Implications of Cost and Bid Format on Electricity Market Studies: Linear Versus Quadratic Costs

Mary B. Cain and Fernando L. Alvarado

Abstract—One important assumption in a model of an electricity market is the format of bids and costs. Most literature on electricity markets uses piecewise linear or quadratic functions to represent costs and bids. Economic theory holds that a firm in a perfectly competitive market maximizes its profit when it sells at marginal cost. This implies that profit-maximizing generators will bid at marginal cost. Different markets have varying rules regarding bid formats. Piecewise linear bid curves are compatible with physical characteristics of electricity generators, but cause difficulties in certain analysis techniques. Quadratic bid curves provide smooth dispatch, revenue, and profit curves that facilitate calculus-based analysis. In profit calculations, bids and marginal costs may either be coupled or independent of each other. The relation between bids and marginal costs impacts the profit-maximizing bid, and thus impacts generator strategies. These assumptions are particularly important for marginal generators, since different bid structures may yield different dispatch results, especially if the system is constrained. We compare markets with all piecewise linear bids, all quadratic bids, and a mixture of bids, and study the impacts of bid format and profit calculation on market outcomes in different scenarios.

***Index Terms*-- Power system economics, Pricing, Market design**

I. Introduction

One of the first assumptions in a model of an electricity market is the format of bids and costs. Most literature on electricity markets uses piecewise linear or quadratic functions, relating money and electric power, to represent costs and bids. Fabra, von der Fehr, and Harbord [1] discuss the impact of discrete and continuous bids on models of different types of electricity auctions, and the importance of understanding the differences between continuous bids used in studies of small, theoretical markets, with discrete bids used in actual markets. According to economic theory, a firm in a perfectly competitive market maximizes its profit when it sells at marginal cost. This implies that profit-maximizing generators will bid at marginal cost, so ideally generators should bid their marginal cost curves. Different markets require generators to submit bids in different formats. Generators with multiple units are well approximated by piecewise linear curves, since there is a jump in cost each time a unit is turned on or off, and then a gradual increase as individual units are ramped up or down. However, piecewise linear bid curves result in jumps in the dispatch, revenue, and profit curves. These jumps make calculus-based analysis difficult, and can result in inconsistencies. Quadratic cost curves result in smooth dispatch, revenue, and profit curves when none of the system constraints (transmission congestion, generator capacities) are active. While quadratic curves facilitate analysis, they are not a perfect characterization of a generator's cost structure.

In some studies, cost and bid are assumed equal, or closely correlated, while in others they are decoupled. This assumption changes the shape of the profit curve, and alters assumptions about generator strategy. In this paper we analyze the impacts of cost and bid assumptions on generator dispatch, revenue, profit, and bidding decisions. These assumptions are particularly important for marginal (pivotal) generators, since different bid structures may yield different dispatch results, especially if the system is constrained. We compare markets with all piecewise linear bids, all quadratic bids, and a mixture of bids, and study the impacts of bid format on market outcomes in different scenarios.

The outline of this paper is as follows: section II discusses reasons for quadratic and piecewise linear cost models in electricity markets, section III explores the impact of cost and bid format on dispatch, revenue and profit curves, and section IV concludes the paper.

II. Choice Of Quadratic or Linear Costs

Market rules determine how generators bid to maximize their profits. Some markets use a one-part bid, while others allow separate bids for startup and shutdown costs. For example, the New York ISO separates energy bids into a startup cost curve (startup cost increases with time offline), and an energy curve, which can be a three-segment staircase or a 6-segment piecewise linear curve [2]. The PJM market

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uses a similar system, but with ten-point energy curves [3]. Most electricity markets have rules forbidding declining bids, so that the first units to be dispatched are the first available to run. However, generators have declining marginal costs since, generators run more efficiently at higher outputs, and due to internalization of fixed costs, such as start-up and shut-down costs. Stobrawe and Haubrich [4] discuss the impacts of internal generator losses and transformer and network losses on an ISO's network operating costs. They calculate the cost of reactive power, and find that it increases quadratically with reactive power output. Rajaraman and Alvarado [5] discuss the impacts of market rules on generator's bidding strategies when cost and operational features of generators are complex and bids are one-part linear cost bids.

Much analysis is based on the assumption that dispatch, revenue, and profit are smooth, differentiable functions of bid. The type of cost curve used affects these functions. Glavitsch and Alvarado [6] note that linear, especially constant, cost curves are incompatible with price signals in congestion pricing. Difficulties arise because bidding blocks of power at constant costs produce jumpy and inconsistent dispatch solutions. Alvarado [7] expands on this, and demonstrates a simple case where linear costs do not allow the power system to reach its optimal solution. He also notes that one way to compensate for the jumpiness of signals from linear costs is to include a price signal for losses, which tend to have a quadratic relation to power injection. He further comments that another way to compensate for the problems associated with linear costs is to rely on the natural delays associated with electricity dispatch.

III. Determining Relationships Between Dispatch, Revenue, Profit, and Bid

To demonstrate the differences between quadratic and piecewise-linear cost models for generators, we use a simple two-generator system. Each generator has a maximum output of 60 MW and there is one load of 70 MW. This means that no single generator is able to meet the load in this example.

A. Purely Quadratic versus Purely Linear Bids

We vary the bid of one generator between \$5/MWh and \$30/MWh, while leaving the second generator's bid constant at \$12/MWh. Both generators bid a constant amount for their entire 60 MW capacities for piecewise linear costs. For the initial purposes of this example, we assume that bids are the result of single-valued bids to an otherwise purely quadratic cost curve going through the corner points (0,0) and (60,bid) as the marginal cost curve (derivative of the total cost curve), so that the quadratic cost equation will have the form in (1).

$$C = 2*bid*P^2/60. \qquad (1)$$

There are, of course, numerous ways to represent segments of quadratic curves with linear approximations. The graphs in Fig. 1 show the variation of dispatch, revenue, and profit for piecewise linear and quadratic costs for the various bid levels.

The piecewise linear graphs, as expected, are not continuous. The quadratic graphs, however, confirm these expectations: Dispatch is approximately a linearly decreasing function of bid. Revenue is approximately a quadratic function of bid. Profits are a quadratic function of bid, proportional to revenue, assuming constant fuel cost, and ignoring startup and shutdown costs, when bids equal marginal costs.

These characteristics of the quadratic graphs are true when the binding constraints are not changing. For example, if a transmission line overloads for higher bids, there will be some non-linearities in the dispatch – bid relationship, causing the revenue and profit curves to deviate more from smooth quadratic functions. In Fig. 1, all of the quadratic curves have a discontinuity at the left where the cheaper generator reaches its maximum output of 60 MW. The discontinuity occurs for the same reason in the linear curves, but at a different point.

Notice that the maxima of the quadratic revenue and profit curves occur very close to a jump point in the linear cost curves, in this case at a bid of \$12. In a perfectly competitive market, maximum profit coincides with a price at marginal cost. Thus it is likely that the maximum profit for this generator will result from bidding in the region of this jumpy point.

The two profit graphs illustrate two different assumptions about costs. In the lower-left graph, we assume that bids are equal to marginal cost. This means that negative bids are associated with negative costs – that is, the cost of production is negative. This is, of course, unrealistic in practice. This is the reason for the high profits at negative bids. In the lower-right graph, we assume that there is an unchanging marginal cost function independent of bid. In other words, bids are decoupled from marginal costs.

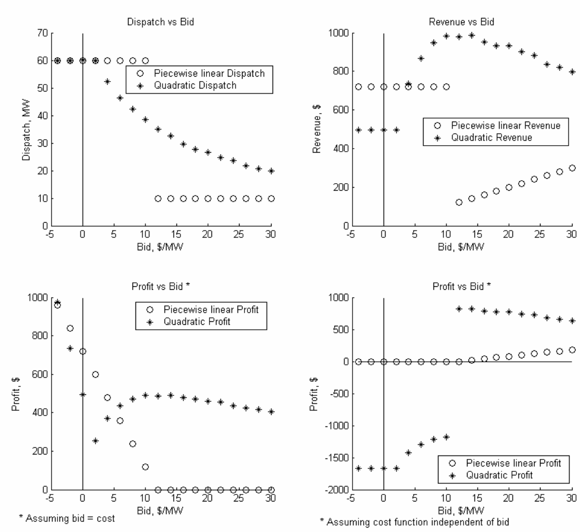

Fig. 1: Comparison of purely quadratic and piecewise linear dispatch, profit and revenue for one generator. The main observation is the fundamentally different nature of the profits depending on whether the cost structures are linear or quadratic, and whether bids correspond to marginal cost or not.

B. Mixed Quadratic and Linear Bids

In converting from the piecewise-linear to the quadratic cost curves, we matched the corner points. Thus the profit and revenue have similar values at bid of $5/MW. However, they have very different behavior for higher bids. For example, the piecewise-linear costs result in a profit and revenue maximum at $5/MW, while the maximum for the quadratic curves occurs around $12/MW. Matching the corner points gives purely quadratic curves. We calculated coefficients for other curves with different levels of linear and quadratic coefficients to see how this affects the results. Fig. 2 shows the different levels of the quadratic coefficient C and the linear coefficient B in the bid curve. We varied the proportions of B and C at the points shown in Fig. 2 to obtain the marginal and total cost (bid) curves shown in Fig. 3.

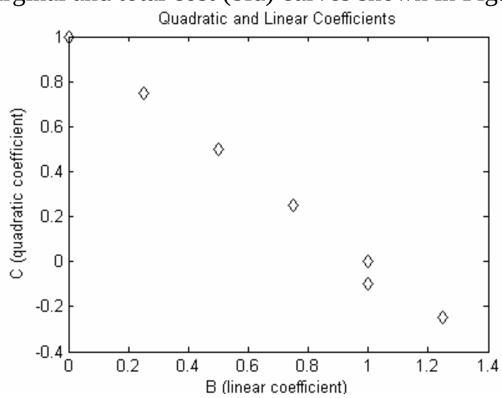

Fig. 2: Quadratic and Linear Coefficients used for comparing results with different bid curves

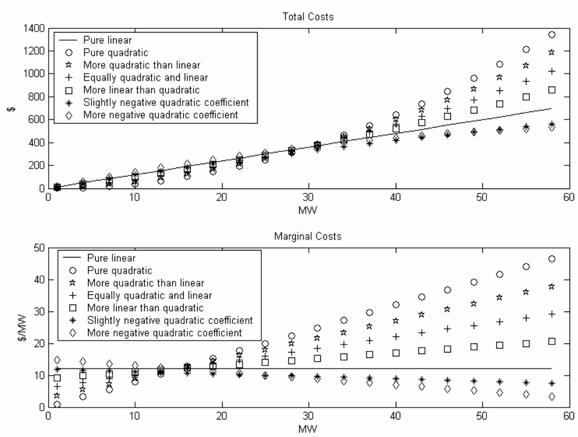

Fig. 3: Total and marginal cost curves, varying quadratic coefficients. All curves intersect at $12/MW.

Up to 30 MW, or half of the generator's capacity, the total cost curves are very similar, but they diverge for higher MW outputs. The curves all intersect at the same point; this is because the curves are all approximations to the same bid ($12/MW for 60 MW). We choose the curves to intersect at this point so that they will be comparable in a range around that point. For both the marginal costs and total costs, the pure quadratic and negative quadratic coefficient are the two extremes. We can view the case of linear costs (quadratic coefficient 0) as an intermediate stage between these two extremes.

Fig. 4 shows the results for dispatch, revenue, and profit from an OPF for the different cost curves. For a very negative quadratic coefficient (steeply declining marginal cost), the generator is always fully dispatched. For a slightly negative quadratic coefficient (gradually declining marginal cost), the drop from full dispatch to 10 MW occurs at a higher bid than in the linear cost case. Making the quadratic coefficient more negative will move this point further and further to the right. If we extend bidding to higher levels in the very negative quadratic coefficient case, the dispatch will eventually drop. The drop in dispatch carries over to the revenue and profit curves, as the point where they begin to decrease.

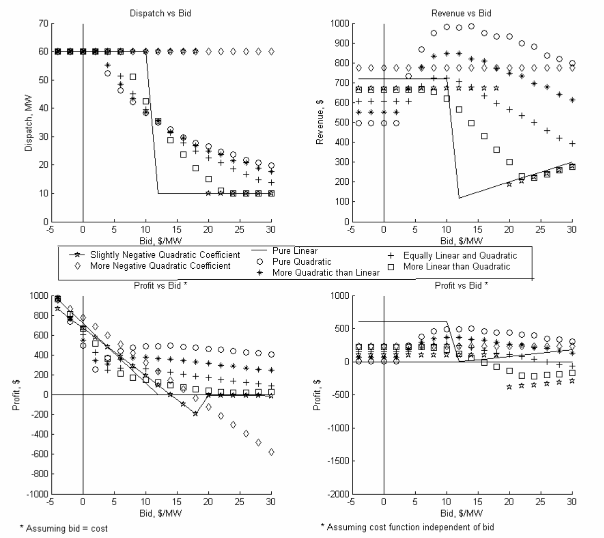

Fig. 4: Dispatch, Revenue and Profit for different quadratic and linear cost coefficients

As seen in Fig. 4, generators that bid declining marginal costs are dispatched at full capacity for higher bids than other generators. However, Fig. 5 shows that the declining marginal cost generator also tends to have lower locational marginal prices than the other generator. This is partly because this generator is fully dispatched at higher bids, so that the other generator which is not changing its bid curve is the pivotal generator and sets the marginal price near $12/MW.

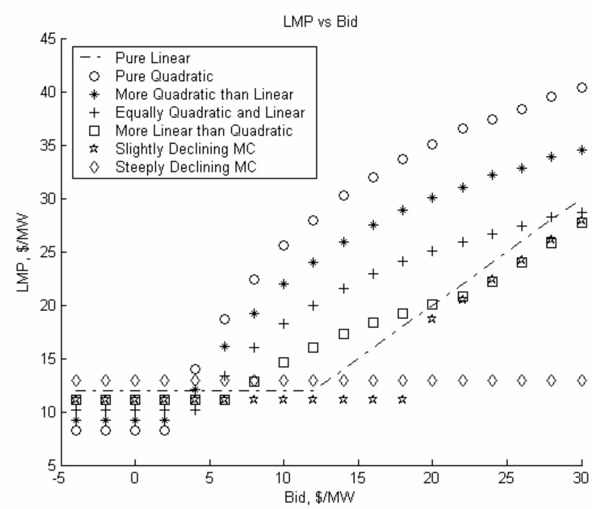

Fig. 5: LMP vs. bid for different bid curves. Note that declining marginal costs have the lowest LMPs.

All of the above figures have the same constant term in quadratic costs/bids. This term affects the unit commitment problem, but does not change the optimal power flow results for a given time period. In this paper we are focusing on optimal power flow for one time period. One possibility for future work is to investigate bidder interactions and behavior over multiple time periods, when unit commitment would have an affect.

C. Markets with Heterogeneous Bid Structures

Fig. 6 shows the dispatch, revenue, and profit for a generator using different bid formats competing with a generator using a constant bid of $12/MW.

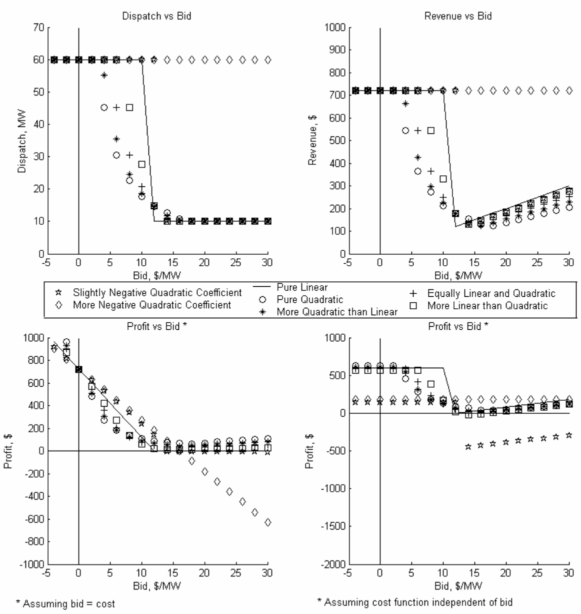

Fig. 6: Dispatch, revenue and profit for mixed bids

The dispatch curve tends to be steeper than the case of generators using the same bid format. The revenue and profit curves for the more quadratic bids all have similar shape to the linear curves in the previous example, but begin to decrease at lower bids than in the homogeneous bid structure example. Thus the quadratic form of the revenue and profit curves only occurs when most of the generators in the market have positive quadratic coefficients. For example, the generator bidding with a purely quadratic curve has decreasing revenue for bids above $5/MW. When this generator bids against another generator with a purely quadratic curve, its revenue begins increasing at this point, until it reaches a maximum at a bid of $12/MW, and then decreases.

Fig. 7 shows the LMP variation for the different bid curves competing against a linear bidder. These curves are all piecewise linear, unlike the quadratic curve shapes seen previously in Fig. 5. The purely linear bid produces an increase in LMP at the lowest bid, while the purely quadratic and steeply declining marginal cost curves do not see an increase in LMP until higher bid levels. The increase in LMP for the more expensive generator coincides with the point where this generator becomes a marginal or pivotal generator in the system. The point where the more expensive generator becomes marginal depends on the bid curve format. The transition occurs at the lowest bid for declining marginal costs, then for purely quadratic, and it occurs at the highest bid in the purely linear case.

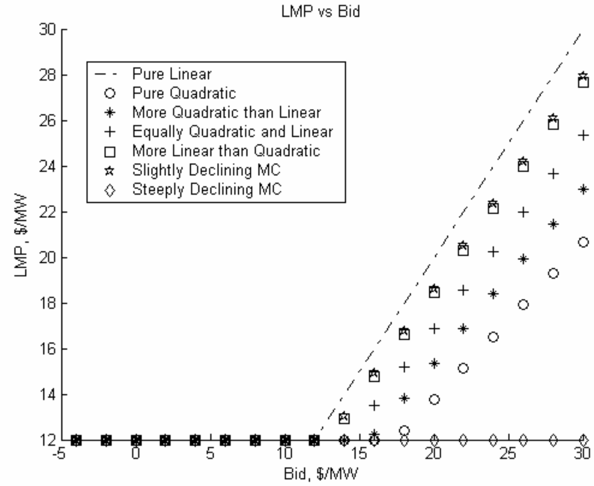

Fig. 7: LMP vs. bid for a generator varying its bid curve competing with a purely linear bidder.

IV. Conclusions

The format of bid curves allowed in a market affects the dispatch, revenue and profit for generators. In a market with purely quadratic bid curves, dispatch, revenue and profit are smooth curves that jump only when system constraints are reached. In a market with purely linear bids, dispatch, revenue and profit curves for different bids have jumps when the system becomes constrained and at points where different generators' bid curves intersect; when two generators have the same bid, the pivotal generator supplying the next unit of power jumps back and forth between the two generators. Within a region around a given operating point, quadratic bid curves produce smooth dispatch, revenue, and profit curves for different bids. As long as active constraints in the system do not change, these curves can be used in traditional calculus-based analysis. Linear bid curves, however, do not produce continuously differentiable profit, revenue or dispatch curves for different bids, requiring different analysis techniques.

Mixed quadratic and linear bid curves produce dispatch, revenue and profits that are between the purely quadratic and purely linear cases. Generators with decreasing marginal cost have negative quadratic coefficients. This causes them to be dispatched at higher bid levels than they would be with positive quadratic bid coefficients, thus increasing profit and revenue. We examine two cases of profit; one where bids are assumed equal to cost and one where cost has no influence on bid. When bids equal costs, negative bids are the most profitable and high bids the least profitable. When bid and cost are independent, negative bids result in zero or low profit, depending on the shape of the bid curve, and medium bids are the most profitable.

The form of bid curve used in analysis depends on the rules in a market, generator characteristics and the type of analysis being conducted.

V. References

[1] N. Fabra, N.H. von der Fehr and D. Harbord. "Modeling Electricity Auctions", *The Electricity Journal*, Elsevier Science, Inc., August/September 2002, pp. 72 – 81.

[2] New York Independent System Operator, NYISO Market Participant User's Guide, March 18, 2003. [Online], available at: http://www.nyiso.com/services/documents/manuals/pdf/admin_manuals/mpug_mnl.pdf

[3] PJM Interconnection, PJM Manual 11: Scheduling Operations, Revision 21, January 31, 2004. [Online], available at: http://www.pjm.com/contributions/pjm-manuals/pdf/m11v21.pdf

[4] M. Stobrawe and H.J. Haubrich. "Real-Time Optimisation of Reactive Power Costs and Transmission Loss Costs", *Proceedings of VIII Symposium of Specialists in Electric Operational and Expansion Planning (VIII SEPOPE)*, Brasilia, Brazil, May 2002.

[5] R. Rajaraman and F. Alvarado. "Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices", PSERC Publication 03-05, Power Systems Engineering Research Center, Madison, WI, April 2003. Available online at http://www.pserc.wisc.edu

[6] H. Glavitsch and F. Alvarado. "Management of Multiple Congested Conditions in Unbundled Operation of a Power System", *IEEE Transactions on Power Systems*, Vol. 13, No. 3, August 1998.

[7] F. Alvarado. "Is System Control Entirely by Price Feasible", *Proceedings of the 36th Hawaii International Conference on System Sciences*, IEEE, 2003.

VI. Biographies

Mary B. Cain received her MS degree in electrical engineering from the University of Wisconsin – Madison in 2003, and her BS in electrical engineering from the University of Illinois at Urbana-Champaign in 2002. Her research interests focus on electricity markets and deregulation. She will start working for the Federal Energy Regulatory Commission in June 2004.

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