	0.098162	-0.003115	5.57E-04
1	10%	0.107574	-0.006990	9.39E-04
2	30%	0.239030	-0.035610	0.002054
3	10%, one non-collocated measurement	4.101085	-0.2225	0.929933
4	30%, one non-collocated measurement	4.101085	-0.2225	0.934167

Table 7 Test bed average Q measurement after 1000 runs compared to actual Q, for all cases

		Q-$Q_{\text{no-error}}$		
Case	Noise	$\lVert r\rVert_2$ of Q	Mean	Variance
0	none	1.676460	0.008759	0.165243
1	10%	1.447761	0.086734	0.115952
2	30%	1.448581	0.084969	0.1149
3	10%, one non-collocated measurement	2.16158	-0.006283	0.274566
4	30%, one non-collocated measurement	2.155998	-0.003878	0.275201

IV. Some Observations drawn from the examples

The complex power resulting from each test Case is shown in Tables 6 and 7. In general, the random measurement error in Cases 1 and 2 increases the 2–norm residual for P-$P_{\text{no-error}}$ and the mean difference for the real power moves away from zero, all relative to the no-error Case 0. For both the real and imaginary power comparisons, the variance increases due to measurement error. Subsequently, the measurement error alone increases overall measurement error and variance. Cases 3 and 4 possess measurement error and a single non-collocated measurement. In Cases 3 and 4 the average P-$P_{\text{no-error}}$ and Q-$Q_{\text{no-error}}$ values result in a larger 2-norm residual, a mean difference further away from zero for P-$P_{\text{no-error}}$, and a larger variance relative to Cases 0, 1 and 2. Because of this, it can be said that a single non-collocated instrument increased the measurement error in the test bed and the variance.

The effect on the estimation of states is more subtle than the direct power measurements. The state variable average differences from the no-error case are shown in Tables 4 and 5. Table 4 shows that Cases 1 and 2, which contain only measurement error, differ little from the no-error Case 0, the only exception being a marked increase in variance in the δ-$\delta_{\text{no-error}}$ calculations in Cases 1 and 2 relative to Case 0. Measurement error alone then only introduces variability into the bus phase angle calculations. When a single non-collocated measurement is added during Cases 3 and 4 the result is a small 2-norm residual increase and variance increase for the δ-$\delta_{\text{no-error}}$ and $|V|$-$|V|_{\text{no-error}}$ measurements relative to Cases 0, 1 and 2. The variance change is smaller for the $|V|$-$|V|_{\text{no-error}}$ measurement than it is for the δ-$\delta_{\text{no-error}}$ measurement. This shows that in power system state estimation, a single non-collocated instrument can widen the variability of calculated bus voltage angles and magnitudes and increase error.

Applying the non-collocated measurement calculations discussed in Section II would eliminate the non-collocated error. In this case, since the non-collocated power is calculated from V_2 and I_1, so Case B from Section II would be the appropriate fix. After the power measurement adjustment, the Case 3 and 4 results would resemble the Cases 1 and 2 results. By comparing Cases 1 and 2 to Cases 3 and 4, the benefits become apparent: there is a lower amount of measurement variance and a lower 2-norm residual in Cases 1 and 2. Therefore, accounting for non-collocated measurement would increase state estimation confidence and will create more accurate measurements system-wide.

V. Conclusion

Non-collocated measurements in a power system can present a source of error. However, with an accurate knowledge of reactance and transformer configuration at the power transducer instrument, it is possible to correct the faulty power measurement and improve state estimation while avoiding costly instrument reinstallation / reconfiguration. In cases where there are non-collocated power measurements, this calculation has the potential to improve state estimation results and system-wide measurement confidence.

VI. ACKNOWLEDGEMENTS

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VIII. BIOGRAPHIES

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