# Measurement based Voltage Stability Monitoring of Power system

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**Abstract:** Many papers discuss the voltage stability assessment of power system using power flow analysis methods. In this paper, a method for online monitoring of a power system based on measurements is proposed, which is aimed at detection of the voltage instability. Thereby an indicator is derived from the fundamental Kirchoff-Laws. Since in the transient process, at any time point, the electric power of the system is in balance, and the Kirchoff-Law is obeyed, this indicator will still work during the transient process. From the indicator, it is allowed to predict the voltage instability or the proximity of a collapse. The advantage of the method lies in the simple numerical calculation and strong adaptation in steady state and transient process. Through the indicator of voltage stability, it is easy to find the most vulnerable area in a system, to find the impacts of other loads, areas and power transactions.

**Keywords**: Voltage Stability, Voltage Collapse, Reactive Power, Steady-State Voltage Stability, Transient Analysis

## I. INTRODUCTION

Voltage stability is a major concern in planning and operations of power systems. It is well known that voltage instability and collapse have led to major system failures; with the development of power markets, more and more electric utilities are facing voltage stability-imposed limits.

The problem of voltage stability may be simply explained as inability of the power system to provide the reactive power or the egregious consumption of the reactive power by the system itself. It is understood as a reactive power problem and is also a dynamic phenomenon. [8,9]

The objective of this paper is to develop a fast and simple method, which can be applied in the power system online, to estimate the voltage stability margin of the power system. In general, the analysis of voltage stability problem of a given power system should cover the examination of these aspects:

– How close is the system to voltage instability or collapse?
– When does the voltage instability occur?
– Where are the vulnerable spots of the system?
– What are the key contributing factors?
– What areas are involved?

Voltage stability analysis often requires examination of lots of system states and many contingency scenarios. For this reason, the approach based on steady state analysis is more feasible, and it can also provide insights of the voltage reactive power problems. A number of special algorithms have been proposed in the literature for voltage stability analysis using static approached [2-4], however these approaches are laborious and does not provide sensitivity information useful in a dynamic process. Voltage stability is indeed a dynamic phenomenon. [8,9]

Some utilities use Q-V curves at some load buses to determine the proximity to voltage instability [6]. One problem with Q-V curve method is that by focusing on a small number buses, the system-wide voltage stability problem will not be readily unveiled.

An approaches, model analysis of the modified load flow Jacobian matrix, has been used as static voltage stability index to determine vulnerable bus's voltage stability problem [7].

This paper explores the online monitoring index of the voltage stability, which is derived from the basic static power flow and Kirchoff law. A derivation will be given. The index of the voltage stability [3] predicts the voltage problem of the system with sufficient accuracy. This voltage stability index can work well in the static state as well as during dynamic process. It can also be used to find the vulnerable spots of the system, the stability margin based on the collapse point, and the key factors for the voltage stability problem, etc.

## II. FUNDAMENTALS: SIGLE GENERATOR AND LOAD SYSTEM

A simple power system is considered, through which the useful index of the voltage stability is derived. As showed in Fig.1, whereby bus 1 is assumed as a generator bus, and bus 2 is a load bus whose voltage behavior will be our interest.

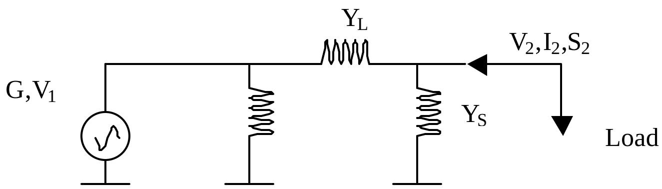


Figure. 1. Single generator and Load system

This simple system can be described by the following equations (where the dot above a letter represents a vector):

$$\dot{I}_2 = \dot{V}_2\dot{Y}_S + (\dot{V}_2 - \dot{V}_1)\dot{Y}_L = \frac{\overset{*}{S}_2}{\overset{*}{V}_2} \tag{1}$$

$$\overset{*}{S}_2 = V_2^{\,2}\dot{Y}_S + V_2^{\,2}\dot{Y}_L - \overset{*}{V}_2\dot{V}_1\dot{Y}_L = V_2^{\,2}\dot{Y}_{22} + \dot{V}_0\overset{*}{V}_2\dot{Y}_{22} \tag{2}$$

$$Here\ \dot{Y}_{22} = \dot{Y}_S + \dot{Y}_L$$

$$and\ \dot{V}_0 = -\frac{\dot{Y}_L}{\dot{Y}_L + \dot{Y}_S}\dot{V}_1 \tag{3}$$

To solve for $\left|\dot{V}_2\right|$, it is needed to solve this complex equation (2), and it is assumed that $\frac{\overset{*}{S}_2}{\dot{Y}_{22}} = a + jb$. Express equation (2):

$$\frac{\overset{*}{S}_2}{\dot{Y}_{22}} = a + jb = V_2^{\,2} + \dot{V}_0\overset{*}{V}_2 = V_2^{\,2} + V_0V_2\cos(d_0 - d_2) + jV_0V_2\sin(d_0 - d_2) \tag{4}$$

Here, $\dot{V}_0 = V_0\angle d_0$

$$\cos(d_0 - d_2) = \frac{a - V_2^{\,2}}{V_0V_2} \tag{5}$$

$$\sin(d_0 - d_1) = \frac{b}{V_0V_2} \tag{6}$$

$(5)^2 + (6)^2$, hence,

$$V_0^{\,2}V_1^{\,2} = (a - V_1^{\,2})^2 + b^2 = a^2 - 2aV_1^{\,2} + V_1^{\,4} + b^2 \tag{7}$$

Solve equation (7):

$$V_2 = \sqrt{\frac{V_0^{\,2}}{2} + a \pm \sqrt{\frac{V_0^{\,4}}{4} + aV_0^{\,2} - b^2}} \tag{8}$$

When $\sqrt{\frac{V_0^{\,4}}{4} + aV_0^{\,2} - b^2} = 0$, the voltage at bus 2 collapses. On the other hand, equation (5) and (6) are considered again.

$$f(V_2, d) = V_0V_2\cos d + V_2^{\,2} = a \tag{5a}$$

$$g(V_2, d) = V_0V_2\sin d = b \tag{6a}$$

It is very easy to get the Jacobian Matrix:

$$J = \begin{bmatrix} 2V_1 + V_0\cos d & -V_1V_0\sin d \\ V_0\sin d & V_1V_0\cos d \end{bmatrix} \tag{9}$$

If the voltage at the bus 2 collapses, there will be no solution for equation (5a) and (6a), it also means that the determinant of the Matrix J should be zero (Jacobian Matrix singular)

$$\det(J) = 2V_2^{\,2}V_0\cos d + V_2V_0^{\,2} = 0$$

$$\Rightarrow \frac{V_2\cos d}{V_0} = \mathrm{Re}\left\{\frac{\dot{V}_2}{\dot{V}_0}\right\} = -\frac{1}{2} \tag{10}$$

When equation (2) is divided by $V_2^{\,2}\dot{Y}_{22}$, it becomes:

$$\frac{\overset{*}{S}_2}{V_2^{\,2}\dot{Y}_{22}} = 1 + \frac{\dot{V}_0}{\dot{V}_2} \tag{11}$$

When the voltage collapses, it is said that $\mathrm{Re}\left\{\frac{\dot{V}_2}{\dot{V}_0}\right\} = -\frac{1}{2}$,

which implies that $\frac{\dot{V}_2}{\dot{V}_0} = -\frac{1}{2} + jc$, hence

$$\left|1 + \frac{\dot{V}_0}{\dot{V}_2}\right| = \left|1 + \frac{1}{-\frac{1}{2} + jc}\right| = \left|\frac{\frac{1}{2} + jc}{-\frac{1}{2} + jc}\right| = 1 \tag{12}$$

From equation (11) and (12), an indicator of the voltage stability is defined as:

$$\left|1 + \frac{\dot{V}_0}{\dot{V}_2}\right| = \left|\frac{\overset{*}{S}_2}{V_2^{\,2}\dot{Y}_{22}}\right| = \frac{S_2}{V_2^{\,2}Y_{22}} \tag{13}$$

Thereby an indicator has been derived which can be used for monitoring the voltage stability problem of the system and for assessing the degree of risk for a potential voltage collapse. When $S_2 = 0$, the indicator will be zero and indicates that there will be no voltage problem. When $S_2 = 1$, the voltage at load bus will collapse.

One example of a single generator and load system was constructed to demonstrate the correctness of the indicator

$$V_1 = 1.0\angle 0°, Y_S = 0, Y_L = -j4(X = j0.25),$$

$$PowerFactor = 0.97 lagging, f = 14.07°$$

Continuously change the load at bus 2, and keep the power factor of the load to find the collapse point of the

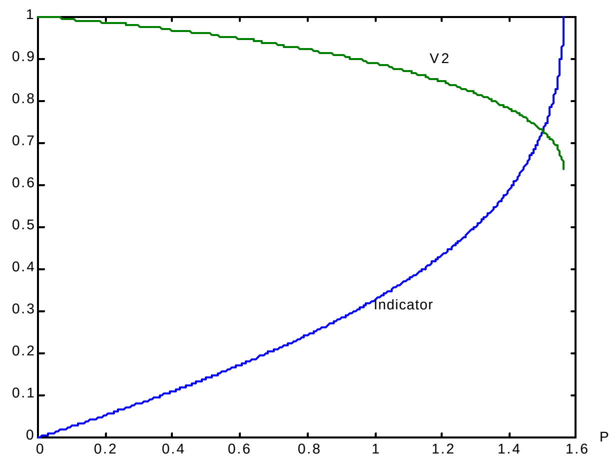


Figure.2 Single generator and load system

## II. GENERALIZATION TO AN N-BUS SYSTEM

As shown in the basic theory of the multi-bus power system, all the buses can be divided into two categories: Generator bus (PV bus and Slack bus) and Load bus (PQ bus). Because the voltage stability problem is reactive power relative problem, and the generator bus can provide the reactive power to support the voltage magnitude of the bus, it is absolutely necessary that the all of buses be distinguished.

The power system can be expressed in the form through Kirchoff Law:

$$I_{System} = \begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} = Y_{System} V_{System} \quad (14)$$

Subscript L means Load bus, and G means Generator bus.

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & -Z_{LL}Y_{LG} \\ Y_{GL}Z_{LL} & Y_{GG} - Y_{GL}Z_{LL}Y_{LG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (15)$$

Here, $Z_{LL} = Y_{LL}^{-1}$

For any load bus $j \in L$, through the equation (15), the voltage of the bus is known as:

$$\dot{V}_j = \sum_{i \in L} Z_{ji} \dot{I}_i + \sum_{k \in G} A_{jk} \dot{V}_k \quad (16)$$

$$A = -Z_{LL} Y_{LG}$$

which can be expressed as the form:

$$V_j^2 + \dot{V}_{0j} \overset{*}{V}_j = \frac{\overset{*}{S}_j{}'}{Y_{jj}{}'} \quad (17)$$

Substituting equivalent $\dot{V}_{0j}$, $\overset{*}{S}_j{}'$ and $Y_{jj}{}'$, we have

$$\dot{V}_{0j} = -\sum_{k \in G} A_{kj} V_k \quad (18)$$

$$Y_{jj}{}' = \frac{1}{Z_{jj}} \quad (19)$$

$$\dot{S}_j{}' = \left(\sum_{i \in L} \frac{\overset{*}{Z}_{ji} \dot{S}_i}{\overset{*}{Z}_{jj} \dot{V}_i}\right) * \dot{V}_j = \dot{S}_j + \left(\sum_{\substack{i \in L \\ i \neq j}} \frac{\overset{*}{Z}_{ji} \dot{S}_i}{\overset{*}{Z}_{jj} \dot{V}_i}\right) * \dot{V}_j \quad (20)$$

Hence, we see that the voltage of the load bus j is affected by an equivalent complex power $\overset{*}{S}_j{}'$ and by an equivalent generator part $\dot{V}_{0j}$.

To compare the equation (17) and (2), we can observe that they have an identical form, and the voltage stability of the multi-bus system has been equivalent to a simple single generator and load system. The indicator of the voltage stability of the load bus j will be easily obtained:

$$Indicator_j = \left|1 + \frac{\dot{V}_{0j}}{\dot{V}_j}\right| = \left|\frac{\overset{*}{S}_j{}'}{V_j^2 \overset{*}{Y}_{jj}{}'}\right| = \frac{S_j{}'}{V_j^2 Y_{jj}} \quad (21)$$

For the system

$$Indicator_{system} = \underset{j \in L}{Max}(Indicator_j) \quad (22)$$

Thereby it is clear that the indicator of the voltage stability at a load bus mainly influenced by the equivalent load $S_j{}'$, which has two parts: the load at bus j itself, and the 'contributions' of the other load buses (showed at equation 20). When the load at a load bus changes, it will influence the indicator. On the other words, the voltage stability problem is a system-wide problem, not a local problem.

Through equation (20), the contribution of any other load bus on the load bus j can be numerically updated and computed. It is a very important concept for the deregulated power market, and will help the customers and ISO to evaluate the responsibility of voltage stability problem.

**Interpretation:**

The indicator is an effective quantitative measurement for the system to find how far is the current state of the system to the voltage collapse point. All the derivations are correct as long as that all the generator bus of the system have the enough reactive power supply to maintain the magnitude of voltage as constant. If some of the generators are unable to maintain the voltage magnitude, these generator buses will become load buses, the load bus congregation will expand; the indicator will have a discrete jump, which will be shown at the following demonstration.

**Demonstration system:**
Western Systems Coordinating Council (WSCC) equivalent nine-bus system is studied. All the steady state flow and voltage measurements are simulated by a power flow program.

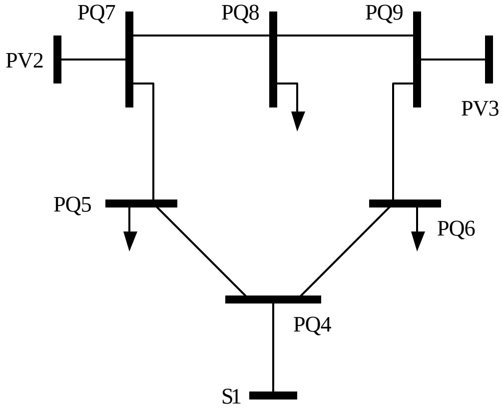


Figure 3 WSCC nine-bus equivalent system

Case one: All the generator buses have enough reactive power support, and the voltage magnitudes are well regulated. The involved event: load increases at PQ bus 5, and we are monitoring the voltage stability margin based on the measurements and computed indicator.

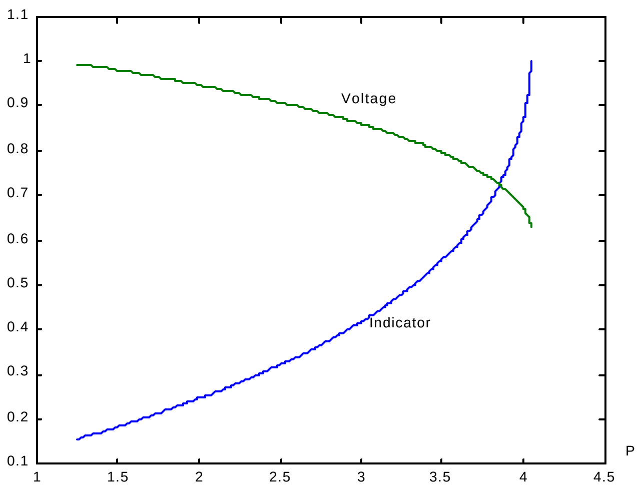


Figure 4. The indicator (case one) of the WSCC Equivalent system

For this case, all the generator buses can maintain the voltage magnitudes, when the load at bus 5 is increasing with a constant power factor. The indicator is continuously increasing as the load increases as shown in Figure4. The voltage stability margin (load) at bus 5 is 4.049 P.U..

Case two: Generator bus PV2 do not have enough reactive power support as the load at bus 5 increases, this PV bus becomes a PQ bus.

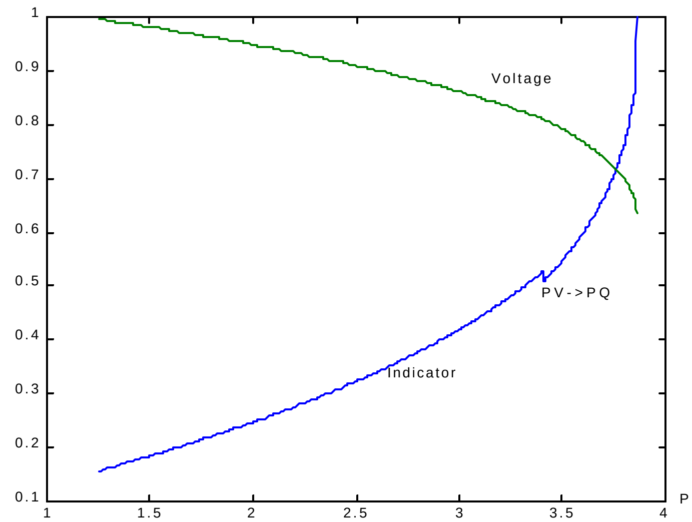


Figure 5. The indicator (case two) of the WSCC Equivalent system

For this case, not all the generator buses can maintain constant voltages. When the load at bus 5 is increased to 3.408 P.U. (the power factor is constant), the PV2 changes to PQ bus, and the indicator have a discrete jump; the stability margin at this case is 3.865 P.U., which is less than the last case.

## III. DYNAMIC MONITORING

Power voltage stability problem is not a simple static problem, but a transient and dynamic problem as shown in my earlier paper [8]. There are many phases of the problem, ranging from generation control, generator exciter controls, network reactive power compensation to load characteristics.

The dynamic process of the power system under the disturbance can be described as a set of algebra and differential equations:

$$\begin{aligned} \dot{x} &= f(x, L(t), G(t), y) \\ I_{System} &= Y_{System} V_{System} \end{aligned} \tag{23}$$

x: Power flow relative variables
L(t): Load of the system
G(t): Generation of the system

y: The variables of the Control devices (power flow un-related variables)

$I_{System} = Y_{System} V_{System}$ : Power flow equations.

To solve this algebra and differential equations, the Time Simulation Method ('step by step') is used. The static power flow is used as an initial solution for the system, then step by step to solve the algebra and state variables of the equations with considering the disturbance of the system, until the system is unstable or the simulation time ends.

It is clear that at every simulation time point, the power flow equations ($I_{System} = Y_{System} V_{System}$) are satisfied. All the theory of the multi-system voltage stability indicator is derived from the equation (14) (Power flow equations). For this means, this indicator can work in the dynamic process too. It is rather difficulty to judge if the generator bus is the PV bus or Slack bus, because the voltage magnitude of the PV bus can not be constant in a transient/dynamic process, although the generator have the enough reactive power support.

In our dynamic demonstration, as long as the voltage of a generator bus is more than a constant pre-set value such as 0.96, and the reactive output of the generator does not exceed its limit, the bus is a PV or Slack bus; otherwise it is a PQ bus.

The WSCC nine-bus equivalent system is studied to check the correctness of the indicator during the dynamic process. Event: continuously to increase the load at bus 5 (keep the power factor as constant)

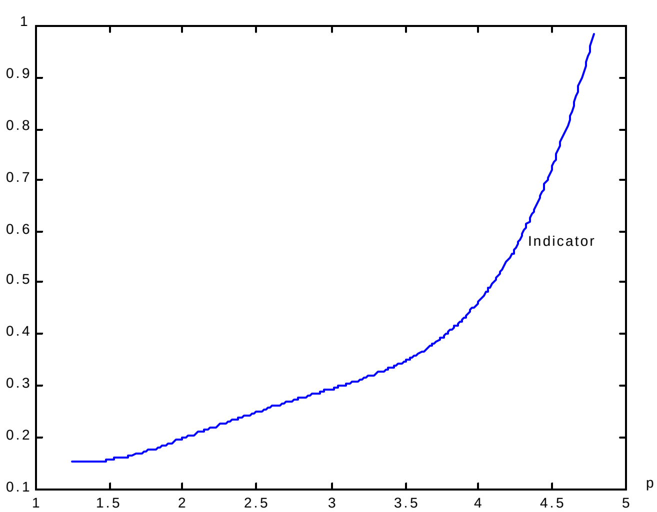


Figure 6. The indicator of WSCC equivalent system During dynamic process

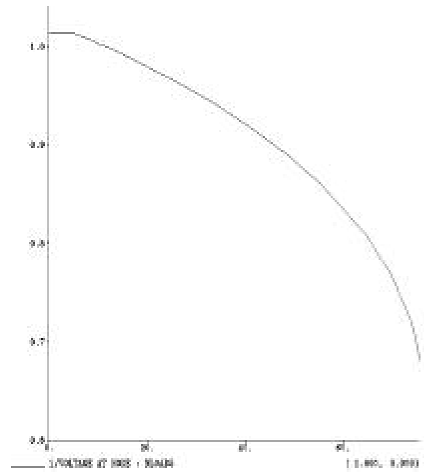

Figure. 7. Voltage at bus 5 of WSCC equivalent system

We simulated the measurement data through an Eurostag dynamic simulation program. Through the simulated measurements, it is clear that the indicator works well during the complicated dynamic process. The involved dynamics include exciter dynamics, governors and tap changing under load transformers.

The advantage of this the proposed dynamic monitoring scheme for voltage stability problem is that it is feasible in the real-time condition. Only the voltage and complex power measurements at the load bus are needed, which can be obtained through local measurements and pre-defined data exchanges.

## IV. CONCLUSION

A real time measurement based voltage stability indicator for monitoring of the power systems is presented. We verify our approach by both static and dynamic simulations. We conclude that:

- The indicator can predict the voltage stability problem correctly and properly by using both steady-state data as well as dynamic data.
- The indicator can be used for both static and dynamic voltage problems.
- Through the indicator, it is very easy to locate the vulnerable locations of the system.
- The indicator can correctly predict the collapse point of the system.
- In the transaction based power system operation, through the indicator, it is easy to find the responsibility and obligation of the every customer and power supplier.
- The indicator has simple structure and can be handled easily. The needed information can be obtained through

local measurements and data exchanges of among pre-set buses.

## V. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support from EPRI/DoD Complex Interactive Networks program.

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## BIOGRAPHIES

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