AN EFFICIENT PROCEDURE FOR THE *RATIONAL BUYER* APPROACH FOR THE ACQUISITION OF CAPACITY-BASED ANCILLARY SERVICES

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Abstract – This paper addresses the competitive procurement of capacity-based ancillary services (AS) in unbundled markets by the Independent Grid Operator (IGO). These AS include upward frequency control, load following and the range of reserve services, which may be procured from unloaded capacity offered by both on-line and off-line sources. The capacity-based AS are prioritized in order of ascending response times. Prioritization allows substitutability of the AS by automatically making the unused capacity of a higher priority AS usable for any lower priority AS without the need of submitting additional offers. This paper discusses the formulation of the auction structures for the acquisition of the prioritizable capacity-based AS and presents an efficient scheme for minimizing the costs incurred by the IGO by using the *rational buyer* procedure. The proposed scheme adopts effective discrete programming techniques that exploit the structural characteristics of the problem for handling the multi-auction formulation. The proposed bounding scheme takes fully advantage of critical physical constraints such as ramp rate, capacity limits, and inter-zonal constraints. The effectiveness and computational efficiency of the proposed scheme are illustrated and discussed with numerical examples.

Keywords: competitive electricity markets, ancillary services, prioritizable capacity-based services, auctions, rational buyer, discrete programming, independent grid operator.

1 INTRODUCTION

A salient characteristic of the restructuring in electricity is the functional unbundling in services with the accompanying advent of new competitive markets in addition to the MWh commodity market [1,2]. We are concerned in this paper with ancillary services (AS) – the essential system support services from generating sources required for the provision of transmission service and without which instantaneous collapse would occur – as defined by FERC [3]. Consequently, we study services provided by generating sources with the focus on capacity-based AS, which include frequency control, load following and various types of reserves.

The provision of AS is the responsibility of the independent grid operator or IGO. We use this term for the entity that operates and controls the network in its generic sense so as to encompass existing implementations such as ISOs and RTOs [4]. The IGO must ensure that there are adequate supplies of the various AS and acquire them cost effectively. Capacity-based AS may be acquired competitively [5]. The objective of this paper is to propose an efficient procedure for the purchase of these services in a way that effectively exploits competition in capacity-based services.

The day-ahead forward energy markets result in the submission of preferred schedules to the IGO. Their feasibility is ensured by the IGO through the congestion management actions that he dispatches to relieve the system congestion. The IGO then proceeds with the acquisition of those AS that are offered in competitive markets. The interrelationships between the various markets are illustrated in Figure 1. In some jurisdictions, such as New Zealand, the joint acquisition of energy and AS is performed in the hour-ahead markets [6]. Such markets allow the simultaneous multi-product optimization of the commodity and AS in each hour. In unbundled markets, however, the acquisition of the AS is performed separately by using various mechanisms. In this paper, we consider the case where the energy schedules and the AS procurement are operated separately and we focus on the acquisition of capacity-based AS by the IGO to meet the requirements of providing transmission service. These capacity-based AS include upward frequency control, load following and the range of reserve services, which may be procured from unloaded capacity of both on- and off-line resources.

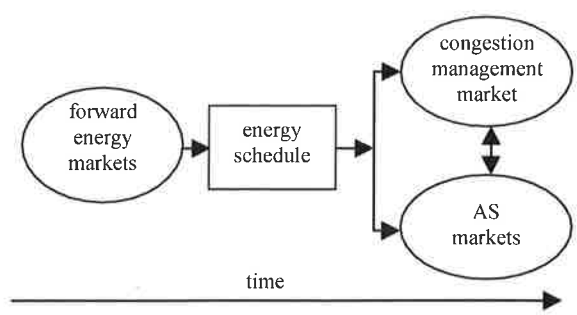

Figure 1: The interrelationships between the unbundled electricity markets

The IGO specifies the time response and amount requirements for each AS. The capacity-based AS are prioritized on the basis of their response times, with the highest (lowest) priority associated with the shortest (longest) response time. To harness the benefits of competition, the IGO holds auctions to procure the capacity-based AS. The prioritization allows *substitutability* in provision of these services [7]. Specifically this means that the unused capacity offered for higher priority prioritizable capacity-based AS (PCAS) may be automatically used for any lower priority PCAS without the need for submitting additional offers. Some work on the development of procurement procedures has been reported in the literature [4,5].

In this paper we discuss the formulation of the auction structures for the acquisition of PCAS and present a proposed scheme for determining the least-cost strategy for the IGO to acquire the required PCAS by using the *rational buyer* procedure [4,7,8]. We propose a solution via effective discrete programming techniques that take full advantage of the structural characteristics of the problem. We give a detailed formulation and discuss the characteristics of the discrete programming approach developed for handling the rational buyer procedure and to render the solution scheme tractable. The proposed scheme effectively incorporates into the rational buyer procedure the critical physical considerations such as ramp-rate, capacity and inter-zonal limits. Two salient aspects of this scheme are the capability to quickly detect infeasible combinations of the offer prices and the ability to identify *avoidable calculations* requiring no further evaluations of feasible combinations. Since the evaluation of the feasible price combinations is the most time consuming task, the detection of *avoidable calculations* results in major reductions in computations for the determination of the optimal combination. We implemented the proposed procedure and illustrate the efficiency of the bounding scheme in the least-cost acquisition of PCAS through representative numerical examples.

This paper has four additional sections. The characteristics of the capacity-based AS and salient aspects of their competitive acquisition by the IGO are discussed in Section 2. We develop an appropriate analytical structure in the section to allow the solution scheme to effectively exploit the structural characteristics of the rational buyer approach to the PCAS acquisition problem. The proposed scheme is presented in Section 3. Numerical results are provided in Section 4 to illustrate the effectiveness of the proposed procedure. Directions for future work are outlined in the last section.

2 ACQUISITION OF PRIORITIZABLE CAPACITY-BASED AS

The *capacity-based* AS are required to maintain secure operation of the power system. The services included in this group are:

- *frequency regulation*: the basic AGC service to track the load with the generation so as to ensure that the frequency stays within a predefined band of the system synchronous frequency; this service requires both up and down shifting of the output level of a participating entity;
- *load following*: non-automatic response of the generating units to the IGO signals to maintain balance with the load in actual operations, with positive and negative variation in the real power generation of the contributing entities;
- *reserves*: unloaded capacity available within a specified response time required for system operations to withstand unexpected generation outages and increases in the forecast demand, used to allow the system operation after outages and/or unexpected variations in the demand; reserves may be provided by either on-line generators loaded below their maximum capacity or off-line generation sources having a sufficiently fast response.

These capacity-based AS are arranged in order of their required response times. For a specified t-minute response, the capacity must be fully operational within the specified t-minutes. The services with the shortest (longest) response time are assigned highest (lowest) priority. Clearly, a service whose required response time is t minutes may be used for all lower priority services with required response time $t' > t$. For this prioritization to work, the use of additional capacity must be in a uniform direction. Consequently, the down direction for frequency control and load following cannot be grouped into the prioritizable capacity-based AS or PCAS aimed at providing additional capacity to the system. Frequency response requires a faster response than load following, which, in turn, is considerably faster than reserve services. As such, frequency control has the highest priority, followed by load following and then the reserve services. The reserve services are themselves ordered on the basis of their response times.

The acquisition of PCAS is considered once the energy markets have cleared and the congestion management issues have been addressed. For each hour h of the day-ahead market, the IGO defines the requirements for each AS. To take advantage of competitive conditions, the PCAS are acquired through day-ahead hourly auctions for each AS. Therefore, for the 24 hour day-ahead market there are hourly auctions for each PCAS. The sellers have no possibility to update their offers once submitted. However, the prioritization allows the relationships among the auctions for the different PCAS for the same hour to be fully exploited.

We develop notation for our discussion of the PCAS acquisition problem. For the auction in a particular hour h we use the notation h to explicitly identify the hour. We consider $a^1, a^2, \ldots, a^N$ to be the PCAS, where a^1 has the highest priority and a^N the lowest. We define the set $\mathcal{M}(h)=\{m : m=1,2,\ldots,M(h)\}$ of the sellers submitting offers for the PCAS in hour h. For each PCAS a^i, for $i = 1, \ldots, N$, we define $r^i(h)$ to be the required capacity for the PCAS a^i in hour h, the set $S^i(h) \subseteq \mathcal{M}(h)$ of the sellers for the PCAS a^i in hour h as $S^i(h)=\left\{s^i_v(h) : v=1,2,\ldots,S^i(h)\right\}$, and the set

$\mathcal{B}^i(h) = \{(c_j^i(h), \rho_j^i(h)) : j \in S^i(h)\}$ of the offers submitted by the sellers for the PCAS a^i in hour h, with capacity $c_j^i(h)$ and price $\rho_j^i(h)$. The total capacity offered, then, for the PCAS a^i in hour h is

$$b^i(h) = \sum_{j \in S^i(h)} c_j^i(h), \qquad i = 1, \ldots, N \tag{1}$$

and is assumed to be sufficient to cover the required demand for each AS

$$b^i(h) \geq r^i(h), \qquad i = 1, \ldots, N \tag{2}$$

Note that $r^i(h)$ need *not* to be a fixed quantity but may be expressed explicitly as function of price. For each PCAS a^i, $i = 1, \ldots, N$, the offers in $\mathcal{B}^i(h)$ are ordered in ascending order of the prices, to construct the *supply curve*. The IGO determines the market clearing price using a uniform price auction [9] with this supply curve, resulting in a clearing quantity $\overline{C}^i(h)$ and a clearing price $\overline{\rho}^i(h)$. We denote by $\overline{c}_j^i(h)$ the offer capacity accepted from the seller $j \in S^i(h)$ for PCAS a^i with

$$\overline{C}^i(h) = \sum_{j \in S^i(h)} \overline{c}_j^i(h) \tag{3}$$

For a given supply curve and a specified price sensitive demand curve, the clearing quantity [price] $\overline{C}^i(h)$ [$\overline{\rho}^i(h)$] is determined by the intersection of the two curves. Such a situation is depicted in Figure 2. We use the Figure 2 construct in the development of the procedure for the IGO to acquire PCAS. The description of the proposed procedure for the rational buyer approach is facilitated by the use of the sequential auction scheme.

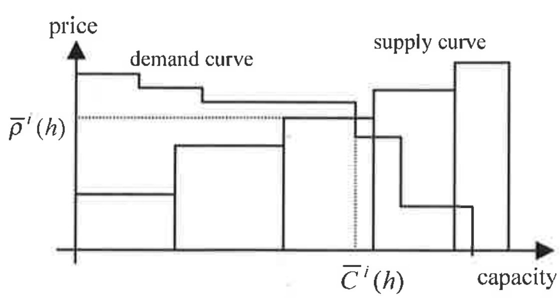

Figure 2: Market clearing price and quantity in the uniform price auction for the capacity-based AS a^i

The sequential auctions approach for the N PCAS leads to N auctions for each hour h in the day-ahead market, i.e., a total of $24N$ separate auctions. We start with the structure of the auction for PCAS a^i in an arbitrary hour h. We determine the selection of the winning offers by constructing a supply curve for the submitted offers $\mathcal{B}^i(h)$. For this auction, the demand is fixed and so the clearing quantity is equal to the required demand $r^i(h)$. The clearing price in the *uniform price* auction is determined from the supply function and is, typically, the price of the highest accepted offer, as shown in Figure 3.

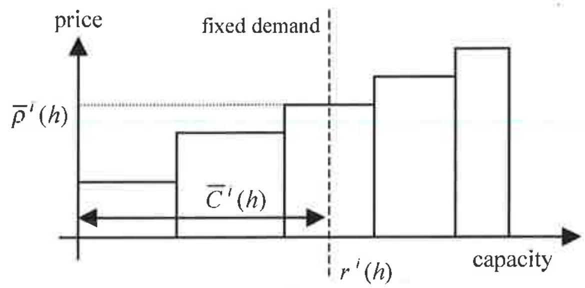

Figure 3: The market clearing price and quantity in the sequential auction construct

When prioritization is brought in, $b^i(h)$ and $r^i(h)$ are no longer fixed. In fact, each seller implicitly uses prioritization in formulating his offers, thereby leading to the submission of strategic offers. We assume that the sequence of auctions for each hour h occurs in descending order of priority. As any higher priority service may be used as a substitute for a lower priority service, the unused portion of the capacity offered for higher priority PCAS becomes automatically useable for any lower priority service, avoiding the need to submit additional offers. The set of initial offers is consequently modified to explicitly include the substitutable quantities. For each period h, we define a modified set of offers recursively, for $i = 1, \ldots, N$:

$$\tilde{\mathcal{B}}^i(h) = \mathcal{B}^i(h) \cup \tilde{\mathcal{B}}^{i-1}(h), \quad \tilde{\mathcal{B}}^0(h) = \phi \tag{4}$$

$$\tilde{\mathcal{B}}^{i-1}(h) = \{(\Delta c_j^{i-1}(h), \rho_j^{i-1}(h)) : \Delta c_j^{i-1}(h) > 0, \ j \in S^{i-1}(h)\} \tag{5}$$

with

$$\Delta c_j^{i-1}(h) = c_j^{i-1}(h) - \overline{c}_j^{i-1}(h) \tag{6}$$

and $S^0(h) = \phi$. In this way, substitutability modifies the supply curve for each PCAS a^i. A simple modification of the auction structure above allows us to explicitly use substitutability in the offers through the use of the modified set of offers $\tilde{\mathcal{B}}^i(h)$ for constructing the supply curve.

In the *rational buyer* approach, substitutability allows the IGO to *overbuy* a PCAS for substituting it for any lower priority PCAS. Substitutability results, however, in the strong coupling among the auctions and consequently of all the PCAS, leading to the need to simultaneously determine the outcomes of all the auctions rather than sequentially determine each auction. This determination may be obtained from the solution of a non-linear mixed discrete optimization problem with the offer prices as discrete variables. The simultaneous determination is formulated to have the following form:

$$min\left\{\sum_{i=1}^{N}\overline{C}^{i}(h)\,\overline{\rho}^{i}(h)\right\} \quad (7)$$

$$\text{s.t.}\ \sum_{k=1}^{i} b^{k}(h) \geq \sum_{k=1}^{i} r^{k}(h) \quad \text{for } i = 1, \ldots, N \quad (8)$$

The optimal $\overline{C}^{i}(h)$ [$\overline{\rho}^{i}(h)$] are the clearing quantity [price] for PCAS a^i with the constraint in (8) replacing the demand part of the market. The constraint in (8) states the requirement to provide sufficient capacity to meet the demand for each PCAS. This requirement may be met by having cumulative offer capacity to be no lower than the cumulative demand capacity for each PCAS. The priority requires that the cumulative terms be computed in the appropriate order starting with the PCAS with the highest priority.

Clearly, the rational buyer approach is more flexible than the sequential auctions approach. In fact, the sequential auctions are constrained by the availability of sufficient capacity to match the demand for *each* PCAS, while the rational buyer procedure is constrained by the availability of sufficient *cumulative* offered capacity to cover the *cumulative* required demands for capacity, and is independent of the total capacity offered for the individual PCAS a^i. Referring to the market equilibrium condition illustrated in Figure 2, substitutability in the rational buyer approach allows the demand of a PCAS to be met by using the offers submitted for higher priority PCAS, resulting in the strong coupling among the PCAS and in an adaptive demand curve. The set of offers $\mathcal{B}^{i}(h)$, however, remains fixed. The optimization is a very challenging due to its combinatorial nature. Consequently, we adopt a discrete programming technique [10] to take advantage of the structural characteristics of the problem. Another formulation using dynamic programming was applied effectively in [4] to solve the rational buyer problem. We developed an efficient bounding scheme which allows the simultaneous determination of successful offers in the multi-auction formulation for the rational buyer procedure. We discuss the proposed approach in the next section.

3 AN EFFICIENT PROCEDURE FOR THE COMPETITIVE ACQUISITION OF PCAS

We consider the uniform price auction construct described above with a finite number of offer prices. For each PCAS, the clearing price, by definition, is equal to at least one of the offer prices. The solution strategy in the rational buyer problem is to consider the offer prices as discrete variables. Due to substitutability, it may happen that no capacity is acquired from the seller of a particular AS a^i for $i > 1$. Hence, the *undefined* offer price x is included in the list of the discrete variables for $i > 1$.

We use a discrete programming approach to determine the optimal combination of the offer prices $[\overline{\rho}^{1}(h), \overline{\rho}^{2}(h), \ldots, \overline{\rho}^{N}(h)]$ for the N PCAS. The bounding scheme we developed makes detailed use of the various constraints such as ramp-rate limits and inter-zonal transfer limits. The auctions of each period is independent of the auctions in any other periods. However, the quantity at the end of the period h-1 determines the effect of these constraints at period h. The limiting capacity $C_{m}^{max}(h)$ that can be physically provided by the seller $m \in \mathcal{M}(h)$ is a function of the capacity $C_m(h\text{-}1)$ used in period h-1 and the ramp rate limit w_m, and also of the inter-zonal constraint $C_{m}^{sup}(h)$ representing the maximum capacity transferable from the seller m without causing network congestion. Mathematically, for a T-minute response capacity-based service

$$C_{m}^{max}(h) = min\left\{C_{m}(h-1) + w_{m}\,T, C_{m}^{sup}(h)\right\} \quad (9)$$

We define the set of the initial offer prices for the PCAS a^i as

$$\mathcal{R}^{i}(h) = \{x\} \cup \left\{\rho_{j}^{i}(h) : \rho_{j}^{i}(h) < \rho_{v}^{i}(h), j < v, \; j \in S^{i}(h)\right\} \quad (10)$$

We start by modifying this set of the offer prices to the subset

$$\tilde{\mathcal{R}}^{i}(h) = \left\{\rho_{j}^{i}(h) \in \mathcal{R}^{i}(h) : \; \rho_{l}^{i}(h) \leq \rho_{j}^{i}(h) \leq \rho_{u}^{i}(h)\right\} \quad (11)$$

where $\rho_{l}^{i}(h)$ [$\rho_{u}^{i}(h)$] is the lower [upper] bound on the discrete variable $\rho_{j}^{i}(h)$ for the PCAS a^i. The lower and upper bounds are computed sequentially for PCAS a^1 to PCAS a^N.

For the highest priority PCAS a^1, $\rho_{l}^{1}(h)$ is the clearing price $\overline{\rho}^{1}(h)$ of the market determined by the offers in $\mathcal{B}^{1}(h)$. For the PCAS a^i with $i > 1$, the lower bound is $\rho_{l}^{i}(h) = x$ if the cumulative demand $\sum_{k=1}^{i} r^{k}(h)$ for the PCAS a^k, $k = 1, \ldots, i$, can be satisfied by using only the offers in $\bigcup_{w<i} \mathcal{B}^{w}(h)$ submitted for all the higher priority PCAS, without using any offer in $\mathcal{B}^{i}(h)$. Otherwise, it is necessary to use at least one offer in $\mathcal{B}^{i}(h)$, and $\rho_{l}^{i}(h)$ becomes the clearing price resulting from satisfying the residual demand $\Delta r^{i}(h) = \sum_{k=1}^{i} r^{k}(h) - \sum_{k=1}^{i-1} b^{k}(h)$ by the offers in $\mathcal{B}^{i}(h)$.

The upper bound $\rho_{u}^{i}(h)$ is computed to be

$$\rho_{u}^{i}(h) = min\left\{\max_{j \in S^{i}(h)}\left\{\rho_{j}^{i}\right\}, \overline{\rho}^{i*}(h)\right\} \quad (12)$$

where $\overline{\rho}^{i*}(h)$ is the clearing price of the market that provides the required demand $\sum_{k=i}^{N} r^k(h)$ for the PCAS of priority not higher than a^i by using only the offers in $\mathcal{B}^i(h)$. For the lowest priority PCAS a^N, the upper bound $\rho_u^N(h)$ is the clearing price $\overline{\rho}^N(h)$ of the market determined by the offers in $\mathcal{B}^N(h)$.

Once the upper and lower bounds on the offer prices are determined, we consider all the possible price combinations $[\tilde{\rho}^1(h), \tilde{\rho}^2(h), \ldots, \tilde{\rho}^N(h)]$ constructed from $\tilde{\rho}^i(h) \in \tilde{\mathcal{R}}^i(h)$, $i = 1, \ldots, N$. We start with the price combination $[\tilde{\rho}_l^1(h), \tilde{\rho}_l^2(h), \ldots, \tilde{\rho}_l^N(h)]$ and construct another price combination by sequentially increasing the offer prices starting with the lowest priority service. Each price combination is tested for infeasibility and whether it may be labeled an *avoidable calculation.*

We consider for the PCAS a^i the set of sellers $\tilde{S}^i(h) = \{s_j^i(h) \in S^i(h): \rho_j^i(h) \le \tilde{\rho}^i(h)\}$ and the set $\tilde{\mathcal{J}}^i(h) = \{\tilde{S}^1(h) \cup \tilde{S}^2(h) \ldots \cup \tilde{S}^i(h)\}$. For infeasibility, we compute the quantities

$$\hat{C}_j^i(h) = \min\left\{ C_j^{max}(h),\ \sum_{\substack{k=1 \\ j \in \tilde{S}^k(h)}}^{i} c_j^k(h) \right\} \tag{13}$$

A price combination is *infeasible* if for any PCAS a^i the capacity $\sum_{j \in \tilde{\mathcal{J}}^i(h)} \hat{C}_j^i(h)$ acquirable from PCAS offers of priority i or higher cannot satisfy the cumulative demand $\sum_{k=1}^{i} r^k(h)$:

$$\sum_{j \in \tilde{\mathcal{J}}^i(h)} \hat{C}_j^i(h) < \sum_{k=1}^{i} r^k(h),\ i = 1, \ldots, N \tag{14}$$

A price combination is *feasible* if for each PCAS a^i, $i = 1, \ldots, N$, the inequality (14) does not hold. Once an infeasible price combination has been detected, the scheme proceeds to construct and verify the next price combination. A feasible price combination is *evaluated* to determine the minimum total costs incurred by the IGO for that price combination. Prior to the evaluation, we check whether a feasible price combination results in an *avoidable calculation*, i.e., a combination with costs incurred by the IGO higher than those of a combination already evaluated. For detecting avoidable calculations, each offer price $\tilde{\rho}^i(h)$ for the PCAS a^i is associated with the lower capacity limit

$$\tilde{C}_l^i(h) = \sum_{\substack{j \in S^i(h) \\ \rho_j^i(h) < \tilde{\rho}^i(h)}} c_j^i(h) \tag{15}$$

For PCAS a^1, $\tilde{C}_l^1(h) \ge r^1(h)$ due to lack of substitutability.

We identified three criteria for automatically recognizing price combinations with a total cost incurred by the IGO higher than the minimum cost $\tilde{\chi}_{min}(h)$ of the combinations already evaluated. The minimum cost $\tilde{\chi}_{min}(h)$ is given an initial value of infinity and it is updated through the use of (26). Whenever one of these criteria is met, the combination is labeled an *avoidable calculation* and requires no further evaluation. The three criteria for a price combination $[\tilde{\rho}^1(h), \tilde{\rho}^2(h), \ldots, \tilde{\rho}^N(h)]$ under test are:

(*i*) we determine the PCAS a^k such that the offer price $\tilde{\rho}^k(h) = \max_{i=1,\ldots,N} \{\tilde{\rho}^i(h)\}$; if $\tilde{\rho}^k(h) > \rho_l^k$, we set the offer price
$$\rho_-^k(h) = \max_{\rho_j^k(h) < \tilde{\rho}^k(h)} \{\rho_j^k(h)\} \tag{16}$$
for the PCAS a^k and consider the price combination $[\tilde{\rho}^1(h), \ldots, \tilde{\rho}^{k-1}(h), \rho_-^k(h), \tilde{\rho}^{k+1}(h), \ldots, \tilde{\rho}^N(h)]$; if this price combination has already been evaluated or identified to be an *avoidable calculation*, the price combination under test would result in higher total costs, so it is labeled as an *avoidable calculation*;

(*ii*) whenever the cost computed from the offer prices and their lower capacity limits $\tilde{C}_l^i(h)$ is higher than the minimum total cost $\tilde{\chi}_{min}(h)$ already computed, i.e.
$$\sum_{i=1}^{N} \tilde{\rho}^i(h)\, \tilde{C}_l^i(h) \ge \tilde{\chi}_{min}(h) \tag{17}$$
the combination corresponds to an *avoidable calculation*;

(*iii*) if for any PCAS a^i, $i=1, \ldots, N$ the sum of the lower capacity limits is equal or higher than the total demand
$$\sum_{k=i}^{N} \tilde{C}_l^i(h) \ge \sum_{k=i}^{N} r^i(h), \tag{18}$$
a combination leading to a lower cost has already been determined and so this combination also corresponds to an *avoidable calculation.*

After weeding out *infeasible combinations* and *avoidable calculations*, the remaining price combinations are *evaluated* to determine the least-cost strategy for each combination. The evaluation requires the solution of a linear programming (LP) problem to determine the least total costs incurred by the IGO for that price combination. The decision variables in the LP are the quantities $\tilde{c}_j^i(h)$ supplied by each offer for the PCAS a^i at a price not higher than $\tilde{\rho}^i(h)$. These variables must satisfy the appropriate constraints.

The LP problem formulation is:

$$min\ \tilde{\chi}(h)=\sum_{i=1}^{N}\left[\tilde{\rho}^{i}(h)\sum_{j\in\tilde{S}^{i}(h)}\tilde{c}_{j}^{i}(h)\right] \quad (19)$$

s.t.

$$0\leq\tilde{c}_{j}^{i}(h)\leq c_{j}^{i}(h),\quad i=1,...,N \text{ and } j\in\tilde{S}^{i}(h) \quad (20)$$

$$\sum_{i=1}^{N}\tilde{c}_{j}^{i}(h)\leq C_{j}^{max}(h),\quad j\in\tilde{\mathcal{J}}^{N}(h) \quad (21)$$

$$\sum_{k=1}^{i}\sum_{j\in\tilde{S}^{k}(h)}\tilde{c}_{j}^{k}(h)\geq\sum_{k=1}^{i}r^{k}(h),\quad i=1,\ldots,N\text{-}1 \quad (22)$$

$$\sum_{i=1}^{N}\sum_{j\in\tilde{S}^{i}(h)}\tilde{c}_{j}^{i}(h)=\sum_{i=1}^{N}r^{i}(h) \quad (23)$$

$$\tilde{C}_{l}^{i}(h)\leq\sum_{j\in\tilde{S}^{i}(h)}\tilde{c}_{j}^{i}(h),\quad i=1,\ldots,N \quad (24)$$

These constraints reflect that the decision variables $\tilde{c}_{j}^{i}(h)$ cannot exceed the submitted offers $c_{j}^{i}(h)$, the need to ensure that capability limits are not violated by any seller, the limits on the substitutable quantities are observed, the balancing of the total capacity and the total demand and the lower and upper limits on the capacity provided for each PCAS a^i are satisfied. The minimum total cost $\tilde{\chi}_{min}(h)$ among the combinations already evaluated is updated using

$$\tilde{\chi}_{min}(h)=:min\{\tilde{\chi}_{min}(h),\tilde{\chi}(h)\} \quad (25)$$

The evaluation of all (unavoidable) feasible combinations determines the optimal price combination with total cost $\tilde{\chi}_{min}(h)$, corresponding for $i = 1, 2, \ldots, N$ to the capacities $\bar{c}_{j}^{i}(h),\ j=1,2,...\tilde{S}^{i}(h)$, clearing quantities $\bar{C}^{i}(h) = \sum_{j\in\tilde{S}^{i}(h)}\bar{c}_{j}^{i}(h)$ and clearing prices $\bar{\rho}^{i}(h)$.

The computational speed of the rational buyer procedure mainly depends on the number of LP evaluations. The proposed scheme allows a fast solution by weeding out most of the price combinations so as to drastically reduce the number of LP evaluations required.

4 NUMERICAL RESULTS

The proposed scheme for solving the rational buyer problem has been tested on several cases and has shown excellent performance. The steps of the bounding scheme for weeding out *infeasible combinations* and *avoidable calculations* are indeed very effective in reducing the computational effort. To illustrate, consider a simple example with three PCAS a^1, a^2 and a^3 and four sellers. We focus on a single hour h. Offer capacities and required demands are expressed in MW. The offer prices are expressed in \$/MW per hour. The submitted offers and the IGO required demands for each PCAS are shown in Table 1. The capacity limits are 130, 120, 100 and 160 MW for the sellers 1, 2, 3 and 4, respectively.

PCAS	r^i	OFFERS							
		seller 1		seller 2		seller 3		seller 4	
		c^i_1	ρ^i_1	c^i_2	ρ^i_2	c^i_3	ρ^i_3	c^i_4	ρ^i_4
a^1	150	60	10	100	12	40	8	60	9
a^2	120	30	7	90	9	50	6	160	7
a^3	200	120	6	40	8	70	4	30	6

Table 1: Offer capacity and price for the set of sellers

The undefined offer price x is included for $i > 1$. The sets of offer prices are:

$$\mathcal{R}^{1}(h)=\{8,9,10,12\};$$
$$\mathcal{R}^{2}(h)=\{x,6,7,9\};$$
$$\mathcal{R}^{3}(h)=\{x,4,6,8\}$$

There are a total of 4*4*4 = 64 possible combinations. For the proposed procedure, after the application of lower and upper bounds, the sets $\tilde{\mathcal{R}}^{i}(h)$ are given in Table 2, and the number of combinations is reduced to 2*3*2 = 12. The exhaustive search is then performed on these combinations, resulting in eight *unfeasible combinations* and one *avoidable calculation*.

PCAS	$\tilde{\mathcal{R}}^{i}(h)$
a^1	{10, 12}
a^2	{6, 7, 9}
a^3	{4, 6}

Table 2: Sets of offer prices resulting after the computation of the lower and upper bounds

The remaining three combinations are evaluated, by solving the LP problem to compute the least acquisition costs incurred by the IGO. Table 3 shows the optimal solution of the rational buyer approach.

PCAS	$\bar{\rho}^i$	$\bar{c}^i_1$	$\bar{c}^i_2$	$\bar{c}^i_3$	$\bar{c}^i_4$	$\bar{C}^i$	costs incurred by the IGO
a^1	12	0	100	0	60	160	1920
a^2	7	10	0	30	70	110	770
a^3	6	120	0	70	10	200	1200
Total IGO costs	--	790	1200	630	1270	--	3890

Table 3: Results of the rational buyer approach

This small system example illustrates the way the proposed procedure reduces the overall computational burden in the discrete programming approach to the rational buyer problem. For the small system reported in [8], the proposed procedure eliminates about 75% of the evaluations – about a fourfold reduction in calculations.

We next discuss the performance of the proposed approach, with the focus on reducing the number of combinations to be evaluated for large number of sellers. We consider four different PCAS for the following three test systems:

(a) 25 sellers and relatively high demand;
(b) 30 sellers and relatively low demand;
(c) 35 sellers and intermediate level of demand.

with the demand referring to the four PCAS.

The results are summarized in Table 4.

test system	*(a)*	*(b)*	*(c)*
number of sellers	25	30	35
exhaustive search	439,400	893,730	1,632,960
bounded search	26,520	15,015	40,698
infeasible combinations	5,349	2,809	9,057
avoidable calculations	20,857	12,135	31,480
LP evaluations	314	71	161

Table 4: Number of combinations for the rational buyer approach for four PCAS for the three test systems

We also show in Figure 4 the convergence of the approach to determine the optimal solution of each of the three test systems.

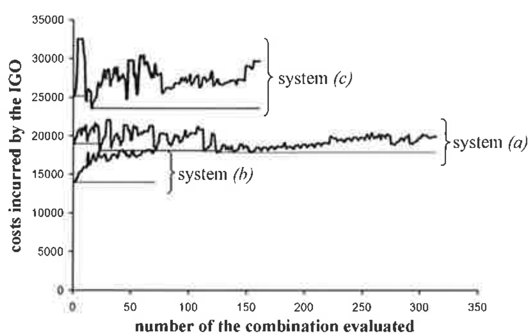

Figure 4: Determination of the least total costs incurred by the IGO.

Each curve in Figure 4 shows the evolution of the IGO-incurred costs to attain the optimal value. System *(a)* requires a relatively larger number of evaluations than the other two systems, due to the large number of capacity limits enforced in providing the quantities required to meet the high demand levels. System *(b)* has a few constraints enforced and the solution is reached with a small number of evaluations. System *(c)* has an intermediate number of evaluations. In all cases, the proposed scheme has shown to be extremely powerful in reducing the number of combinations to be evaluated.

5 CONCLUSIONS

Some key concepts in the effective procurement of prioritizable capacity-based on a competitive basis AS have been exploited, for the purposes of developing a computationally efficient scheme for the rational buyer approach used by the IGO for the acquisition of PCAS. A salient feature of the proposed procedure is its ability to accommodate constraints such as ramp-rate, capacity and inter-zonal limits.

Future work will address the formulation of optimal offers by sellers of capacity-based AS, with focus on the interrelationships between the forward energy and AS markets. An equally important problem is the contribution of a load as a resource. With that in mind, we will study the customers' participation in the AS procurement, by explicitly considering the impacts of customer load elasticity on the required demand of capacity. From the IGO viewpoint, an important problem that we will report on is our study of the possible incorporation of the AS in a bilateral trading framework. Work on the incorporation of geographic diversity in the provision of capacity-based AS will be reported in a future paper.

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REFERENCES

[1] M.Ilic, F.Galiana and L.Fink, "Power Systems Restructuring: Engineering and Economics", Power Electronics and Power Systems Series, Kluwer Academic Publishers, May 1998, ISBN 0-7923-8163-7.

[2] K.Bhattacharya, M.H.J.Bollen and J.E.Daalder, "Operation of Restructured Power Systems", Kluwer Academic Publishers, June 2001, ISBN 0-7923-7397-9.

[3] North American Electric Reliability Council IOS Working Group, "Defining Interconnected Operations Services under Open Access: Final Report", NERC, March 1997.

[4] S.S.Oren, "Design of Ancillary Service Markets", Proc. 34th Hawaii International Conference on System Sciences, Maui, Hawaii, January 3-6, 2001, ISBN 0-7695-0981-9, pp.769-777.

[5] H.Singh and A.Papalexopoulos, "Competitive Procurement of Ancillary Services by an Independent System Operator", IEEE Trans. on Power Systems, Vol.14, No.2, May 1999, pp.498-504.

[6] T.Alvey, D.Goodwin, X.Ma, D.Streiffert and D.Sun, "A Security-constrained Bid-clearing System for New Zealand Wholesale Electricity Market", IEEE Trans. on Power Systems, Vol.13, No.2, May 1998, pp.340-346.

[7] G.Chicco and G.Gross, "The Acquisition of Prioritizable Capacity-based Ancillary Services", Proc. Int. Symposium "Electric Power Engineering at the beginning of the Third Millennium", Naples–Capri, Italy, May 12-18, 2000, ISBN 88-495-0158-7, Vol.2, pp.63-82.

[8] Y.Liu, Z.Alaywan, M.Rothleder, S.Liu and M.Assadian, "A Rational buyer's Algorithm used for Ancillary Service Procurement", Proc. IEEE/PES 2000 Winter Meeting, Singapore, January 23-27, 2000, ISBN 0-7803-5935-6, Vol.2, pp.855-860.

[9] G.Gross and D.Finlay, "Generation Supply Bidding in Perfectly Competitive Electricity Markets", Computational and Mathematical Organization Theory special issue on User Competition in Operations Management, May 2000, Vol.6, pp.83-98.

[10] F.S.Hillier, "Introduction to Operations Research", McGraw Hill, New York, 2000, ISBN 0071181636.