A Concurrent Non-Recursive Textured Algorithm for Distributed Multi-Utility State Estimation

Garng M. Huang, *Senior Member, IEEE*, and Jiansheng Lei, *Student Member, IEEE*

Abstract: **During power deregulation, power companies are releasing their transmission grids to form ISOs/RTOs while maintaining their own state estimators over their own areas. A recent trend for these ISOs/RTOs is to further combine and enlarge to become a bigger Mega-RTO grid for a better market efficiency. The determination of state over the whole system becomes challenging due to its size. Instead of a totally new estimator over the whole grid, we propose a distributed textured algorithm to determine the whole state; in our algorithm, the existing state estimators in local companies/ISOs/RTOs are fully utilized and the new estimator is no longer required. We need only some extra communication for some instrumentation or estimated data exchange. In addition, such an algorithm has the following advantages: 1) The distributed textured algorithm is non-recursive, asynchronous and avoids central controlling node. Therefore, it is fast and practical. 2) Based on exchanging data with neighboring companies/ISOs/RTOs, textured overlapped areas become part of the process. With the developed textured decomposition method, bad data detection and identification ability is better than existing distributed state estimation algorithm, especially when bad data occur around the boundary of individual estimators. 3) Discrepancy on the boundary buses of different estimators decreases and the result over whole grid become more consistent. Moreover, when updating local estimation through estimated data exchanges, matrix modification techniques that utilize sparse techniques are developed to accelerate the computation speed. Detailed numerical tests are given to verify the efficiency and validity of the new approach.**

Key words: **Power Market, Concurrent Textured Algorithm, Distributed State Estimation, Bad Data Analysis, Sparse Matrix Technique.**

I. INTRODUCTION

State estimation (SE) is an essential function in energy management systems (EMS) for monitoring and control of a power system. In a traditional regulated environment, the whole power system is owned by a limited number of locally monopolistic organizations. These utilities have the responsibility and the ownership of the instrumentation in their local own region to meet their own needs. There is

almost no need to exchange data with other organizations.

However, during power deregulation, power companies are releasing their transmission grids to form ISOs/RTOs [1] while still maintaining their own local state estimators. In other words, companies run their own SE's and focus on the quality of SE in their own area. Therefore, there are multiple state estimators distributed with different owners in one ISO/RTO.

Furthermore, a recent trend for these ISOs/RTOs is to further cooperate and to run the power market on even a bigger grid as a Mega-RTO for a better market efficiency [2]. The grid of an ISO/RTO could be large. The size of Mega-RTO is even bigger, as concluded recently by Federal Energy Regulatory Commission (FREC), that only four Mega-RTOs should cover the entire nation [2]. The state estimation over the whole grid becomes very challenging just for its size.

One possible scheme is to implement a totally new estimator over the whole grid, and one state estimator (OSE) is executed over the whole system. However, OSE approach has many disadvantages such as:

1) The investment on the new estimator could be enormous. The maintenance cost over such a huge area is also high.

2) The size of system is extremely large, which raises the scalability issue. The system matrix becomes more ill-conditioned, and the computation speed and convergence performance becomes slower and poor.

3). The existing local state estimators distributed in different entities are wasted.

Because of the above disadvantages of OSE, a new concurrent non-recursive textured algorithm is developed as an alternative to determine the state of whole grid, where the currently existing state estimators are fully utilized without using a new estimator. This textured algorithm is a distributed state estimation (DSE) algorithm, which overcomes the disadvantages of OSE.

Concurrent textured algorithm has been well developed to deal with the optimization problem of power systems by our team led by Dr. Huang [3,4,5]. The basic idea of a textured algorithm is as follows [3]. First, the problem on a large system is decomposed into several smaller and more tractable sub-problems for concurrent computation by fixing some boundary variables. Then by rotating the fixed variables, a recursive sequence of concurrent sub-problems are solved and the original high dimensional problem is solved by divide-and-conquer. The origin of term 'texture' is because there are overlapping areas between the neighboring sub-systems, which are just like texture. And the boundary variables are

located on these overlapping areas.

The introduction of such a concurrent textured algorithm in state estimation problem avoids the disadvantages of OSE. Furthermore, compared with existing DSE algorithm [6,7,8], the performance of new algorithm improves greatly in respect of bad data detection and identification ability and avoiding discrepancy on boundary buses.

This paper is organized as the following: the main flowchart and advantages of the new algorithm are discussed in Section II. The selection of data exchange scheme, as the center issue of textured decomposition method, is described in Section III. Furthermore, in Section IV the sparse matrix technique and its application are discussed. The determination of the state over the whole system are given in Section V. Numerical tests are studied in Section VI. In the last section, a conclusion is drawn.

II. CONCURRENT TEXTURED DSE ALGORITHM

A. Existing DSE Algorithms And Their Drawback

Assume multiple entities such as companies, ISOs and RTOs connected physically and cooperate to run the whole system as Fig.1. Accordingly, there are multiple existing estimators distributed in the subsystems like Company A, Company B, ISO A, RTO A and RTO B. And every entity will maintain and execute their local state estimation on their own areas. These entities are connected through tie lines near the boundary buses.

With the development of Information Technology (IT), DSE algorithms, especially those without central controlling node [6,7], become more and more applicable.

The main drawback of the existing DSE algorithms [6,7,8] is that bad data detection and identification ability decreases greatly compared to OSE who is over the whole system, especially when bad data is close to the boundary of individual estimators. Moreover, the estimation accuracy on boundary buses are much lower than OSE, which decrease the accuracy in determination of global reference bus and make the whole result inconsistent.

B. Introduction of a New Algorithm

The objective of our new algorithm is to remove the drawback of existing DSE algorithm while preserve the beneficial characteristics.

In the new algorithm, there are some overlapping areas in the neighboring estimators as shown in Fig.2, where some information are shared.

Furthermore, the point here is to extend the sharing information: not only the boundary buses are shared formally in the estimation sub-problems as in [6], but also instrumentation data (real time measurement information before execution of local estimator) and estimated data (estimation results after execution of local estimator) are exchanged among neighboring entities. Such a data exchange are introduced simultaneously between multiple entities, such as Company A and Company B, ISO A and Company B,
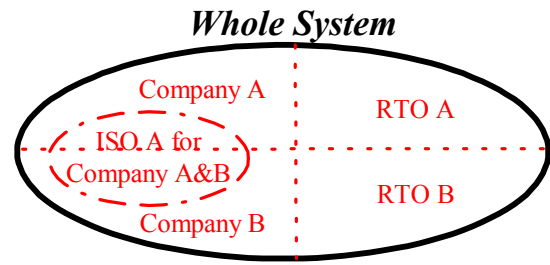

Fig.1 Multiple Companies/ISOs/RTOs connected physically
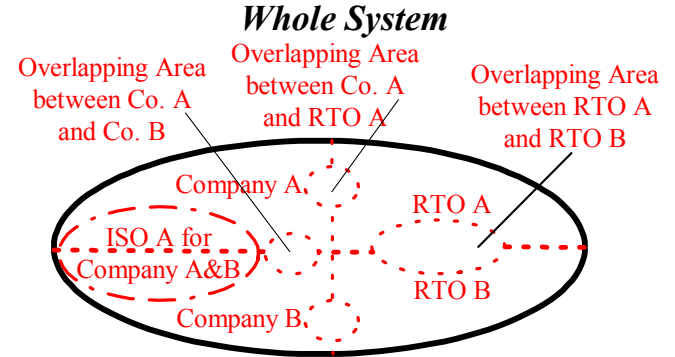

Fig.2 Overlapping areas come into being after data exchange

Company B and ROT B, and so on. Accordingly, textured network is formed.

C. Main Algorithm

The new algorithm is described as follows:

Step1. Select a set of real time instrumentation data to be exchanged between neighboring entities.

Step2. Select a set of estimated data to be exchanged between neighboring entities.

Step3. Taking the exchanged instrumentation data into account, the multiple local estimators distributed in different entities are executed simultaneously and asynchronously until they converge individually to the desired tolerance.

Step4. In view of the exchanged estimated data, modify the estimation result of local estimators accordingly and re-run bad data analysis.

Step5. Based on the modified results of local estimators, finally determine the state of whole system according to the different accuracy and reliability of estimators.

D. Advantages of the new algorithm

Advantage 1: Bad data detection and identification ability in the new algorithm is higher than existing DSE algorithm, especially when bad data appear close to the boundary of individual estimators. Such an improvement is because of the cooperation between estimated data exchange scheme and textured network formed by instrumentation data exchange.

For example, in existing DSE algorithm, if bad data appears in the boundary of one local estimator *A*, then it is hard to be detected in *A*. However, in a textured decomposition environment, the boundary buses in A are internal buses of another estimator (for example *B*) at the same time, where the

bad data can be detected. And the corrected information on these buses will be exchanged from *B* to *A* via estimated data exchange, which will finally make *A* also capable to detect the bad data. This capability suits well for an industrial environment, in which it is better to obtain SE results with good enough accuracy without bad data than the results with higher accuracy but with possible undetected bad data.

Advantage 2: The estimation accuracy on boundary buses is much higher than existing DSE and is comparable to OSE. Our approach decreases the discrepancy on the boundary buses and makes the whole result more consistent.

Advantage 3: Many earlier methods of DSE assume a star-like function network [8], where the communications between the multiple remote processors and the central computer are critical during iteration processes. Such a hierarchical approach suffers from the bottleneck and reliability issues because of the central controlling node.

On the contrary, our concurrent textured algorithm is asynchronous without a central controlling node. As a consequence, the new algorithm becomes very fast and practical.

Advantage 4: Utilizing the instrumentation decoupling nature in SE, we removed the recursion process in our original optimization. At the same time, the performance of estimation is still satisfied, which is verified in our numerical tests.

As a consequence, the speed of our new algorithm gets even faster. This further advances our original textured algorithm.

Advantage 5: The multiple local estimators can use different SE algorithms. Furthermore, the convergence tolerance can be different based on different quality of local measurement system.

Accordingly, our new algorithm becomes very flexible in which current existing estimators can be included easily.

Advantage 6: The performance of bad data detection and estimation accuracy in individual existing estimators improves as well, which benefits individual companies/ISOs/RTOs. Accordingly, they are more willing to share the information for their own benefits.

III. DSE TEXTURED DECOMPOSITION METHOD

A. Introduction

As discussed, we need to determine an instrumentation data and estimated data exchange scheme to be used in Step 1 and Step 2. As special cases, if all the information is exchanged and shared, the estimation becomes one estimator over the whole system, an OSE but not a DSE. On the other hand, if no measurement is exchanged, it becomes the existing DSE algorithm, which has the drawbacks described before. Therefore, a trade off in the selection of data exchange is necessary to make the overlapping areas moderate, not too large nor too small. Then an appropriate texture can be formed, which is a critical precondition for our new algorithm.

In addition, in the original textured decomposition method [3-5], the decomposition is based on the requirement of algorithm. However, in DSE problem here, the range of individual estimators has been determined in advance from the actual industry ownership. And the hardware/software cost on data exchange implement should be minimized, which implies schemes with smaller overlapping areas are preferred if all the other performance remains the same.

Note that not all the data exchange is beneficial. In fact, some data exchange may harm the local estimators and thus the exchange has to be carefully designed. Experience alone cannot resolve the design issues. In particular, for big mega-RTOs, no one has any experience.

Therefore, it is critical to develop a systematic approach to search for appropriate data exchange schemes, which can also help the design automation process.

B. A Systematic Textured Decomposition Method

Effective data exchange scheme design method and the corresponding software have been proposed to determine the textured decomposition by us in [9]. A simple description is attached as the follows:

The strength of bus *b* in estimator *A* is defined as the bad data detection and identification ability and estimation accuracy on bus *b* in this particular estimator *A*. A newly developed concept of Bus Redundancy Descriptor (BRD) is used to numerically evaluate the strength of every bus in different estimators. Details of the definition of BRD and determination of strength are given in [9].

Accordingly, two main principles are proposed by us [9] to search for the estimated data/instrumentation exchange, which is a critical part of textured decomposition.

After data exchange, the strength of buses improves greatly. And the ultimate objective is that the strength of every bus in the local estimators is almost as high as that in OSE. As for the buses in the overlapping areas of different local estimators, the objective is to ensure that the strength of these buses is high enough in at least one local estimator. The accomplishment of such an objective is critical to guarantee the advantage 1 and 2 of the new algorithm.

In addition, artificial intelligence (AI) is widely used to deal with problems described with uncertain terms like 'almost'. Therefore, the application of AI in textured decomposition problem is quite natural. Further studies to combine AI with the textured decomposition method are still under way.

C. Numerical Examples

Different distribution of local estimators will leads to different textured decomposition schemes.

For example, in an IEEE-14 Bus system as shown in Fig.3, RTO-A and RTO-B will merge into one Mega-RTO. And there are already two local estimators *A* and *B*, distributed in RTO A and RTO B, with neither overlapping areas nor data exchange. In other words, no texture exists in local estimators.

A suggested textured decomposition as shown in Fig. 4 is:
Instrumentation Data Exchange:
Estimator *A* expands to include bus 9. Furthermore, the instrumentation data on these buses, such as *9-10* (power flow measurements from Bus9 to Bus10), *9-14* and *9* (power
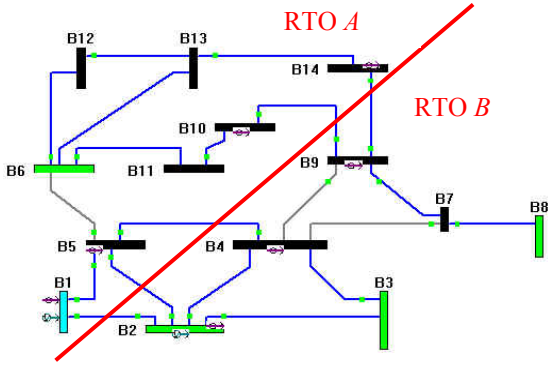
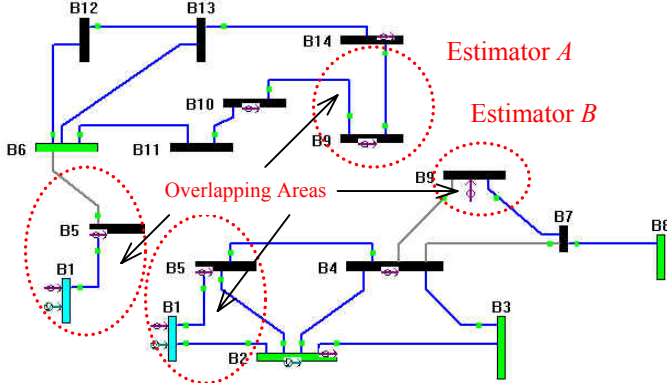Fig.3 Two RTOs merge into one Mega-RTO

Fig.4 Local estimators after instrumentation data exchange

injection measurement on Bus9), are also exchanged from *B* to *A* in a real time manner. And estimator *A* is executed in such an expanded sub-system.

Similarly, estimator *B* expands to include bus1 and bus 5. Instrumentation data, such as *1-5* and *1-5*, are also exchanged from *A* to *B*.

Therefore, the textured estimator consists of two independent estimators *A* and *B* with the overlapping areas including bus 1, 2, and 9. However, estimated data exchange needs extra updating as described below.

Estimated Data Exchange:

After local estimator *A* and *B* have been executed simultaneously and asynchronously, selected estimation result in *A*, such as *9-14*, *9-14* and *9*, are exchanged from *A* to *B*. And in view of these estimated data, *B* modifies its own estimation result accordingly and re-run bad data analysis.

Similarly, selected estimation result in *B*, such as *1-5*, *5-1* and *1*, are exchanged from *B* to *A*. And taking these estimated data into account, *A* also modifies its own estimation result accordingly and re-run bad data analysis.

IV. ESTIMATED DATA EXCHANGE

A. Sparse Technique for Matrix Modification

Sparse technique for matrix modification is widely used in power system computation, and the main idea is as follows: instead of re-computing a new sparse matrix, a modification on the old one is processed to reduce its computation complexity.

One major technique about the inverse matrix of sparse

matrix $(A+MaN^T)$ is well known as:

$$(A+MaN^T)^{-1} = A^{-1} - A^{-1}M(a^{-1}+N^TA^{-1}M)^{-1}N^TA^{-1} \quad (1)$$

where A is a $n \times n$ high-dimension sparse square matrix whose inverse matrix A^{-1} is already computed in advance, a is a $m \times m$ square matrix and m is much less than n.

And the computation complexity of re-computing $(A+MaN^T)^{-1}$ is much higher than that of right side of (1).

B. Application of Sparse Technique

In Step 4 of the flowchart, after the estimated data exchange, the succeeding modification on the estimation result of local estimators can be time-consuming if local estimation is executed again from the very beginning.

When a sequential SE algorithm, such as orthogonal method based on row-wise Givens rotations [10], is used in local estimator, the speed of such a modification process is fast even without other special techniques because of the nature of the sequential SE algorithm itself.

However, if the conventional Gauss Newton method is utilized in local estimator, it is time-consuming to execute SE again. Therefore, a sparse matrix modification technique is developed to modify the estimation result of local estimators and to avoid re-computing from the very beginning when some estimated data are newly added from other neighboring estimators. Such a technique can significantly accelerate the process in Step 4. Details are discussed as follows:

SE problem is based on the model [11]:

$$z = h(x) + e \quad (2)$$

Where

z represents measurements,

e is the measurement noise vector,

x is the state vector composed of the phase angles and the magnitudes of the voltages on network buses,

$h(\bullet)$ stands for the nonlinear measurement functions.

Traditionally a nonlinear iterative algorithm is widely used to solve the SE problem. At each iteration i, the following equations is solved:

$$\Delta x_i = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z_i = G^{-1} H^T R^{-1} \Delta z_i \quad (3)$$

Where

R is the measurement covariance matrix

H is the Jacobian matrix $\partial h / \partial x$,

$\Delta x_i = x_{i+1} - x_i$,

$\Delta z_i = z - h(x_i)$,

$G = H^T R^{-1} H$ is the gain matrix.

The most time-consuming computation in solving (3) is the determination of G and G^{-1}.

Suppose SE result based on current measurements is already obtained, and then some new data are introduced while the observable island maintains same, that is, the dimension of state variables is fixed.

Then the following equations hold:

$$\begin{bmatrix} z \\ z_{new} \end{bmatrix} = \begin{bmatrix} h(x) \\ h_{new}(x) \end{bmatrix} + \begin{bmatrix} e \\ e_{new} \end{bmatrix} \Rightarrow$$

$$\Delta x_i = \left(\begin{bmatrix} H^T & H_{new}^T \end{bmatrix} \begin{bmatrix} R^{-1} & 0 \\ 0 & R_{new}^{-1} \end{bmatrix} \begin{bmatrix} H \\ H_{new} \end{bmatrix} \right)^{-1} \bullet$$

$$\begin{bmatrix} H^T & H_{new}^T \end{bmatrix} \begin{bmatrix} R^{-1} & 0 \\ 0 & R_{new}^{-1} \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta z_{new} \end{bmatrix}$$

$$= \left(G + H_{new}^T R_{new}^{-1} H_{new} \right)^{-1} (H^T R^{-1} \Delta z + H_{new}^T R_{new}^{-1} \Delta z_{new}) \quad (4)$$

where the subscript 'new' stands for the newly introduced exchanged estimated data, whose dimension is very low.

Since the new measurements in Step 4 are estimated data with high accuracy, it is reasonable to fix H as a constant during modification process. Therefore, $(H^T R^{-1} \Delta z)$ and G^{-1} in (4) are the same as those in the old SE result (3), and they are known before modification process.

Accordingly, considering the dimension of R_{new} is much lower than that of G, sparse matrix technique described in (1) can be utilized here to determine $\left(G + H_{new}^T R_{new}^{-1} H_{new} \right)^{-1}$ in (4). Consequently, the modification is no longer time-consuming.

V. DETERMINATION OF STATE OVER WHOLE GRID

After the first four steps of the flowchart have been executed, the state over whole grid is determined as follows:

Step 5.1: Determine the angle difference of reference buses between any two local estimators. A reasonable scheme is based on the estimation accuracy of different local estimators, and the scheme is formulated as:

$$\Delta \theta_{AB} = \sum_{i \in I} (\theta_{i,A} - \theta_{i,B})(c_{i,A}^{-1} + c_{i,B}^{-1}) \Big/ \sum_{i \in I} (c_{i,A}^{-1} + c_{i,B}^{-1}) \quad (5)$$

where $\Delta \theta_{AB}$ is the angle difference of reference buses between local estimator A and B,

I is the set of all the overlapping buses of estimator A and B,

$\theta_{i,A}$ is the estimated angle on bus i in estimator A,

$c_{i,A}$ is the i-th diagonal element of covariance matrix $C = G^{-1}$.

Matrix C stands for the variances of estimation errors on bus i in estimator A. Therefore, the magnitude of $c_{i,A}$ is proportional to the estimation error on bus i in estimator A.

Step 5.2: Select a reference bus of one estimator (e.g. A) as the global reference bus for the whole grid.

Step 5.3: Determine the angle difference between this global reference bus and reference bus in every local estimator. For local estimators (e.g. B) who connect directly with A, (5) can give the angle difference directly. However, for local estimators (e.g. C) who only connects the neighboring estimators of A (e.g. B) while estimator C itself does not connects A directly, then the following equation is utilized:

$$\Delta \theta_{AC} = \Delta \theta_{AB} + \Delta \theta_{BC}.$$

Step 5.4: The estimated angle of each local estimator will be subtracted with the angle difference between the global reference bus and the local difference bus.

Step 5.5: For non-overlapping buses, the state variables are finally determined by now, which is just the current estimation result in local estimators.

Step 5.6: For overlapping bus i belonging to multiple local estimators $K_j, j = 1, 2, ..., m$, the state variables x_i are finally determined as:

$$x_i = \sum_{j=1}^{m} (x_{i,K_j} c_{i,K_j}^{-1}) \Big/ \sum_{i=1}^{m} c_{i,K_j}^{-1}$$

where x_{i,K_j} is the state variable of bus i in estimator K_j.

VI. NUMERICAL RESULTS

IEEE 14-bus system mentioned before is used here as an explanatory example to verify that:

1. The accuracy and discrepancy performance is satisfied compared to OSE, and higher than existing DSE algorithm.

2. The bad data detection ability improves greatly than existing DSE algorithm where bad data analysis is executed only in individual local estimators.

Algorithm speed will be testified in a larger practical system, which will be reported in following papers soon.

Case1: Accuracy and Discrepancy

Suppose there is a derivation 0.01 p.u. on power flow measurement *5-2*, which is still in the range of tolerance, and no bad data is detected. Table 1 shows concurrent textured DSE algorithm is more accurate than existing DSE algorithm.

Accordingly, the discrepancy decreases from 0.004 in existing DSE to 0.002 in textured DSE.

TABLE 1. ESTIMATION RESULT DERIVATION

Algorithm	OSE	Existing DSE	Textured DSE
Deviation on θ_2	0.003	0.007	0.004

Case2: Effect of textured instrumentation data exchange (1)

Bad Data In RTO *A*: Suppose that *11-10* is bad instrumentation data (sign is reversed). Note that measurements with largest normalized residues will be selected as bad data according to WLS algorithm for SE.

Without Instrumentation Data Exchange (Non-Textured):

For estimator A in Fig.3, 11-10 can only be detected as bad data but can not be identified based on Table 2.

With Instrumentation Data Exchange (Textured):

For estimator A in Fig.4, *11-10* is identified successfully according to Table 2.

TABLE 2. NORMALIZED RESIDUES FOR LOCAL ESTIMATOR *A*

Order	Estimator A in Fig.3		Estimator A in Fig.4	
	Meas.	Max.Residue	Meas.	Residue
	10	53.38660	*11-10*	52.40
1	11-10	53.38660	*10*	46.42

Case3: Effect of textured instrumentation data exchange (2)

Bad Data in RTO *B*: Suppose that both *2-3* and *2-4* are bad data (all increase by 0.1 p.u.).

Without Instrumentation Data Exchange (Non-Textured):

For estimator B in Fig.3, *2* and *4* are selected incorrectly as bad data based on Table 3.

With Instrumentation Data Exchange (Textured):

For estimator B in Fig.4, *2-4* and *2-3* are identified successfully one by one based on Table 3.