A New Bifurcation Analysis for Power System Dynamic Voltage Stability Studies

Garng M. Huang, *Senior Member, IEEE*, Liang Zhao, Xuefeng Song

Abstract: The dynamic of a large class of power systems can be represented by parameter dependent differential-algebraic models of the form $\dot{x} = f(x, y, p)$ and $0 = g(x, y, p)$. When the parameter p of the system (such as load of the system) changes, the stable equilibrium points may lose its dynamic stability at local bifurcation points. The systems will lose its stability at the feasibility boundary, which is caused by one of three different local bifurcations: the singularity induced bifurcation, saddle-node and Hopf bifurcation. In this paper the dynamic voltage stability of power system will be introduced and analyzed. Both the reduced and unreduced Jacobian matrix of the system are studied and compared. It is shown that the unreduced Jacobian matrix, whose eigen-structure matches well with the reduced one; and thus can be used for bifurcation analysis. In addition, the analysis avoids the singularity induced infinity problem, which may happen at reduced Jacobian matrix analysis, and is more computationally attractive.

Keywords: Voltage Collapse, Voltage Stability, Bifurcation, Differential -algebraic equations, Singularity, Voltage Collapse

I. INTRODUCTION

The dynamics of a physical system can be modeled by parameter dependent differential-algebraic equations as:

$$\dot{x} = f(x, y, p), \qquad f : \Re^{n+m+q} \to \Re^{n} \qquad (1)$$

$$0 = g(x, y, p), \qquad g : \Re^{n+m+q} \to \Re^{m} \qquad (2)$$

$$x \in X \subset \Re^{n},\ y \in Y \subset \Re^{m},\ p \in P \subset \Re^{q}$$

In the state space $X \times Y$, dynamic state variables x and instantaneous variables y are distinguished. The dynamics of the states x is defined by equation (1), and the dynamics of the y variables is such that system satisfies the constraints equations (2); the parameter p defines a specific system configuration and the operation condition.

For a power system, the typical state variables are the time dependent generator voltages (For different generator models, the variables of generator voltages will be different, such as $E', E_d', E_q', E_d'', E_q''$), the rotor variables of the generator (such as w, d), as well as the variables of the exciter, speed governor and so on; sometimes the dynamics of the load behavior will also be considered. The instantaneous variables are the power flow variables such as magnitudes and the angles of bus voltages. The parameter space p is composed of the system parameter (which describe the system topography, i.e., which lines, buses are energized, and equipment constants such as inductance, capacitor, transformer ratio, etc.) and operating parameters (such as load, generations and voltage set-point etc.). The dynamics of the generator, exciter, load dynamic and some other control devices together form the differential equations (1), and the power flow balance form the equations (2). For different objectives, some part of the differential or algebraic equations can be ignored. For example, when the dynamics of the voltage stability is studied, some equations of the generator rotor will not be considered.

For a fixed parameters p_0, the state variables describe the dynamics of the system, and the constraint equations $0 = g(x, y, p_0)$, which is the power balance equation, limit the state to a constraint set, L:

$$L = \{(x, y) \in X \times Y : g(x, y, p_0) = 0\} \qquad (3)$$

In the set L, the singularity set of the system, S, is:

$$S = \{(x, y) \in L : \det(D_y g(x, y, p_0)) = 0\} \qquad (4)$$

Here $D_y g$ denotes the matrix of partial derivatives of the components of g with respect to instantaneous variables y. This singular set S is defined as the set where the conditions of the implicit function theorem for eliminating y from the algebraic constraint are violated. And the behavior of the reduced matrix of the system becomes unpredictable.

The aim of this paper is to introduce a new bifurcation analysis method, unreduced Jacobian matrix analysis, which analyzes the power system voltage stability as effectively as the traditional reduced Jacobian matrix analysis. In addition, our new method can avoid the "singularity induced infinity" problem, which may happen at traditional analysis around singular points. Thus the method is more computationally attractive and conceptually easier to understand. Detailed procedures and conditions of the new method is described.

II STRUCTURE STABILTY for DIFFERENTIAL SYSTEMS

The authors gratefully acknowledge the support from EPRI/DoD Complex Interactive Networks program and PSERC.

Garng M. Huang, Department of Electrical EngineeringTexas A & M UniversityCollege Station, TX 77843-3128 (huang@ee.tamu.edu)
Liang Zhao, Department of Electrical EngineeringTexas A & M UniversityCollege Station, TX 77843-3128 (bennyz@ee.tamu.edu)
Xuefeng Song, Department of Electrical EngineeringTexas A & M UniversityCollege Station, TX 77843-3128 (xuefengsong@yahoo.com)

The structure stability problem for a system described by ordinary differential equations can be denoted as:

$$\dot{x} = h(x, p) \quad (5)$$

Where x is $n\times 1$ state vector and p is $q\times 1$ parameters vector. For every value p, the system equilibrium points are given by the solution of

$$h(x^*, p) = 0 \quad (6)$$

The equation (6) defines a q-dimensional equilibrium manifold in the (n+q)-dimensional space of states and parameters. For structure stability analysis, there are two types of bifurcation points:

Saddle-node bifurcation point, where two equilibrium coalesce and then disappear, at this point the Jacobian has a zero eigenvalue;

Hopf bifurcation point, where there is an emergence of oscillatory instability, at this point, two complex conjugate eigenvalues of Jacobian cross the imaginary axis.

These two bifurcation sets are the boundary of the feasible region of the system (5). When an equilibrium point passes through the boundary, the system will lose its stability. [9,13]

III. STRUCTURAL STABILITY OF DIFFERENTIAL-ALGEBRAIC SYSTEMS

3.1 Reduction of algebraic equations

Differential-algebraic systems are analyzed using the implicit function theorem. Consider a point (x,y,p) for which the algebraic Jacobian $D_y g$ is nonsingular. According to the implicit function theorem, there exists a locally unique, smooth function F in the form:

$$\dot{x} = F(x, p) \quad (7)$$

where the algebraic variables have been eliminated.

For a fixed value of p, an equilibrium is a solution of the system:

$$f(x, y, p) = 0 \quad (8)$$

$$g(x, y, p) = 0 \quad (9)$$

The stability of equilibrium points can be determined by liberalizing (8) and (9) around the equilibrium point:

$$\begin{bmatrix} \Delta \dot{x} \\ 0 \end{bmatrix} = J \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (10)$$

where J is the unreduced Jacobian of the differential-algebraic system:

$$J = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} \quad (11)$$

Assuming g_y is nonsingular we can eliminate Δy from (10):

$$\Delta \dot{x} = \left[f_x - f_y g_y^{-1} g_x \right] \Delta x \quad (12)$$

Hence:

$$A = F_x = \left[f_x - f_y g_y^{-1} g_x \right] \quad (13)$$

In the power system literature, **A** is often called the reduced Jacobian associated with the unreduced one J.

For a structural stability problem, there are three different kinds of bifurcation points:

Saddle-node bifurcation point, where two equilibrium coalesce and then disappear, at this point the reduced Jacobian has a zero eigenvalue;

Hopf bifurcation point, where there is an emergence of oscillatory instability, at this point, two complex conjugate eigenvalues of reduced Jacobian cross the imaginary axis;

Singularity induced bifurcation, at this point, g_y is singular, through the equation (13), we know that the inverse of g_y will become infinity, which is called "singularity induced infinity", where it is not easy to compute and analyze the stability of the system.

These three bifurcation sets are the boundary of the feasible region of the system (1)&(2). When one equilibrium point passes through the boundary, the system will lose its stability. [9,13]

3.2 Relation between Singular Perturbation and Differential-Algebraic Equations

Many differential systems have dynamics evolving in different time scales. Some are fast, others slow. In most cases it is not practical to handle both dynamics in a single model. The multiple time scales was used to solve this problem, when a multi-time-scale model is available, one can derive accurate, reduced-order models suitable for each time scale. This process is called time-scale decomposition [6,13]. Thus the system can be rewrite as:

$$\begin{cases} \dot{x} = f(x, y_d, p, \boldsymbol{x}) \\ \boldsymbol{x}\,\dot{y}_d = g(x, y_d, p, \boldsymbol{x}) \end{cases} \quad (14)$$

The question arises if we can use (14) to study equation (1)-(2). The advantage is that we do not need to apply implicit function theorem and thus we may avoid the singularity-induced problem. Through unreduced Jacobian analysis, the singularity-induced infinity problem can be avoided; the computation and analysis will be easier.

An Issue behind equations (1)(2) and (14)

Note that a necessary condition to replace the fast mode with algebraic equation is that the fast convergence of the equation:

$$x\dot{y}_d = g(x, y_d, p, x) \qquad (15)$$

Otherwise, equation (15) will not be zero since the equilibrium point is not reached.

On the other hand, note that the algebraic equation $g(x, y, p) = 0$ is the same as $-g(x, y, p) = 0$; but the corresponding differential equation is not the same. In order to keep the same dynamic behavior of the system (1)(2) and (14), some adjustment of the sign of the algebraic equation is necessary.

Here is a simple example to demonstrate this issue:

$$\begin{cases} \dot{x} = -x + y \\ 0 = -0.5x + y \end{cases}$$

The reduced A matrix is -0.5, the system is stable;

$$\begin{cases} \dot{x} = -x + y \\ 0.1\dot{y} = -0.5x + y \end{cases}$$

the unreduced Jacobian of the system is $[-1,1;-5,10]$, the eigenvalues are –0.5249 and 9.5249, where the fasted mode is diverging. Accordingly, the system is not stable. However, the original differential-algebraic equation is stable. It is clear that these two systems are totally different. But if we change the sign of the algebraic equation, then the dynamic behavior of the system

$$\begin{cases} \dot{x} = -x + y \\ 0.1\dot{y} = 0.5x - y \end{cases}$$

is the same as the original system. Note that the unreduced Jacobian is $[-1,1;5,-10]$ and its eigenvalues are -.04751 and -10.5249, in which the fastest mode is converging.

In power system modeling, the algebraic equations are power flower equations. In order to make sure that the unreduced system will have the same dynamic behavior as the reduced one, some adjustments on the sign of the power equations are necessary.

IV. VOLTAGE STABILITY CASES

The results in the previous sections are valid for common dynamic physical systems, as well as in large power system. The only requirement is that the functions f and g in the models should be smooth. In this section, the voltage stability of a simple one generator and one load bus system will be studied to demonstrate our analysis. [14]

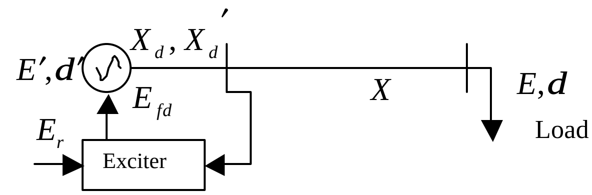

Fig. 1. Simple two bus system

In this example, the voltage stability of the system is studied, and it is assumed that the power factor of the load is constant as the load changes. It is also assumed that the voltage dynamic is isolated from the angle dynamic, so the so-called "classical" generator model is used and the angle dynamic is ignored at this scenario. Using an excitation system, which is a simplification of the IEEE Type 1 excitation dynamics [15], the model of the system can be stated as [14]:

$$\dot{E}' = \frac{1}{T'_{d0}} \cdot \left[-\frac{x+x_d}{x'}E' + \frac{x_d - x'_d}{x'} \cdot \frac{(E^2 + x'Q)}{E'} + E_{fd} \right] \qquad (16)$$

$$\dot{E}_{fd}' = \frac{1}{T}\left\{ -(E_{fd} - E_{fd}^0) - K\left[\frac{1}{E}\sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right] \right\} \qquad (17)$$

$$0 = E'^2E^2 - (x'P)^2 - (x'Q + E^2)^2 \qquad (18)$$

Through the analysis above, we can re-write the equation (18) as:

$$x\dot{E} = E'^2E^2 - (x'P)^2 - (x'Q + E^2)^2 \qquad (19)$$

For this case, $x = [E\,, E_{fd}]^T,\ y = E,\ p = [P, Q]^T$.

$$f_x = \begin{bmatrix} \frac{1}{T'_{d0}}\left[-\frac{x+x_d}{x'} - \frac{x_d - x'_d}{x'} \cdot \frac{(E^2 + x'Q)}{E'^2} \right] & \frac{1}{T'_{d0}} \\ 0 & -\frac{1}{T} \end{bmatrix} \qquad (20)$$

$$f_y = \begin{bmatrix} \frac{1}{T'_{d0}}\left[\frac{x_d - x'_d}{x'} \cdot \frac{2E}{E'^2} \right] \\ -\frac{1}{T} \cdot K \cdot [-\frac{1}{E^2}\sqrt{(xp)^2 + (xQ + E^2)^2} + \\ \frac{1}{E} \cdot \frac{(xQ + E^2) \cdot 2E}{\sqrt{(xp)^2 + (xQ + E^2)^2}}] \end{bmatrix} \qquad (21)$$

$$g_x = \left[2E'E^2/x \quad 0 \right] \qquad (22)$$

$$g_y = (2E'^2E - 2(x'Q + E^2) \cdot 2E)/x \qquad (23)$$

Through equation (11) and (13), we can get J and A matrix. Through equation (13), it is clear that A matrix is the same as the original system.

Scenario

$T'_{d0} = 5, T = 1.5, E_{fd}^0 = 1.0, x_d = 1.2, x = 0.1, x'_d = 0.2, Q = 0.5P$

$K = 2.5, E_r = 1.0, x = 0.0001$

Note that the voltage E is an instantaneous variable. When there is a disturbance, E will converge to the steady state instantaneously. To make the system (16,17,18) have the same behavior as system (16,17,19), a very small singular factor χ is introduced, where we select $\chi = 0.0001$. When χ is small enough, systems (16,17,18) and (16,17,19) are expected to have similar system behaviors.

Sign Adjustment

As discussed at last section, the sign of algebraic equation may need some adjustment. The fast mode of the system is the mode associated with the power flow (algebraic) equation. When the real part of the eigenvalue of the fast mode, which has the maximum absolute real part of all eigenvalues, is positive, we need to adjust the sign of the algebraic equation to make the fastest mode converge.

In this example, at some part of the lower part of the P-V curve (Figure 2. at lower part of the P-V curve, from p=0.5 to C point), the sign adjustment is introduced:

At equilibrium point $E' = 0.8505, E_{fd} = 2.9830, E = 0.2986$ $P = 0.5$, the eigenvalues of J matrix are -1.534, -.49, and 2179.8. The eigenvalue of the fast mode is 2179.8, which implies a sign adjustment of algebraic equation is necessary. The adjusted eigenvalues are -1.535,-0.4900, and -2179.8, which matches well with the eigenvalues of A matrix, which are -1.535, and -0.4897.

In this scenario, there are three types of bifurcation points in the figures below: Hopf Bifurcation (A Point); Saddle-Node Bifurcation (B Point); Singularity induced bifurcation (C Point).

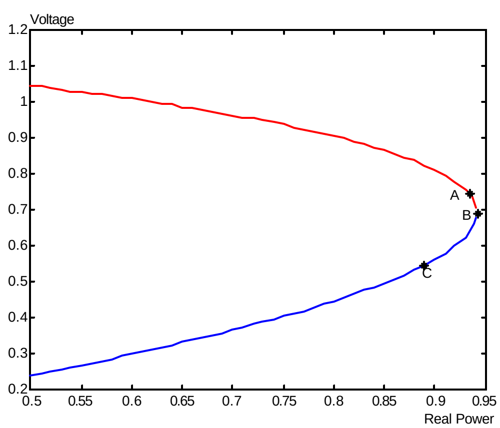

Fig. 2. P-V Curve of the system

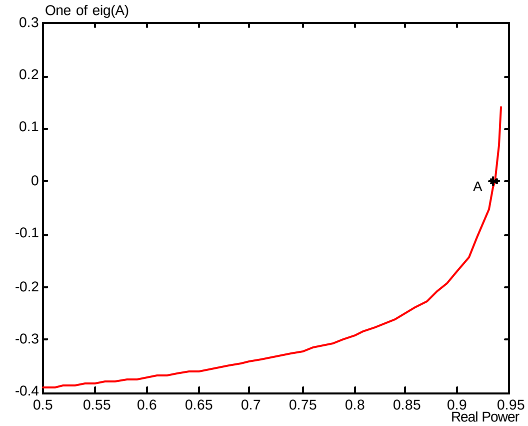

Fig. 3. One of eigenvalues of the reduced A matrix at upper part of the P-V curve as p varies. Note that at point A, the matrix A becomes singular.

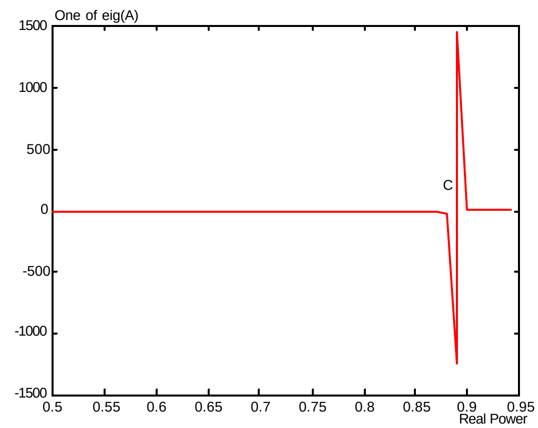

Fig. 4. One of eigenvalues of the reduced A matrix at lower part of the P-V curve as p varies, where a singularity occurred at point c.

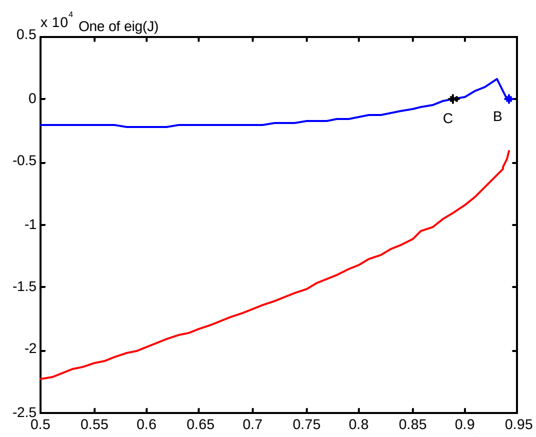

Fig. 5. One of eigenvalues of the unreduced J matrix; where the lower curve represents the eigenvalue at upper part of PV curve.

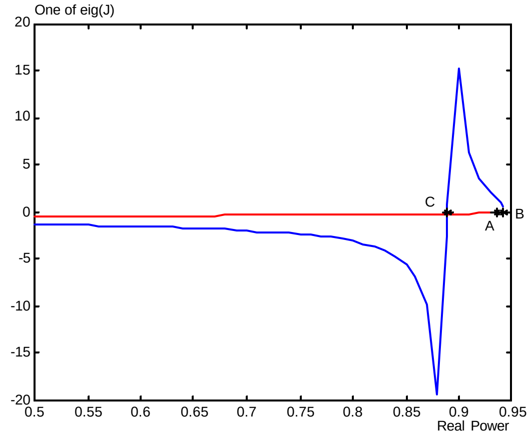

Fig. 6. One of eigenvalues of unreduced J matrix (Note that all the value is continuous, even at c, and no singular induced infinity occurs)

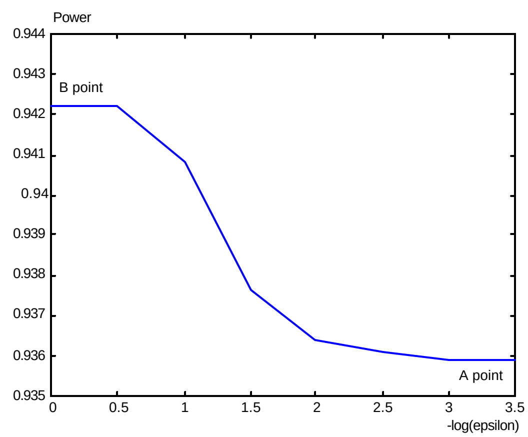

Figure. 7. The influence of the coefficient x

At this scenario, when the parameter p of (1) and (2) change, the equilibrium point will also change. Our analysis indicates that

1) This Case includes all three bifurcation points in reduced Jacobian analysis: Hopf (A point), Saddle-Node (B point) and Singularity-induced (C point) bifurcations as shown in figure 2;
2) As discussed in section II, there are only two types of bifurcation points in unreduced Jacobian analysis: Hopf (point A) and Saddle-Node (Points B and C) bifurcations.
3) The unreduced Jacobian Matrix matches the result well as reduced one: A,B and C point in unreduced Jacobian analysis is consistent with them in reduced Jacobian analysis;
4) At this case, Hopf bifurcation point (A point) is caused by the control device itself (too big a gain K of the exciter; if the value of K is smaller, there will be no Hopf bifurcation point). Mathematically, at least one pair of the eigenvalues of reduced and unreduced Jacobian will become a imaginary number (real part will change from negative to positive); physically, it implies that system will oscillate and lose its stability;
5) The Saddle-Node bifurcation point (B point) is the voltage collapse point of the system, determinant of the reduced and unreduced Jacobian will be zero. At least one of the eigenvalues of the A and J matrix will become zero, one eigenvalue will change the sign, and system will lose its dynamic voltage stability monotonously;
6) At the singularity induced bifurcation point C, at least one eigenvalue of A matrix will change from negative infinity to positive infinity, a singularity-induced infinity, which is rather messy to compute and analyze (as shown in figure 4). In our newly introduced unreduced Jacobian analysis, no singularity-induced infinity occurs; only one eigenvalue change the sign, and the stability of the system will change (as shown in figure 6). Note that at singular point, determinant of A also goes to infinity.
7) The instability feasible region of this sample system is the point set defined by Arc(A,B,C) as shown in figure 2.
8) Note that to make two systems have a similar dynamic behavior, a small enough x is introduced. In this example, $x = 0.0001$. When x is not small enough (for example $x = 0.1$), our calculation shows that "A point" of the system (16,17,19) will not match the "A point" of the system (16,17,18) precisely: at the case $x = 0.1$, "A point" of system (16,17,19) is very close to "B point". When the x decreases, the tendency of the "A point" of the system (16,17,19) is to move from B point and at last when x is small enough it will match the "A point" of system (16,17,18) more precisely. The figure 7 shows this tendency, where the x-axis of the figure is $-\log_{10} x$. Note that B remains fixed when A changes.
9) How to select suitable x value depends on the precision of the power step. For this case, if the precision of the power step is 0.001, the maximum x value is 0.00286 to demonstrate the effects around point A, which changes from stability to instability in one step.

From the above calculation, we know that the unreduced Jacobian analysis can match the stability behavior well as the reduced one. In addition, it bypasses singularity induced infinity problem, which happened in the reduced Jacobian analysis.

V. CONCLUSION

This paper analyzes the bifurcation of differential-algebraic systems (power system dynamic voltage stability) using unreduced Jacobian analysis based on singular perturbation. With proper sign adjustments to make the fast modes converge,

- The unreduced Jacobian J of differential-algebraic matches well as the reduced one.
- The unreduced Jacobian analysis can avoid the singularity induced infinity problem, which may happen at reduced Jacobian analysis.

Through a simple example (A power system dynamic voltage stability example), it is demonstrated that our analysis matches well with the reduced Jacobian analysis.

VI. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the supports from EPRI/DoD Complex Interactive Networks program and PSERC.

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BIOGRAPHIES

Dr. Garng M. Huang received his B.S. and M.S. in E.E. from national Chiao Tung University, Hsinchu, Taiwan, R.O.C. in 1975, 1977 respectively. He received his doctorate degree in System Science and Mathematics form Washington University, St. Louis in 1980. He had been teaching there since then until 1984. He joined Texas A&M University, Department of Electrical Engineering in 1984. He is currently a professor and the director of graduate studies there. He have been working on many funded research projects, such as Emergency Control of Large Interconnected Power System, HVDC Systems, Restoration of Large Scale Power Systems, On-line Detection of System Instabilities and On-line stabilization of Large Power System, Fast Parallel/Distributed Textured Algorithm for Data Network Routing Problem, etc. His current interest is the large scale systems theory, large scale parallel/distributed computing and control and their applications.

Dr. Huang is a senior Member of IEEE, and a Registered Professional Engineer of Texas. He has served as the Technical Committee Chairman of Energy System Control Committee and an associated editor in the IEEE Automatic Control Society; he has also been serving in a number of committees and subcommittees of IEEE PAS Society. Dr. Huang has published more than one hundred papers and reports in the areas of nonlinear, distributed control systems, parallel/distributed computing and their applications to power systems, data networks and flexible structures.

Liang Zhao received his BS and MS degrees in Electrical Engineering from Tsinghua University, Beijing, China, respectively in 1994 and 1998. He is presently a Ph.D. student at the Department of Electrical Engineering, Texas A&M University, College Station, Texas.

Xuefeng Song received his BS degree in Mathematics Department of Shangdong University and MS degrees in Electrical Engineering from Dalian Maritime University, China, respectively in 1984 and 1996. He is presently a Ph.D. student at the Department of Electrical Engineering, Texas A&M University, College Station, Texas.