Post-Contingency Equilibrium Analysis of Power Systems

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Abstract

This paper presents alternative methods to compute the equilibrium condition immediately following a disturbance to an electric power system. The first uses the brute force method of simply integrating the dynamic model until it reaches steady state. The second uses the straightforward analytical choice of setting all time derivatives of the dynamic model to zero and solving the remaining algebraic equations for the equilibrium values of the dynamic states. This method requires the creation of new commercial software to solve the large-scale network algebraic equations simultaneously with the dynamic equilibrium equations. The third uses a method that takes advantage of existing commercial load flow software to perform the major portion of the solution process. These load flow solutions then iterate with the de-coupled algebraic equations for each generator.

Keywords: *Dynamic equilibrium, contingency analysis, load flow, steady-state stability.*

1. Introduction

Traditional power system security analysis includes the simulation of static as well as dynamic performance of a power system in response to a list of possible disturbances [1-3]. The static analysis normally relies on load flow as the primary commercial software to predict the post-contingency equilibrium condition. However, traditional load flow analysis makes a variety of assumptions that are not suitable for computing the equilibrium immediately following a disturbance. For example, normal load flow assumes the system is operating at rated frequency which implies that all synchronous machines are turning at rated synchronous speed. In virtually all disturbances, the post-contingency equilibrium will include a change in generator speeds as governors respond to mismatches between system load and generation. Similarly, generator controlled voltages are assumed to be the desired values as determined by control set points. In the actual post-contingency condition, the voltage regulators will provide controls which are based on the pre-contingency reference set points. This may be different from the assumption of some desired bus voltage. This difference can be considerable when the desired bus voltage is remote from the generator.

To see the issues associated with this calculation, consider the traditional basic load flow program that has the following computational procedure for a system with m generator buses and n-m load buses:

a. Specify the swing bus voltage magnitude and angle.
b. Specify the m-1 generator bus voltage magnitudes and real power generation.
c. Specify the loads at each load bus in terms of real and reactive power
d. Solve the appropriate Kirchhoff current law equations for the n-m unknown load bus voltages (magnitude and angle), and the m-1 generator bus voltage angles.
e. Compute the m generator reactive power requirements.
f. Compute the swing bus real power requirement.

Implicit in this formulation is the assumption that the frequency is nominal and therefore all generators are turning at rated synchronous speed. In this formulation, it is also possible to specify bus voltages other than generator bus voltages (i.e. remote buses that are more critical than the generator terminal buses). As such, the load flow solution represents the desired equilibrium assuming all controls are set to provide the desired values of frequency and various bus voltages (determined by the system operator). The important point here is that the load flow solution does not solve for the equilibrium condition for fixed control inputs, rather for specified conditions throughout the network.

In the actual power system, the controllers will respond to a disturbance by performing their intended function of returning sensed quantities near their pre-contingency set points. This may or may not result in the desired network conditions that would normally be specified in a post-contingency load flow. That is, a post-contingency load flow would specify the same desired conditions as the pre-contingency load flow. The difference between this post-contingency load flow and the actual post-contingency equilibrium can be significant. This means that the post-contingency load flow solution does not compute the correct dynamic equilibrium needed to perform steady-state stability analysis. The remainder of this paper addresses this issue and alternative methods to compute the post-contingency equilibrium.

2. The dynamic model

To see how dynamic steady-state equilibrium compares to "load flow" analysis, it is necessary to examine a typical dynamic model. As in the load flow model discussed above, the following dynamic model assumes that the generator buses are numbered 1 to m and load buses are numbered m+1 to n. The model neglects the very fast "stator/network" transients, but includes field flux decay, one damper winding, shaft inertial dynamics, automatic voltage control and speed control through a turbine/governor [4]. Each variable and parameter could contain a subscript i which ranges from 1 to m.

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - \left(X'_q - X'_d\right)I_d I_q - T_{FW}$$

$$T_E\frac{dE_{fd}}{dt} = -\left(K_E + S_E\left(E_{fd}\right)\right)E_{fd} + V_R$$

$$T_F\frac{dR_f}{dt} = -R_f + \frac{K_F}{T_F}E_{fd}$$

$$T_A\frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F}E_{fd} + K_A\left(V_{ref} - V_{controlled}\right)$$

$$T_{CH}\frac{dT_M}{dt} = -T_M + P_{SV}$$

$$T_{SV}\frac{dP_{SV}}{dt} = -P_{SV} + P_C - \frac{1}{R_D}\frac{\omega - \omega_s}{\omega_s}$$

The stator model consists of Kirchhoff's voltage law for each generator bus. There are m such sets of equations.

$$0 = Ve^{j\theta} + \left(R_s + jX'_d\right)\left(I_d + jI_q\right)e^{j\left(\delta - \frac{\pi}{2}\right)} - \left(\left(X_q - X'_d\right)I_q + jE'_q\right)e^{j\left(\delta - \frac{\pi}{2}\right)}$$

Similarly, the following equations are the standard Kirchhoff's current law equations for the m generator buses. They include possible loads (injected notation) at the generator bus.

$$\left(I_{di} + jI_{qi}\right)e^{j\left(\delta_i - \frac{\pi}{2}\right)} + \left(\frac{P_{Li} - jQ_{Li}}{V_i e^{-j\theta_i}}\right) = \sum_{k=1}^{n} Y_{ik}e^{j\alpha_{ik}}V_k e^{j\theta_k}$$

The following equations are the standard Kirchhoff's current law equations for the n-m pure load buses.

$$\left(\frac{P_{Li} - jQ_{Li}}{V_i e^{-j\theta_i}}\right) = \sum_{k=1}^{n} Y_{ik}e^{j\alpha_{ik}}V_k e^{j\theta_k}$$

This model includes 9m differential equations with the following dynamic states:

$x = [E_q', E_d', \delta, \omega, E_{fd}, R_f, V_R, T_M, P_{SV}]^t$

Each entry of x is a vector of m variables (one for each generator). At this point it is important to point out that one angle in the system model can be eliminated. Since all of the angles in the model eventually appear as the difference between two angles, one may be explicitly removed. For example, δ_1 may be subtracted from all other angles (creating "difference angeles") and the complete model without δ_1 remains correct. This is discussed again later in the paper.

The model also includes 2m+2n real algebraic equations with the following algebraic states:

$y = [I_d, I_q]^t$

$z = [V_G, V_L, \theta_G, \theta_L]^t$

Where the subscripts indicate Generator or Load voltage magnitude and angles.

The control inputs are:

$u = [V_{ref}, P_C]^t$

In most dynamic analysis, the control inputs are maintained as constants.

Using this notation, the above dynamic model is of the form:

$$\frac{dx}{dt} = f(x, y, z, u)$$
$$0 = g(x, y, z)$$
$$0 = h(x, y, z)$$

While this form corresponds exactly to the equations given above, it is not in a form that identifies the "load flow" portion of the model. The model can be rewritten in the following form:

$$\frac{dx}{dt} = F(x, y, z_m, z_{lf}, u)$$
$$0 = G(x, y, z_m, z_{lf})$$
$$0 = H(z_{lf})$$

where the standard load flow variables are:

$z_{lf} = [V_{m+1}, \ldots, V_n, \theta_2, \ldots, \theta_n]^t$

As such, H contains exactly 2n-m-1 equations that are solved for these 2n-m-1 variables.

The vector G includes the 2m stator algebraic equations and m+1 equations not used in load flow (normally used to compute the generator reactive powers and the swing bus real power). Thus G is 3m+1 equations.

3. Pre-contingency equilibrium

The pre-contingency equilibrium is normally a steady-state condition at nominal frequency (which assumes perfect real-power generation balance with loads and losses). In addition, the voltage control inputs in the above dynamic model are not specified a-priori. Alternatively, various system conditions are specified in their place. For example, a generator terminal bus (or remote bus) voltage magnitude may be specified under the assumption that the system operators have adjusted the generator voltage regulator references (V_{ref}) to provide this condition. The load-flow algorithm discussed in section 1 is used to provide the resulting generator reactive power outputs and swing bus real power. From this, the control inputs u are computed by setting the time derivatives of the dynamic model to zero and computing V_{ref} and P_c. Once these inputs are computed, they must be maintained constant when computing the post-contingency equilibrium.

4. Post contingency load flow

If a post-contingency load flow is performed using the new system conditions, the solution is not correct until operator action has restored the voltage and speed controllers to values which give the desired voltages and speed that are assumed in the load flow analysis. While this may be correct for "long term" equilibrium following a disturbance, it is not correct for the immediate equilibrium following a disturbance. Hence it is not correct for determining the stability of the post-contingency equilibrium.

5. Analysis by integration

The first method that produces the correct post-contingency equilibrium is numerical integration until steady state is reached. By definition, this computation produces the correct immediate post-disturbance equilibrium. However, this is a very expensive method to compute this result.

6. Analysis by simultaneous iteration

The second method that produces the correct post-contingency equilibrium is direct solution of the full set of algebraic equations obtained by setting the time derivatives of the dynamic model to zero. This involves solving the following equations for x, y, z, given u:

$$0 = f(x, y, z, u)$$
$$0 = g(x, y, z)$$
$$0 = h(x, y, z)$$

While this method should agree with the analysis by numerical integration, it also is expensive in the sense that new commercial software code would have to be written to perform this analysis.

7. Analysis by partitioned iteration

The third method that should produce the correct immediate post-contingency equilibrium is the partitioned iterative solution that utilizes existing commercial load flow software for the bulk of the analysis. This analysis would involve the following sequential steps:

a. Start with the base case steady-state equilibrium as determined by a standard load flow and all associated dynamic variable and input values.
b. Create changes in the dynamic model (i.e. remove a line which changes the Ybus matrix).
c. Evaluate the new generator real and reactive powers.
d. Solve a standard load flow using the new Ybus matrix and generator powers. This is equivalent to solving the 2n-m-1 equations of H for the 2n-m+1 variables in z_{lf} defined above.
e. Set F to zero and solve the 12m+1 equations in F and G for the 12m+1 variables x, y, z_m. Notice that these equations are all in de-coupled sets and hence can be solved by a very low-order solution program. Actually this could be reduced by 1 equation by using angles referenced to the machine 1 angle.
f. Compute new load flow input quantities (swing bus voltage magnitude and angle, generator voltage magnitudes and real power generation.
g. Resolve the load flow using the new input quantities, and repeat this iterative process until convergence.

This method has the advantage that existing commercial software code can be used. And, the additional software needed to compute the equilibrium involves de-coupled algebraic equations for the generators. As such, the size of this software code should be small.

8. Discussion and conclusions

This concept has not been tested yet to see any problems which may arise in solution methods or convergence. There are several sticky issues which may enter the process. The first is the issue of eliminating one generator angle and using difference angles. The choice of angle to be eliminated may affect the convergence. A second is the issue of VAR limits in load flow vs the control limits in the dynamic model. These do not have a perfect correlation and may impact the solution process. A third is the issue of load models. While the load flow model for loads is usually constant power, the dynamic model may be different (i.e. constant impedance or current etc.). A fourth is the issue of variable frequency. It may be necessary to include this variable frequency in the load flow algorithm. The method needs to be tested for computational feasibility and convergence properties.

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