Observability Analysis and Measurement Placement for Systems with PMUs

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***Abstract*—This paper is concerned about the analysis of network observability and phasor measurement unit (PMU) placement when using a mixed measurement set. The measurements will include conventional power flows and injections as well as phasor measurements for voltages and line currents provided by phasor measurement units. The observability analysis is followed by an optimal meter placement strategy for the PMUs.**

***Index Terms*—State estimation, network observability, meter placement, phasor measurement units.**

I. Introduction

Secure operation of power systems requires close monitoring of the system operating conditions. This is traditionally accomplished by the state estimator which resides in the control center computer and has access to the measurements received from numerous substations in the monitored system. These measurements are commonly provided by the remote terminal units (RTU) at the substations and include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. More recently, synchronized phasor measurements have started to become available at selected substations in the system. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the positive sequence phasor voltages and currents measured at a given substation. These types of measurements will in turn improve the performance of the state estimators.

Pioneering work in PMU development and utilization is done by Phadke et al. [1-2]. It is argued that the use of PMUs at each bus will lead to a simplified linear state estimator. This requirement is later relaxed in [3] due to the fact that each PMU can measure not only the bus voltage but also the currents along all the lines incident to the bus. Hence, furnishing a selected subset of buses with PMUs can make the entire system observable. This will only be possible by proper placement of PMUs among the system buses. This problem is formulated and solved using graph theoretic observability analysis and an optimization method based on Simulated Annealing in [3]. In this paper, a different formulation which is numerical and uses integer programming will be presented. This formulation allows easy analysis of network observability for mixed measurement sets, which may include conventional power flow and injection measurements in addition to the PMUs. Determination of optimal locations of PMUs with or without existing conventional measurements is also accomplished using the same formulation.

II. PMU placement Problem formulation

A PMU placed at a given bus is capable of measuring the voltage phasor of the bus as well as the phasor currents for all lines incident to that bus. Thus, the entire system can be made observable by placing PMUs at strategic buses in the system. The objective of the PMU placement problem is to accomplish this task by using a minimum number of PMUs. In this paper, the problem is formulated and solved as an Integer Programming problem as shown below.

For an n-bus system, the PMU placement problem can be formulated as follows:

$$\begin{aligned} &\min \sum_{i}^{n} w_i \cdot x_i \\ &s.t. \quad f(X) \geq \hat{1} \end{aligned} \tag{1}$$

where

X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 & if\ \ a\ PMU\ \ is\ installed\ at\ bus\ i \\ 0 & \qquad\qquad otherwise \end{cases}$$

w_i is the cost of the PMU installed at bus i;

$f(X)$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$ is a vector whose entries are all ones.

The expressions for the nonlinear constraints can be formed based on the knowledge about the locations and types of existing measurements. Given a PMU at a bus, it is assumed that the bus voltage phasor and all current phasors along lines connected to that bus will be available. This also implies that this bus voltage, along with all adjacent bus voltages will also be available (solvable).

The procedure for building the constraint equations will be described for three possible cases where there are (1) only

PMU measurements, (2) PMU measurements and injections (they may be zero injections or measured injections) or (3) PMU measurements, injections and flows. Description of the procedure for each case will be given using a small 7-bus tutorial example for clarification. However, the entire procedure is actually programmed and successfully tested on different size systems with diverse measurement configurations.

Consider the 7-bus system and its measurement configuration shown in Figure 1.

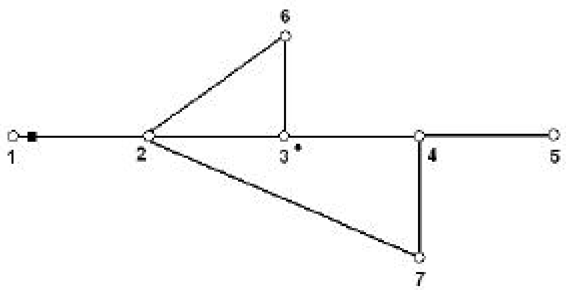

Figure 1. 7-Bus Example System

Case 1: A system which has no conventional measurements and/or zero injections.

First, form the binary connectivity matrix A. The entries of A are defined as follows:

$$A_{k,m} = \begin{cases} 1 & if \quad k = m \\ 1 & if \quad k \text{ and } m \text{ are connected} \\ 0 & if \quad otherwise \end{cases}$$

Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form. Building the A matrix for the 7-bus system of Figure 1 yields:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The constraints for this case can be formed as:

$$f(X) = \begin{cases} f_1 = x_1 + x_2 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 + x_6 & \geq 1 \\ f_4 = x_3 + x_4 + x_5 + x_7 & \geq 1 \\ f_5 = x_4 + x_5 & \geq 1 \\ f_6 = x_2 + x_3 + x_6 & \geq 1 \\ f_7 = x_2 + x_4 + x_7 & \geq 1 \end{cases} \tag{3}$$

The operator "+" serves as the logical "OR" and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero. For example, consider the constraints associated with bus 1 and 2 as given below:

$$f_1 = x_1 + x_2 \geq 1$$
$$f_2 = x_1 + x_2 + x_3 + x_6 + x_7 \geq 1$$

The first constraint $f_1 \geq 1$ implies that at least one PMU must be placed at either one of buses 1 or 2 (or both) in order to make bus 1 observable. Similarly, the second constraint $f_2 \geq 1$ indicates that at least one PMU should be installed at any one of the buses 1, 2, 3, 6, or 7 in order to make bus 2 observable.

Case 2: A system which contains injection measurements some of which may be zero injection pseudo-measurements.

Injection measurements whether they are real measurements or zero injections, are treated the same way. Consider again the 7-bus system shown in Figure 1, where bus 3 is assumed to be a zero injection bus. In this case, it is easy to see that if the phasor voltages at any three out of the four buses 2, 3, 4 and 6 are known, then the fourth one can be calculated using the Kirchhoff's Current Law applied at bus 3 where the net injected current is known. Hence, the constraints associated with these buses will have to be modified accordingly as shown below:

$$f_2 = x_1 + x_2 + x_3 + x_6 + x_7 + f_3 \cdot f_4 \cdot f_6 \geq 1$$
$$f_4 = x_3 + x_4 + x_5 + x_7 + f_2 \cdot f_3 \cdot f_6 \geq 1$$
$$f_6 = x_2 + x_3 + x_6 + f_2 \cdot f_3 \cdot f_4 \geq 1$$

Note that the operator '.' serves as the logical "AND" in the above equations.

The expressions for f_i can be further simplified by using the following properties of the logical AND (.) and OR (+) operators:

Given two sets A and B, where set A is a subset of set B, then $A + B = B$ and $A \cdot B = A$.

For instance, substituting the expression for f_3 in the expression for f_2, f_2 can be written as:

$$\begin{aligned} & x_1 + x_2 + x_3 + x_6 + x_7 + f_3 \cdot f_4 \cdot f_6 \\ &= x_1 + x_2 + x_3 + x_6 + x_7 + (x_2 + x_3 + x_4 + x_6) \cdot f_4 \cdot f_6 \\ &= x_1 + x_2 + x_3 + x_6 + x_7 \\ &\quad + x_2 \cdot f_4 \cdot f_6 + x_3 \cdot f_4 \cdot f_6 + x_4 \cdot f_4 \cdot f_6 + x_6 \cdot f_4 \cdot f_6 \\ &= x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot f_4 \cdot f_6 \end{aligned}$$

Note that the expression for f_3 should also include an extra product term given by $f_2 \cdot f_4 \cdot f_6$, however this term will be neglected. In all our simulated cases, this approximation is found to have no effect on the optimization.

Carrying on with the simplifications, the product $x_2 \cdot f_4 \cdot f_6$ is eliminated because it is the subset of x_2, which already exists in the expression. Using similar reasoning, $x_3 \cdot f_4 \cdot f_6$ and $x_6 \cdot f_4 \cdot f_6$ are also eliminated. Then, substituting the expression of f_4 yields:

$$\begin{aligned}
&x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot f_4 \cdot f_6 \\
&= x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot (x_3 + x_4 + x_5 + x_7) \cdot f_6 \\
&= x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot f_6
\end{aligned}$$

Since x_4 is a subset of $(x_3 + x_4 + x_5 + x_7)$, one can write: $x_4 \cdot (x_3 + x_4 + x_5 + x_7) \cdot f_6 = x_4 \cdot f_6$.

Now, substitute for f_6:

$$\begin{aligned}
&x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot f_6 \\
&= x_1 + x_2 + x_3 + x_6 + x_7 + x_4 \cdot (x_2 + x_3 + x_6) \\
&= x_1 + x_2 + x_3 + x_6 + x_7
\end{aligned}$$

Furthermore, having $x_4 \cdot (x_2 + x_3 + x_6)$ as a subset of $(x_2 + x_3 + x_6)$, allows its elimination from the expression. Finally, the expression for f_2 simplifies to the following: $f_2 = x_1 + x_2 + x_3 + x_6 + x_7$.

Applying similar simplification logic to all other expressions will yield:

$$\begin{aligned}
f_2 &= x_1 + x_2 + x_3 + x_6 + x_7 && \geq 1 \\
f_4 &= x_2 + x_3 + x_4 + x_5 + x_6 + x_7 && \geq 1 \\
f_6 &= x_2 + x_3 + x_6 + x_1 \cdot x_4 + x_4 \cdot x_7 && \geq 1
\end{aligned}$$

Note that the constraints corresponding to all other buses will remain the same as given in equation (3). One exception is the constraint for bus 3 where the injection is measured (or known). This constraint will be eliminated from the constraint set. The reason for removing the constraints associated with injection buses is that their effects are indirectly taken into account by the product terms augmented to the constraints associated with the neighboring buses.

<u>Case 3</u>: A system which contains injections as well as flow measurements.

The modifications needed in the formulation for this case will again be illustrated using the 7-bus example, where a flow measurement (P and Q) is added for branch 1-2. In this case, the constraints for bus 1 and 2 will have to be modified accordingly.

Note that having a flow measurement along a given branch, allows the calculation one of the terminal bus voltage phasors when the other one is known. Hence, the constraint equations associated with the terminal buses of the measured branch can be merged into a single constraint.

For this example the constraints for buses 1 and 2 are merged into a joint constraint as follows,

$$\begin{cases} f_1 = x_1 + x_2 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \end{cases} \Rightarrow$$

$$f_{1\_new} = f_1 + f_2 = x_1 + x_2 + x_3 + x_6 + x_7 \geq 1$$

which implies that if either one of the voltage phasors at bus 1 or 2 is observable, the other one will be observable.

Applying this modification to the constraints for the 7-bus system, the following set of final constraints will be obtained:

$$f(X) = \begin{cases} f_{1\_new} = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & \geq 1 \\ f_5 = x_4 + x_5 & \geq 1 \\ f_6 = x_2 + x_3 + x_6 + x_1 \cdot x_4 + x_4 \cdot x_7 & \geq 1 \\ f_7 = x_2 + x_4 + x_7 & \geq 1 \end{cases}$$

Note that, the constraints corresponding to buses 1 and 2 are merged into a single constraint. The constraint associated with bus 3 where there is an injection measurement, is eliminated as explained in case 2.

III. Simulation results

Simulations are carried out on the IEEE 14-bus, IEEE 57-bus and IEEE 118-bus systems. The technical specifications of the computer used for these simulations are given in Table I. Integer programming problem is solved using the TOMLAB Optimization Toolbox [4].

Figure 2 shows the results of optimal PMU placement for the IEEE 14-bus system, which has one zero injection at bus 7 and no other conventional power flow or injection measurements. There are three PMUs installed at bus 2, bus 6 and bus 9, which can make the whole system observable.

Simulation results of different cases are shown in Table II. In case 1, 2 and 3, there are no conventional power flow and injection measurements installed in the system. The numbers of zero injections are 1, 15 and 10 respectively. Case 4 is carried out using the same system as in case 3, however 6 power flow measurements (P and Q) whose locations are given in Table III, are assumed to exist in the system. Comparing cases 3 and 4 in Table II, the IP problem takes longer CPU time to solve case 4, however the required number of PMUs are reduced from 29 to 26. Hence, as expected, having conventional measurements will reduce the number of required PMUs to make the entire system observable.

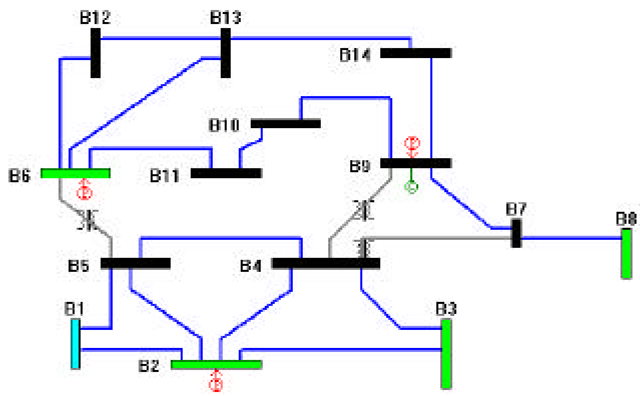

Figure 2. PMU Placement on IEEE 14-Bus System

TABLE I.
COMPUTER CONFIGURATION

CPU	LEVEL 2 CACHE	SYSTEM MEMERY
PENTIUM II 333 MHZ	512 KB	128 MB SDRAM

TABLE II.
SIMULATION RESULTS

CASE	POWER SYSTEM	ZERO INJECT.	FLOW MEAS.	PMU	CPU TIME (S)
1	IEEE 14-BUS	1	0	3	2.04
2	IEEE 57-BUS	15	0	12	4.15
3	IEEE 118-BUS	10	0	29	43.96
4	IEEE 118-BUS	10	6	26	45.25

TABLE III.
FLOW MEASUREMENTS POSITIONS FOR CASE 4

BRANCHES OF FLOW MEASUREMENTS					
1-2	4-5	20-21	21-22	17-113	86-87

IV. Conclusions

This paper presents an integer programming based formulation and the associated solution to the problem of PMU placement in power systems. The problem formulation has some attractive properties where conventional measurements such as injections and flows can also be taken into account if they already exist in the system.

Numerical results are given for different size systems where a minimum number of PMUs are placed with and without other conventional measurements. As the power systems become more populated by PMUs, the presented approach may assist the system planners in deciding on the location of new PMU installments for maximum network observability.

V. References

[1] A. G. Phadke, "Synchronized phasor measurements in power systems", IEEE Computer Applications in Power, Vol. 6, Issue 2, pp. 10-15, April 1993.

[2] A. G. Phadke, J. S. Thorp, and K. J. Karimi, "State Estimation with Phasor Measurements", IEEE Transactions on Power Systems, Vol. 1, No. 1, pp. 233-241, February 1986.

[3] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power System Observability With Minimal Phasor Measurement Placement", IEEE Transactions on Power Systems, Vol. 8, No. 2, pp. 707-715, May 1993

[4] The TOMLAB Optimization Environment, Web page URL: http://tomlab.biz/

VI. Biographies

Bei Xu (S'2004) received her B.S. and M.S. degree from Shanghai JiaoTong University, China in 1998 and 2001 respectively. She is currently a Ph.D student at the Department of Electrical Engineering at Texas A&M University, College Station, Texas.

Ali Abur (F'2003) received his Ph.D. from The Ohio State University, in 1985. He joined the Department of Electrical Engineering at Texas A&M University, where he is currently a Professor. His research interests are in power system state estimation, visualization of system operation, optimal utilization of phasor measurement units for system monitoring and analysis of network transients for identification of causes of disturbances.