The influence of futures markets on real time price stabilization in electricity markets

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Abstract

Markets can interact with power systems in ways that can render an otherwise stable market and an otherwise stable power system into an unstable overall system. This unstable system will be characterized not only by fluctuating prices that do not settle to constant values, but, more worrisome, it creates the possibility of inducing slow electromechanical oscillations if left unchecked. This will tend to happen as a result of "price chasing" on the part of suppliers that can react (and over-react) to changing system prices. This paper examines the role that futures markets may have on the clearing prices and on altering the volatility and potential instability of real time prices and generator output.

1. Nomenclature

The following notation will be used throughout this paper:

Pg_{RTi}: Power supply of generator i sold in real time market.
Pg_{DAi}: Power supply of generator i sold in day-ahead market.
Pg_{Ti}: Total power supply of generator *i* (Pg_{RTi} + Pg_{DAi}).
Pd_{RTj}: Power demand of consumer *j* in real time market.
Pd_{DAj}: Power demand of consumer *j* in day-ahead market.
Pd_{Tj}: Total power demand of consumer *j*.
Pg_{RT}, Pd_{RT}, P_{RT}: Real time total market sales/purchases
Pg_{DA}, Pd_{DA}, P_{DA}: Day-ahead total market sales/purchases
λ_{RT}: Real time market price
λ_{DA}: Day-ahead market price
Πg_i: Total benefit of supplier i
$MgCg_i$: Marginal cost generator i
$MgIg_i$: Marginal income generator i
$MgBd_j$: Marginal benefit consumer j
TIg_i: Total income generator i (from D.A. and R.T. market)

2. Introduction

Power system markets have fundamentally altered the manner in which we must view and analyze power systems. From 1982, when Chile introduced the first Poolco based electricity market, to the most recent developments in the U.S. and elsewhere, power system markets and market design have become of fundamental importance in the operation and planning of power systems. Different market models have been used (Poolco, bilateral contracts, power exchanges, etc.). Markets are becoming more transparent and price updates within these markets are becoming more frequent. Therefore, when designing electricity markets it is important to study not only the impact of a particular market design on the resulting equilibrium point, but also on the stability of the resulting market.

Many researchers have studied real time markets, the exercise of market power exercise, optimal bidding strategies, optimal contracting, and more. Game theory models (such as Cournot, Bertrand and Stackelberg.) are often applied to simulate the strategic interactions between different market participants. Much less research has been directed towards the understanding of the interactions between forward and real time markets. Williams [7] studied how futures market positions affects firm's cost in the spot market. Allaz and Vila [8] studied the interaction between the two markets using a Cournot duopoly model. They found that the market equilibrium tends toward a competitive equilibrium when the number of trading periods in futures market increases. In contrast, Ferreira [9] found that when the number of periods prior to spot market where firms can contract tends to infinite, price differences between competitive and Cournot prices can be sustained in the market.

There has been much less research in the field of dynamic behavior and stability in electricity markets. Some tangential mention of the topic is made in [10]. The main research in this field is the one led by Alvarado [1] and Alvarado, Meng, Mota and DeMarco [2,3]. They studied the stability of the power system in a single market under several assumptions. They also evaluated the impact of congestion, and, through an energy imbalance market, the interaction with power systems.

Prior reference that have considered market dynamics [1,2,3] have assumed that both suppliers and consumers participate only in a real time market. In this paper we expand these models to include the possibility of a multi-settlement market, where a fraction of the energy sales/purchases takes place in a forward market (e.g., in a day-ahead market). This allows power suppliers and consumers to protect themselves from real time price fluctuations for part of their output. Day-ahead sales and prices are exogenous variables in our model.

We re-state the assumptions used by Alvarado in [1]:

- Production costs are quadratic functions of generated power, thus marginal production costs are a linear

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functions of generator output.

- Consumers' marginal benefit functions are negatively sloping linear functions of power consumption.
- Demand is a function of marginal benefit and power price.

References [1,2,3] also assumed that a generator's power output set-point is only a function of its marginal cost and the market price for power. We show here that this simple and apparently innocuous assumption may lead to wrong conclusions when trying to extend the model to study strategic interactions between a reduced number of firms in the market. The model may also be inadequate when trying to assess the impact of forward markets on real time markets.

We assume electricity markets work under a self dispatch (or self commitment or self scheduling) scheme, where both power suppliers and consumers are exposed to real time market prices that are continuously changing and they respond by increasing or reducing their sales and purchases. These dynamic responses are assumed to be continuous and based on first order linear differential equations. The conclusions drawn in this paper are applicable to markets where real time prices are computed and updated "fast enough."

3. Market Dynamics

A. Basic Model: Price Taker

We start from the model developed by Alvarado in [1] to represent a single market. We extend this model to represent the effect of introducing a forward market. We assume that all generation is sold in the two markets, day-ahead and real time. Likewise for consumption:

$$Pg_{Ti} = Pg_{RTi} + Pg_{DAi}\,, \qquad i = 1..m \qquad (1)$$

$$Pd_{Tj} = Pd_{RTj} + Pd_{DAj}\,, \qquad j = 1..n \qquad (2)$$

Since sales in forward markets commit in advance a supplier and a producer, there is no energy imbalance in this market. For the case of m suppliers and n consumers:

$$\sum_{i=1}^{m} Pg_{DAi} = \sum_{j=1}^{n} Pd_{DAj} \qquad (3)$$

Assuming no energy imbalance in the real time market and assuming that the generator's power output decision is only a function of its marginal cost and the market price for power, we obtain the following equations:

$$\tau_{g_i}\dot{Pg}_{Ti} = \tau_{g_i}\dot{Pg}_{RTi} = \lambda_{RT} - MgCg_i \qquad (4)$$

$$\tau_{d_j}\dot{Pd}_{Tj} = \tau_{d_j}\dot{Pd}_{RTj} = MgBd_j - \lambda_{RT} \qquad (5)$$

$$\sum_{i=1}^{m} Pg_{RTi} = \sum_{j=1}^{n} Pd_{RTj} \qquad (6)$$

Replacing the linear expressions for generators' marginal cost ($MgCg_i$) and consumers' marginal benefit ($MgBd_j$), we obtain the following expressions:

$$\tau_{g_i}\dot{Pg}_{RTi} = \lambda_{RT} - b_{gi} - c_{gi}(Pg_{RTi} + Pg_{DAi}) \qquad i = 1..m$$

$$\tau_{d_j}\dot{Pd}_{RTj} = b_{dj} + c_{dj}(Pd_{RTj} + Pd_{DAj}) - \lambda_{RT} \qquad j = 1..n$$

$$\sum_{i=1}^{m} Pg_{RTi} = \sum_{j=1}^{n} Pd_{RTj}$$

Writing these equations in matrix form we obtain:

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P} + \tilde{D} \qquad (7)$$

Where:

$$\tilde{T} = \begin{bmatrix} \tau_{g1}+\tau_{g2} & \cdots & \tau_{g1} & -\tau_{g1} & \cdots & -\tau_{g1} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \tau_{g1} & \cdots & \tau_{g1}+\tau_{gm} & -\tau_{g1} & \cdots & -\tau_{g1} \\ -\tau_{g1} & \cdots & -\tau_{g1} & \tau_{g1}+\tau_{d1} & \cdots & \tau_{g1} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ -\tau_{g1} & \cdots & -\tau_{g1} & \tau_{g1} & \cdots & \tau_{g1}+\tau_{dn} \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} -(c_{g1}+c_{g2}) & \cdots & -c_{g1} & c_{g1} & \cdots & c_{g1} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ -c_{g1} & \cdots & -(c_{g1}+c_{gm}) & c_{g1} & \cdots & c_{g1} \\ c_{g1} & \cdots & c_{g1} & -c_{g1}+c_{d1} & \cdots & -c_{g1} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ c_{g1} & \cdots & c_{g1} & -c_{g1} & \cdots & -c_{g1}+c_{dn} \end{bmatrix}$$

$$\tilde{D} = \begin{bmatrix} (b_{g1}-b_{g2}) + (c_{g1}Pg_{DA1} - c_{g2}Pg_{DA2}) \\ \vdots \\ (b_{g1}-b_{gm}) + (c_{g1}Pg_{DA1} - c_{gm}Pg_{DAm}) \\ (-b_{g1}+b_{d1}) + (-c_{g1}Pg_{DA1} + c_{d1}Pd_{DA1}) \\ \vdots \\ (-b_{g1}+b_{dn}) + (-c_{g1}Pg_{DA1} + c_{dn}Pd_{DAn}) \end{bmatrix} \quad \tilde{P} = \begin{bmatrix} Pg_2 \\ \vdots \\ Pg_m \\ Pd_1 \\ \vdots \\ Pd_n \end{bmatrix}$$

The equilibrium (real time price and real time sales) can be obtained setting the derivative terms to zero and solving the following linear system:

$$\tilde{C}\tilde{P} = -\tilde{D} \qquad (8)$$

The stability conditions can be obtained through the analysis of the eigenvalues of the following homogenous equation:

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P} \qquad (9)$$

In order to have a stable system, these eigenvalues must be negative.

1) Application to a One Supplier - One Consumer Case

Solving the set of equations (8) for the case of one supplier – one consumer, the equilibrium can be written as follows:

$$P_{RT} = \frac{b_d - b_g}{c_g - c_d} - P_{DA} \qquad (10)$$

$$\lambda_{RT} = \frac{-b_g c_d + c_g b_d}{c_g - c_d} \qquad (11)$$

This result is the same as in [1]. The result in [1] was obtained without considering sales in the forward market. This means that, subject to our assumptions, the presence or absence of a day-ahead market makes no difference. The total quantity traded is also the same, but now it is divided among the two existing markets, forward and real time.

The stability conditions (from the eigenvalues of equation 9) indicate that this equilibrium is stable when:

$$\frac{c_d - c_g}{\tau_{gT} + \tau_{dT}} < 0 \qquad (12)$$

which simply means that $c_d < c_g$. This condition is almost always assured because c_d is negative and c_g is usually assumed to be positive (increasing marginal cost of generation). Thus, the stability condition for the market dynamics are once again the same as those obtained by Alvarado in [1] without considering a forward market.

This model doesn't show any incentives to participate in the forward market and it says that the equilibrium price in the real time market, the total generation, and the stability of the market are unaffected by the existence of a day-ahead market.

The model used assumes that generators increase generation when market prices exceed their marginal cost, and consumers increase consumption when market prices become lower than their marginal benefit, achieving an equilibrium where market prices perfectly match with marginal cost. Hidden in the generators' behavior is the assumption that generators act as **price takers** and that the market achieves a perfectly competitive equilibrium. In a perfect market, as described here, real time prices and day-ahead prices would converge. The fact that the suppliers "behave" in the real time market implies no additional price risks to consumers, no withholding in real time, and no exploitation of low demand elasticity of consumers in the real time market.

A similar analysis could be done for the demand side. Because of the nature of demand, it is reasonable in most cases to assume that demand exhibits a pure price taker behavior.

In order to incorporate the possible market power component of the generators' decision making process, we now analyze the structure of the supplier benefit in the market.

B. Market Power and Generators´ Behavior

The benefit of a supplier i (Πg_i) selling in day-ahead and real time markets, will be given by the total income generated in both forward (TIg_{DAi}) and real time markets (TIg_{RTi}), minus the total cost of generation (TCg_i), then we can write the following expression for the Benefit:

$$\prod g_i = TIg_{DAi} + TIg_{RTi}(Pg_{RTi}) - TCg_i(Pg_{RTi})$$

$$\Pi g_i = Pg_{DA}\lambda_{DA} + Pg_{RTi}\lambda_{RT}(Pg_{RTi}) - TCg_i(Pg_{RTi}) \qquad (13)$$

Both the generation cost and the real time price have now been expressed as functions of the supplier sales. Indeed, the output level of one supplier will have some effect on the real time price.

To maximize the generator's benefit, we compute the marginal benefit ($d\Pi g_i/dPg_{RTi}$) as follows:

$$\frac{\partial \prod g_i}{\partial Pg_{RTi}} = \frac{\partial (Pg_{DAi}\lambda_{DA})}{\partial Pg_{RTi}} + \frac{\partial (Pg_{RTi}\lambda_{RT}(Pg_{RTi}))}{\partial Pg_{RTi}} - \frac{\partial TCg_i(Pg_{RTi})}{\partial Pg_{RTi}}$$

$$\frac{\partial \prod g_i}{\partial Pg_{RTi}} = \lambda_{RT}(Pg_{RTi}) + Pg_{RTi}\frac{\partial \lambda_{RT}(Pg_{RTi})}{\partial Pg_{RTi}} - MgCg_i(Pg_{RTi}) \qquad (14)$$

In *(14)* the marginal benefit can also be decomposed into marginal income and marginal cost components:

$$\frac{\partial \prod g_i}{\partial Pg_{RTi}} = MgIg_i(Pg_{RTi}) - MgCg_i(Pg_{RTi}) \qquad (15\text{-}a)$$

Here the marginal income ($MgIg_i$) is given by:

$$MgIg_i(Pg_{RTi}) = \lambda_{RT} + Pg_{RTi}\frac{\partial \lambda_{RT}(Pg_{RTi})}{\partial Pg_{RTi}} \qquad (15\text{-}b)$$

Here marginal income is no longer equal to real time price λ_{RT}. The marginal income is now smaller due to the price reducing effect caused by its own output ($d\lambda_{RT}/dPg_{RTi}$). The more you sell, the lower the market price you face. Therefore a rational firm would behave as a profit maximizing agent making selling decisions based on real time prices λ_{RT} and considering the price reducing effect caused of its own output ($d\lambda_{RT}/dPg_{RTi}$). The firm would not just follow the price.

Thus, power suppliers increase their output as long as their marginal benefit remain positive. In other words, until marginal income equals the marginal cost. Comparing equations (15-a) and (15-b) with equation (4) from the basic model, we notice the appearance of a new term proportional to the derivative of the real time price with respect to its the power output ($d\lambda_{RT}/dP_{RTi}$), or in other words the impact of the supplier on the real time price (market power effect).

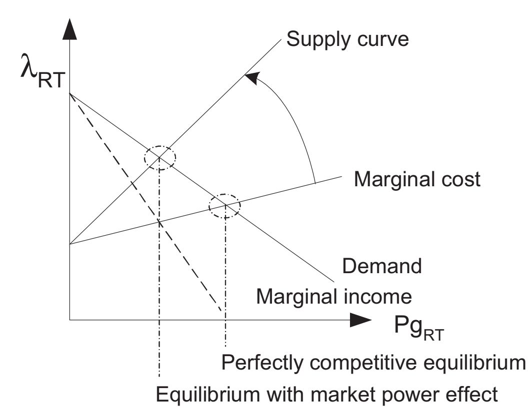


Fig. 1. Market Equilibrium

Figure 1 describes clearly this situation for a one consumer-one supplier case. The equilibrium quantity sold when marginal income equals marginal cost (zero marginal benefit) result in lower market quantity sales and higher prices. Companies withhold to keep prices higher. The prices are given by the consumers willingness to pay (supply curve) instead of by the marginal costs.

Using these new behavioral patterns of power suppliers, we can restate our model as follows:

$$\tau_{g_i} \dot{Pg}_{RTi} = \lambda_{RT} - b_{gi} - c_{gi}(Pg_{RTi} + Pg_{DAi}) + Pg_{RTi}\frac{\partial \lambda_{RT}(Pg_{RTi})}{\partial Pg_{RTi}}$$

$$\tau_{d_j} \dot{Pd}_{RTj} = b_{dj} + c_{dj}(Pd_{RTj} + Pd_{DAj}) - \lambda_{RT} \qquad (16\text{-}17)$$

$$\sum_{i=1}^{m} Pg_{DAi} = \sum_{j=1}^{n} Pd_{DAj} \qquad \sum_{i=1}^{m} Pg_{RTi} = \sum_{j=1}^{n} Pd_{RTj} \qquad (18)$$

In this model consumers compare their marginal benefit with the real time prices and adjust their consumption with a rate proportional to this difference. Generators increase their generation according to their marginal benefit.

The equilibria conditions are obtained setting the time derivatives of equation (16) and (17) equal to zero:

$$\lambda_{RT} - b_{gi} - c_{gi}(Pg_{RTi} + Pg_{DAi}) + Pg_{RTi}\frac{\partial \lambda_{RT}(Pg_{RTi})}{\partial Pg_{RTi}} = 0$$

$$b_{dj} + c_{dj}(Pd_{RTj} + Pd_{DAj}) - \lambda_{RT} = 0 \qquad (19\text{-}20)$$

$$\sum_{i=1}^{m} Pg_{DAi} = \sum_{j=1}^{n} Pd_{DAj} \qquad \sum_{i=1}^{m} Pg_{RTi} = \sum_{j=1}^{n} Pd_{RTj} \qquad (21)$$

C. Real Time Price to Generation Sensitivity

Since the new set of equations (19-21) require the computation of $d\lambda_{RT}/dPg_{RTi}$ (derivative of the real time price λ_{RT} with respect to the power output Pg_{RTi}), we develop a general expression for this term. We name it ro_i (ρ_i) and define it as follows:

$$\rho_i = \frac{\partial \lambda_{RT}(Pg_{RTi})}{\partial Pg_{RTi}} \qquad (22)$$

Using equation (20) we can express the total real time consumption or demand as follows (eq. 23):

$$Pd_{RT} = \sum_{j \in \text{Active Consumers}} Pd_{RTj} = \lambda_{RT} \sum_{j \in \text{Active Consumers}} \frac{1}{c_{dj}} - \sum_{j \in \text{Active Consumers}} \frac{b_{dj}}{c_{dj}} - Pd_{DA}$$

Active consumers are those how are trading some part of their energy in the real time market, therefore they satisfy the following relation:

$$c_{dj} \cdot Pd_{DAj} > \lambda_{RT}$$

Using equations (19, 21, 22) we can express the total real time generation of an arbitrary generator (let's say 1) as:

$$Pg_{RT1} = Pd_{RT} - \sum_{i \in ActiveGenerator, i \neq 1} Pg_{RTi} \qquad (24\text{-}25)$$

$$Pg_{RT1} = Pd_{RT} - \left[\lambda_{RT} \sum_{\substack{i \in \text{Active Generator} \\ i \neq 1}} \frac{1}{c_{gi} - \rho_i} - \sum_{\substack{i \in \text{Active Generators} \\ i \neq 1}} \frac{b_{gi} + c_{gi} \cdot Pg_{DAi}}{c_{gi} - \rho_i} \right]$$

Active generators are those who are trading some part of their energy in the real time market, therefore they satisfy the following relation:

$$b_{gi} + c_{gi} \cdot Pg_{DAi} < \lambda_{RT}$$

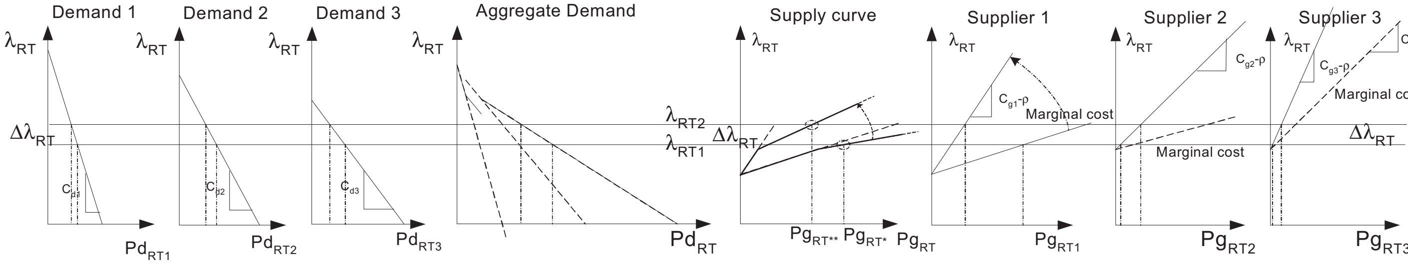
Then, from (23) we substitute Pd_{RT} into (25) and then determine:

$$\rho_1 = \frac{\partial \lambda_{RT}(Pg_{RT1})}{\partial Pg_{RT1}} = \frac{1}{\displaystyle\sum_{j \in \text{Active Consumers}} \frac{1}{c_{dj}} - \sum_{\substack{i \in \text{Active Generators} \\ i \neq 1}} \frac{1}{c_{gi} - \rho_i}} \qquad (26)$$

This expression has been calculated for the most general case, where all generator exercise some market power. It can be applied to other cases simply using $\rho_i = 0$ for all generators i that behave as price takers.

Equation (26) simply says that the price sensitivity of the real time market to the output of generator 1 is given by the elasticity of the residual demand generator 1 is facing.

As we see in (26), ρ_i depends not only on the demand sensitivities to price changes but also on the supply sensitivities to price changes. As we can see in figure 2, all suppliers and consumers are sensitive to price changes, but in different degrees, depending on the slope of their supply or demand curve respectively. For instance, if supplier 1 starts exercising some market power by withholding generation (this is equivalent to use $\rho_1 \neq 0$ increasing the slope of supplier's 1 curve), we face a raise in market prices equal to $\Delta\lambda_{RT}$. Energy sales would adjust in the following way: consumer j would reduce his consumption in $\Delta\lambda_{RT}/C_{dj}$ (it is a reduction because C_{dj} is negative) and generator i would increase his generation in $\Delta\lambda_{RT}/(C_{gi} - \rho_i)$ (it is usually an increase because C_{gj} is typically positive). Finally the new equilibrium is achieved (λ_{RT2} and $Pg_{RT^{**}}$ in fig. 2). Generators working as price takers will increase their generation more than the ones that have similar cost structure but are exercising market power. Even



b_{dj} +

Fig. 2. Effect of market power of supplier 1 on market equilibrium.

in the case where demand is fixed, participation of other generators limits the market power of the generator.

The expressions we have used to estimate the derivative of the real time price with respect to the power output of generator i ($d\lambda_{RT}/dPg_{RTi}$) were obtained assuming equilibrium. This means that time derivatives of power sales and purchases are equal to zero. Demand curves and offer curves we used to compute the equilibrium also assume time derivatives equal to zero. However. it is possible that the market power effect may have a time dependent component that vanishes when a new equilibrium is achieved. In that case ρ_i ($\rho_i = d\lambda_{RT}/dPg_{RTi}$) would be constant only under equilibrium.

D. Model Applications

We now apply the model to specific market scenarios:

1) m Suppliers with Market Power - n Consumers:

For the case of n consumers and m suppliers we can write equations (16-18) as:

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P} + \tilde{D}$$

Where:

$$\tilde{C} = \begin{bmatrix} -(c_{g1}+c_{g2})+\rho_1+\rho_2 & \cdots & -c_{g1}+\rho_1 & c_{g1}-\rho_1 & \cdots & c_{g1}-\rho_1 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ -c_{g1}+\rho_1 & \cdots & -(c_{g1}+c_{gm})+\rho_1+\rho_m & c_{g1}-\rho_1 & \cdots & c_{g1}-\rho_1 \\ c_{g1}-\rho_1 & \cdots & c_{g1}-\rho_1 & -c_{g1}+\rho_1+c_{d1} & \cdots & -c_{g1}+\rho_1 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ c_{g1}-\rho_1 & \cdots & c_{g1}-\rho_1 & -c_{g1}+\rho_1 & \cdots & -c_{g1}+\rho_1+c_{dn} \end{bmatrix}$$

$$\tilde{T} = \begin{bmatrix} \tau_{g1}+\tau_{g2} & \cdots & \tau_{g1} & -\tau_{g1} & \cdots & -\tau_{g1} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \tau_{g1} & \cdots & \tau_{g1}+\tau_{gm} & -\tau_{g1} & \cdots & -\tau_{g1} \\ -\tau_{g1} & \cdots & -\tau_{g1} & \tau_{g1}+\tau_{d1} & \cdots & \tau_{g1} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ -\tau_{g1} & \cdots & -\tau_{g1} & \tau_{g1} & \cdots & \tau_{g1}+\tau_{dn} \end{bmatrix}$$

$$\tilde{D} = \begin{bmatrix} (b_{g1}-b_{g2})+(c_{g1}Pg_{DA1}-c_{g2}Pg_{DA2}) \\ \vdots \\ (b_{g1}-b_{gm})+(c_{g1}Pg_{DA1}-c_{gm}Pg_{DAm}) \\ (-b_{g1}+b_{d1})+(-c_{g1}Pg_{DA1}+c_{d1}Pd_{DA1}) \\ \vdots \\ (-b_{g1}+b_{dn})+(-c_{g1}Pg_{DA1}+c_{dn}Pd_{DAn}) \end{bmatrix} \quad \tilde{P} = \begin{bmatrix} Pg_2 \\ \vdots \\ Pg_m \\ Pd_1 \\ \vdots \\ Pd_n \end{bmatrix}$$

The equilibrium can be obtained setting the derivative terms to zero and solving the following linear system:

$$\tilde{C}\tilde{P} = -\tilde{D}$$

The stability conditions require eigenvalues of the following homogenous equation to be negative.

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P}$$

2) m Generators with Fixed Demand:

For the case of fixed demand and m suppliers we can write equations (16-18) as follows:

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P} + \tilde{D}$$

Where:

$$\tilde{C} = \begin{bmatrix} -(c_{g1}+c_{g2})+\rho_1+\rho_2 & \cdots & -c_{g1}+\rho_1 \\ \vdots & \ddots & \vdots \\ -c_{g1}+\rho_1 & \cdots & -(c_{g1}+c_{gm})+\rho_1+\rho_m \end{bmatrix}$$

$$\tilde{T} = \begin{bmatrix} \tau_{g1}+\tau_{g2} & \cdots & \tau_{g1} \\ \vdots & \ddots & \vdots \\ \tau_{g1} & \cdots & \tau_{g1}+\tau_{gm} \end{bmatrix} \qquad \tilde{P} = \begin{bmatrix} Pg_2 \\ \vdots \\ Pg_m \end{bmatrix}$$

$$\tilde{D} = \begin{bmatrix} Pd_{RT}(Cg_1-\rho_1)+(b_{g1}-b_{g2})+(c_{g1}Pg_{DA1}-c_{g2}Pg_{DA2}) \\ \vdots \\ Pd_{RT}(Cg_1-\rho_1)+(b_{g1}-b_{gm})+(c_{g1}Pg_{DA1}-c_{gm}Pg_{DAm}) \end{bmatrix}$$

The equilibrium can be obtained setting the derivative terms to zero and then solving the following linear system:

$$\tilde{C}\tilde{P} = -\tilde{D}$$

The stability conditions require eigenvalues of the following homogenous equation to be negative.

$$\tilde{T}\,\dot{\tilde{P}} = \tilde{C}\tilde{P}$$

As stated in (26), the price sensitivities can be computed as follows:

$$\rho_1 = \frac{\partial \lambda_{RT}(Pg_{RT1})}{\partial Pg_{RT1}} = \frac{-1}{\sum\limits_{\substack{i \in \text{Active} \\ \text{Generators} \\ i \neq 1}} \dfrac{1}{c_{gi}-\rho_i}}$$

If only one generator (GEN1) has market power, then this expression reduces to:

$$\rho_1 = \frac{-1}{\sum\limits_{i=2..m} \dfrac{1}{c_{gi}}}$$

As stated in [1], in the absence of congestion equilibrium is achieved when all unconstrained generators operate at the same marginal price. There is no assurance of positive output in this formulation. If it turns out to be negative, then you have to take this generator out manually. The stability of the solution is independent of the linear cost coefficients b_{gi}. Since T is a diagonal dominant all positive matrix, it is enough to have $-C_{g1}-C_{g2}+\rho_1+\rho_K < 0$ to ensure stability. In this case you could have $C_{g1}+C_{g2}$ slightly negative and still be stable. It means you could have some generator with economies of scale ($C_{g1}<0$) and still have a stable market. However, too many generators with this property will likely result in instability.

E. Example: One Supplier – One Consumer Case Results

Let's analyze in details the case of one supplier and one consumer, The expressions for the supply, demand and power balance are the following:

$$\tau_g \dot{Pg}_{RT} = \lambda_{RT} - b_g - c_g (Pg_{RT} + Pg_{DA}) + Pg_{RT} \frac{\partial \lambda_{RT}(Pg_{RT})}{\partial Pg_{RT}}$$

$$\tau_d \dot{Pd}_{RT} = b_d + c_d (Pd_{RT} + Pd_{DA}) - \lambda_{RT}$$

$$Pg_{DAi} = Pd_{DAj} = P_{DA} \quad Pg_{RTi} = Pd_{RTj} = P_{RT}$$

The equilibrium point is obtained by setting the derivatives equal to zero and solving the following linear system:

$$0 = \lambda_{RT} - b_g - c_g (Pg_{RT} + Pg_{DA}) + Pg_{RT} \cdot \rho$$

$$0 = b_d + c_d (Pd_{RT} + Pd_{DA}) - \lambda_{RT}$$

$$Pg_{RTi} = Pd_{RTj} = P_{RT}$$

From equation (26) the price sensitivities can be computed as follows:

$$\frac{\partial \lambda_{RT} \{Pg_{RT})}{\partial Pg_{RT}} = \rho = c_d$$

Adding these equations to eliminate the real time price we obtain the real time sales in the market:

$$Pg_{RT} = Pd_{RT} = P_{RT} = \frac{b_d - b_g}{c_g - 2c_d} - P_{DA} \frac{c_g - c_d}{c_g - 2c_d}$$

Evaluating this expression in a scenario without day-ahead sales ($P_{DA} = 0$), we obtain the following real time sales:

$$P_{RT} = \frac{b_d - b_g}{c_g - 2c_d}, \quad \text{where} \quad \frac{b_d - b_g}{c_g - 2c_d} < \frac{b_d - b_g}{c_g - c_d}$$

Comparing this result with the one obtained through the simple model (the one with only one market), we identify a reduction in the total energy sales. We conclude that the inclusion of the market power term in generators' behavior brings about a reduction in their energy output. Suppliers withhold to increase market prices.

Observe further that:

$$\frac{c_g - c_d}{c_g - 2c_d} < 1$$

This indicates that day-ahead sales bring about a reduction in the suppliers' real time sales, but this reduction is smaller than the increase in the day-ahead sales.[1] Overall, considering both markets, there is an increase in the total energy available in the market. The bigger the day-ahead market sale, the bigger the total energy market (day-ahead + real time) sale and the lower the real time prices. This implies that *the application of forward markets helps limit market power, increasing total sales and reducing market prices*.

Consequently we obtain the following market price:

$$\lambda_{RT} = b_g + \frac{(c_g - c_d)(b_d - b_g)}{c_g - 2c_d} - \frac{c_d^{\ 2}}{c_g - 2c_d} P_{DA}$$

or

$$\lambda_{RT} = b_d + \frac{c_d (b_d - b_g)}{c_g - 2c_d} - \frac{c_d^{\ 2}}{c_g - 2c_d} P_{DA}$$

In order to provide more insight consider Figure 3:

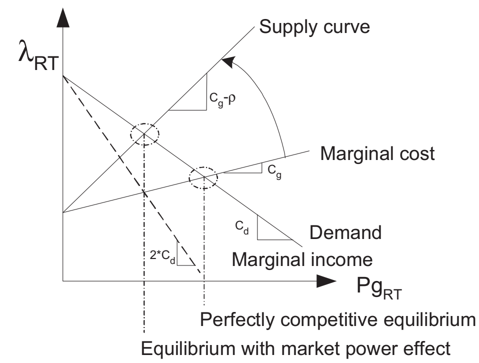


Fig. 3: Market Equilibrium

Since the total income of generator 1 (*TIg*) is given by the sales in both day-ahead and real time markets:

$$TIg = TIg_{DA} + TIg_{RT} = \lambda_{RT} Pg_{RT} + \lambda_{DA} Pg_{DA}$$

Then, its marginal income (*MgIg*) is given by:

$$MgIg = d\lambda_{RT}/dPg_{RT}\, Pg_{RT} + \lambda_{RT}$$

Under equilibrium, and for one consumer – one supplier, $d\lambda_{RT}/dPg_{RT} = c_d$, the marginal income becomes:

$$MgIg = c_d\, Pg_{RT} + \lambda_{RT.}\ .$$

Marginal income has the same shape as the demand ($\lambda_{RT} = b_d + c_d P_{dDA} + c_d P_{dRT}$) but with a different slope ($\lambda_{RT} = b_d + c_d P_{dDA} + 2c_d P_{dRT}$) .

Similarly, supplier marginal cost is described by the following expression: $\lambda_{RT} = b_g + c_g P_{gDA} + c_g P_{gRT}$. The supplier's offer to the market is given by $\lambda_{RT} = b_g + c_g P_{gDA} + (c_g\text{-}\ \rho) P_{gRT}$. Since ρ is negative, a generator´s offer is similar to its marginal cost but with an increased slope, reducing at each price the power the generator is willing to sell to the market (Generator withholds to raise prices).

Now we analyze the stability of the equilibrium. The stability conditions require eigenvalues of the following homogenous equation to be negative.

$$\widetilde{T}\,\dot{\widetilde{P}} = \widetilde{C}\widetilde{P}$$

Then the stability conditions for this model are :

$$\tau_g \dot{Pg}_{RT} = \lambda_{RT} - b_g - c_g (Pg_{RT} + Pg_{DA}) + Pg_{RT} \cdot c_d$$

$$\tau_d \dot{Pd}_{RT} = b_d + c_d (Pd_{RT} + Pd_{DA}) - \lambda_{RT}$$

Adding these two equations to eliminate the real time price:

$$(\tau_g + \tau_d)\, \dot{P}_{RT} = (2c_d - c_g) P_{RT} + (b_d - b_g) + P_{DA}(c_d - c_g)$$

The resulting homogeneous equation is:

[1] It means that for every MW traded in the day-ahead market, the real time market will experience a reduction in less than 1 MW (1-δ MW for instance) and the total market sales will increase by this difference (δ MW in our example).

$$(\tau_g + \tau_d)\dot{P}_{RT} = (2c_d - c_g)P_{RT}$$

The equilibrium is stable when:

$$s^* = \frac{2c_d - c_g}{\tau_{gT} + \tau_{dT}} < 0 \quad \text{or} \quad 2C_d < C_g$$

This means that the stability of the real time market is again unaffected by day-ahead sales. Remarkably, even when day-ahead sales affect the market equilibrium (by reducing real time prices and increasing total market sales), they don't affect the stability of the market.

The eigenvalue s* can also be written in a more general way as follows:

$$s^* = \frac{c_d - c_g}{\tau_g + \tau_d} + \frac{\dfrac{\partial \lambda_{RT}\{Pg_{RTi}\}}{\partial Pg_{RTi}}}{\tau_g + \tau_d}$$

The eigenvalue s* has the following characteristic:

$$\underset{With\_Mkt\_Power}{s^*} = \underset{Without\_Mkt\_Power}{s^{**}} + \underset{\substack{Eigenvalue\_displacement \\ due\_to\_Mkt\_Power\_effect}}{\Delta s}$$

It has two components. The first term is the original eigenvalue computed in [1] without considering market power. The second term is the market power effect on the stability of the problem.

Since $d\lambda_{RT}/dPg_{RTi}$ (derivative of the real time price λ_{RT} with respect to the power output Pg_{RTi}) is negative, market power exercise tends to improve the stability of the system by making generators less responsive to real time price variations (driven by their tendency to withhold).

Therefore, some degree of market power could be beneficial to improve the dynamics and volatility of the real time market. This finding raises a new dimension in the market power analysis, a tradeoff of between market efficiency and market dynamic performance.

4. Conclusions

A good market design requires not only a static simulation of the market equilibrium but also a study of the market dynamics and the stability of the equilibria. Here we present an approach to simulate the market dynamics considering a more accurate modeling of the behavior of market agents coupled with a more realistic representation of the market.

The experiences of California [4], Colombia [5], England and Wales [6] and others, have brought market power exercise as a matter a primary concern when designing or monitoring electricity markets. We presented a model were market power considerations are part of the core of suppliers' behavior and based on that, we have shown how market outcomes deviate from the perfectly competitive case.

A two settlement system was simulated and we have shown that forward positions affect the market equilibrium mitigating market power exercise, increasing total power sales in the market and reducing real time prices. We have also shown that day-ahead positions don't affect market stability.

We have shown that market power exercise affects the market dynamics, making suppliers less responsive to prices (their output get stiffer to price increases – withholding). Based on that, we have shown that some degree of market power could be beneficial to improve the dynamic response of the market, raising a new tradeoff between market outcome (efficiency) and market dynamics.

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Biographies

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