Estimating the Actual Cost of Transmission System Congestion

Thomas J. Overbye
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801 USA
overbye@ece.uiuc.edu

Abstract

This paper describes a methodology that could be used by a utility to estimate the actual cost of congestion on its transmission system using limited, non-state estimator data. The assumed problem inputs are a power flow model of an entire interconnected grid (i.e., the Eastern Interconnect), costs for the utility's generators, and then hourly values of the utility's generation, load and tie-line flows over the study time period. Due to the common lack by most utilities of external measurements, the system is first equivalenced to retain only the utility's own internal buses and a small subset of the external buses. Then, for each hour, the utility's load and generation is set to match their historical values, while the external generation is adjusted to match the tie-line flows. Next, an economic dispatch is performed to determine the unconstrained cost. Finally, a security constrained OPF (SCOPF) is solved to take into account base case and contingent constraints. The methodology uses a complete ac power flow formulation to accurately estimate the impact of voltage constraints and the incremental impact of system losses. The inclusion of hydro generation is also considered. For illustrative purposes only, the methodology demonstrated on the TVA system using publicly available data transmission system data.

1. Introduction

As the electric power industry continues to restructure there is an increased desire by many industry participants to accurately cost the various components associated with electric transmission. One such component is transmission system congestion, defined here as the short-term costs associated with having to redispatch the system generation (and possibly other controls) to avoid exceeding transmission system limits in either base case or contingent system operation. The impact of transmission congestion and the identification of transmission system bottlenecks was a key focus of the recent U.S. Department of Energy National Transmission Grid Study [1]

In its simplest form determination of the cost of transmission system congestion is rather straightforward. Assuming complete knowledge of the power system inputs, such as the loads at all system buses, and that the generators bid in their actual costs (i.e., they are not taking advantage of localized market power), then the hourly short-term cost of transmission congestion is the difference between the hourly cost of an optimal power flow (OPF) solution that includes transmission constraints and one that does not. If contingency constraints are to be included then transmission congestion is the difference between the security constrained OPF (SCOPF) solution and the unconstrained OPF. In the absence of energy-constrained generation (such as hydro) or generator ramping/shutdown/startup constraints, then congestion costs could be calculated by repeating the above analysis for each hour of the year.

A simple example of this approach is shown in Figures 1 and 2 using a three bus system that contains lossless lines with equal impedance and equal MVA limits, here set to 100 MVA. With a single 180 MW load at bus 3, a generator at bus 1 with a cost of \$10/MWh and a generator at bus 2 with a cost of \$12/MWh, the unconstrained and constrained dispatches are as shown in the figures. The hourly cost of transmission system congestion is then \$120/hr, while the marginal cost of enforcing the bus 1 to bus 3 line constraint is \$6/hr/MVA.

However, when this approach is applied to estimate the actual congestion on a large-scale power system, such as the North American Eastern Interconnect, the simplicity of the methodology is quickly overwhelmed by the devil in the details. Currently, nobody knows the actual, annual cost of transmission congestion for this system, with perhaps the best guesses documented in [1]. Complicating factors to calculating this value include the following: First, there is a lack of coordinated, historical system data. While individual utilities or independent system operators (ISOs) may have detailed historical data for their own transmission system, they have little access to data about their neighbors. Necessary data would include bus loads,

Presented at 36th Hawaii International Conference on System Sciences, January 2003, Big Island, Hawaii

generator bids (or cost data), and various control setpoints, such as the generator terminal voltage setpoints and setpoints of LTC and phase shifting transformers. While detailed snapshot models of the entire transmission are available through the FERC Form 715 filings, nobody currently sees the complete, real-time status.

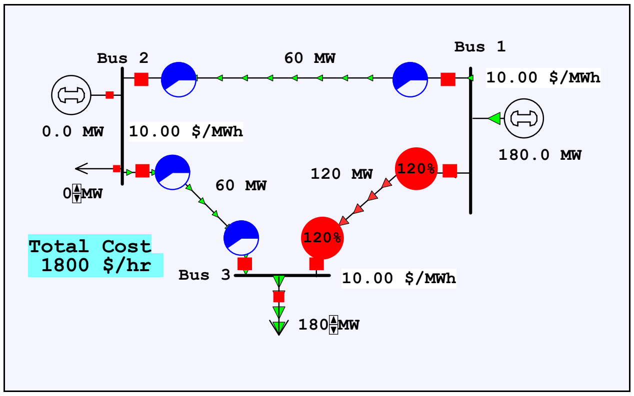

Figure 1: Unconstrained Three Bus Dispatch

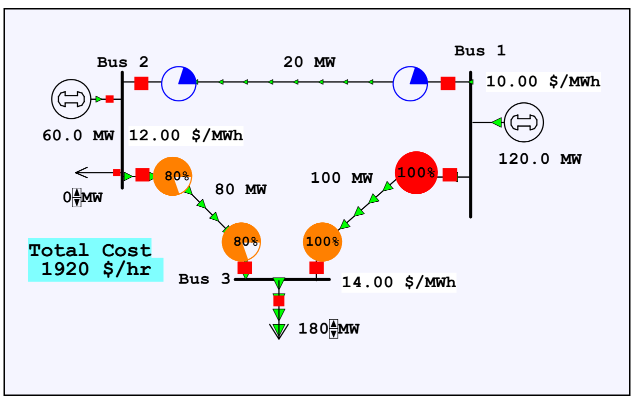

Figure 2: Constrained Three Bus Dispatch

Second, an SCOPF of the entire Eastern Interconnect for even one hour could be quite computationally quite demanding. The current Eastern Interconnect model has approximately 37,000 buses, 5800 generators and 50,000 transmission lines and transformers. While a single power flow can be solved in several seconds, including full contingency solutions for even a small percentage of the potential single element contingencies would vastly increase this value. Linear techniques, such as the use of line outage distribution factors (LODFs), can help, but a significant number of contingencies involve voltage constraints and/or operating procedures which must be considered using a more full ac analysis.

A final complicating factor is such an hourly snapshot analysis completely ignores the longer term generator constraints such as ramp limits, minimum up/down times, and energy constraints. Such limitations can be particularly important in systems with substantial hydroelectric generation and/or pumped storage.

Currently, the industry lacks comprehensive analysis tools to address this problem, with the development of such tools one of the recommendations in [1]. Generation operation and planning tools lack adequately detailed models of the transmission system, while most transmission analysis software lacks time-domain analysis capability.

The purpose of this paper is to at least partially bridge this gap by presenting a software methodology that could be used by a utility or other market participant to estimate the cost of transmission congestion. The approach, which is rooted firmly in the transmission analysis software domain, utilizes a time-domain based SCOPF solution. The paper begins with a brief review of the linear programming (LP) based OPF and SCOPF. Section 3 then shows how the LP algorithm, coupled with power flow information, can be used to approximate the hourly power system state for an equivalenced portion of the network. Section 4 then demonstrates the algorithm using the TVA system.

2. LP-Based SCOPF

The OPF algorithm, which was first formulated in the 1960's [2], [3], involves the minimization of some objective function subject to a number of equality and inequality constraints:

$$\text{Minimize } F(\mathbf{x},\mathbf{u}) \tag{1}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

$$\mathbf{h}_{\mathbf{min}} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\mathbf{max}}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where $\mathbf{x}$ is a vector of the dependent variables (such as the bus voltages), $\mathbf{u}$ is a vector of the control variables, $F(\mathbf{x},\mathbf{u})$ is the scalar objective function, $\mathbf{g}(\mathbf{x},\mathbf{u})$ is the set of equality constraints (e.g., the power flow equations), and $\mathbf{h}(\mathbf{x},\mathbf{u})$ is the set of inequality constraints. Originally the OPF only considered base case violations, but was later augmented to include contingency constraints in a formulation now know as the security constrained OPF (SCOPF) [4], [5].

Over the years several different OPF and SCOPF solution approaches have been proposed, with an excellent literature survey recently presented in [6] and a tutorial in [7]. These approaches can be broadly classified as either linear programming (LP) based methods or non-linear programming based methods. The algorithm utilized here is based upon the LP approach [8]. This section briefly describes this algorithm.

Overall, the LP SCOPF implemented here iterates between solving the power flow to take into account system non-linearities and using an LP with a linearized model of system constraints to redispatch the control variables subject to certain equality and inequality constraints. The key to the LP approach is to minimize the number of constraints included in the LP tableau. Practically all the constraints of (1) are considered by either enforcing them using the power flow, or, in the case of most nonbonding inequality constraints, monitoring but not enforcing them as long as they remain nonbonding. Table 1 summarizes the enforcement of the various constraints.

Table 1: Location of Constraint Enforcement

Constraint	**Enforcement**
Real/reactive bus power balance	Power flow
Generator voltage setpoint	Power flow
Generator reactive power limits	Power flow
LTC transformer voltage or reactive power control	Power flow
Phase shifter real power	Power flow
Switched shunt voltage setpoint	Power flow
Area real power balance	LP or Power flow
Line flow limits (MW or MVA)	LP (if binding)
Interface limits (MW)	LP (if binding)
Bus voltage magnitude limits	LP (if binding)

For the main optimization the LP itself utilizes a primal simplex algorithm with explicitly bounded variables [9]:

$$\begin{aligned} &\text{Minimize } \mathbf{c}^{\mathrm{T}}\mathbf{u} \\ &\text{s.t.} \quad \mathbf{Au} = \mathbf{b} \\ &\qquad \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \end{aligned} \tag{2}$$

where $\mathbf{u}$ is the vector of control variables from (1) augmented to include the LP slack variables, $\mathbf{c}$ is the vector of the current control incremental costs, $\mathbf{A}$ contains the active linearized constraints, and $\mathbf{b}$ is the vector of limit violations.

Since one of the main issues with the SCOPF is lack of feasibility (i.e., the available controls are insufficient to enforce all constraints), all of the LP slack variables are implemented as unbounded variables, with high incremental penalty costs used to push these variables to a feasible solution. Unbounded slack variables insure there is always an LP solution, albeit one with some unenforceable constraints if the problem itself is infeasible. Figure 3 shows representative costs for slack variables used with equality constraints and inequality constraints. Note the equality constraint slack variables only incur no penalty when they are zero, whereas the inequality slack variables incur a penalty when they are greater than zero. The advantage of this approach is it allows very explicit control over how strongly to enforce the various constraints. This directly prevents the application of ineffective and hence high marginal cost controls. To better illustrate the slack variables the equality constraint from (2) is rewritten below in expanded form:

$$\begin{bmatrix} \mathbf{A}_u & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{controls} \\ \mathbf{u}_{slack} \end{bmatrix} = \mathbf{b} \tag{3}$$

Note that $\mathbf{u}_{\text{slack}} = \mathbf{b}$ always provides an initial basic feasible solution.

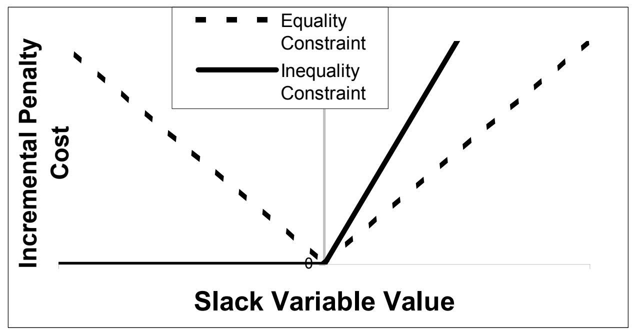

Figure 3: Slack Variable Penalty (Cost) Functions

The elements of each row in $\mathbf{A}$ can be calculated quite efficiently using the approach from [10]. That is, let h_j be the pertinent system constraint. Then the elements of the row associated with this constraint, $\partial h_j/\partial \mathbf{u}$, are

$$\frac{\partial h_j}{\partial \mathbf{u}} = \left[\frac{\partial h_j}{\partial \mathbf{x}}\right]^T \left[\mathbf{J}(\mathbf{x})\right]^{-1} \left[\frac{\partial \mathbf{p}}{\partial \mathbf{u}}\right] + \frac{\partial h_j}{\partial \mathbf{u}} \tag{4}$$

where $\mathbf{J}(\mathbf{x})$ is the power flow Jacobian and $\mathbf{p}$ is the vector of bus power injections. Note, the matrix $\partial\mathbf{p}/\partial\mathbf{u}$ is quite sparse, with most of the rows requiring trivial computation.

The constraints are only included in the tableau if they are binding or likely to become binding. The marginal impact of area losses can be included simply by including the losses in the area power balance equation

$$\sum_{j=1}^{n} P_{Gen,j} - P_{Load} - P_{Losses}(\mathbf{P}_{\mathbf{Gen}}) = Transactions \tag{5}$$

where the summation is over all the generators in the operating area.

The basic steps in the SCOPF are then as follows:

1. Solve the base case power flow and perform a full contingency analysis. Determine the set of constraint violations, ranked by severity. Also, for each contingent violation store its linearized control sensitivities and its original value.

2. Check for constraint violations. If none goto 5; otherwise select the worst constraint and add it to the LP tableau.
3. Solve the LP to determine the new control variable values.
4. Update the control variables, and then update the power system state using a linearized network model and update the violating constraint values using the linearized control sensitivities. Goto 2.
5. Resolve the power flow, and (optionally) resolve selected contingencies (e.g., those with the worst violations). If there are still violations that can be enforced goto 2.
6. The solution has been reached; calculate the final solution cost and the bus/constraint marginal prices.

Once an optimal solution has been determined, the marginal costs for enforcing the different constraints can be determined from the control costs and the final LP basis matrix:

$$\lambda^T = \mathbf{c}_B^T \; \mathbf{A}_B^{-1} \tag{6}$$

where

λ^T = marginal costs of enforcing constraints

$\mathbf{c}_B^T$ = control costs

$\mathbf{A_B}$ = LP basis matrix

The bus MW marginal costs (also known as the locational marginal prices or LMPs) are then computed as

$$\lambda_{buses}^T = \lambda^T \mathbf{S} \tag{7}$$

where

λ_{buses}^T = bus MW marginal costs

$\mathbf{S}$ = matrix of sensitivity of bus MW injections to the set of constraints

The results of this algorithm are demonstrated using a 1443 bus model in Section 4.

3. Hourly Power System State Approximation

Estimation of transmission congestion costs requires a model of the system state, which includes the bus voltage magnitudes and angles, transformer tap positions, and the status of switched devices such as shunts. Ideally, this state information would be available from the energy management system (EMS) state estimator (SE). Typically, this is done using a weighted least squares (WLS) algorithm [11] in which actual, real-time measurements and pseudo-measurements are used to estimate the state for an internal system, and a portion of the surrounding external system. A power flow model can then derived from the SE solution, and used as an input to SCOPF.

However, in the problem considered here an SE solution was not available. Rather, the assumed inputs were a planning model of the entire Eastern Interconnect, the hourly bus real and reactive loads for the internal system, the hourly internal system real power generation, the hourly real power tie-line flows, and cost information for all the internal system generators. Due to the lack of external system measurements, the full system model was initially equivalenced to retain all the internal buses, but only a small fraction of the external buses. The Ward injection method was utilized to retain the transfer admittance of the external network for accurate contingency modeling in the SCOPF [12]. To match the hourly tie-line flows, the external model was augmented to include fictitious generators at many of the boundary buses.

To approximate the hourly state an SE algorithm could have been employed, with the measurement set augmented by the base case power injections for the external system to achieve full observability. However, because the measurement set corresponded to power flow inputs, with the exception of the tie-line flows, the following, power flow based approach was used instead:

1. Set the internal generation and load to their historical values.
2. Solve the power flow.
3. Use the primal simplex algorithm from (2) and (3) to adjust the external generation to match the historical tie-line flows.
4. Solve the power flow and check the change in the tie-line flows from the previous power flow solution. If they are above a tolerance goto 2; otherwise done.

In step 3 the control set, $\mathbf{u}$, consisted of most of the external generator real power outputs and the constraint slack variables, while $\mathbf{A}$ contained the linearized tie-line sensitivities enforced as equality constraints, and $\mathbf{b}$ was the difference between the actual tie flows and the measured values. The cost vector, $\mathbf{c}$, can be set to either the generator's actual cost values, or to a piecewise linear quadratic function to minimize the change in generation. Again, $\mathbf{u}_{slack} = \mathbf{b}$ provides the initial basic feasible solution.

With the internal generation and load treated as power flow inputs, any errors or time skew in the hourly input values will appear as tie-line flow errors. Assuming no errors and sufficiently high slack variable penalty costs, all of the slack variables should ultimately be removed from the LP basis. In practice, if all of the tie-line flows are specified the problem is over-determined since generation minus load plus losses is equal to net tie flow. How the error appears depends upon the slack variable penalty functions. If they are "V-shaped", as in Figure 3, the error tends to concentrate on several tie-lines. The error can be more evenly distributed by using a quadratic cost function as shown in Figure 4.

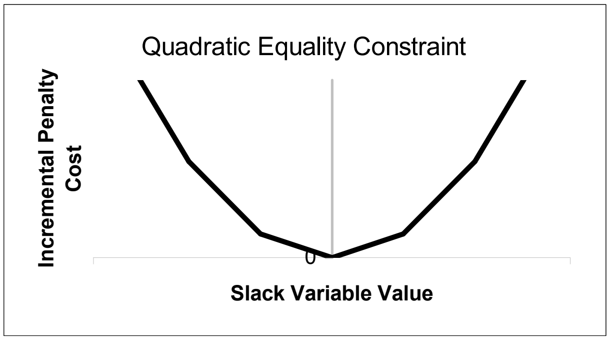

Figure 4: Piecewise Quadratic Equality Penalty Function

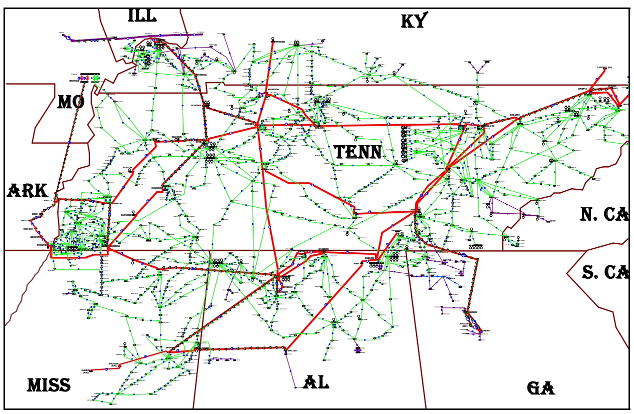

Figure 5: TVA One-line Diagram

The advantage of preceding algorithm is it provides SE like functionality using only a power flow and slightly modified LP-based OPF. Computationally, the algorithm will outperform the SE, provided the number of tie-line measurements is low. Since **A** in (2) is not sparse, each LP iteration is of order m^2 where m is the number of tie flow measurements. To completely remove the m slack variables will take at least m iterations. Therefore the LP portion of algorithm is of order m^3, while the remainder of the computation is spend in the power flow solution.

The algorithm is demonstrated using the TVA system shown in Figure 5. The original 37,000 bus Eastern Interconnect model was reduced to 1443 buses, of which 964 were internal TVA buses and the remainder external. Hourly data was then matched using TVA supplied data for approximately 120 generators (MW only), 600 loads (MW and Mvar) and 48 tie lines (MW only) for all hours of August 2000. The vast majority of the tie-lines were at either 500 or 161 kV, while the average TVA load in August 2000 was approximately 20,000 MW.

The accuracy of the algorithm can be accessed by comparing the error between the reported real power tie-line flow and the matched tie-line flow. For the 744 hours in August the average of the sum of the absolute value of the errors for the 48 tie-lines was 324 MW, or about 6.8 MW per tie-line. This compares to an average sum of the absolute value of the flow on the tie-lines of 7833 MW, so the error was slightly above 4%. The upper line in Figure 6 shows the hourly sum of the absolute value of the tie-line flows, while the bottom line shows the error. Figure 7 compares the reported flows to the matched flows for each of the 48 lines for a representative hour. Note, at the end of each hourly simulation the values of the external generators were stored, allowing each power flow state to be restored for subsequent studies without having to resolve the tie-line matching algorithm[i].

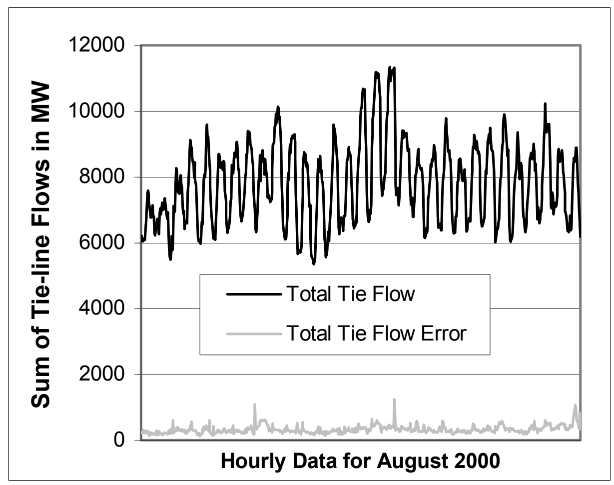

Figure 6: TVA Tie-line Flow Data for August 2000

The result from this analysis is 744 hourly power flow snapshots that should provide a reasonable approximation of the actual state of the TVA system during August 2000. The only significant shortcoming in this analysis was lack of historical reactive power/voltage values (except Mvar were provided for the loads). In particular, the actual generator voltage setpoint values, and the tie-line Mvar flows would have been helpful. The generator setpoint values could have been used directly as power flow inputs, while the tie-line reactive power flows could have been used to adjust the voltage setpoints of the external generators. In the absence of this information the generator voltage setpoints were maintained at the values

[i] Because of the presence of non-zero reactive control deadbands, such as the voltage range on LTC transformers or switched shunts, obtaining the exact same power flow solution with a varying initial voltage guess required the additional storage of the reactive power control values, such as the previously solved LTC tap position.

specified in the original model. Nevertheless, full reactive power modeling was maintained in the power flow, including the enforcement of generator reactive power limits, the tap movement of LTC transformers and the switching of shunts on automatic voltage regulation. While additional reactive power measurements would have been useful, the resultant hourly snapshot solutions should provide an excellent basis for the time-domain based SCOPF analysis.

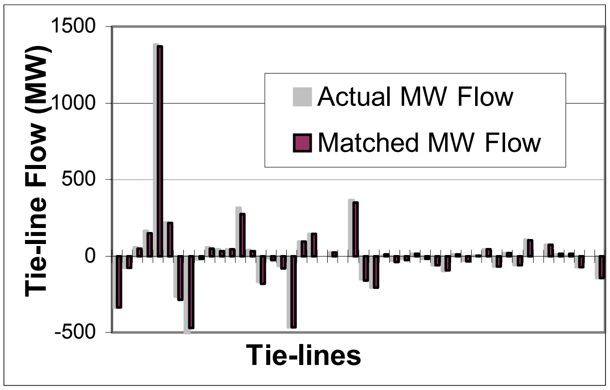

Figure 7: Representative Tie-line Flow Differences

4. Time-Domain Based SCOPF Analysis

The inputs to the SCOPF analysis are 1) a set of hourly power flow snapshots, 2) cost information and control range information for the SCOPF controls, and 3) a set of plausible contingencies. Here, the hourly snapshots were provided using the approach from Section 3, but they could have been supplied from an on-line SE.

This section demonstrates the application of the time-domain based SCOPF analysis using the reduced TVA model from Section 3 with a set of 134 plausible contingencies. In order to correctly model the impact of voltages constraints on system operation, full ac processing was performed for each of these contingencies for each hour. Also, the impact of operating procedures were included for the contingencies. For example, if a particular line is overloaded the operating procedure might be to simply open the line. The hourly cost of transmission congestion could then be estimated by comparing the cost between an unconstrained OPF solution and the SCOPF solution.

However, a problem with such a snapshot approach is the correct modeling of energy constrained generation. This was particularly important for the TVA case since their system contains a significant amount of reservoir controlled hydro, along with a 1600 MW pumped storage unit at Raccoon Mountain. Since the implementation of long-term hydro scheduling was beyond the scope of this study, the approach used was to keep the hydro and pumped storage generation fixed during the unconstrained OPF. Then, in the SCOPF, the hydro generation was priced at its bus marginal cost determined during the unconstrained OPF solution. This allowed the hydro generation to be redispatched to correct security violations, while still maintaining (at least to some extent) the energy constraints considered in the actual system dispatch.

One of the advantages of such a full, detailed, time-domain SCOPF simulation is the ability to precisely cost the impact of proposed changes to the transmission and/or generation systems. Using the set of hourly power flow solutions from Section 3, the cost impact of system changes can be determined by first running the month (or other time period) with the original system configuration, and then rerunning the time period with the changes. The difference in costs then provides a very precise estimate of the net benefit. Since the SCOPF is performed at each hour, changing the output of the various system controls, the modified system solutions should provide a good estimate of how the system would have been operated with the proposed additions. This allows users to easily perform detailed “what if” types of analysis.

The remainder of this section demonstrates the time-domain SCOPF approach on the TVA system. The starting point for this study was the actual hourly power flow snapshots for August 2000 and the actual generation costs. However, to maintain data confidentiality yet still present reasonable results, the system data associated with the examples presented here was modified in two ways. First, a non-linear scaling was applied to the actual generator costs to insure that the costs reported here do not represent actual costs, nor are they a simply a scaling of actual costs. Second, to disguise transmission system flows, a “what if” analysis was first performed by rerunning the data from Section 3 with significant modifications to the TVA transmission system. Therefore the constraints identified here do not necessarily correspond to actual system constraints.

In the first example case the modified case was solved without considering the marginal impact of system losses. That is, the losses were excluded from (5), resulting in all generator control sensitivity entries in the row of **A** corresponding to the power balance equation being equal to one. Snapshot analysis was then performed for each of the 744 hours in August 2000. Using a 1.5 GHz PC each hour solved in slightly less than 20 seconds, with about 90% of the time spent solving the 134 contingencies. Hence solving a month of data required about 4 hours. Figure 8 plots the variation in the hourly congestion costs over the course of the month.

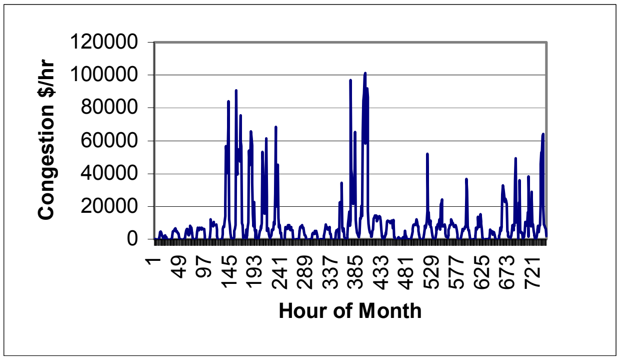

Figure 8: Hourly Variation in Transmission Congestion

Because of the large amount of data generated by each study contour analysis was used to help interpret the results [13]. For example, Figure 9 shows a contour of the variation in the LMPs across the TVA system for a sample hour, with the color mapping running from \$0/MWh (gray) to \$100/MWh (magenta). By creating contours for each hour of the study it is possible to generate "movies" showing the hourly variation in the system LMPs. In the absence of congestion and when the impact of marginal losses are excluded, all the LMPs should be identical.

Next, the August 2000 data was resolved including the impact of marginal losses. This required the calculation of the loss sensitivities for each bus in the TVA area, $\partial P_{Losses}/\partial P_i$. Since the losses are dependent upon the generation dispatch, these values needed to be recomputed as the SCOPF is iteratively solved. Figure 10 shows an example contour of the loss sensitivities across the TVA system, with the sign convention being positive values correspond to locations in which increased generation results in increased area losses. Here the color mapping runs from -0.08 (gray) to 0.02 (magenta).

In comparing the two studies, the inclusion of the marginal impact of area losses increased the estimate of transmission system congestion by about 20%. Therefore the inclusion of these impacts is probably warranted, particularly if one is interested in fairly exact congestion estimates. Certainly the impact of marginal losses in transmission system congestion requires further study.

5. Conclusion

This paper has presented an approach to estimating the actual cost of transmission system congestion using limited knowledge about the system state. The approach assumed historical load, generation and tie-line data for an internal portion of the system. An LP-based approach was then utilized to adjust external controls to match the internal measurements. Congestion can then be estimated by comparing the cost associated with an SCOPF solution to an unconstrained solution. The paper demonstrated that the inclusion of the marginal impact of system losses can have a significant impact on the final estimate of transmission system congestion.

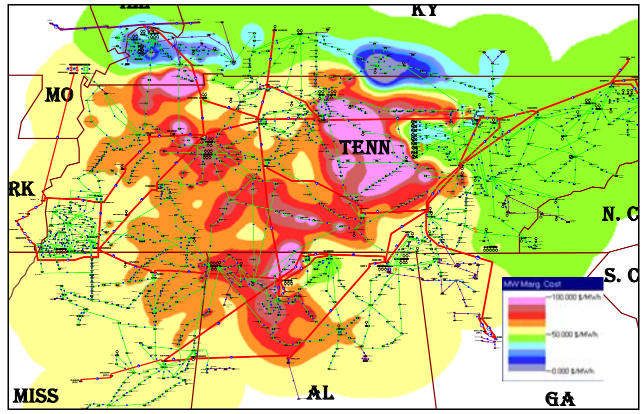

Figure 9: Example Contour of LMPs

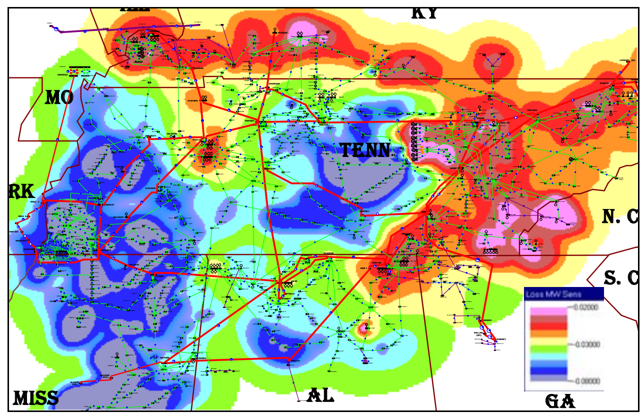

Figure 10: Example Variation in Loss Sensitivity

6. Acknowledgement

The author would like to acknowledge the support of TVA, the U.S. Department of Energy through the Consortium for Electric Reliability Technology Solutions (CERTS) program, PSERC (Power System Engineering Research Center), and PowerWorld Corporation.

7. References

[1] U.S. Department of Energy, *National Transmission Grid Study*, May 2002, [Online]. Available: http://www.energy.gov/NTGS/MAIN_SCREEN.PDF

[2] J. Carpienterm, "Contribution e l'étude do Dispatching Economique," *Bulletin Society Francaise Electriciens*, Vol. 3, August 1962.

[3] H.W. Dommel, W.F. Tinney, "Optimal power flow solutions," *IEEE Trans. Power Apparatus and Systems*, Oct. 1968, pp. 1866-1876.

[4] B. Stott, O. Alsac, A.J. Monticelli, "Security analysis and optimization," *Proc. of IEEE*, Dec. 1987, pp. 1623-1644.
[5] T. Bertram, K.D. Demaree, L.C. Dangelmaier, "An integrated package for real-time security enhancement," *IEEE Trans. on Power Systems*, May 1990, pp. 592-600.
[6] J.A. Momoh, M.E. El-Hawary, R. Adapa, "A review of selected optimal power flow literature to 1993," *IEEE Trans. on Power Systems*, February 1999, pp. 96-111.
[7] M. E. El-Hawary et. al., *IEEE Tutorial Course, Optimal Power Flow: Solution Techniques, Requirements and Challenges*, IEEE 96 TP 110-0, 1996.
[8] B. Stott and J.L. Marinho, "Linear programming for power system network security applications," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-98, May/June 1979, pp. 837-848.
[9] G.B. Dantzig, M.N. Thapa, *Linear Programming*, Springer, New York, NY, 1997.
[10] O. Alsac, J. Bright, M. Prais, B. Stott, "Further developments in LP-based optimal power flow," *IEEE Trans. on Power Systems*, August 1990, pp. 697-711.
[11] F.C. Schweppe, J. Wildes, "Power system static state estimations, Part I, II and III," *IEEE Trans. Power Apparatus and Systems*, January 1970, pp. 120-135.
[12] S. Deckmann et. al., "Studies on power system load flow equivalencing," *IEEE Trans. Power Apparatus and Systems*, Nov/Dec 1980, pp. 2301-2310.
[13] J.D. Weber, T.J. Overbye, "Voltage Contours for Power System Visualization," *IEEE Trans. on Power Systems*, February, 2000, pp. 404-409.