Competitive Acquisition of Prioritizable Capacity-Based Ancillary Services

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***Abstract*—The rational buyer procedure provides the competitive procurement of capacity-based ancillary services (AS) in unbundled markets by the independent grid operator (IGO). The capacity-based AS are prioritized in order of ascending response times. Prioritization allows substitutability of the AS by automatically making the unused capacity of a higher priority AS usable for any lower priority AS without the need of submitting additional offers. We develop an efficient scheme for the rational buyer procedure for the acquisition of the prioritizable capacity-based AS. The scheme allows the simultaneous determination of the successful offers in the multiauction procedure through the effective deployment of discrete programming notions and the exploitation of the structural characteristics of the formulation. A key feature is the incorporation of physical constraints such as capacity, ramp-rate, and interzonal constraints. The use of bounding techniques combined with procedures for the quick detection of infeasible combinations of the offer prices and the identification of avoidable calculations leads to reducing the computational burden. The effectiveness and computational efficiency of the scheme are illustrated with representative numerical results including case studies based on the IEEE 118-bus network.**

***Index Terms*—Ancillary service provision, discrete programming, independent grid operator, prioritizable capacity-based services, ramp-rate constraints, rational buyer, simultaneous auctions.**

I. Introduction

THE experience of restructuring in California [1] and in other jurisdictions, such as in England and Wales [2], shows that the acquisition of ancillary services can have a critical impact on wholesale electricity prices. This fact is clearly recognized in the standard market design (SMD) area [3], where a key objective is to effectively integrate the markets for ancillary services into the wholesale electricity market through the formulation of workable rules and frameworks. The provision of AS is the responsibility of the independent grid operator (IGO). We use this term for the entity that operates and controls the network in its generic sense, so as to encompass existing implementations such as independent system operators (ISOs) [4], regional transmission organizations (RTOs) [5], independent transmission providers (ITPs) introduced in the SMD proposal [3], and transmission system operators (TSOs) [6]. The IGO must ensure that there are adequate supplies of the various AS and to acquire them cost effectively.

In this paper, we address the market-based acquisition of a subset of the ancillary services (AS) that are capacity based. Capacity-based AS may be acquired competitively [7]. The objective of this paper is to propose an efficient procedure for the purchase of these services in a way that effectively exploits competition in capacity-based services. We focus on the acquisition of capacity-based AS by the IGO to meet the requirements of providing transmission service. These capacity-based AS include upward frequency control, reserve services supplied by unloaded capacity of both on and offline resources, and load following. The IGO specifies the time response and amount requirements for each AS. The capacity-based AS are *prioritized* on the basis of their response times, with the highest (lowest) priority associated with the shortest (longest) response time. To harness the benefits of competition, the IGO holds auctions to procure the capacity-based AS. The prioritization allows *substitutability* in the provision of these services [9]. Specifically, this means that the unused capacity offered for higher priority prioritizable capacity-based AS (PCAS) may be automatically used for any lower priority PCAS without the need for submitting additional offers. The development of procurement procedures has been reported recently in [7], [8], [10], and [11].

In this paper, we discuss the formulation of the auction structures for the acquisition of PCAS. We propose a scheme for determining the least-cost strategy for the IGO to acquire the required PCAS by using the *rational buyer* (RB) approach. The superiority of RB over other procedures for the acquisition of PCAS has been established [8], [9], [11]–[14]. Our focus is on the development of a computationally efficient scheme for the RB approach. We apply discrete programming notions to determine simultaneously the market outcomes of the multiple auctions for the prioritized services. The proposed scheme provides the solution at a specified time and as such may be used for the solution of both the hour-ahead auction and a single hour auction in the day-ahead market. A salient feature is the ability of the scheme to incorporate physical considerations such as seller capacity, unit ramp-rate limits and interzonal constraints, which are critical in the determination of the sellers' limiting capacities. The proposed scheme may be incorporated as the basic scheme in a general procedure for AS procurement with time-dependent coupling such as when the effects of ramp-rate constraints are taken into account. In addition, it may be easily adapted to solve the problem of the simultaneous auctions of energy services and AS. The scheme is implemented by applying good bounding techniques combined with procedures for the

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quick detection of infeasible combinations of the offer prices and the identification of *avoidable calculations* requiring no further evaluations of feasible combinations. These procedures lead to major reductions in computation for the determination of the optimal combination. The paper presents significant improvements of the earlier work by the authors reported in [9] and [14]. These initial efforts are extended in two important ways: the simplification and increased efficiency of the computational procedure and the presentation of appropriate illustrative examples in the application of the proposed scheme. We present in this paper the effective discrete programming procedure we developed to exploit extensively the structural characteristics of the problem formulation. We illustrate the key elements of the scheme using a simple system and provide numerical results on various systems including studies based on the IEEE 118-bus network.

The body of this paper is contained in the next three sections. We start with a description of the characteristics of the capacity-based AS and the salient aspects of their competitive acquisition by the IGO in Section II. We develop an appropriate analytical structure to allow the solution scheme to effectively exploit the structural characteristics of the rational buyer approach to the capacity-based AS acquisition problem. We discuss the proposed scheme in Section III and illustrate it with a simple example. Numerical results are provided in Section IV to pinpoint the effectiveness of the proposed procedure. We conclude with a brief summary in the last section.

II. Provision of Prioritizable Capacity-Based AS

The SMD must accommodate the acquisition of AS on a competitive basis, whenever these services can be provided in this way. The *capacity-based* AS are required to maintain secure operation of the power system. The services included in this group are:

- *frequency regulation*: the basic AGC service to track the load with the generation so as to ensure that the frequency stays within a predefined band of the system synchronous frequency; this service requires both the up and the down shifting of the output level of the unit that provides the service;
- *reserves*: unloaded capacity available within the specified response time required for system operations to withstand unexpected generation outages and increases in the forecast demand, and used to allow the continued operation after system undergoes outages and/or unexpected variations in the demand; reserves may be provided by either online generators loaded below their maximum capacity or offline generation sources having a response capability to meet requirements.

These capacity-based AS may be classified in terms of their required response times. For a specified t-minute response, the capacity must be fully operational within the specified t-minutes. The services with the shortest (longest) response time are assigned highest (lowest) priority. Clearly, a service whose required response time is t minutes may be used for all lower priority services whose required response time $t' > t$. For this prioritization to work, the use of additional capacity must be in a uniform direction. Consequently, the *down direction* for frequency control and load following cannot be grouped into the prioritizable capacity-based AS or PCAS aimed at providing additional capacity to the system. The PCAS are ordered on the basis of their required response times. Frequency control has the highest priority, followed by the other services in ascending order of their response times.[1]

Typically, the acquisition of PCAS is considered once the energy markets have cleared and the congestion management issues have been addressed. For each hour h of the day-ahead market, the IGO defines the requirements for each AS. To take advantage of competitive conditions, the PCAS are acquired through day-ahead hourly auctions for each AS. Therefore, for the 24-h day-ahead market, there are hourly auctions for each PCAS. The sellers have no possibility to update their offers once submitted. However, the prioritization allows the relationships among the auctions for the different PCAS for the same hour to be fully exploited.

We develop notation for our discussion of the PCAS acquisition problem. We indicate explicitly the hour h in considering the auction in that hour. We consider $a^1, a^2, \ldots, a^N$ to be the PCAS, where a^1 has the highest priority and a^N the lowest. We define the set $\mathcal{M}(h) = \{m : m = 1, 2, \ldots, M(h)\}$ of the sellers submitting offers for the PCAS in hour h. For each PCAS a^i, $i = 1, \ldots, N$, we define $r^i(h)$ to be the required capacity for the PCAS a^i in hour h, the subset $\mathcal{S}^i(h) \subseteq \mathcal{M}(h)$ of the sellers submitting offers to provide the PCAS a^i in hour h, with $\mathcal{S}^i(h) = \{s^i_\nu(h) : \nu = 1, 2, \ldots, S^i(h)\}$, and the set of submitted offers $\mathcal{B}^i(h) = \{(c^i_j(h), \rho^i_j(h)) : j \in \mathcal{S}^i(h)\}$ for the capacity $c^i_j(h)$ at the price $\rho^i_j(h)$. The total capacity offer for the PCAS a^i in hour h is

$$b^i(h) = \sum_{j \in \mathcal{S}^i(h)} c^i_j(h), \quad i = 1, \ldots, N \tag{1}$$

and is assumed to be sufficient to cover the required demand for the AS a^i

$$b^i(h) \geq r^i(h), \quad i = 1, \ldots, N. \tag{2}$$

Note that in (2), $r^i(h)$ is constant and is independent of price. For each PCAS a^i, $i = 1, \ldots, N$, the offers in $\mathcal{B}^i(h)$ are ordered in ascending order of the prices, to construct the *supply curve*. The IGO then determines the market clearing price using a *uniform price* auction [15] with this supply curve, resulting in the clearing quantity $\bar{C}^i(h)$ and the clearing price $\bar{\rho}^i(h)$. We denote by $\bar{c}^i_j(h)$ the offer capacity for PCAS a^i accepted from the seller $j \in \mathcal{S}^i(h)$. Then, the accepted total capacity offer for a^i in hour h is

$$\bar{C}^i(h) = \sum_{j \in \mathcal{S}^i(h)} \bar{c}^i_j(h), \quad i = 1, \ldots, N. \tag{3}$$

[1]The so-called load following services, that involve nonautomatic response of the generating units to the IGO signals to maintain supply-demand balance in actual operations, with positive and negative variation in the real power generation of the contributing units, may be included in PCAS once their response times are specified.

To express the contribution of seller $m = 1, \ldots, M(h)$ to the provision of PCAS a^i, we use the *participation indicator*

$$\delta_m^i(h) = \begin{cases} 1 & m \in \mathcal{S}^i(h) \\ 0 & m \notin \mathcal{S}^i(h) \end{cases} \tag{4}$$

The constraints are set up in terms of limiting capacities. The limiting capacity $C_m^{\max}(h)$ that the seller $m \in \mathcal{M}(h)$ can physically provide is a function of the capacity $C_m^{en}(h)$ committed to provide megawatt-hours (MWh) at hour h, the interzonal constraint $C_m^{sup}(h)$ representing the maximum allowable capacity transferable from the seller m without causing network congestion, and the ramp-rate constraint $R_m(h)$ expressed in megawatts per hour, related to the capacity variation from hour $h-1$ to hour h. The time-varying nature of the ramp-rate constraint is due to the units used by each seller to provide the ramping. The capacity used by seller m at hour $(h-1)$ is

$$C_m(h-1) = C_m^{en}(h-1) + \sum_{i=1}^{N} c_m^i(h-1)\delta_m^i(h-1) \tag{5}$$

and the limiting capacity of seller m at hour h becomes

$$C_m^{\max}(h) = \max\{\min\{\mathrm{C_m(h-1)} + R_m(h), C_m^{\sup}(h)\} - C_m^{en}(h), 0\}. \tag{6}$$

The proposed scheme may be easily embedded into a general procedure used for simultaneously determining the outcomes of the ES and AS markets, in which the determination of the ES and the AS are coupled through their values $C_m^{sup}(h)$ and $C_m^{en}(h)$. In addition, it is possible to include the scheme into a day-ahead market framework, in which the auctions at each hour are coupled in time by the effects of ramp-rate constraints, by specifying the values of $C_m^{sup}(h)$, $C_m^{en}(h-1)$, $C_m^{en}(h)$, and $R_m(h)$ in the RB procedure. Appendix A provides a comprehensive list of the notation used in the paper.

We start with the structure of the auction for PCAS a^i in an arbitrary hour h. We determine the selection of the successful offers by constructing a supply curve for the submitted offers $\mathcal{B}^i(h)$. For this auction, the demand is fixed and so the clearing quantity equals the required demand $r^i(h)$. The clearing price in the *uniform price* auction is determined from the supply function and is, typically, the price of the highest accepted offer, as shown in Fig. 1. The construct discussed may be generalized to the case of a price-dependent demand, in which case we have a monotonically nonincreasing demand curve $r^i(h,\rho)$ as a function of price ρ to replace the vertical line $r^i(h)$. The determination of $\bar{C}^i(h)$ and $\bar{\rho}^i(h)$ is provided by the intersection of the demand and supply curves, as shown in Fig. 2. We use the Fig. 2 construct in the development of the procedure for the IGO to acquire PCAS. The description of the proposed procedure for the rational buyer approach is facilitated by the use of the sequential auction scheme.

When prioritization is considered, the subsets $\mathcal{B}^i(h)$ and the demands $r^i(h)$ are no longer fixed. In fact, each seller implicitly uses prioritization in formulating his (or her) offers, thereby leading to the submission of strategic offers. We assume that the sequence of auctions for each hour h occurs in descending order of priority. As any higher priority service may be used as a substitute for a lower priority service, the unused portion of the capacity offered for higher priority PCAS becomes automatically usable for any lower priority service. The use of substitutability avoids the need to submit additional offers. The set of initial offers becomes consequently modified to explicitly account for the substitutable quantities. For each hour h, we define modified subsets of offers recursively, for $i = 1, \ldots, N$

$$\tilde{\mathcal{B}}^i(h) = \mathcal{B}^i(h) \cup \tilde{\mathcal{B}}^{i-1}(h), \quad \tilde{\mathcal{B}}^0(h) = \phi \tag{7}$$

$$\tilde{\mathcal{B}}^{i-1}(h) = \left\{\left(\Delta c_j^{i-1}(h), \rho_j^{i-1}(h)\right) : \Delta c_j^{i-1}(h) > 0, \; j \in \mathcal{S}^{i-1}(h)\right\} \tag{8}$$

with

$$\Delta c_j^{i-1}(h) = c_j^{i-1}(h) - \bar{c}_j^{i-1}(h) \tag{9}$$

and $\mathcal{S}^0(h) = \phi$. In this way, substitutability modifies the supply curve for each PCAS a^i. A simple modification of the auction structure above allows us to explicitly use substitutability in the offers through the use of the modified set of offers $\tilde{\mathcal{B}}^i(h)$ for constructing the supply curve.

In the *rational buyer* approach, substitutability allows the IGO to *overbuy* a PCAS and substitute it for use for any lower priority PCAS. Substitutability results, however, in the strong coupling among the auctions and, consequently, of all the PCAS. In this way, we determine *simultaneously* the outcomes of the N auctions rather than each auction sequentially. This simultaneous determination may be obtained from the solution of the following discrete optimization problem:

$$\min\left\{\sum_{i=1}^{N} \bar{C}^i(h)\bar{\rho}^i(h)\right\} \tag{10}$$

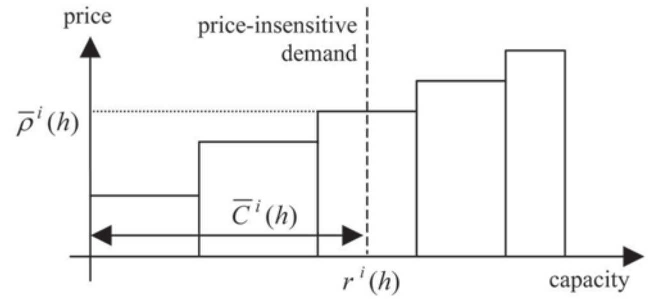

Fig. 1. Market clearing price and quantity in the *uniform price* auction for a price-insensitive demand.

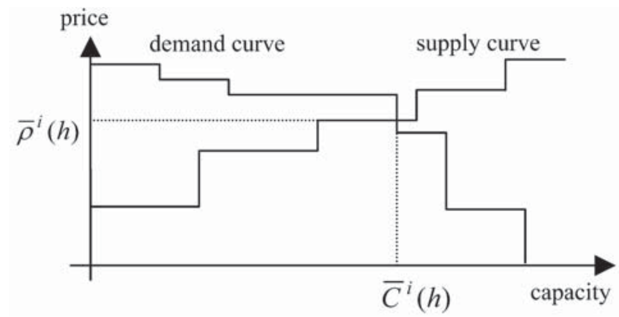

Fig. 2. Market clearing price and quantity in the uniform price auction for the capacity-based AS a^i.

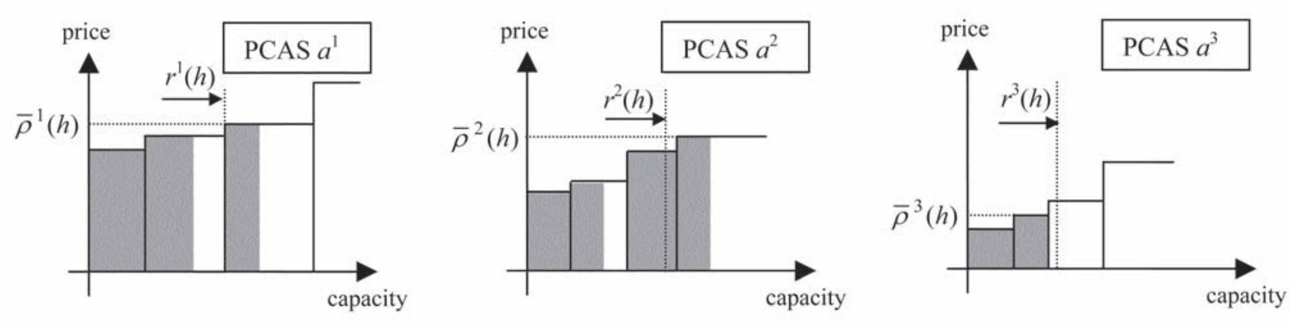

Fig. 3. Optimal solution for the three PCAS at hour h with the accepted offers being shaded.

$$\text{s.t} \quad \sum_{k=1}^{i} b^k(h) \geq \sum_{k=1}^{i} r^k(h), \quad i = 1, \ldots, N \tag{11}$$

$$\sum_{i=1}^{N} \bar{c}_m^i(h) \leq C_m^{\max}(h), \quad m = 1, \ldots, M(h). \tag{12}$$

The optimal $\bar{C}^i(h)[\bar{\rho}^i(h)]$ is the clearing quantity (price) for PCAS a^i with the constraint in (11) replacing the demand part of the market. The constraint in (11) replaces the requirement to provide sufficient capacity to meet the demand for each PCAS. This requirement may be met by having cumulative offer capacity to be no lower than the cumulative demand capacity for each PCAS. Prioritization requires that the cumulative terms be computed in the appropriate order starting with the PCAS with the highest priority.

Clearly, the rational buyer approach is more flexible than the sequential auctions approach. In fact, the sequential auctions are constrained by the availability of sufficient capacity to match the demand for *each* PCAS, while the rational buyer procedure is constrained by the availability of sufficient *cumulative* offered capacity to cover the *cumulative* required demands for capacity, rather than the total capacity offered for each individual PCAS a^i. Referring to the market equilibrium condition illustrated in Fig. 2, substitutability in the rational buyer approach allows the demand of a PCAS to be met by using the offers submitted for higher priority PCAS, resulting in the strong coupling among the PCAS and in an adaptively constructed demand curve. Though the set of offers $\mathcal{B}^i(h)$ remains fixed, the optimization is very challenging due to its combinatorial nature. Consequently, we construct a solution scheme that makes use of discrete programming notions [16] and takes advantage of the structural characteristics of the problem. An important feature of the proposed scheme is its lower computational burden than that of the dynamic programming formulation used in [8] and [11]. We describe the proposed approach in the next section.

III. Proposed Procedure

To make it easier for the reader to understand the complexities of the simultaneous determination of the successful offers in the multiauction formulation, we use a simple example of three PCAS a^1, a^2, and a^3, and four sellers. We focus on a single hour h. The offer and constraint capacities $c_m^i(h)$ and $C_m^{\max}(h)$, respectively, and the required demands $r^i(h)$ in megawatts are given in Table I together with the corresponding offer prices in dollars per megawatt per hour. We do not show here the steps of deriving the optimal solution, but the interested reader may duplicate the results after completing this section. We summarize the results in Table II and Fig. 3.

TABLE I
Offer Capacity and Price Data of the Sellers

PCAS	r^i	offers							
		seller 1		seller 2		seller 3		seller 4	
		c_1^i	ρ_1^i	c_2^i	ρ_2^i	c_3^i	ρ_3^i	c_4^i	ρ_4^i
a^1	120	50	9	70	10	80	11	40	14
a^2	120	50	7	30	6	50	9	80	10
a^3	80	50	5	30	4	40	3	70	8
limiting capacity		80		100		120		100	

TABLE II
Results of the Rational Buyer Approach

PCAS	$\bar{\rho}^i$	$\bar{c}_1^i$	$\bar{c}_2^i$	$\bar{c}_3^i$	$\bar{c}_4^i$	$\bar{C}^i$	payments incurred by the IGO
a^1	11	50	40	30	0	120	1320
a^2	10	30	30	50	20	130	1300
a^3	4	0	30	40	0	70	280
total IGO payments		850	860	990	200	--	2900

We next consider a sensitivity case using the identical data in Table I with the single change of the offer price $\rho_4^3(h)$, reduced to 7 from 8 U.S.$/MW. The results of this sensitivity case are given in Table III and illustrated in Fig. 4. The reduction of the offer price $\rho_4^3(h)$ renders the offer $c_4^3(h)$ competitive and the solution changes in such a way that a portion of that offer is now acquired by the IGO. Using the results of this simple example illustrates the lack of intuition in the determination of the optimum, due to the cross-coupling among the PCAS in terms of the simultaneous auctions and of the sellers' limiting capacities. In fact, it is precisely this cross-coupling that results in lower overall costs for the rational buyer approach than the sequential auction scheme. The two cases show that even the variation of the highest price of the lowest priority PCAS can impact the entire solution; such an effect cannot, of course, be captured in the sequential auction scheme. The example shows well how the IGO overbuys higher priority PCAS to meet the requirements of

TABLE III
RESULTS OF THE RATIONAL BUYER APPROACH FOR THE SENSITIVITY CASE

PCAS	$\bar{\rho}^i$	$\bar{c}_1^i$	$\bar{c}_2^i$	$\bar{c}_3^i$	$\bar{c}_4^i$	$\bar{C}^i$	payments incurred by the IGO
a^1	11	10	70	80	0	160	1760
a^2	7	50	30	0	0	80	560
a^3	7	0	0	40	40	80	560
total IGO payments		460	980	1160	280	--	2880

lower priority ones by accepting the offers for PCAS a^3 in the base case and PCAS a^2 in the sensitivity case, even though the respective required demands $r^3(h)$ and $r^2(h)$ are lower.

The scheme we develop exploits effectively the cross-coupling and the substitutability effects illustrated in the example. We consider the double-sided uniform price auction construct with a finite number of offer prices. By definition, the clearing price for each PCAS is equal to at least one of the offer prices. The solution strategy in our approach is based on the explicit consideration of the offer prices as discrete variables. Due to substitutability, it may happen that no capacity is acquired from the seller of a particular AS a^i for $i > 1$. Hence, we include the *undefined* offer price x as one of the discrete variables for $i > 1$. We make use of discrete programming notions and a numerically efficient bounding scheme to determine the optimal combination of the offer prices $[\bar{\rho}^1(h), \bar{\rho}^2(h), \ldots, \bar{\rho}^N(h)]$ for the N PCAS.

We start out with the set of the initial offer prices for the PCAS a^i by including for $i > 1$ the undefined offer price x as the first component and the different submitted offer prices arranged in ascending order

$$\mathcal{R}^i(h) = \{x, i > 1\} \cup \left\{\rho_j^i(h) : \rho_j^i(h) < \rho_\nu^i(h), j < \nu, j \in \mathcal{S}^i(h)\right\}. \tag{13}$$

We modify this set of offer prices and construct

$$\tilde{\mathcal{R}}^i(h) = \left\{\rho_j^i(h) \in \mathcal{R}^i(h) : \rho_l^i(h) \le \rho_j^i(h) \le \rho_u^i(h)\right\} \tag{14}$$

where $\rho_l^i(h)[\rho_u^i(h)]$ is the lower (upper) bound on the discrete variable $\rho_j^i(h)$ for the PCAS a^i. The lower and upper bounds are computed individually for each PCAS. For the highest priority PCAS a^1, $\rho_l^1(h)$ is the clearing price $\bar{\rho}^1(h)$ of the market determined by the offers in $\mathcal{B}^1(h)$. For the PCAS a^i, $i > 1$, we compute the residual demand

$$\Delta r^i(h) = \sum_{k=1}^{i} r^k(h) - \sum_{m=1}^{M(h)} \left[\min\left\{C_m^{\max}(h), \sum_{k=1}^{i-1} c_m^k(h)\, \delta_m^k(h)\right\}\right]. \tag{15}$$

If $\Delta r^i(h) < 0$, the lower bound $\rho_l^i(h) = x$, that is, the cumulative demand $\sum_{k=1}^{i} r^k(h)$ for the PCAS a^k, $k = 1, \ldots, i$ can be satisfied by using only the offers in $\bigcup_{w<i} \mathcal{B}^w(h)$ submitted for all of the higher priority PCAS, without using any offer in $\mathcal{B}^i(h)$. Otherwise, it is necessary to use at least one offer in $\mathcal{B}^i(h)$, and $\rho_l^i(h)$ becomes the clearing price resulting from satisfying the residual demand $\Delta r^i(h)$ by the offers in $\mathcal{B}^i(h)$.

The upper bound $\rho_u^i(h)$ is computed by using the reduced set of offers $\mathcal{B}_u^i(h) = \{(\Delta c_j^i(h), \rho_j^i(h)) : \Delta c_j^i(h) > 0, j \in \mathcal{S}^i(h)\}$, in which each seller can only provide the residual offer

$$\Delta c_j^i(h) = \min\left\{c_j^i(h), \max\left\{0, C_j^{\max}(h) - \sum_{k=1}^{i-1} c_j^k(h)\delta_j^k(h)\right\}\right\} \tag{16}$$

without exceeding its limiting capacity to serve the PCAS a^k with $k < i$. The upper bound $\rho_u^i(h)$ is the clearing price of the market for the provision of the required demand $\sum_{k=i}^{N} r^k(h)$ for the PCAS of priority not higher than a^i by using the offers in $\mathcal{B}_u^i(h)$. If the capacity available in $\mathcal{B}_u^i(h)$ cannot satisfy the required demand, the upper bound is set to the maximum price of the offers in $\mathcal{B}^i(h)$ (i.e., $\rho_u^i(h) = \max_{j \in \mathcal{S}^i(h)} \{\rho_j^i(h)\}$).

Once the upper and lower bounds on the offer prices are determined, we consider all of the possible price combinations $[\tilde{\rho}^1(h), \tilde{\rho}^2(h), \ldots, \tilde{\rho}^N(h)]$ constructed from $\tilde{\rho}^i(h) \in \tilde{\mathcal{R}}^i(h)$, $i = 1, \ldots, N$. We start with the price combination $[\tilde{\rho}_l^1(h), \tilde{\rho}_l^2(h), \ldots, \tilde{\rho}_l^N(h)]$ and construct another price combination by sequentially increasing the offer prices starting with the lowest priority service. Each price combination is tested for infeasibility and whether it may be labeled an *avoidable calculation*.

We consider for the PCAS a^i, the reduced subsets of sellers $\tilde{\mathcal{S}}^i(h) = \{s_j^i(h) \in \mathcal{S}^i(h) : \rho_j^i(h) \le \tilde{\rho}^i(h)\}$, associated with the participation indicators

$$\tilde{\delta}_m^i(h) = \begin{cases} 1 & m \in \tilde{\mathcal{S}}^i(h) \\ 0 & m \notin \tilde{\mathcal{S}}^i(h) \end{cases} \tag{17}$$

and $\hat{\mathcal{S}}^i(h) = \{s_j^i(h) \in \mathcal{S}^i(h) : \rho_j^i(h) < \tilde{\rho}^i(h)\}$, associated with the participation indicators

$$\hat{\delta}_m^i(h) = \begin{cases} 1 & m \in \hat{\mathcal{S}}^i(h) \\ 0 & m \notin \tilde{\mathcal{S}}^i(h) \end{cases}. \tag{18}$$

A price combination is *infeasible* if for any PCAS a^i, the capacity acquirable from PCAS offers of priority $k \le i$ at prices not higher than $\tilde{\rho}^k(h)$ for the offers of PCAS k cannot satisfy the cumulative demand $\sum_{k=1}^{i} r^k(h)$

$$\sum_{m=1}^{M(h)} \min\left\{C_m^{\max}(h), \sum_{k=1}^{i} c_m^k(h)\tilde{\delta}_m^k(h)\right\} < \sum_{k=1}^{i} r^k(h), \quad i = 1, \ldots, N. \tag{19}$$

A price combination is *feasible* if for each PCAS a^i, $i = 1, \ldots, N$, the inequality (19) does not hold. Once an infeasible price combination has been detected, the scheme proceeds to construct and verify the next price combination. A feasible price combination is *evaluated* to determine the minimum total costs incurred by the IGO for that price combination. Prior to the evaluation, we check whether a feasible price combination results in an *avoidable calculation*, since the combination has costs incurred by the IGO that are higher than those of a combination already evaluated. For detecting avoidable calculations,

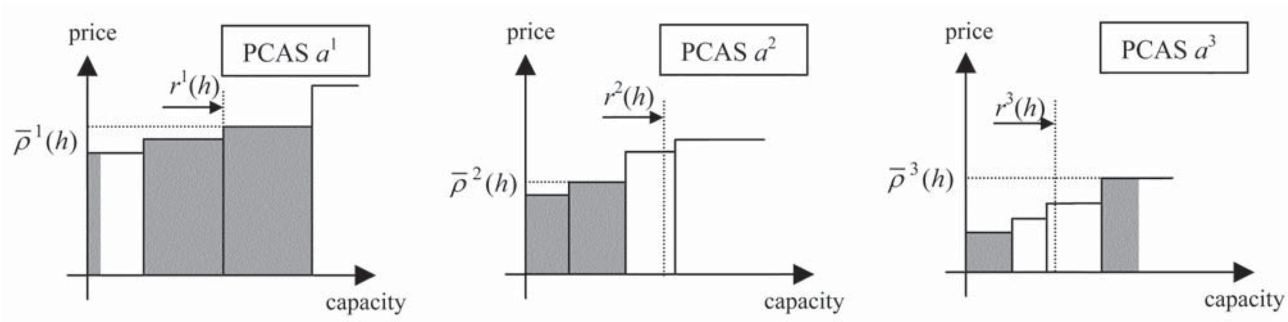

Fig. 4. Optimal solution for the sensitivity case with the accepted offers being shaded.

each offer price $\tilde{\rho}^i(h)$ for the PCAS is associated with the lower capacity limit $\tilde{C}_l^1(h)$, whose computation requires the definition of a new residual offer, for $j \in \hat{\mathcal{S}}^i(h)$

$$\Delta\tilde{c}_j^i(h)=\min\left\{c_j^i(h), \max\left\{0, C_m^{\max}(h)-\sum_{k=1}^{i-1}c_m^k(h)\hat{\delta}_m^k(h)\right\}\right\} \tag{20}$$

and to compute the lower capacity limit as

$$\tilde{C}_l^i(h) = \sum_{m=1}^{M(h)} \Delta\tilde{c}_m^i(h)\hat{\delta}_m^i(h). \tag{21}$$

For PCAS a^1, $\tilde{C}_l^1(h) \geq r^1(h)$ due to lack of substitutability.

We identified three criteria for automatically recognizing price combinations with a total cost incurred by the IGO higher than the minimum cost $\tilde{\chi}_{\min}(h)$ of the combinations already evaluated. The minimum cost $\tilde{\chi}_{\min}(h)$ is given an initial value of infinity and it is updated through the use of (31). The price combination under test is $[\tilde{\rho}^1(h), \tilde{\rho}^2(h), \ldots, \tilde{\rho}^N(h)]$. In addition, we determine the PCAS a^k such that the offer price $\tilde{\rho}^k(h) = \max_{i=1,\ldots,N}\{\tilde{\rho}^i(h)\}$. If $\tilde{\rho}^k(h) > \rho_l^k(h)$, we set for the PCAS a^k the offer price

$$\rho_{-}^k(h) = \max_{\rho_j^k(h) < \tilde{\rho}^k(h)} \left\{\rho_j^k(h)\right\}. \tag{22}$$

Whenever one of the following criteria is met, the price combination $[\tilde{\rho}^1(h), \tilde{\rho}^2(h), \ldots, \tilde{\rho}^N(h)]$ under test is labeled an *avoidable calculation* and requires no further evaluation:

i) for some $k = 1, \ldots, N$, the price combination $[\tilde{\rho}^1(h), \ldots, \tilde{\rho}^{k-1}(h), \rho_{-}^k(h), \tilde{\rho}^{k+1}(h), \ldots, \tilde{\rho}^N(h)]$ has already been evaluated or identified as an *avoidable calculation*, so that the price combination under test would result in higher total costs;

ii) the cost computed from the offer prices and their lower capacity limits $\tilde{C}_l^i(h)$ is higher than the minimum total cost $\tilde{\chi}_{\min}(h)$ already computed

$$\sum_{i=1}^{N} \tilde{\rho}^i(h)\tilde{C}_l^i(h) \geq \tilde{\chi}_{\min}(h). \tag{23}$$

iii) for any PCAS a^i, $i = 1, \ldots, N$, the sum of the lower capacity limits is equal or higher than the total demand

$$\sum_{k=i}^{N} \tilde{C}_l^k(h) \geq \sum_{k=i}^{N} r^k(h) \tag{24}$$

so that a combination leading to a lower cost has already been determined.

After weeding out *infeasible combinations* and *avoidable calculations*, the remaining price combinations are *evaluated* to determine the least-cost strategy for each combination. The evaluation requires the solution of a linear programming (LP) problem to determine the least total costs incurred by the IGO for that price combination. The decision variables for the PCAS a^i in the LP are the quantities $\tilde{c}_j^i(h)$ supplied by each seller $j \in \tilde{\mathcal{S}}^i(h)$. These variables must satisfy the appropriate constraints.

The LP problem formulation is

$$\min \tilde{\chi}(h) = \sum_{i=1}^{N}\left[\tilde{\rho}^i(h) \sum_{j \in \tilde{\mathcal{S}}^i(h)} \tilde{c}_j^i(h)\right] \tag{25}$$

s.t.

$$0 \leq \tilde{c}_j^i(h) \leq c_j^i(h), \quad i=1,\ldots,N; \quad j \in \tilde{\mathcal{S}}^i(h) \tag{26}$$

$$\sum_{i=1}^{N} \tilde{c}_m^i(h)\tilde{\delta}_m^i(h) \leq C_m^{\max}(h), \quad m = 1, \ldots, M(h) \tag{27}$$

$$\sum_{k=1}^{i} \sum_{j \in \tilde{\mathcal{S}}^k(h)} \tilde{c}_j^k(h) \geq \sum_{k=1}^{i} r^k(h), \quad i = 1, \ldots, N-1 \tag{28}$$

$$\sum_{i=1}^{N} \sum_{j \in \tilde{\mathcal{S}}^i(h)} \tilde{c}_j^i(h) = \sum_{i=1}^{N} r^i(h) \tag{29}$$

$$\tilde{C}_l^i(h) \leq \sum_{j \in \tilde{\mathcal{S}}^i(h)} \tilde{c}_j^i(h), \quad i = 1, \ldots, N. \tag{30}$$

These constraints reflect that the decision variables $\tilde{c}_j^i(h)$ cannot exceed the submitted offers $c_j^i(h)$, the capability limits are not violated by any seller, the limits on the substitutable quantities are observed, the balancing of the total capacity and the total demand and the lower and upper limits on the capacity provided for each PCAS a^i are satisfied. The minimum total cost $\tilde{\chi}_{min}(h)$ among the combinations already evaluated is updated using

$$\tilde{\chi}_{\min}(h) =: \min\left\{\tilde{\chi}_{\min}(h), \tilde{\chi}(h)\right\}. \tag{31}$$

The evaluation of all (unavoidable) feasible combinations determines the optimal price combination with total cost $\widetilde{\chi}_{\min}(h)$, corresponding to $i = 1, 2, \ldots, N$ to the capacities $\bar{c}^i_j(h)$, $j = 1, 2, \ldots, S^i(h)$, clearing quantities $\bar{C}^i(h) = \sum_{j \in \tilde{\mathcal{S}}^i(h)} \bar{c}^i_j(h)$ and clearing prices $\bar{\rho}^i(h)$.

The computational speed of the rational buyer procedure depends principally on the number of LP evaluations. The proposed scheme allows a fast solution by weeding out most of the possible price combinations so as to drastically reduce the number of LP evaluations required. This reduction is a distinct advantage over the dynamic programming approach in [8] and [11] since the proposed scheme requires no preventive discretization of the capacity.

IV. Numerical Results

The proposed scheme for solving the rational buyer problem has been tested on several cases and results indicate excellent performance. The steps of the bounding scheme for weeding out *infeasible combinations* and *avoidable calculations* are indeed very effective in reducing the computational effort.

We illustrate the workings of the proposed scheme on the simple example of Section III. The three sets of the initial offer prices are

$$\begin{aligned}\mathcal{R}^1(h) &= \{9, 10, 11, 14\}\\ \mathcal{R}^2(h) &= \{x, 6, 7, 9, 10\}\\ \mathcal{R}^3(h) &= \{x, 3, 4, 5, 8\}\end{aligned}$$

with a total of $4 * 5 * 5 = 100$ possible combinations. The application of the computed lower and upper bounds results in the modified sets

$$\begin{aligned}\tilde{\mathcal{R}}^1(h) &= \{11, 14\}\\ \tilde{\mathcal{R}}^2(h) &= \{6, 7, 9, 10\}\\ \tilde{\mathcal{R}}^3(h) &= \{x, 3, 4, 5, 8\}\end{aligned}$$

reducing the number of combinations to $2 * 4 * 5 = 40$. The exhaustive search is then performed on these combinations, resulting in 17 *infeasible combinations* and 16 *avoidable calculations*, 7 and 9 of which satisfy the criterion *i)* and *ii)*, respectively. The remaining seven combinations require the solution of the LP problem to compute the least acquisition costs incurred by the IGO. The optimal solution of Table II is obtained. This small system example illustrates the way the proposed procedure reduces overall computational burden in the discrete programming approach to the rational buyer problem. For the small system reported in [12], with four PCAS and four sellers, the proposed procedure reaches a solution with total IGO payments of U.S.$ 1700 rather than U.S.$ 1780 determined in [12], eliminating about 75% of the evaluations and resulting in about a fourfold reduction in calculations.

The results of testing the proposed approach on the IEEE 118-bus system with 16 sellers for four PCAS shows the performance characteristics on a larger system. We consider two cases, with the base case having relatively high loading and one with lower loading in which the loads and the generations are uniformly halved. Table IV summarizes the results of the proposed approach. The theoretical maximum number of combinations would be used in an exhaustive search. The application of upper and lower bounds limits the number of combinations, resulting in the number indicated as bounded search. By weeding out the infeasible combinations and the avoidable calculations under the three proposed criteria, the number of LP evaluations is drastically reduced. The curves in Fig. 5 show the evolution of the IGO payments as the successive LP evaluations are made. The results shown are representative of the attractive feature of the proposed scheme of reducing the number of combinations requiring evaluation.

TABLE IV
NUMBER OF COMBINATIONS FOR THE RATIONAL BUYER APPROACH WITH 16 GENERATORS

		base case	low-loading case
exhaustive search		23760	43875
bounded search		1008	13440
infeasible combinations		34	4669
avoidable calculations under *criterion*	(*i*)	806	7675
	(*ii*)	37	659
	(*iii*)	61	266
LP evaluations		70	171

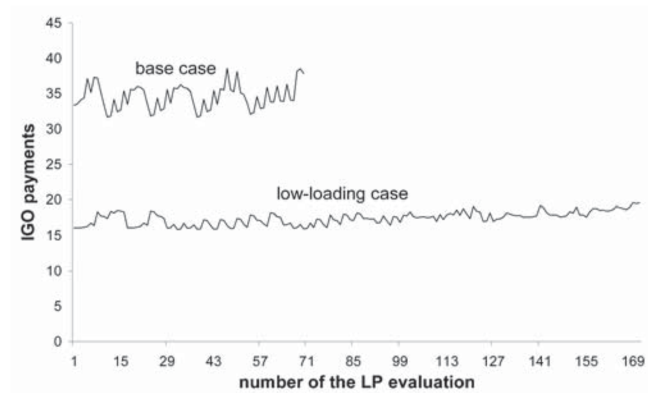

Fig. 5. LP evaluations for the base case and the low-loading case.

V. Conclusion

We developed a computationally efficient scheme to simultaneously determine the set of successful offers in the multiauction framework of the rational buyer procedure for the acquisition of PCAS. A salient feature of the proposed procedure is its ability to accommodate physical constraints such as capacity, ramp-rate, and interzonal limits. In addition, the proposed procedure provides a good tool for market monitoring, since it establishes a reference basis for comparison purposes. Even if the rational buyer approach is not implemented, the market monitor can assess how large the gap is between the actual prices and those resulting from the reference ones produced by this scheme.

There are several extensions of the work reported here. We are investigating the formulation of optimal offers by sellers of capacity-based AS, the customers' participation in the AS procurement, and the incorporation of geographic diversity in the provision of capacity-based AS. Reports on our work will appear in future papers.

APPENDIX
LIST OF NOTATIONS

a^i	Prioritizable capacity-based ancillary service (PCAS) with priority i with $i = 1$ being highest priority.
$\mathcal{S}$	Set of the sellers submitting offers for a specified PCAS.
b	Total capacity offer.
$\mathcal{B}$	Set of the offers.
c	Offer quantity.
C	Limiting capacity.
$\bar{C}$	Clearing quantity.
h	Hour.
M	Total number of sellers.
$\mathcal{M}$	Set of the sellers.
N	Total numbers of PCAS.
r	Required capacity demand.
R	Ramp-rate constraint.
$\mathcal{R}$	Set of the offer prices.
x	Undefined offer price.
χ	Total cost.
δ	Participation indicator.
ρ	Offer price.
$\bar{\rho}$	Clearing price.

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