

## (DIS)PROVING MARKET POWER

Rajesh Rajaraman<sup>1</sup> and Fernando Alvarado<sup>2,3</sup>

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### 1. INTRODUCTION

Market power and allegations of market power abuse are perhaps the most contentious contemporary issues concerning electricity markets. A number of industry observers and participants have made allegations regarding the abuse of market power. While some of these allegations may be well-founded, many others may be nothing more than an attempt by some to retroactively correct for their own bad business decisions. Some commentators appear willing to believe that market power exists and is exercised without substantiation.

For example, *New York Times* columnist Paul Krugman asserts that it is natural for generators to exercise market power by withholding output to drive the price of electricity higher.

*The generators didn't have to conspire: the logic of the situation made it easy, almost irresistible, for each individual company to manipulate the market. In fact, to believe that the generators didn't engage in market manipulation, you have to believe that they are either saints or very bad businessmen, because they would have been passing up an obvious opportunity to increase their profits.*

*Imagine the situation: it's a hot summer, and the California electricity market is very tight. You are one of only a handful of major players selling wholesale electricity. Surely the thought has to occur to you: what would happen to prices if one of my plants just happened to go off line? And when companies act on that thought . . . well, you get the picture.<sup>5</sup>*

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<sup>1</sup> Rajesh Rajaraman is with Laurits R. Christensen Associates, Inc. ([www.LRCA.com](http://www.LRCA.com)), an economics and engineering consulting firm. Comments about this article can be sent to [rrajaraman@LRCA.com](mailto:rrajaraman@LRCA.com).

<sup>2</sup> Fernando L. Alvarado is with Laurits R. Christensen Associates ([www.LRCA.com](http://www.LRCA.com)). He is also a Professor of Electrical and Computer Engineering at The University of Wisconsin-Madison and is affiliated with PSerc (Power Systems Engineering Research Center [www.pserc.wisc.edu](http://www.pserc.wisc.edu)).

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<sup>4</sup> This paper differs from the original version dated January 24, 2002 in the following ways. We have modified text in several places to make the exposition clearer. For example, we have significantly modified Example 3 and have added Appendix B as a companion to this example. We have also added a new example (Example 4), and have re-numbered the original Examples 4, 5 to Examples 5, 6. We have also removed a couple of errors and typos.

<sup>5</sup> P. Krugman, *New York Times* op-ed article, March 24, 2001. Krugman's statement was made in reference to the California market and was a commentary on the California crisis of 2000-2001.

It is a generally accepted economic principle that the potential to exercise market power exists when the supply of a product is limited/scarce and the demand is inelastic. However, one interpretation of Krugman’s statement is that market power in electricity markets is *always* going to be exercised during the hours when supply is “very tight.” This is not necessarily true. Typically, firms hedge some output in the forward (and other derivative) markets; firms can also speculate by taking positions in the forward markets. A firm’s incentive to withhold output, therefore, depends on its financial positions in addition to its physical assets. Moreover, there may be particular market rules (e.g., the imposition of punitive or regulatory penalties) that may affect a firm’s incentives to exercise market power. Therefore, even for the relatively simple case involving periods of tight supply, it is important to fully investigate before drawing any conclusion about whether market power has been abused.

The purpose of this paper is to establish an accurate, systematic and rigorous methodology and framework for proving or disproving the exercise of market power in the electricity markets under specific circumstances, and to do so taking full consideration of all of the realities of power markets.

The debate on market power has been influenced by a slew of recent articles<sup>6</sup> that claim to show that generating firms have exercised market power in deregulated markets (especially in California in 2000-2001). However, empirical studies that purport to show market power abuses suffer from some significant shortcomings. Scott Harvey and William Hogan have described these shortcomings in great detail in their recent publications.<sup>7</sup> The empirical studies that analyze market power typically use hourly simulation models to estimate competitive prices. These prices are then compared with actual historical prices. Market power abuse is suspected when the simulated prices are substantially below the observed market clearing prices, and the discrepancies between the two cannot be easily explained. However, there are assumptions and approximations implicit in the simulation studies, such as ignoring inter-temporal constraints, which have the potential to significantly affect the simulated prices. Harvey and Hogan

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<sup>6</sup> A partial list includes S. Borenstein, J. Bushnell, and F. Wolak, “Diagnosing Market Power in California’s Restructured Wholesale Electricity Market,” Working Paper PWP-064, University of California Energy Institute, August 2000, <http://www.ucei.berkeley.edu/ucei/PDF/pwp064.pdf>; E. Hildebrandt, “Further Analyses of the Exercise and Cost Impacts of Market Power in California’s Wholesale Energy Market,” California Independent System Operator, March 2001, <http://www1.caiso.com/docs/2001/04/13/200104131157168307.pdf>; Attachment A in FERC Docket Nos. EL00-95-017; A. Sheffrin, “Empirical Evidence of Strategic Bidding in California ISO Real Time Market,” California Independent System Operator, March 21, 2001, <http://www1.caiso.com/docs/2001/04/13/200104131158188342.pdf>; Attachment B in FERC Docket Nos. EL00-95-017). See also P. L. Joskow, “California’s Electricity Crisis,” MIT Working Paper, (forthcoming *Oxford Review of Economic Policy*), November 2001; P. L. Joskow and E. Kahn, “A Quantitative Analysis of Pricing Behavior in California’s Electricity Market During Summer 2000,” January 2001; “Identifying the Exercise of Market Power: Refining the Estimates,” July 2001. Papers by Joskow can be found on the website: <http://econ-www.mit.edu/faculty/pjoskow/papers.htm>.

<sup>7</sup> See S. M. Harvey and W. W. Hogan, “Issues in the Analysis of Market Power in California,” October 27, 2000; “On the Exercise of Market Power Through Strategic Withholding in California,” April 2001; “Further Analysis of the Exercise of Market Power in the California Electricity Market,” November 2001; “Market Power and Withholding,” December 2001; and “Identifying the Exercise of Market Power in California,” December 2001. For simplicity, these publications will be collectively referred to as Harvey and Hogan (2000, 2001). These papers can be found on the website: <http://ksghome.harvard.edu/~whogan.cb.g.Ksg/>.

persuasively argue that quantifying market power by such simulation studies is a difficult problem.

An alternative to simulation as a means to understand market power is the use of experiments and experimental economics techniques. Such work was pioneered by V. Smith<sup>8</sup> and has been elaborated upon further by Mount, Thomas and their colleagues at Cornell<sup>9</sup>. Experimental economic analysis has brought about a much better understanding of the fundamentals for the influence of behavior in the face of complex decisions on the ability to exercise market power. However, such techniques lack generality, as each experiment is constructed to test a specific set of rules in a specific environment and not to come up with a general procedure for the assessment of markets in which complex inter-temporal constraints and other such factors play a major role.

Given these shortcomings of the typical simulation approach and of the limited scope of the applicability of the experimental approach, there ought to be a higher and more readily applicable standard of proof for demonstrating that market power has been exercised. Taking price as a given exogenous input is an accepted method to examine optimal generator behavior in market power analysis. For example, Joskow and Kahn (2001) perform a simple version of such an analysis for very high priced hours. In this paper, we significantly extend this methodology by fully accounting for forecast uncertainty (e.g., due to market design), inter-temporal constraints, non-convex costs, and multiple markets. We estimate whether each generator<sup>10</sup> in a market participant's portfolio behaves as one would expect if the generator were a price taker, given the market design rules, multiple markets, non-convex operational constraints, and non-convex cost structure, in the presence of forecast uncertainty. This approach not only has the advantage of being much more practical and manageable than simulation studies, but also deals with each generator in the market participant's portfolio individually.

The test that we propose for the detection of market power by a market participant has two main parts:

- a quantitative model-based market test that can be used in most cases to help determine whether or not the market participant has exercised market power
- a qualitative analysis part (for those cases that cannot be resolved by the model-based test) that examines the incentives (or perceived incentives) of the market participant to exercise market power.

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<sup>8</sup> V. Smith and S. Rassenti, "Deregulating Electric Power: Market Design Issues and Experiments," in "Designing Competitive Electricity Markets," edited by Hung-po Chao and Hillard G. Huntington., Boston, MA: Kluwer Academic Publishers, 1998

<sup>9</sup> See, for example, R. D. Zimmerman, J. Bernard, R. J. Thomas and W. Schulte "Energy Auctions and Market Power: An Experimental Examination," Proceedings of the 32<sup>nd</sup> Hawaii International Conference on Systems Science, Maui, Hawaii, January 1999, or B. C. Lesieutre and R. J. Thomas and T. D. Mount, "Identification of Load Pockets and Market Power in Electric Power Systems," Submitted to Elsevier Journal on Decision Support Systems Special Issue on Competitive Electricity Markets, January 2003.

<sup>10</sup> In this paper, a single generator is one that does not directly constrain the output of other generators. For example, a hydroelectric system with three hydro units cascaded in series would be considered a single generator.

The model-based market power test solves an optimization problem that fully incorporates inter-temporal constraints, generator cost information, and forecast uncertainty for a single generator’s profit-maximizing commitment and dispatch **policy** given uncertain exogenous locational prices. This optimal generation dispatch and commitment **policy** can be used to formulate the optimal generator bidding strategy, and to understand generator behavior. This approach has the advantage of requiring minimal data — in particular, only data related to the generator(s) suspected of having exercised market power are needed. Based on this optimization, we provide a test that can be used to show that a market participant is “not guilty” of exercising market power. A more stringent test must be met to show that market power was exercised. These tests avoid one of the main shortcomings of the simulation approach concerning inter-temporal effects, non-convex costs and constraints, market design, and the effect of forecast uncertainty.

There are certain market participant behaviors that are beyond the scope of the model. These behaviors include the withholding of output by derating the capacity of a generator, either by falsely reporting an operational problem or by exaggerating the severity of an operational problem. To identify such behaviors, we examine the market participant’s incentives to exercise market power. For example, if the market participant’s profits are partially decoupled from spot market prices, then there is less of an incentive for the market participant to manipulate spot prices. Furthermore, if there are multiple ways to exercise market power, a rational generating firm may choose the least costly method of doing so. Our method can be applied, for example, to the California market to assist in resolving controversial questions about whether—and which—market participants exercised market power in 2000-2001.

This paper is organized as follows. We first define market power. Next we discuss the theoretical shortcomings of the empirical studies that use hourly price-prediction models to test for market power. We then propose a two-step approach that can help determine whether a market participant has exercised market power. We use a number of examples to illustrate the basic concepts behind our approach.

## **2. MARKET POWER**

Market power “signifies the degree of control that a single firm or a small number of firms has over the price and production decisions in an industry.”<sup>11</sup> An example of the exercise of market power is when a market participant profitably withholds output to raise prices above competitive levels. Harvey and Hogan give the following definition of market power to account for transmission constraints: “... to reduce profits from production on some units in order to change market prices and profit more from production on other units.”<sup>12</sup> Because a market participant may have other financial positions with payouts<sup>13</sup> tied to spot prices, we use the following definition.

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<sup>11</sup> P. A. Samuelson and W. D. Nordhaus, *Economics*, McGraw Hill, 13<sup>th</sup> edition, 1998 (Chapter 24).

<sup>12</sup> S. M. Harvey and W. W. Hogan, “Market Power and Withholding,” December 2001.

<sup>13</sup> For example, if a market participant has a net “long” (combined financial and physical) position in the spot markets, there may be an incentive to raise prices by withholding output. A market participant may also exercise market power by lowering prices through overproduction, though this has not been the main focus of market power

**DEFINITION:** A profit-maximizing market participant exercises market power if, for any generator in the market participant’s portfolio, its output is shown to be significantly different from that of a profit-maximizing price-taking hypothetical generator with identical cost and operating characteristics at the same location.

As discussed below, a direct application of this definition as a test of market power is complicated by a number of factors, including price uncertainty, multiple markets, and market design rules<sup>14</sup>.

### **3. SHORTCOMINGS OF EXISTING METHODOLOGIES**

A number of commentators (see footnote 6) have used hourly simulation models to empirically test for market power. Empirical studies that have analyzed market power typically use hourly simulation models to estimate competitive prices. In these studies, each generator is implicitly assumed to be “bidding” its incremental costs; using these incremental costs, a market supply curve is constructed. The intersection of the supply curve and demand curve sets the competitive market-clearing price which is then compared with actual, historical price. If there are substantial discrepancies between the two that cannot be easily justified, then there is a strong presumption of the exercise of market power.<sup>15</sup>

As elaborated in considerable detail in Harvey and Hogan (2000, 2001), these simulation studies generally make critical approximations that can significantly affect their conclusions. In particular, these studies tend to ignore:

1. Inter-temporal constraints such as ramp rate constraints
2. Interplay between the energy and reserves markets
3. The non-convex cost structure that results from startup and shutdown costs, “valve points,” and other features of actual generators
4. The complexity of hydroelectric dispatch
5. Demand and price forecast uncertainty
6. Transmission constraints.

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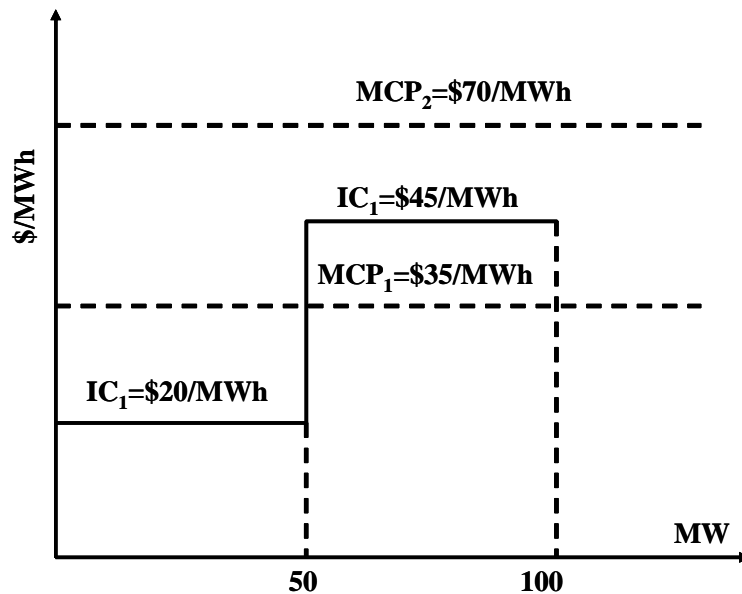
analysis in the electric power industry. For example, if a market participant has a net “short” financial position in the spot markets that more than offsets the “long” physical positions, there may be an incentive to lower prices by overproducing. Moreover, a market participant may keep prices lower than competitive levels to deter entry by other players into the market in order to profit from the reduced competition in future periods.

<sup>14</sup> Several reviewers have commented that this definition of market power must be used with care. For example, Scott M. Harvey cautions that under this definition, bidding mistakes could be categorized as market power. He gives the following example: a net buyer that was excessively cautious in bidding its unit into the market may be in danger of being characterized as having exercised market power rather than simply being dumb. We discuss this in more detail at the end of Section 4.1.

<sup>15</sup> Simulation studies typically check for evidence of the exercise of market power but do not identify the market participants who exercised market power (or how they exercised it).

We discuss each of these problems in turn below.

1. Ignoring inter-temporal and ramp rate constraints. Hourly simulation models typically tend to be non-chronological, i.e., each hour is considered independently of the other hours. However, this can lead to incorrect dispatch of generators. For example, consider a generator having a minimum capacity of 0 MW, a maximum of 100 MW, and whose incremental costs are as shown in Figure 1. Suppose that ramp rate for the generator is 30 MW/hr. Assume that for hour 1 the simulated market-clearing price for energy is  $MCP_1 = \$35/MWh$ , for hour 2 it is  $MCP_2 = \$70/MWh$ , and that the simulation is non-chronological. If ramp rates are ignored in the hourly simulation model, the generator will produce 50 MW in hour 1 and 100 MW in hour 2. However, this violates the ramp rate constraint that the generator output cannot change by more than 30 MW between the 2 hours.
2. Ignoring interplay between energy and reserve markets. When markets for reserves exist, the interaction between energy and reserve markets tends to be complex. Prices in one market tend to affect prices in the other market because of ramping constraints, startup times, etc. Hourly simulation models do not capture such interactions.



**FIGURE 1: INCREMENTAL COSTS OF A GENERATOR AND MARKET CLEARING PRICES FOR EACH OF TWO PERIODS**

3. Ignoring non-convex cost structure. Typically, non-convex costs (such as commitment costs, increased generator efficiencies at higher loading levels, “valve point” effects and other such effects) are ignored in hourly simulation models. Commitment costs and minimum load costs are assumed to be “sunk” costs in the hourly simulation models, which only include generator incremental costs. However, at the time of commitment, these costs are “variable” costs—the generators will not commit the resources if the total expected revenues over the commitment cycle are lower than the total commitment and

dispatch costs. Commitment costs could affect the commitment of generators near the margin; if so, generators may have to adjust their incremental costs upward to account for these commitment costs. Moreover, this adjustment may vary over time; on days when commitment costs are non-binding, incremental costs need not be adjusted at all; on days when commitment costs are binding, incremental costs need to be adjusted by an amount (based on expected prices) that ensures cost recovery.

4. Complexity of energy-limited generator dispatch rules: Hourly simulation models typically assume that energy-limited generators such as hydroelectric generators are infra-marginal. This is not always true. For hydroelectric generators that have storage capacity, the optimal dispatch of energy tends to be a complicated function of many parameters, including (uncertain) forecasts of energy and reserve prices in the future, expectation of reservoir inflows, and other operational constraints such as environmental limits, reservoir limits, etc. Other energy-limited generators can have their dispatch constrained by environmental operating conditions, total emissions limits within a period, etc. Appendix A discusses the challenges of modeling energy-limited generator behavior and shows that optimal dispatch rules are complex. Energy-limited generators can also be marginal generators in some periods. Hourly simulation models tend to ignore these complications.
5. Forecast uncertainty: A profit-maximizing generator will optimize its commitment and dispatch decisions based on its expectation of future spot energy and reserve prices. In markets where the optimal commitment, dispatch, and cost-recovery are guaranteed by the ISO, a generator will simply bid its costs and let the ISO do the optimization. However, as Harvey and Hogan (2000, 2001) point out, in markets that do not have this feature, and where generators have to estimate the future spot market-clearing prices in order to optimize on dispatch and commitment decisions, forecast uncertainty affects bidding decisions in a complicated manner. Hourly simulation models tend to ignore these complications.
6. Transmission constraints. Transmission constraints are an important consideration in electricity markets. Hourly simulation models tend to ignore these constraints, thereby affecting simulated clearing prices. For example, consider a market with 2 nodes: East and West, with a link connecting them. If there is congestion from West to East, then East prices will be higher than West prices, and a load-weighted average of West and East prices would tend to be higher than the prices implied by an unconstrained dispatch. When hourly models do consider the transmission constraints, a simple radial model is assumed. Such an assumption could be misleading in typical power systems; indeed, even in California, which models its system as being radial, there are loop flow considerations. The best way to analyze transmission constraints is to use Optimal Power Flow software with detailed representation of the transmission system; however, in such a case, data requirements become stringent. For example, one would need load data by bus, information of transmission line outages, etc.

In summary, simulation models tend to ignore some very important factors when estimating market-clearing prices. Importantly, these factors tend to be the ones that could most influence

the commitment and dispatch decisions of the marginal generator(s) setting market-clearing prices. These factors also have a significant effect on the simulated prices that are used by traditional market power analysis techniques. According to Harvey and Hogan (2000, 2001), the approximations made in the hourly simulation models tend to have a downward bias on the estimated market clearing prices and hence overstate market power.

Simulation studies also have a more pressing problem: data needs are stringent. To run such a model, one would need operational information from all generation plants, including costs, energy and reserve capabilities, load information, etc. Some of this information is easy to obtain; for example, system load information is generally available in the public domain. However, getting all of the right data and getting a model to use these data correctly would require herculean efforts; there exists no model today that can currently run a full unit-commitment, with an optimal load flow *plus* generators’ behavioral uncertainty factored in. Therefore, even ISOs that have the data *on all resources* would have a hard time using the data to run such simulations.

Hourly simulation models have their uses. For example, they can be used to get “ballpark” estimates of market-clearing prices. Such estimates often provide qualitative insight into behavior of market prices under various conditions and scenarios. However, hourly simulation models are too blunt of a tool to be used for market power analysis. It would be hard to make the case that the results from such a simulation study provide the proverbial “smoking gun.” The standards of proof ought to be much higher. The next sections propose a better approach.

#### **4. MARKET POWER TEST, PART ONE: MODEL-BASED**

We have discussed the practical difficulty of using a model that could simulate not just competitive market prices given a large volume of generator and load data, but also simulate market participants’ behavior in response to uncertain forecasts. In this section, we discuss another approach that is much more robust and practical. We turn the problem on its head: instead of using information about *all* generators and loads to simulate competitive prices, we propose to use historical prices as a *given exogenous input* and simulate a generator’s profit-maximizing (competitive) dispatch **policy**. This dispatch **policy** can then be compared with the generator’s actual dispatch to see if there are significant discrepancies between the two.

To the extent that a generator was in a position to make a reasonable estimate of future prices at any given time, the generator would have been expected to engage in behavior that is consistent with its forecast prices and its desire to optimize profits. We propose using *actual* location-based historical prices (including an uncertainty component to account for the fact that these prices may not have been perfectly predictable *ex-ante*) to find the profit-maximizing dispatch of each generator available to operate in the market participant’s portfolio of generators. In particular, we can state Problem P below for a price taking and profit-maximizing generator:

**PROBLEM P:** *For each generator, solve for the expected<sup>16</sup> profit-maximizing commitment and dispatch policy (or equivalently the optimal bidding policy) given the following exogenous*

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<sup>16</sup> The expected value formulation could allow for risk aversion by including a variance of the uncertainty term.



*parameters:*

- *operating horizon*<sup>17</sup> *over which the optimization will happen, including initial and ending boundary conditions (if any)*
- *inter-temporal constraints (startup/shutdown times, ramp rates, aggregate limits on fuel, water, etc.),*
- *cost parameters (startup/shutdown costs, no-load costs, ramping costs, incremental costs, etc.)*
- *exogenous, uncertain,*<sup>18</sup> *location-based price forecasts of energy and ancillary services (perfect foresight of prices is not required).*

The output of Problem P is an optimal commitment and dispatch **policy** and not just an expected value estimate of the dispatch. That is, the output of the optimization is not just a single number or set of numbers, but rather a **function**, which depends on the generator’s state (such as status) and exogenous parameters (such as prices); this function guides the generator’s dispatch and commitment decisions and its optimal bidding policies. The difference between the actual dispatch (e.g., in MW) and the dispatch **policy** (a function) is as follows. The actual dispatched MW and generator commitments over time are a result of following the optimal commitment and dispatch **policy** in response to *one particular (actual) realization* of prices. In other words, for a generator following the optimal **policy**, its observed dispatch and commitment would be “contained in” or “in the range of” the optimal commitment and dispatch **policy**. We will illustrate the concepts underlying the solution of Problem P using examples in Section 4.2.

Problem P is, in effect, the “self-commitment” problem that must be solved by every self-scheduling generator in the system. The mathematics of finding the **optimal bidding policy** over multiple periods for energy and ancillary services is described in Rajaraman and Alvarado

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<sup>17</sup> Some hydroelectric generators may operate on a very long-term cycle, say a yearly cycle. Forecasting hourly prices over a year or longer becomes a challenge. Even if a price forecast is available over a long-term horizon, the optimization to allocate water on an hourly schedule may become numerically intractable. To get around this problem, a nested optimization approach may be used. First the long term horizon is broken down into smaller sub-intervals, and water is optimally allocated into these sub-intervals; then the first sub-interval is broken down into still smaller sub-intervals, and the water allocated over the first sub-interval is further optimally sub-divided, and so on.

<sup>18</sup> Two points must be noted. One, the uncertain price forecasts of energy and ancillary services would be based on actual market-clearing prices, with the uncertainty component included to account for the fact that the market participant may not have had perfect foresight of the future spot prices at the time of commitment and/or dispatch. The uncertainty component would depend on market design; in markets such as NY and PJM, where the ISO does a joint optimization of energy and reserve markets, the uncertainty component would be applicable only to energy-limited generators (such as hydroelectric generators). The second point is that markets for reserves do not exist everywhere, and even where they do, there are differences in the ways that reserves are priced. Reserve incremental costs typically have two components: availability and usage. A generator offering reserves incurs a reserve availability cost (influenced mostly by opportunity costs in the energy markets), and, if the reserves are actually called, incurs an additional usage cost (covering mostly an energy component, but could also include heat rate degradation and wear-and-tear costs). [The original version of the paper stated that in markets such as NY and PJM, the uncertainty component also “enters via the option to sell in different markets ... and their market rules”. However, Scott M. Harvey has correctly pointed out (email correspondence to the authors, May 4, 2002) that the decision to export power to a neighboring market is irrelevant to the generator’s bidding problem. This is because, in NY and PJM, the export decision is independent of the generator bidding decision for a price-taker.]

(2003).<sup>19</sup> Related references are Rajaraman and Alvarado (2002) and Rajaraman et al. (2001).<sup>20</sup> The optimization involves backward Dynamic Programming techniques and the use of tree-like structures to capture price uncertainty (i.e., lack of perfect foresight). The optimization commits a generator only if the expected profits from optimally operating the generator over its commitment cycle are non-negative. In the optimization, the model considers all opportunity costs for the generator to sell part of its output to other markets. For example, these opportunity costs include the anticipated prices of energy and reserves in other periods and total energy restrictions that the generator may face (a particularly important consideration for hydro and pumped hydro generators.) The model also accounts for uncertainty using an option-like valuation methodology, rather than "averaging" methods (including Monte-Carlo methods based on averaging.) By permitting the use of locational prices rather than a single system price, the model also accounts for the effects of transmission constraints. Theoretically, this is the optimal way to dispatch and commit any generator operating in any market. Our contention is that such a method should reasonably mimic the actions of a profit-maximizing, price-taking generator.<sup>21</sup>

#### 4.1 "GUILTY" OR "NOT GUILTY" TESTS FOR EXERCISE OF MARKET POWER

A market participant can exercise market power even when a generator in the market participant's portfolio is producing *more* than its theoretically optimal dispatch, not just when it is producing *less* than its theoretically optimal dispatch. The latter can happen when a generator in the market participant's portfolio is deliberately withholding output to drive up prices. The former can happen when a market participant produces output upstream, even if the market price there is *below* the incremental costs of production, in order to cause transmission congestion and profit from it; the congestion can also exacerbate market power in the downstream market.<sup>22</sup> It is important to devise a test that accounts for both kinds of behavior.

We first give a sufficient condition for testing for market power.

***Proposition 1 [Not Guilty]:*** *Suppose that there exists a "credible" price forecast such that the market participant's simulated optimal commitment and dispatch policy of energy and ancillary*

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<sup>19</sup> R. Rajaraman and F. L. Alvarado, "Optimal Bidding Strategies in Electricity Markets Under Uncertain Energy and Reserve Prices", PSERC Report 03-05, April 2003 ([http://www.pserc.wisc.edu/ecow/get/publicatio/reports/2003report/alvarado\\_bidding\\_report\\_0411.pdf](http://www.pserc.wisc.edu/ecow/get/publicatio/reports/2003report/alvarado_bidding_report_0411.pdf)) (henceforth Rajaraman and Alvarado (2003)).

<sup>20</sup> R. Rajaraman and F. L. Alvarado, "Testing for Market Power in Hydro-Dominant Regions", in *Proceedings of VIII SEPOPE*, Paper SP-2002, 2002; R. Rajaraman, L. D. Kirsch, F. L. Alvarado, and C. Clark, "Optimal Self-Commitment Under Uncertain Energy and Reserve Prices," *Next Generation of Electric Power Unit Commitment Models* (eds. B. F. Hobbs et al.), International Series in Operations Research and Management Science, vol. 36, Kluwer Academic Publishers, Boston, April 2001 (henceforth Rajaraman et al. 2001).

<sup>21</sup> The optimization problem is tricky when the output of one unit within a generator influences the output of other units within the generator; e.g., in the case of hydroelectric systems where such units are connected in "series." The problem is further complicated if water transit times must be considered. While a full discussion of this problem is beyond the scope of this paper, one can use "relaxation" techniques to perform the optimization. See, for example, D. P. Bertsekas et al. "Optimal Short-Term Scheduling of Large-Scale Power Systems," *IEEE Transactions on Automatic Control*, AC-28(1): 1983.

<sup>22</sup> See P. L. Joskow and J. Tirole, "Transmission Rights and Market Power in Electric Power Networks," *RAND Journal of Economics*, vol. 31, no. 3, Autumn 2000, pp. 450-487.

*services found by solving Problem P (see Rajaraman and Alvarado 2003) is consistent with ("contains") the actual historical dispatch (or bids) for each generator in the market participant's portfolio. Then the market participant is not guilty of the exercise of market power.*

Thus to absolve itself of market manipulation charges, each generator in the market participant's portfolio must show that its historical dispatch is consistent with a profit-maximizing commitment and dispatch **policy** (and hence bidding **policy**) after accounting for inter-temporal constraints and "credible" price forecasts. The price forecast has to be credible in the sense that it has to be acceptable to auditors investigating the generator's behavior, and must be consistent with historical behavior, market data, and market design rules. To be sure, there may be cases when what may seem to be credible to one party may not seem credible to a different party. For example, a regulator may suspect that a market participant is using a "randomized" withholding strategy and may not believe the market participant's price forecast that satisfies Proposition 1 is a "credible" forecast. While such cases may not be easy to resolve, solving Problem P will yield important clues about a generator's behavior in response to prices; these insights could be used to design appropriate market power mitigation rules for the generator.

The test described in Proposition 1 could also be extended to include planned maintenance outages. The model would merely be used over a longer time horizon (say a year), with the smallest period within the horizon itself being on the order of one or more weeks.<sup>23</sup> In such a case, the market participant has to show that one credible price forecast exists where the simulated optimal maintenance outage scheduling **policy** is consistent with the actual schedule (given constraints on scheduling the maintenance) to prove that the market participant is not guilty.

We give a stricter test to show that market power has been exercised:

***Proposition 2 [Guilty of market manipulation?]:*** *Suppose that no "credible" price forecast from the market participant's simulated optimal commitment and dispatch **policy** of energy and ancillary services found by solving Problem P (see Rajaraman and Alvarado 2003) is consistent with ("contains") the actual historical dispatch<sup>24</sup> (or bids) for at least one generator in the market participant's portfolio. Then the market participant is either guilty of the exercise of market power or the market participant is not a profit-maximizer.*

If the test in Proposition 2 is satisfied, it does not necessarily mean that the market participant has exercised market power; an alternative possibility is that the market participant is *not* a

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<sup>23</sup> See also footnote 17.

<sup>24</sup> Scott M. Harvey adds the following caveats (email correspondence received by the authors, May 4, 2002). "... in every market relying on telephone based dispatch, some economic generation goes unused, despite low bids and a willingness to run, because the ISO simply did not have time to make lots of phone calls. This is an infrastructure problem, not a market power problem. .... There are a variety of other situations involving ISO communications, system operator judgment, and operating conditions that may result in economic generation remaining undischarged while higher cost generation is instructed to meet load. This is also not market power. The system operators dispatch process will hopefully reveal why a unit was not dispatched during high priced intervals...." Thus we must account for the ISO dispatch instructions in applying the tests in both Propositions 1 and 2.

profit-maximizer. It is therefore important to rule out this alternative possibility by examining the market participant's incentives. For example, a net buyer that withholds output due to a sub-optimal bidding strategy and raises prices is automatically penalized by his actions; therefore when the net buyer raises prices due to bidding mistakes, it is clear that he is not exercising market power. On the other hand, if a net buyer succeeds in *lowering* prices by, for example, producing from one generator at a loss but still profiting on his overall portfolio, then market manipulation by the net buyer cannot be ruled out. As another example, if a market participant with a net long position in the market withholds output, and if this behavior meets the test of Proposition 2, then the market participant's claims of "dumb" bidding mistakes will not be credible.

We examine the issue of market participant's incentives in more detail in Section 5.

## 4.2 MODEL EXAMPLES

We illustrate the concepts underlying Problem P and its solution using six examples. In the first example, we show the effects of inter-temporal constraints on profit-maximizing dispatch. The second example illustrates how to properly account for the effect of uncertainty on profit-maximizing commitment. The third, fourth, and fifth examples illustrate the effect of market design on bidding strategy, and how this effect is captured in Problem P. The examples also show that the output of Problem P is a **functional policy**, rather than simply an expected value estimate of commitment and dispatch. Finally, these examples show that it is possible for a generator to have an *ex-post* sub-optimal dispatch, even if such a dispatch is optimal *ex-ante* given the market design rules and forecast uncertainty.

**Example 1:** (*A multi-period problem*). Assume a single market for energy (no reserve markets), and assume that the analysis is confined to 2 periods with no price uncertainty. Consider the (price-taking) generator whose incremental cost characteristics are as described in Figure 1, with marginal costs of \$20/MWh and \$45/MWh depending on the level of output. Further assume that the generator predicts (with perfect foresight) market clearing prices for each period  $i = 1, 2$ . In this example,  $MCP_1 = \$35/MWh$ , and  $MCP_2 = \$70/MWh$ , as indicated in Figure 1. Assume that, initially (during period 0), the generator is operating at 45 MW. The generator has an inter-temporal (ramping) constraint such that it cannot change its output by more than 30 MW between any two periods (between period 0 and period 1 and between period 1 and period 2). Assuming that there are no other costs or constraints and the objective of the generator is to optimize its profit over the 2 periods, we can find the profit-maximizing dispatch. The generator will produce at least 50 MW in both periods because its incremental costs over that range are lower than market-clearing prices in both periods. However, the generator could produce *more than 50 MW* in hour 1 (but no more than 75 MW because of ramping constraints), and incur a loss in hour 1 if that loss is more than compensated for from additional profits in hour 2. More precisely, given the ramp rate considerations and taking into account the 100 MW maximum limit, the total profits from generating at  $X$  MW ( $X > 50$ ) in hour 1, and  $\min(X + 30, 100)$  in hour 2 are:<sup>25</sup>

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<sup>25</sup> Within each square bracket, the first term refers to the profits of the block whose incremental costs are \$20/MWh, and the second one refers to the profits of the block whose incremental costs are \$45/MWh.

$$\text{Total Profits} = \underbrace{[50*15-(X-50)*10]}_{\text{Period 1 Profits}} + \underbrace{[50*50 + \min[X+30-50, 100-50]*25]}_{\text{Period 2 profits}}$$

These profits are maximized when  $X = 70$  MW. That is, the generator will produce 70 MW in period 1 (incurring an apparent reduction of profits during this period) and 100 MW in period 2. This result is in contrast with the result that would be produced by a typical hourly simulation model (see previous section), where, ignoring ramp rates, the generator will produce 50 MW in period 1 and 100 MW in period 2.

The objectives of the following illustrative examples are to show:

- how “typical” Monte Carlo methods do not capture the effects of uncertainty (or lack of perfect foresight in forecasts)
- how to correctly account for uncertainty
- how inter-temporal constraints in combination with non-convex costs and uncertainty of future prices affect the commitment decisions of generators
- that the output of Problem P is a **functional policy**, rather than simply an expected value estimate (e.g., of dispatch MW).

**Example 2:**<sup>26</sup> (*The effect of price uncertainty and inter-temporal constraints on generator commitment.*) Assume a 2-period case. Consider a price-taking generator that has a capacity of 100 MW, a minimum generation constraint of 90 MW, a constant incremental cost of \$30/MWh over this range, and an additional inter-temporal constraint that, once online, the generator has to stay online for two consecutive periods. The boundary conditions are that the generator is offline initially, and must be offline at the end of the two periods. There are no other constraints or costs. Assume that the generator forecasts that market clearing prices are such that each period has a 50% chance of HIGH price (\$35/MWh) and a 50% chance of LOW price (\$10/MWh), regardless of the previous period price. We give two solutions to the problem, one a wrong answer and the other a right answer.

**Solution #1 (wrong answer):** One plausible way of finding the expected generator dispatch is to generate a large number of random price scenarios for the time interval by Monte Carlo methods, then use a deterministic self-commitment model to find the profit-maximizing dispatch corresponding to each ensemble of price scenarios and then average the dispatch over the Monte Carlo runs. Thus, in the present example, we could *first* generate all the price scenarios, and *then* run a deterministic optimal unit commitment on each possible price sequence. The four equally likely price sequences for the two periods are {HIGH, HIGH}, {HIGH, LOW}, {LOW, LOW}, and {LOW, HIGH}. If we make four deterministic unit commitment runs on these four price sequences, the deterministic unit commitment will only run the generator at maximum output (100 MW) for both periods when the price sequence is {HIGH, HIGH}. The profit for this price sequence is \$1000, and the dispatch will be 100 MW in both periods. For all other price sequences, the generator will not run, and the profit will be zero. Hence expected profits using this method will be  $0.25*1000 + 0.75*0 = \$250$ , and the expected dispatch over the four

<sup>26</sup> This example was originally presented in Rajaraman et al. (2001).

scenarios is similarly 25 MW in both periods. *This answer is wrong. As we shall see below, the expected profits that result from committing the unit during period 1 are not \$250, but are actually negative.*

**Solution #2 (right answer):** The optimal **policy** (of maximizing expected generator profits) is ***not to commit the generator regardless of what the period 1 price is.*** The reasoning is as follows. If period 1 price is HIGH, and the generator commits, the generator would produce 100 MW in period 1 to make a profit of \$500. However, there is a 50/50 chance that the period 2 price is HIGH or LOW. If the period 2 price is HIGH, the generator's two-period profit will be \$1000. If period 2 had a LOW price, the generator would produce the minimum 90 MW and lose  $90 \times 20 = \$1800$  in period 2 for a net two-period loss of \$1300. Therefore, if the generator commits to be online when the period 1 price is HIGH, the expected two-period payoff is  $1000 \times 0.5 - \$1300 \times 0.5 = (\$150)$ , for an expected loss of \$150. Therefore a profit-maximizing generator will not commit to be online even when the period 1 price is HIGH. Obviously, it would want even less to be online when the period 1 price is LOW. *Therefore the expected dispatch (and profit) under an optimal self-commitment policy<sup>27</sup> under uncertainty is zero.*

Example 2 shows that a generator that would have found it profitable to commit itself with perfect foresight of future prices (assuming that it was {HIGH, HIGH}) would find it to be a losing proposition (from an expected value viewpoint) with sufficiently imperfect foresight and would not commit itself.

A more formal and numerically much more efficient way to solve such kinds of multi-period problems is to recognize that this problem is akin to a complex option valuation problem and to use tree-like methods, starting from the terminal time-period (backward dynamic programming). Rajaraman and Alvarado (2003) illustrate how one can use such methods; we also illustrate the backward DP method in detail in Example 3.

**Example 3:** *(The effect of price uncertainty and inter-temporal constraints on bidding behavior.)* Assume a 2-period case. Most assumptions are as in the previous example. That is, consider a price-taking generator that has a capacity of 100 MW, a minimum generation constraint of 90 MW, a constant incremental cost of \$30/MWh over this range, and an additional inter-temporal constraint that, once online, the generator has to stay online for two consecutive periods. The boundary conditions are that the generator is offline initially, and must be offline at the end of the two periods. There are no other constraints or costs. As before, assume that that the generator forecasts that market clearing prices are such that each period has a 50% chance of HIGH price and a 50% chance of LOW price, regardless of the previous period prices. However, assume this time around that the HIGH price period 1 is \$45/MWh, and the HIGH price in period 2 is \$65/MWh while the LOW price in period 1 is \$33/MWh, and in period 2 is

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<sup>27</sup> The reason for the wrong answer when using *this* particular Monte Carlo method is that in each Monte Carlo run, the generator "peeped ahead" and "knew" the future prices and therefore chose the profit-maximizing commitment accordingly. Typical Monte Carlo methods are very efficient when one needs to simulate a large number of different random outcomes and find the expected value (or some other statistic) of some function based on these random outcomes. They are much more complicated to implement, and prohibitively expensive, when the value of a function at any given time *t* itself depends on what may happen *in the future*, as in optimal self-commitment policy problems that have inter-temporal constraints.

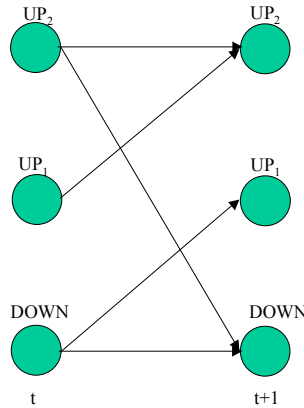
\$5/MWh. What is the generator bidding **policy**, assuming a uniform clearing price auction structure (and no other payments), if the objective is to maximize expected profits?

**Solution:** We go through the exercise of solving Problem P to find the optimal commitment and dispatch strategy to illustrate the method. We use a tree to find the optimal **policy** as a *decision rule* that commits and dispatches the generator for the next period, as a function of price level and generator state at the beginning of the period.

Because of the requirement that the generator has to stay online for at least two periods, we define two generator online states,  $UP_1$  and  $UP_2$ , and one generator offline state DOWN. The generator will be in state  $UP_2$  if it has stayed online for at least one period, and will be in state  $UP_1$  if it has come online from an offline state. It is not permissible for the generator to transition to the off-line state from the  $UP_1$  state. The permissible generator state transitions are shown in Figure 2a.

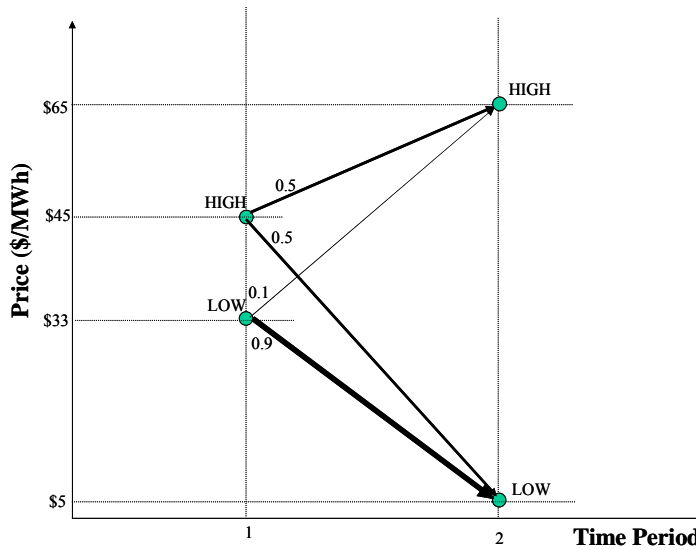
The price levels can be either HIGH or LOW in either period. Moreover, the price in period 2 is independent of period 1. Figure 2b shows the price state transition possibilities.

The price states are exogenous (cannot be controlled) while the generator states are controlled by the optimal **policy**.



**a. Generator State Transition Possibilities**

There are two UP states (to model the minimum UP time of 2 hours) and one DOWN state. When the generator is in state  $UP_1$  at time  $t+1$ , it means that the generator was DOWN at time  $t$ . When the generator is in state  $UP_2$  at time  $t+1$ , it means that at time  $t$ , the generator was either in state  $UP_1$  (it has been online for one hour) or in state  $UP_2$  (it has been online for more than one hour). When the generator is in state DOWN at time  $t+1$ , it means that at time  $t$ , the generator was either in state  $UP_2$  (it had been online for more than one hour) or in state DOWN (it was offline the previous hour).



**b. Price Transition Probabilities**

If the price is LOW in period 1, then there is probability of 0.1 of price being HIGH in period 2 and a probability of 0.9 of price being LOW in period 2. Similarly, if the price is HIGH in period 1, then there is probability of 0.5 of price being HIGH in period 2 and a probability of 0.5 of price being LOW in period 2. The HIGH and LOW prices themselves vary by time.

**FIGURE 2: TRANSITION DIAGRAMS**



In order to solve the problem, we introduce *Value* as a function of generator state, price state, and period  $t$ ; *Value* is defined as the expected cumulative profits from period  $t$  to the terminal period, provided the optimal commitment and dispatch policy is followed during this interval. During period 1, the *Value* function gives us the optimal expected cumulative profits from  $t=1$  to the terminal period. We find the optimal commitment policy by maximizing *Value*.

We first fix the boundary conditions for *Value*. The requirement of the problem is that after 2 periods, the generator has to be DOWN. That is, the generator has to be DOWN at during period 3. The generator is forbidden to be in either the  $UP_1$  or  $UP_2$  states during period 3. We therefore assign a prohibitively low *Value* for these states for  $t=3$ ; *Value* is (arbitrarily) set equal to negative infinity for the forbidden states. For the allowed DOWN state during period 3, we set  $Value=0$ , since there is no market for this period (we have assumed a 2-period market) and therefore profits will be 0 in this period.

We use the following recursive formula to compute  $Value_t$  at time  $t$ :

$Value_t(\text{state}_t, \text{price}_t) = \text{profits}_t(\text{state}_t, \text{price}_t) + \text{Max} [E(Value_{t+1}(\text{state}_{t+1}, \text{price}_{t+1}) | \text{price}_t)]$  (1)  
where  $E$  is the expected value operator and the Max refers to the maximum over all possible state transitions.

Tables 1(a)-(b) give the solution to the optimal dispatch policy problem. Table 1(a) shows the optimal dispatch for each (price level, generator state) pair. Table 1(b) shows the optimal profits made at time  $t$  corresponding to the optimal dispatch.

Table 2 gives the entries of *Value* as a function of generator state, price level, and time. Based on this table, we can derive the optimal commitment decision made at the end of each period; this is shown in Figure 3. Appendix B shows in much greater detail how Tables 1(a)-(b) and 2 and Figure 3 are constructed.

Table 1a. Optimal Dispatch Policies  $d^*$  (in MW)

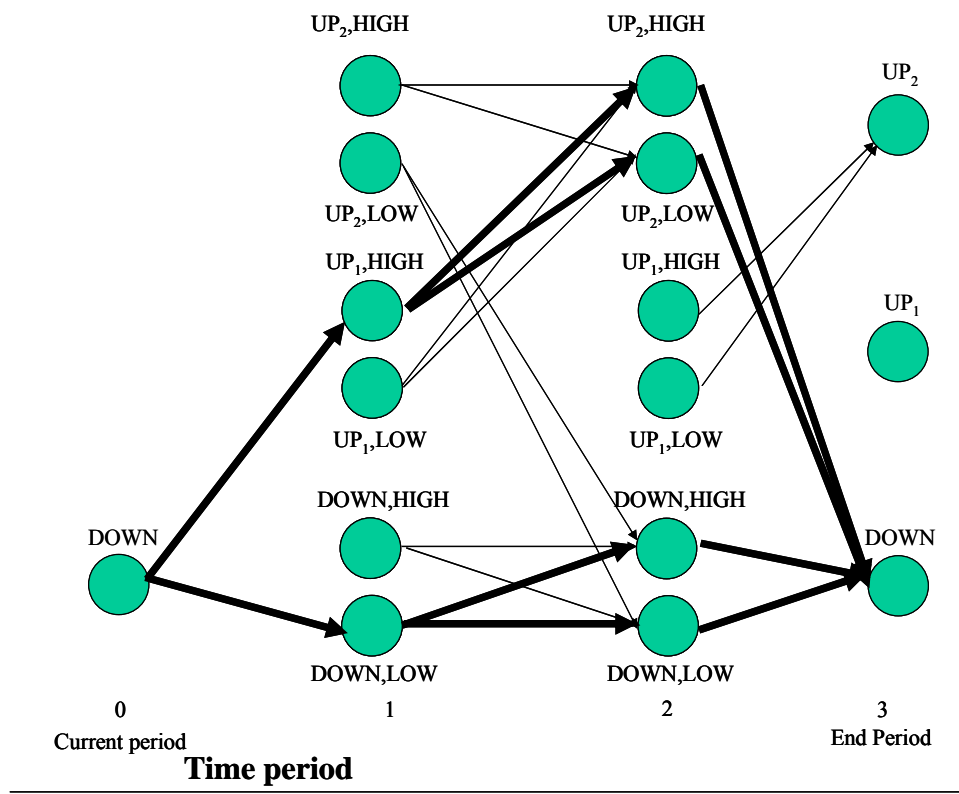
(State, Price level)	Period 1	Period 2
UP <sub>2</sub> , HIGH	100	100
UP <sub>2</sub> , LOW	100	90
<b>UP<sub>1</sub>, HIGH</b>	<b>100</b>	100
<b>UP<sub>1</sub>, LOW</b>	<b>100</b>	90
DOWN, HIGH	0	0
DOWN, LOW	0	0

Table 1b. Profits Made in Time  $t$  (in \$)

(State, Price level)	Period 1	Period 2
UP <sub>2</sub> , HIGH	1500	3500
UP <sub>2</sub> , LOW	300	(2250)
<b>UP<sub>1</sub>, HIGH</b>	<b>1500</b>	3500
<b>UP<sub>1</sub>, LOW</b>	<b>300</b>	(2250)
DOWN, HIGH	0	0
DOWN, LOW	0	0

Table 2: *Value* (in \$) as a function of generator state, price state and period  $t$  is the expected cumulative profits from period  $t$  to the terminal period corresponding to the optimal commitment and dispatch policy

(Generator State, Price Level)	Period 1	Period 2
UP <sub>2</sub> , HIGH	2125	3500
UP <sub>2</sub> , LOW	300	(2250)
UP <sub>1</sub> , HIGH	2125	$-\infty$
UP <sub>1</sub> , LOW	(1375)	$-\infty$
DOWN, HIGH	0	0
DOWN, LOW	0	0



**FIGURE 3. OPTIMAL STATE TRANSITIONS OR COMMITMENT POLICY.**

*The optimal commitment policy at any time  $t$  depends on both generator state and price level. For example, in period  $t=1$ , if the generator is  $UP_2$  and if the price  $p_1$  is HIGH, then it is optimal for the generator to stay  $UP_2$  in period  $t=2$ . On the other hand, in period  $t=1$ , if the generator is  $UP_2$  and if the price  $p_1$  is LOW, then it is optimal for the generator to go DOWN in period  $t=2$ . The boldfaced arrows show all possible optimal state transitions starting from the initial DOWN state in period  $t=0$ .*

Given the results in Tables 1(a)-(b), 2, and Figure 3, the bidding strategy can be deduced as follows. The generator is initially DOWN, and the state that it can occupy in period 1 is either  $UP_1$  or DOWN. The generator will find it preferable to be DOWN if the price is LOW in period 1 because the ( $UP_1$ , LOW) entry of (\$1375) in column 1 (and row 4) of Table 2 is lower than the (DOWN, LOW) entry of \$0 in the same column (and row 6). However, the ( $UP_1$ , HIGH) entry of \$2125 in column 1 (and row 3) of Table 2 is higher than the (DOWN, HIGH) entry of \$0 in the same column (and row 5). Therefore the optimal transition from the current period  $t=0$  can be stated as a function of price in period 1; if price is HIGH in period 1, then it is optimal to go to the  $UP_1$  state in period 1; if price is LOW in period 1, then it is optimal to stay in the DOWN state in period 1. This is shown in Figure 3 (state transitions for period  $t=0$  from the DOWN state).

Therefore it will reap expected profits of \$2125 by being online if the price is HIGH in period 1. If the generator is selected in period 1, the generator will find itself in the UP<sub>2</sub> state in period 2 (from Figure 3). Its minimum dispatch in the UP<sub>2</sub> state from column 2 of Table 1(a) is 90 MW. This leads us to the following optimal bidding strategy:

- in period 1: bid 100 MW at any value between \$33.01/MWh to \$44.99/MWh
- in period 2: (only) *if* selected in period 1, bid 90 MW at a low enough price (less than or equal to \$4.99/MWh) to guarantee selection, and bid the remaining 10 MW at its incremental cost of \$30/MWh. If not selected in period 1, the generator will not make any bid in period 2.

Since the chances of being selected in period 1 are 50%, *expected* profits from this strategy are  $0.5 \cdot 0 + 0.5 \cdot 2125 = \$1062.50$ . However, even with an optimal bidding strategy, the generator will lose \$750 (= 1500 - 2250) if the prices are HIGH (\$45/MWh) in period 1 and LOW (\$5/MWh) in period 2.

The moral of this example is that the optimal strategy for the generator is to bid above cost in the first period (and below cost in the second period), and that such behavior has nothing to do with market power. It is the rational expected behavior of a profit-maximizing, price-taking generator.

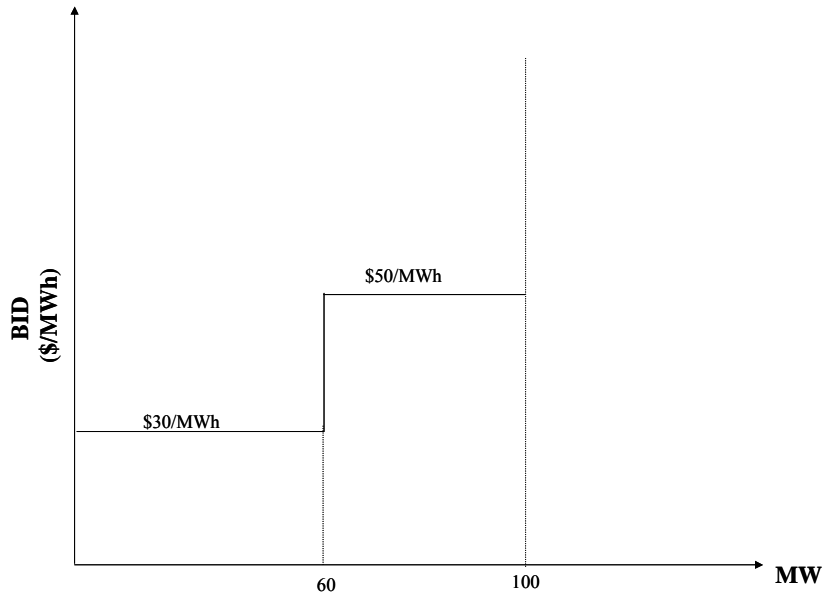
**Example 4:** (*The effect of market design and combined energy and reserve markets; necessity of varying bids by time.*) Assume two markets, one for energy and one for reserves. Suppose that a price-taking generator has a capacity of 100 MW, constant incremental costs of \$30/MWh (and no other costs), and a reserve capability of 40 MW; suppose that there are no direct costs for offering reserves. Suppose that the market design is such that the energy market clears first.<sup>28</sup> Then reserve bids are accepted and the reserve market clears. Both markets have uniform clearing prices. Suppose that the generator has **perfect foresight** of reserve availability price.<sup>29</sup> It forecasts accurately that, for an **on-peak period**, the reserve availability price is \$20/MW/h; for an **off-peak period**, reserve availability price is forecast to be \$2/MWh. What are the generator's optimal bids in the energy market for both the off-peak and on-peak periods if the objective is to maximize expected profits?

**Solution:** The generator bids two stairs in the energy market, as shown in Figure 4. In stair one, the generator bids its incremental costs for the capacity (60 MW) that cannot be sold into the reserve market. In stair two, the generator bids incremental costs *plus* foregone profits in the reserve availability market for the capacity (40 MW) that can be made available in the reserve markets. In the on-peak period, the foregone profits are \$20/MW/h; in the off-peak period, the foregone profits are \$2/MW/h. The optimal bidding strategy for the price-taking generator is to *vary its bids* by period; it has a steeper sloped bid curve for the on-peak period. A cursory glance at the bid curves may invite suspicions that the generator is withholding capacity in the on-peak period; however, they are a perfectly legitimate strategy in this example.

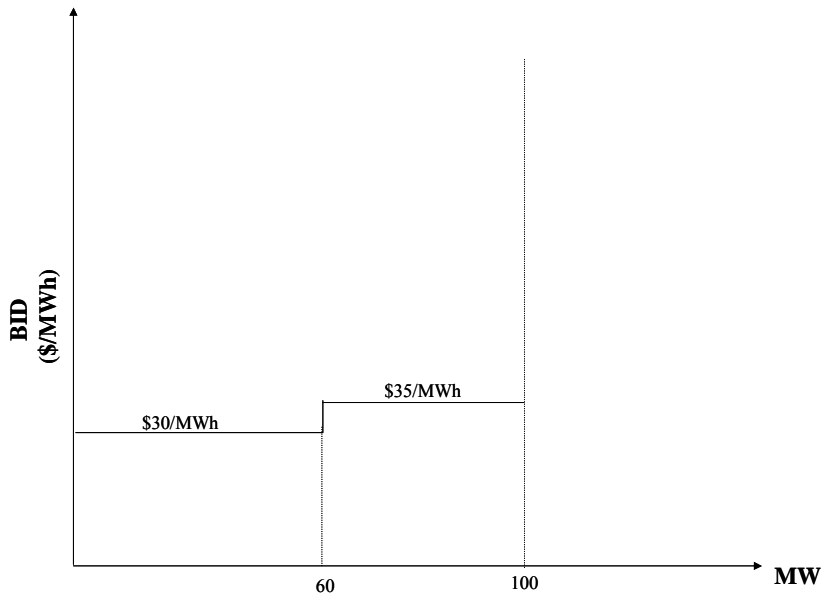
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<sup>28</sup> This design is similar to the method by which the California ISO clears its markets today (September 2003).

<sup>29</sup> We also make the simplifying assumption here that the reserves are not called; hence the generator only gets the reserve availability price.



**a. OPTIMAL ON-PEAK BIDS.**



**b. OPTIMAL OFF-PEAK BIDS.**

**FIGURE 4. OPTIMAL BIDS FOR THE GENERATOR IN EXAMPLE 6.**

*The generator bids vary with time because the opportunity costs of the generator in the reserve markets vary with time.*

**Example 5:** (The effect of market design, forecast uncertainty, and combined energy and reserve markets on bidding behavior.) Assume a single-period case and two markets, one for energy and one for reserves. Suppose that a price-taking generator has a capacity of 100 MW, constant incremental costs of \$30/MWh (and no other costs), and a reserve capability of 40 MW; suppose that there are no direct costs for offering reserves. Suppose that the market design is such that the energy market clears first. Then reserve bids are accepted and the reserve market clears. Both markets have uniform clearing prices. Suppose that the generator forecasts that energy price would be either \$35/MWh or \$40/MWh (with equal probability) and, independent of energy prices, the reserve availability prices would be either \$4/MWh or \$12/MWh (with equal probability). What is the generator's optimal bidding strategy if the objective is to maximize expected profits?

**Solution:** Since the energy market clears first and only the energy price is observable after this market clearing, the model described in this paper would solve such problems by finding the optimal dispatch **policy** as a function  $g(P_E)$  of energy price ( $P_E$ ) only; the effect of the reserve price would be "internalized" in the function  $g(P_E)$ . We find the optimal function  $g(P_E)$  as follows. If the energy price is \$35/MWh, it is optimal to produce 60 MW of energy and offer 40 MW of reserves. This is because (a) the expected profit margin on reserves is \$8/MWh ( $= 0.5*4 + 0.5*12$ ), while the profit margin from energy sale is only \$5/MWh ( $= 35 - 30$ ); (b) at most 40 MW of reserves can be offered; and (c) energy sales are profitable ( $\$35/\text{MWh} > \$30/\text{MWh}$ ). If the energy price is \$40/MWh, it is optimal to produce 100 MW of energy. This is because the expected profit margin on reserves is \$8/MWh ( $= 0.5*4 + 0.5*12$ ), which is lower than the profit margin of \$10/MWh ( $= 40 - 30$ ) from energy sale. Therefore the optimal bidding strategy is to offer the first 60 MW at \$30/MWh and to offer the remaining 40 MW at any bid between \$35.01/MWh and \$39.99/MWh; any unaccepted capacity<sup>30</sup> in the energy market is then offered into the reserve markets. Expected profits from this strategy would be<sup>31</sup>  $0.25*100*10 + 0.25*100*10 + [0.25*(60*5 + 40*4)] + [0.25*(60*5 + 40*12)] = \$810$ . With this bidding strategy, when the market prices for energy and reserve availability are \$35/MWh and \$4/MWh respectively, the generator will (sub-optimally) produce 60 MW of energy and offer 40 MW of reserves; with perfect hindsight, it would have been optimal to produce 100 MW of energy. Similarly, when the market prices for energy and reserve availability are \$40/MWh and \$12/MWh respectively, the generator will (sub-optimally) produce 100 MW of energy and 0 MW of reserves; with perfect hindsight, it would have been optimal to produce 60 MW of energy and offer 40 MW of reserves.

**Example 6:** (The effect of market design, forecast uncertainty, combined energy and reserve markets, with correlation between energy and reserve price forecasts.) Consider the same

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<sup>30</sup> Because of the nature of the sequential market clearing, the bidding strategy for offering reserves is as follows. Generators near or at the margin in the reserve market should generally offer their reserve output at their expectation of the market clearing prices (which should closely approximate their forgone opportunity costs in the energy market); infra-marginal generators can offer their output at a lower bid to guarantee selection.

<sup>31</sup> The explanation is as follows. There are four equally possible events: energy price = \$40/MWh, reserve price is \$4/MWh; energy price = \$40/MWh, reserve price is \$12/MWh; energy price = \$35/MWh, reserve price is \$4/MWh; energy price = \$35/MWh, reserve price is \$12/MWh. In the first two events, the generator's bids will be accepted in the energy markets, and the generator will produce 100 MW of energy; in the latter two, the generator will dispatch 60 MW in the energy market and 40 MW in the reserve market.

problem as in Example 4, but with a different price forecast. Suppose that the generator forecasts that energy price would be either \$35/MWh or \$40/MWh (with equal probability). However, the reserve availability prices are dependent on energy prices as follows. If the energy price is \$35/MWh, reserve availability price is \$4/MW/h. If the energy price is \$40/MWh, reserve availability price is \$12/MW/h. What is the generator's optimal bidding strategy if the objective is to maximize expected profits?

**Solution:** We again find the optimal dispatch  $g(P_E)$  as follows. If the energy price is \$35/MWh, it is optimal to produce 100 MW of energy. This is because the expected profit margin on reserves is \$4/MW/h, while the profit margin from energy sale is \$5/MWh (= 35 - 30). If the energy price is \$40/MWh, it is optimal to produce 60 MW of energy and 40 MW of reserves. This is because (a) the expected profit margin on reserves is \$12/MW/h, which is higher than the profit margin of \$10/MWh (= 40 - 30) from energy sale, and (b) at most 40 MW of reserves can be offered. Therefore when the energy price is high, it is optimal to offer less in the energy market and *vice-versa*. This implies that the optimal strategy is to bid discretionary MW (up to 40 MW) in the energy market only if it less profitable to take part in the reserve market. The expected profit margin in the reserve market is \$8/MW/h (=  $0.5 \cdot 4 + 0.5 \cdot 12$ ), which is more than the expected profit margin of \$7.5/MWh (=  $0.5 \cdot (35 - 30) + 0.5 \cdot (40 - 30)$ ) in the energy market. Therefore, the optimal bidding strategy is to bid 60 MW of energy at \$30/MWh and to withhold the remaining 40 MW from the energy market (e.g., by bidding them above \$40/MWh). The remaining 40 MW is offered in the reserve market at \$3.99/MW/h or lower. With this bidding strategy, when the market prices for energy and reserve availability are \$35/MWh and \$4/MW/h respectively, the generator will (sub-optimally) produce 60 MW of energy and offer 40 MW of reserves; with perfect hindsight, it would have been optimal to produce 100 MW of energy.

The implication of the last four examples for market power analysis is that the use of simplistic methods can lead to the conclusion that a generator was manipulating markets by bidding above incremental costs for at least part of its output (and thus practicing "economic withholding"), whereas the generator was actually acting like a price taker and correctly maximizing profits based on market design and price uncertainty. Thus, in these examples, the generator is not guilty of market manipulation concerns.

### **4.3 DATA REQUIREMENTS**

One of the main advantages of our approach is that the data requirements for running such a model are modest. However, two caveats are in order. First, all data may not be available in the public domain.<sup>32</sup> Second, we assume here that data on generator cost and operating characteristics can be independently verified, though this may not always be the case; it is therefore conceivable that a generator could manipulate market prices by manipulating these data.

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<sup>32</sup> In particular, according to W. W. Hogan (email correspondence to authors, Feb. 4, 2002), "... there are data about ancillary services requirements, instructions to individual generators, and environmental limitations such as the "Delta" dispatch, and so on, which involve detailed communication between the system operator and the generators and result in significant effects on the level of energy and ancillary service output. However, with the exception of the actual energy output that we can measure, we do not know how much these other decisions were relevant, without getting the information from the system operator..." See also Harvey and Hogan (2000, 2001) for a detailed elaboration.

We discuss the data needs below.

1. The relevant data on individual generator(s) suspected of exercising market power are generator costs (startup and shutdown costs, ramping costs, no-load costs, incremental costs, emission costs), operational limits<sup>33</sup> (minimum and maximum generation, reserve capability by type of reserve, startup and shutdown times, ramp rates, or other constraints).
2. Historical location-based data on market prices for energy and (where applicable) reserve prices are also needed.
3. We need an uncertainty component that quantifies the lack of perfect foresight to forecast prices and the necessity to forecast these prices. In other words, we cannot simply assume that because certain prices did develop, the generator, operating without the benefit of hindsight, should have been able to predict them exactly as they occurred. Moreover, as previously mentioned, in markets with pricing rules such as NY and PJM, uncertainty would matter less than in other markets for *thermal* generators because non-negative profits are guaranteed for every generator. In Rajaraman and Alvarado (2003), price uncertainty is represented as a Markov chain; for numerical calculations, one can discretize price states at some time  $t$  and have a transition probability between price states at time  $t$  and time  $t + 1$ .
4. Importantly, the observed prices may need to be adjusted by any changes that the generator's bid may have had on the price itself. In other words, if there was a \$200/MWh price spike in the market at a given hour, it is possible that if the generator had been in service the spike would have been only \$80/MWh. Thus, it would be disingenuous, if the generator was out of service, to argue that the generator, if it had decided to operate, would have ever actually seen and been paid the historical observed price. Thus the input price "forecast" of the model must appropriately account for this "feedback effect."

In summary, the market power test presented in this section incorporates the effect of inter-temporal constraints, uncertainty, and the effect of transmission constraints on the optimal commitment and dispatch; moreover, one can measure its sensitivity to various parameters of interest. Therefore the test is a robust way of checking for market power. Such a market power test also addresses the major concerns in the use of simulation models as discussed in Harvey and Hogan (2000, 2001).

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<sup>33</sup> For hydroelectric generators, one needs information on reservoir inflow forecasts, storage capacity, environmental restrictions, pumping efficiency (if applicable), etc.



The "guilty/not guilty" tests in Propositions 1 and 2 in Section 4.1 are *direct* tests in the sense that these tests can be applied to particular generators to rule out (or prove) that they exercised market power. Given the public outcry over market power in the California crisis of 2000-2001, and talks about large refunds<sup>34</sup> for exercising market power, we suggest that such tests could be used to check whether and how some generators<sup>35</sup> exercised market power. These tests are more focused and effective than the present more nebulous simulation-based approach of “retroactive price forecasting” in present use.

While the model-based test outlined described in this section can be helpful in resolving questions about market power, there are some questions that it cannot answer. For example, if a generator claims to be on forced full or partial outage, and claims that the discrepancy between simulated and observed behavior can be attributed to the outage, the validity of the claim could be difficult to verify. We deal with this situation next.

## **5. MARKET POWER TEST, PART TWO: CHECKING FOR INCENTIVES**

In this section, we attempt to answer those questions that are unanswered by the model-based test—for example, is a generator’s forced (full or partial) outage an artificial one and an attempt to exercise market power? Devising a simple test to check market participants’ incentives to withhold output is difficult because forced outages could have a genuine cause. The discussion below gives conditions under which we can rule on the possibility that market power was exercised by keeping generation partially or fully off-line due to alleged operational problems.

In this section, we will mainly concentrate on the incentive to withhold by derating either partially or fully the capacity of the generator by falsely reporting an operational problem or by exaggerating the severity of an operational problem. (Our discussion on incentives can also be used to analyze other market participant claims of deviating from the profit-maximizing output; for example, due to claimed poor bidding decisions.) Exercising market power by overproducing

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<sup>34</sup> While our tests prove or disprove the exercise of market power, we do not directly measure the welfare losses or indices such as Lerner Index associated with the market power. In our opinion, obtaining exact estimates of these quantities would be a very difficult data intensive task requiring highly sophisticated optimization analysis. Instead we suggest the following simpler “first-order” approximation. Suppose that it can be shown via Propositions 1 and 2 that market power was exercised by a handful of generators. Then one can solve Problem P for these generators (assuming a “credible” price forecast) to estimate their “ideal competitive” dispatch. The difference in output between the actual dispatch and “ideal competitive” dispatch could be used in conjunction with estimates of supply elasticity for energy and reserve markets (from the historical bids), to estimate competitive market prices. This would yield approximate estimates of the competitive prices for the historical periods, from which welfare losses, Lerner Index, etc. can be estimated. (This is only an approximate approach, and ignores, among other factors, the potential feedback effect of at least some generators — e.g., energy limited generators — changing their behavior in response to the new competitive prices.)

<sup>35</sup> Marketers who deal in pure financial transactions without any physical control of any assets in the spot markets generally lack the ability to exercise market power in the spot markets; therefore there is no need to develop tests for such entities. Even PJM, an otherwise sensible organization, has gotten tripped up over this point. PJM has developed measures (including a system of financial penalties) that purport to mitigate market power in the Financial Transmission Rights (FTR) markets, without any regard to whether an FTR holder has control of physical assets in the spot market (PJM Open Access Tariff, Section 5.2.1(b)-(c)). At best these flawed measures are developed without thinking the matter through; at worst, the financial penalties represent a confiscation of property rights.

or by withholding output without claiming operational problems or poor bidding decisions can be resolved by the market power tests in Propositions 1 and 2.

Genuine operational problems are random events with low probabilities, though the probability of generator outages could increase significantly under certain conditions, e.g., in weather storms such as tornadoes. Given the random nature of these problems, it may be difficult to consistently (falsely) claim operational problems to withhold capacity without getting “caught.” For example, it will be easy to spot those generators that seem to develop operational problems with clockwork regularity under certain conditions (e.g., when loads are high). Statistical tests such as hypothesis testing can also be used to detect more complicated cases of withholding output due to falsely claimed operational problems. Therefore market participants who behave in such “easy to detect” manner can be identified and could be subject to appropriate mitigation measures. If the mitigation measures are punitive and/or significantly constrain the market participants’ behavior, they will likely be undesirable and the market participants will therefore have some incentive not to behave in an “easy to detect” manner.

However, there could be other more subtle cases of claimed operational problems that are more difficult to resolve by statistical or other tests. We will analyze such cases by:

- taking account of the market participant’s financial positions
- examining whether the market participant is withholding output in the least-costly way.

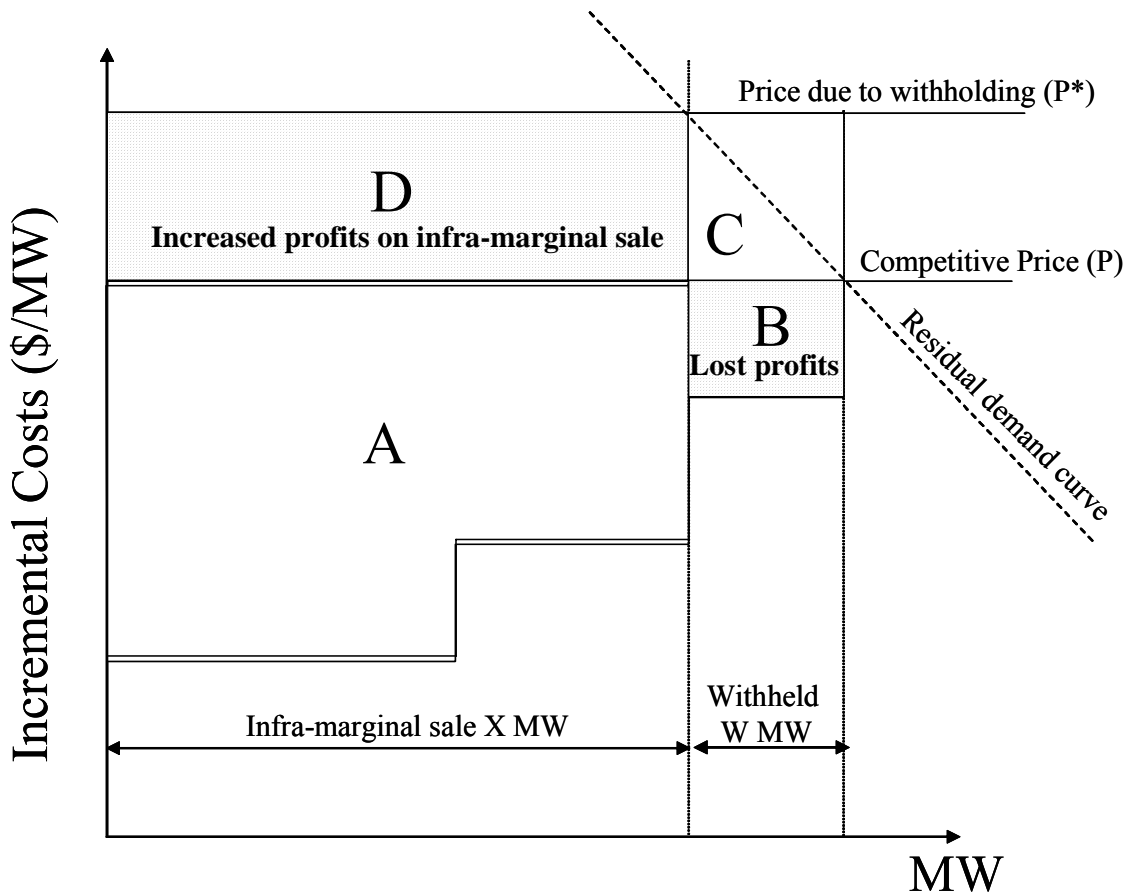
## **5.1 FINANCIAL POSITIONS**

As we noted in footnote 13 and in our opening remarks, the firm’s incentives to exercise market power also depend on its financial positions<sup>36</sup> in addition to its physical assets. Typically, firms tend to hedge at least some output in the forward (and other derivative) markets; firms could also take speculative positions in forward markets.

We informally discuss the increasingly complex incentives faced by the market participant. These examples refer to Figure 5 showing a market participant’s incremental costs of generation and its profits as a function of the market participant’s withholding.

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<sup>36</sup> Having the obligation to serve a fixed load is similar in principle to entering into a forward contract.



**FIGURE 5: PROFITS FROM WITHHOLDING.**

**If the shaded area B is less than the shaded area D, it is profitable to withhold.**

1. *No forward market sales; objective is to maximize current period profits only:* This situation is illustrated in Figure 5. Assuming that the optimization is over the current spot market only, the market participant who owns generation with incremental costs shown in Figure 5 is withholding some amount W MW that raises the market-clearing price from P to P\* (as given by the residual demand curve facing the market participant). The shaded region B shows the market participant's lost profits on the withheld amount W MW. On the other hand, the shaded region D shows that the market participant has increased profits on infra-marginal sales (X MW). If  $D > B$ , it is profitable to withhold. The market participant would produce<sup>37</sup> X MW, and total profits from the strategy would be the area represented by A + D.
2. *Some output had been sold in the forward market; objective is to maximize current period profits only:* Assume that the market participant had already locked in X MW in forward market sales. Since the market participant maximizes profits for the current spot market only, it can be seen that it is not optimal to withhold W MW from the spot markets because

<sup>37</sup> The optimal level X to produce (or W to withhold) would be obtained by optimizing profits given the residual demand curve facing the market participant.

area B is sacrificed without any offsetting compensation from infra-marginal sales. This is because profits from infra-marginal sales are already locked in, regardless of spot market prices. Therefore, the market participant will attempt to capture an additional profit equal to at least area B; this means that the amount withheld will be less than W MW.

3. *All output had already been sold on the forward markets; objective is to maximize current period profits only:* In this case, the profit-maximizing outcome is to produce  $X + W$  MW. The market participant withholds no amount of output; otherwise lost profits have no offsetting compensation.
4. *Objective is to maximize current period and future period profits:* This is a much more complicated optimization<sup>38</sup> but the *only* relevant one for the market participant. It depends on a number of factors: the time horizon of the optimization; the market participant's ability to raise forward prices by manipulating the spot markets today and hence the market's expectations of future spot prices; the market participant's forecast of future spot prices, as a function of the market participant's withholding in those periods; and the market participant's risk preferences. Moreover, it must be kept in mind that the claimed operational problems could bind the market participant into withholding output over a number of periods.<sup>39</sup> Given the complicated game-theoretic aspects of the problem, the generator may develop over time some rules of thumb to withhold an amount in the current period spot market in conjunction with plans to take positions in forward markets<sup>40</sup> as part of an attempt to maximize multi-period profits.

The above cases demonstrate that a market participant's incentive to *withhold* output by derating the capacity of a generator (either by falsely reporting an operational problem or by exaggerating the severity of an operational problem) tends to lessen:

1. When the net amount of output hedged in the forward markets is higher (or equivalently, when the market participant has load obligations) and over a longer time horizon.
2. When the market participant is unable to manipulate the forward markets to his/her advantage.

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<sup>38</sup> See, for example, B. L. Allaz and J. Vila, "Cournot Competition, Forward Markets and Efficiency," *Journal of Economic Theory*, vol. 59, 1993, pp. 1-16. See also S. M. Harvey and W. W. Hogan, "California Market Prices and Forward Market Hedging," October 2000 for a discussion on this topic.

<sup>39</sup> For example, if, during one period, a generator's capacity is derated by 20% because of a falsely claimed boiler leak, then this 20% derate will continue to apply to other periods until the "boiler problem" is "fixed."

<sup>40</sup> For example, a market participant may take a long position in the forward markets (for delivery at a future time T) with the intention of manipulating the spot prices at time T. Alternatively, the market participant may unwind the forward position prior to time T, if, by manipulating the spot markets at times prior to T, the market's expectation of forward prices at time T are also affected. These examples show that market participants could have incentives to exercise market power even if they hedge by selling output forward because they can take additional positions in the forward markets and therefore change their objective function for profit-maximization. Conversely, a market participant who, by manipulating prices in the spot market is able to manipulate forward prices, may decide to lock-in profits by selling generation in the forward markets; in this case, the market participant will have less incentive to withhold output in future periods.

3. When, in order to profitably exercise market power in *some* periods by withholding generation due to falsely claimed operational problems, the market participant is also committed (to maintain the "credibility" of the falsely claimed operational problem) to withhold that generation over longer periods.

## **5.2 WITHHOLDING OUTPUT IN A LEAST COST WAY**

All other things being equal, a market participant would recognize that withholding Y MW from slightly "in-the-money" generators whose incremental costs are nearest to the market price is less expensive (in exercising market power) than withholding the same Y MW from generators with cheaper incremental costs.<sup>41</sup>

However, at least theoretically, such a strategy may not always be followed. It is conceivable that a market participant may deliberately withhold output in a sub-optimal fashion (withholding from an intermediate or baseload plant rather than a peaking plant) to avoid detection. Such a strategy may be successful in increasing the profits of the market participant above the corresponding competitive case, but the profits would be lower than that from the theoretically optimal withholding strategy.

In spite of this theoretical possibility, we are skeptical that a market participant will consistently leave money on the table by behaving in sub-optimal ways. Therefore, if a market participant is withholding output from a generator that has very inexpensive incremental costs (e.g., by shutting it down because of a claimed operational problem), but is running a more expensive generator instead, chances are that the outage is legitimate after all. As another example, one would not find it more profitable to withhold from a reservoir with lower opportunity costs and produce output from one with higher opportunity costs.

## **6. CONCLUSIONS**

This paper has presented a practical and accurate method to test for the exercise of market power in the electricity market. The common practice for showing that market power has been exercised is to use hourly simulation models to simulate competitive market prices given a large volume of generator and load data. This paper has elaborated upon the many shortcomings that affect the simulated market clearing prices model for market power analysis.

The paper has also described the issue of data requirements and limitations for market power analysis using price simulation models. Even with more sophisticated and comprehensive simulation models, that one cannot hope to have much better estimates of competitive prices.

As an alternative, this paper has described a more robust and practical approach that is based on an accepted method for market power analysis, which is to examine optimal generator behavior by taking price as a given exogenous input. This paper has substantially extended this method by taking full account of forecast uncertainty, market design, inter-temporal constraints, non-

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<sup>41</sup> It is possible that the generator that is slightly "in-the-money" during some hours could be relatively deep "in-the-money" during other hours. If this generator derates capacity (by falsely reporting operational problems) during a multi-hour period, offsetting profits must compensate for lost profits over this time interval.

convex costs, multiple markets, multiple periods, etc. That is, instead of simulating competitive market prices and asking what the price should have been under competitive conditions, this paper takes historical location-based prices (adjusted for imperfect foresight and other factors) as *given exogenous input* along with generator operational parameter characteristics, and proceeds to simulate a generator’s profit-maximizing (competitive) commitment and dispatch **policy**. This optimal generation dispatch and commitment **policy** can be used to formulate the optimal generator bidding strategy and to understand generator behavior. Based on the optimization, the paper has given a test that can be used to show that a market participant is “not guilty” of exercising market power. That is, if certain conditions are met, a market participant can exonerate himself or herself from any accusations of market power abuse. A stricter “guilty” test must be met to show that market power has been exercised.

In certain cases, however, these tests may be inconclusive. To resolve such questions, and for a more complete test, this paper has also discussed the market participant’s incentives to exercise market power. The method in the paper can be applied, for example, to the controversies that have arisen in the California market to assist in resolving questions about whether—and which—generators exercised market power during the crisis of 2000-2001.

To summarize, the important features of the proposed approach are that:

- it fully incorporates locational effects as well as individualized characteristics of specific generators to help provide a “guilty” or “not guilty” answer to the market power question,
- it also gives us the optimal commitment and dispatch **policy** for the generator from which an optimal bidding strategy can be formulated, and the generator’s behavior can therefore be better understood, and,
- it can also give guidance on how to design appropriate market mitigation rules for a given generator if the exercise of market power is indeed detected under some conditions.

## **7. APPENDIX A: ENERGY LIMITED GENERATORS**

In this appendix, we deal with the problem of optimally allocating the output of energy-limited generators. As an illustrative example, we analyze the allocation of hydroelectric output; however, the principles in this appendix are general and can be used for the analysis of any energy-limited generator.<sup>42</sup> Examples of energy-limited generators include thermal generators constrained by emissions limits on total output over a period of time, environmental restrictions, etc.

Imagine a hydroelectric reservoir with an initial amount of water storage. For simplicity, assume that no more water will ever be added to this reservoir, assume that there is a limit to how much water we can spend in any single period, and that the direct production costs of producing electricity is zero. Finally, assume that we are given a perfect forecast of energy prices (assume perfect foresight) for as far away into the future as the eye can see.<sup>43</sup> Given these conditions,

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<sup>42</sup> The problem stated in this Appendix can be formally solved by solving Problem P.

<sup>43</sup> To make meaningful comparisons assume that all forward prices are expressed in terms of prices today.

finding a profit-maximizing dispatch is easy: we would plan to dispatch first in the highest priced hour up to our allowed hourly limit, then in the next highest priced hour, and so on, until all the water in the reservoir is exhausted. Therefore there may be many hours for which we would not be producing any electricity. In particular, it is possible that we may not have allocated any water for today, because the future prices looked much more attractive compared to today’s prices, given the water storage level today; therefore electricity will be withheld from the spot market.<sup>44</sup> This is not an exercise of market power, but a legitimate allocation of water to the highest valued periods.

Now suppose that we have to deal with the fact that there may be rain in some future periods to fill up the reservoir. In such a case, we would still do a similar exercise, excepting that we would include an additional inter-temporal variable that keeps track of the water level of the reservoir at any given time to account for water inflows and outflows.<sup>45</sup>

In general, exactly how much water one would allocate among the different periods of the year would depend mainly on two factors (besides the operational constraints): the forecast of the future electricity prices and ancillary services prices and the forecast of future rain (or water inflow into the reservoir). Of course there is a lot of uncertainty in the forecasts, and we would certainly be able to optimize the dispatch better if there is less uncertainty and we are sure of the future prices and precipitation. Even if we can get protection from uncertainty (say by hedging price risk by engaging in forward market transactions),<sup>46</sup> we would still have an “option” feature<sup>47</sup> in the water allocation process. The water allocation process would lead to a decision rule (that may need to be updated continuously) that would allocate water among the different periods of the year based on the best available forecasts and information today.

In reality, of course, hydroelectric generators deal with more complex operational conditions.<sup>48</sup> While the optimization problem is more challenging, the basic principles remain the same. The bottom line on discretionary hydro dispatch is that we would need to consider how much profit we would make in the *future* before committing to selling today. As a general rule, the strategy is to withhold discretionary output in the periods of relatively lower demand hours (because prices would be relatively lower in such periods) and produce in periods with relatively higher demands (because prices would be higher in such periods). In a nutshell, allocation of water

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<sup>44</sup> The problem would be more complicated if ancillary services markets are also included, but the basic principles would remain the same. In particular, when making reserves available, there is an element of “eating your cake and having it too,” because there is a likelihood that reserves will not be called, and hence water will not be spent.

<sup>45</sup> For pumped hydroelectric generators, the problem is more complicated; the pumping action is equivalent to water inflow into a reservoir, with the water inflow itself being a decision variable.

<sup>46</sup> It is harder to hedge weather risks such as drought.

<sup>47</sup> We would retain this option value *even if all output were sold* in the forward markets. The reasoning is that we would still want to optimize the dispatch in the spot markets; for example, if the opportunity costs were higher than the current period market price, we would find it more profitable to cover the short position by buying from the market. The option value will depend on the uncertainty of forecast and the generator’s risk preferences; it must be included in the cost calculations because it reflects a true cost of dispatch.

<sup>48</sup> For example, the hydro system may be required to operate under minimum runoff constraints, environmental constraints, and constraints of irrigation, navigation, etc.

across different periods is a difficult problem, made more complex by the uncertainty in the forecast.

As this strategy has the tendency to shift the aggregate supply curve to the left as the demand curve is also shifting to the left, it could result in less pronounced price spreads between peak and off-peak periods. Therefore if there are significant hydro resources in the market, then the optimal water allocation process tends to "levelize" prices over time. That is, the prospect of high prices in some future or peak periods can and do get reflected as high prices during the current or off-peak periods.

How would one bid hydroelectric output into the market for any given time-period, assuming a uniform-price auction?<sup>49</sup> It would depend on the optimal **policy** found by solving Problem P. For example, suppose that the optimal **policy** is to withhold all the output if the market-clearing price is lower than, say, \$20/MWh; conversely, the optimal **policy** may be to sell the maximum possible hourly output if the market-clearing price is higher than, say, \$30/MWh. The band of bid prices could reflect uncertainty of price forecast for this and other periods. The optimal **policy** may then guide us into bidding the following: bid some MW at \$20/MWh, some more at \$22/MWh; and so on, until all allocated output is exhausted at \$30/MWh.<sup>50</sup> Therefore, there may be a tendency to bid not only relatively *high* prices (reflecting opportunity costs in other periods) during some periods, but the bids may also include an option value that reflects the uncertainty in our forecast. (Conversely, when we are *sure* that the uncertain opportunity costs in other periods will be lower than this period's market clearing price, we may choose to bid relatively *low* prices to guarantee selection.)

The behavior of other market participants could also affect hydroelectric bids. Suppose that it turns out that a large baseload generating plant has a sudden forced outage in the spring and that the outage is expected to last through the summer. Then this may lead to increase in expectation of higher prices in the summer; consequently, this could get reflected as a sudden and legitimate increase in the hydro generator's bids in the spring.

Therefore, hydroelectric bids could be complicated curves, based on expectations and uncertainty of profits from current and future periods.

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<sup>49</sup> For the underlying mathematical theory, see R. Rajaraman and F. L. Alvarado, "Optimal Bidding Strategies in Electricity Markets Under Uncertain Energy and Reserve Prices", PSERC Report 03-05, April 2003 ([http://www.pserc.wisc.edu/ecow/get/publicatio/reports/2003report/alvarado\\_bidding\\_report\\_0411.pdf](http://www.pserc.wisc.edu/ecow/get/publicatio/reports/2003report/alvarado_bidding_report_0411.pdf)).

<sup>50</sup> See examples 4.1, 4.2, and 4.3 of the reference Rajaraman and Alvarado (2003).



## 8. APPENDIX B: CALCULATION DETAILS FOR EXAMPLE 3

This section derives the entries in Tables 1(a)-(b) and Table 2 for Example 3.

### DERIVATION OF OPTIMAL DISPATCH POLICY (TABLES 1(a)-1(b)):

The question we address to solve the **optimal dispatch problem** is: **if** the generator is in some state  $g$  in period  $t$ , and **if** the price is  $p$ , then what is the optimal dispatch and what are the optimal profits?

In period 2, **if** the generator is  $UP_2$  and **if** the price is HIGH (\$65/MWh) — row 1, column 2 of tables 1(a) and 1(b) — it is optimal to produce 100 MW for a profit of  $100*(65 - 30) = \$3500$ . (The generator has incremental costs of \$30/MWh, while the price is \$65/MWh; therefore it is optimal for the generator to maximize its output.)

Similarly, in period 2, **if** the generator is  $UP_2$  and **if** the price is LOW (\$10/MWh) — row 2 column 2 of tables 1(a) and 1(b) — it is optimal to produce as little as possible (the required minimum 90 MW) at a loss of  $90*(5 - 30) = (\$2250)$ .

As another example, in period 1, **if** the generator is  $UP_1$  and **if** the price is HIGH (\$45/MWh) — row 3, column 1 of tables 1(a) and 1(b) — it is optimal to produce 100 MW to make a profit of  $100*(45 - 30) = \$1500$ .

All other individual entries of Tables 1(a) and (b) are similarly filled.

### DERIVATION OF OPTIMAL COMMITMENT POLICY (TABLE 2 AND FIGURE 3):

Now we use the results of Tables 1(a)-(b) to derive the optimal commitment strategy. The commitment strategy is derived by working *backwards* from the final period to the current period (this is the backward DP method).<sup>51</sup>

The optimal commitment strategy is found by calculating  $Value_t$  for period  $t$ , as given in Table 2. Recall that  $Value_t$  is the expected cumulative profit from period  $t$  to the terminal period, provided the optimal commitment and dispatch policy is followed during this interval. The reader is referred to equation (1) for a formal definition of  $Value_t$ .

The optimal commitment policy is given in Figure 3. The optimal state transitions are given by bold arrows. The state transitions depend on price level.

We now show how to derive Figure 3 and Table 2 by working backwards from the terminal period to the initial period using the recursive equation 1.

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<sup>51</sup> This is also the same "tree" method used to value financial options.

### COMPUTATION OF VALUE<sub>2</sub>

We start by computing the *Value*<sub>2</sub> entries corresponding to period 2 (second column of Table 2). We illustrate a few sample calculations.

- **If** the generator is UP<sub>2</sub> and **if** the price is LOW (\$5/MWh) in period 2, then profits from this period are obtained from Table 1b. For this case, the value is -\$2250 (column 2, row 2). From Figure 2A, the generator can go to either UP<sub>2</sub> or DOWN in the next period, t=3. However, because *Value* is negative infinity for the UP<sub>2</sub> state in period 3, it is optimal to go DOWN in period 3. This optimal transition is shown in Figure 3 (see arrow from UP<sub>2</sub> to DOWN in period 2). Therefore *Value* for period 2 for generator state UP<sub>2</sub> and price LOW is the *Value* of the DOWN state in period 3 + current period profits = 0 – 2250 = -\$2250 and this is entered in column 2, row 2 in Table 2.
- **If** the generator is DOWN, and **if** the price is HIGH (\$65/MWh) in period 2, then the generator will not run (see row 5, column 2 of Table 1a) and profits in this period are zero (see row 5, column 2 of Table 1b). From Figure 2A, the generator can go to either UP<sub>1</sub> or DOWN in the next period, period 3. However *Value* is negative infinity for the UP<sub>1</sub> state in period 3, so it is optimal to go DOWN in period 3. This optimal transition is shown in Figure 3 (see arrow from DOWN to DOWN in period t=2). Therefore *Value* for period 2 for generator state DOWN and price HIGH is the *Value* of the DOWN state in period 3 + current period profits = 0 + 0 = \$0 and this is entered in column 2, row 5 in Table 2.
- **If** the generator is UP<sub>1</sub>, and **if** the price is HIGH (\$65/MWh) in period 2, then the generator will produce 100 MW run (see row 3, column 2 of Table 1a) and profits in this period will be \$3500 (see row 3, column 2 of Table 1b). From Figure 2A, the generator can only go to UP<sub>2</sub> in the next period, period 3. However *Value* is negative infinity for the UP<sub>2</sub> state in period 3. This optimal transition is shown in Figure 3 (see arrow from UP<sub>1</sub> to UP<sub>2</sub> in period t=2). Therefore *Value* for period 2 for generator state UP<sub>1</sub> and price HIGH is the *Value* of the UP<sub>2</sub> state in period 3 + current period profits = -∞+ 3500 = -∞ and this is entered in column 2, row 3 in Table 2.
- Similarly all the remaining entries in column 2 of Table 2 and the optimal state transitions in period 2 (Figure 3) can be derived.

### COMPUTATION OF VALUE<sub>1</sub>

We now derive *Value*<sub>1</sub> for period 1 (column 1 of Table 2). We illustrate some sample calculations.

- **If** the generator is UP<sub>2</sub> and **if** the price is LOW (\$33/MWh) in period 1, then profits from this period are obtained from Table 1b, which is \$300 (column 1, row 2). From Figure 2A, the generator can go to either UP<sub>2</sub> or DOWN in the next period, period 2.
  - a. If it goes to UP<sub>2</sub> in period 2, then  $Value_2(UP_2, LOW) = -2250$  while  $Value_2(UP_2, HIGH) = 3500$ . Hence  $E(Value_2(UP_2, p_2) | p_1 = LOW) = 0.1 * 3500 - 0.9 * 2250 = -\$1675$ .<sup>52</sup>

<sup>52</sup> From Figure 2B, probability( $p_2 = LOW | p_1 = LOW$ ) = 0.9, while probability( $p_2 = HIGH | p_1 = LOW$ ) = 0.1.

- b. If it goes to DOWN in period 2, then  $Value_2(DOWN,LOW)=0$  and  $Value_2(DOWN,HIGH)=0$ . Hence  $E(Value_2(DOWN,p_2)|p_1=LOW) = 0.9*0+0.1*0 = \$0$ .<sup>53</sup> Therefore  $E(Value_2(DOWN,p_2)|p_1=LOW) > E(Value_2(UP_2,p_2)|p_1=LOW)$
  - c. Clearly, then it is optimal to go DOWN in period 2 **if** price is LOW in period 1. This optimal transition is shown in Figure 3 (see arrow from  $UP_2$  to DOWN in period 1 corresponding to price in period 1 being LOW). Therefore *Value* for period 1 for generator state  $UP_2$  and price LOW is the expected *Value* of the DOWN state in period 2 + current period profits =  $0 + 300 = \$300$  and this is entered in column 1, row 2 in Table 2.
- **If** the generator is  $UP_2$  and **if** the price is HIGH (\$45/MWh) in period 1, then profits from this period are obtained from Table 1b, which is \$1500 (column 1, row 1). From Figure 2A, the generator can go to either  $UP_2$  or DOWN in the next period, period 2.
  - a. If it goes to  $UP_2$  in period 2, then  $Value_2(UP_2,LOW)=-2250$  while  $Value_2(UP_2,HIGH)=3500$ . Hence  $E(Value_2(UP_2,p_2)|p_1=LOW) = 0.5*3500-0.5*2250 = \$625$ .<sup>54</sup>
  - b. If it goes to DOWN in period 2, then  $Value_2(DOWN,LOW)=0$  and  $Value_2(DOWN,HIGH)=0$ . Hence  $E(Value_2(DOWN,p_2)|p_1=HIGH) = 0.5*0+0.5*0 = \$0$ .<sup>55</sup> Therefore  $E(Value_2(UP_2,p_2)|p_1=HIGH) > E(Value_2(DOWN,p_2)|p_1=HIGH)$ .
  - c. Clearly then it is optimal to go  $UP_2$  in period 2 **if** price is HIGH in period 1. This optimal transition is shown in Figure 3 (see arrow from  $UP_2$  to  $UP_2$  in period 1 corresponding to price in period 1 being HIGH). Therefore *Value* for period 1 for generator state  $UP_2$  and price HIGH is the expected *Value* of the DOWN state in period 2 + current period profits =  $625 + 1500 = \$2125$  and this is entered in column 1, row 1 in Table 2.
- **If** the generator is  $UP_1$  and **if** the price is HIGH (\$45/MWh) in period 1, then profits from this period are obtained from Table 1b, which is \$1500 (column 1, row 3). From Figure 2A, the generator can only go to  $UP_2$  in the next period, period 2.
  - a. If it goes to  $UP_2$  in period 2, then  $Value_2(UP_2,LOW)=-2250$  while  $Value_2(UP_2,HIGH)=3500$ . Hence  $E(Value_2(UP_2,p_2)|p_1=HIGH) = 0.5*3500-0.5*2250 = \$625$ .<sup>56</sup>
  - b. Clearly then it is optimal (indeed, this is the only choice) to go  $UP_2$  in period 2 **if** price is HIGH in period 1. This optimal transition is shown in Figure 3 (see arrow from  $UP_1$  to  $UP_2$  in period 1 corresponding to price in period 1 being HIGH). Therefore *Value* for period 1 for generator state  $UP_1$  and price HIGH is the expected *Value* of the  $UP_2$  state in period 2 + current period profits =  $1500 + 625 = \$2125$  and this is entered in column 1, row 3 in Table 2.
- **If** the generator is  $UP_1$  and **if** the price is LOW (\$33/MWh) in period 1, then profits from this period are obtained from Table 1b, which is \$300 (column 1, row 4). From Figure 2A, the generator can only go to  $UP_2$  in the next period, period 2.

<sup>53</sup> From Figure 2B, probability( $p_2=LOW|p_1=LOW$ ) = 0.9, while probability( $p_2=HIGH|p_1=LOW$ ) = 0.1.

<sup>54</sup> From Figure 2B, probability( $p_2=LOW|p_1=HIGH$ ) = 0.5, while probability( $p_2=HIGH|p_1=HIGH$ ) = 0.5.

<sup>55</sup> From Figure 2B, probability( $p_2=LOW|p_1=HIGH$ ) = 0.5, while probability( $p_2=HIGH|p_1=HIGH$ ) = 0.5.

<sup>56</sup> From Figure 2B, probability( $p_2=LOW|p_1=HIGH$ ) = 0.5, while probability( $p_2=HIGH|p_1=HIGH$ ) = 0.5.

- a. If it goes to  $UP_2$  in period 2, then  $Value_2(UP_2,LOW)=-2250$  while  $Value_2(UP_2,HIGH)=3500$ . Hence  $E(Value_2(UP_2,p_2)|p_1=LOW) = 0.1*3500-0.9*2250 = -\$1675$ .<sup>57</sup>
- b. Clearly then it is optimal (indeed, this is the only choice) to go  $UP_2$  in period 2 **if** price is LOW in period 1. This optimal transition is shown in Figure 3 (see arrow from  $UP_1$  to  $UP_2$  in period 1 corresponding to price in period 1 being LOW). Therefore *Value* for period 1 for generator state  $UP_1$  and price LOW is the expected *Value* of the  $UP_2$  state in period 2 + current period profits =  $300 - 1675 = -\$1375$  and this is given in column 1, row 4 in Table 2.
- **If** the generator is DOWN, and **if** the price is HIGH (\$45/MWh) in period 1, then the generator will not run (see row 5, column 1 of Table 1a) and profits in this period will be zero (see row 5, column 2 of Table 1b). From Figure 2A, the generator can go to either  $UP_1$  or DOWN in the next period, period 2.
  - a. If it goes to  $UP_1$  in period 2, then  $Value_2(UP_1,LOW)=-\infty$  and  $Value_2(UP_1,HIGH)=-\infty$ . Hence  $E(Value_2(UP_1,p_2)|p_1=HIGH) = 0.5*(-\infty)+0.5*(-\infty)=-\infty$ .<sup>58</sup>
  - b. If it goes to DOWN in period 2, then  $Value_2(DOWN,LOW)=0$  while  $Value_2(DOWN,HIGH)=0$ . Hence  $E(Value_2(DOWN,p_2)|p_1=HIGH) = 0.5*0+0.5*0 = \$0$ .<sup>59</sup> Therefore  $E(Value_2(DOWN,p_2)|p_1=HIGH) > E(Value_2(UP_1,p_2)|p_1=HIGH)$ .
  - c. Clearly, then it is optimal to go DOWN in period 2 **if** price is LOW in period 1. This optimal transition is shown in Figure 3 (see arrow from DOWN to DOWN in period 1 corresponding to price in period 1 being LOW). Therefore *Value* for period 1 for generator state DOWN and price LOW is the expected *Value* of the DOWN state in period 2 + current period profits =  $0 + 0 = \$0$  and this is given in column 1, row 5 in Table 2.

<sup>57</sup> From Figure 2B, probability( $p_2=LOW|p_1=LOW$ ) = 0.9, while probability( $p_2=HIGH|p_1=LOW$ ) = 0.1.

<sup>58</sup> From Figure 2B, probability( $p_2=LOW|p_1=HIGH$ ) = 0.5, while probability( $p_2=HIGH|p_1=HIGH$ ) = 0.5.

<sup>59</sup> From Figure 2B, probability( $p_2=LOW|p_1=HIGH$ ) = 0.5, while probability( $p_2=HIGH|p_1=HIGH$ ) = 0.5.