

Adaptive Power Flow Method for Distribution Systems with Dispersed Generation¹

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Abstract: Recently, there has been great interest in the integration of dispersed generation units at the distribution level. This requires new analysis tools for understanding system performance. This paper presents an adaptive distributed power flow solution method based on the compensation-based method. The comprehensive distributed system model includes three-phase nonlinear loads, lines, capacitors, transformers, and dispersed generation units. The numerical properties of the compensation-based power flow method are compared and analyzed under different situations, such as load unbalance, sudden increase of one-phase loads, degree of meshed loops, number of generator nodes and so on. Based on these analyses, an adaptive compensation-based power flow method is proposed that is fast and reliable while maintaining necessary accuracy. It is illustrated that this adaptive method is especially appropriate for simulation of slow dynamics.

Keywords: distribution power flow, distribution system modeling, weakly meshed, dispersed generation, compensation-based method, adaptive.

I. INTRODUCTION

Recent technology improvements in micro-turbines, fuel cells and energy storage devices have provided the opportunity for dispersed generation at the distribution level. With the possibility of significant penetration of distributed generation, more studies are needed on dynamic analysis of distribution systems. For dynamic simulations considering network effects, a load flow calculations must be performed at appropriate time steps. The system operational conditions may vary widely during dynamic events. Together, this requires that the load flow algorithm be more robust and faster than that required for static studies. Previous distribution power flow methods have primarily looked at static power flow analysis for planning studies. When these methods are used in dynamic simulations, they tend to be neither fast nor robust enough. This paper develops a load flow algorithm appropriate for the simulation of slow dynamics in the distribution system.

The traditional distribution system differs from the transmission system in the following aspects:

- It is typically radial or weakly meshed.
- Distribution lines usually have a larger R/X ratio.
- There may be significant three-phase unbalance including unbalanced loads and single phase or two-phase lines.

In [1], a compensation-based power flow method was first presented and used for solution of weakly meshed systems. It was extended to a dispersed generation system with PV node compensation in [2]. By adding voltage correction, it was illustrated that the iterative process of power flow calculation is faster and more reliable [3]. The compensation-based method was extended to three-phase unbalanced system in [4]. For short circuit analysis, fault compensation was combined with breakpoint compensation and PV node compensation into a hybrid compensation [5].

The adaptive method presented in this paper is for fast and accurate calculation of power flow considering general device models and varied operational conditions. It is based on the compensation-based method for a three-phase unbalanced system proposed in [4]. Appropriate for the application of interest here, the following extensions are made:

- Distribution system elements, including lines, capacitors, dispersed generation units, nonlinear loads, and transformer connections are comprehensively modeled.
- The convergence rate of the compensation-based power flow method is exploited based on different situations, including load unbalance, sudden change in one-phase loads, the number and position of mesh breakpoints and the number and position of PV nodes.
- The adaptive compensation-based method is presented and implemented for dynamic simulations.

The organization of this paper is as follows. Model of distribution elements is given in the next section. The compensation-based method [4] is introduced in section III. In section IV, the convergence property of compensation-

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based method is exploited and illustrated for different situations. The adaptive compensation-based method is developed and several improvements are shown in section V. Conclusions are given in section VI.

II. SYSTEM MODELING

Distribution system is viewed as a network of lines, shunt capacitors, dispersed generators, loads, and transformers. Comprehensive modeling issues are addressed in this section.

2.1 Lines

As in [4], the series impedance of a line section l is represented by a 3×3 matrix:

$$Z_l = \begin{bmatrix} z_{aa,l} & z_{ab,l} & z_{ac,l} \\ z_{ab,l} & z_{bb,l} & z_{bc,l} \\ z_{ac,l} & z_{bc,l} & z_{cc,l} \end{bmatrix} \quad (1)$$

2.2 Loads

Loads can be a combination of constant power, constant current and constant impedance. They may be either Y connected or Δ connected. Suppose load Li is connected to bus i , V_i is the three-phase voltage in bus i , I_{Li} is three-phase load current, then there are the following load models:

- Grounded- Y : All grounded- Y connected loads can be modeled as a combination of the following:

- 1) constant power

$$I_{Li} = (S_{Li} / V_i)^* \quad (2)$$

$$\text{where } S_{Li} = \begin{bmatrix} s_{a,Li} & 0 & 0 \\ 0 & s_{b,Li} & 0 \\ 0 & 0 & s_{c,Li} \end{bmatrix}$$

- 2) constant current

$$I_{Li} = [I_{a,Li} \quad I_{b,Li} \quad I_{c,Li}]^T \quad (3)$$

- 3) constant impedance

$$I_{Li} = Y_{Li} V_i \quad (4)$$

$$\text{where } Y_{Li} = \begin{bmatrix} y_{a,Li} & 0 & 0 \\ 0 & y_{b,Li} & 0 \\ 0 & 0 & y_{c,Li} \end{bmatrix}$$

- Ungrounded- Δ : All Ungrounded- Δ connected loads can be modeled as a combination of the following:

- 1) constant power

$$I_{Li} = T^T [S_{Lppi} (TV_i)^{-1}]^* \quad (5)$$

$$\text{where } T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, \quad S_{Lppi} = \begin{bmatrix} s_{ab,Li} & 0 & 0 \\ 0 & s_{bc,Li} & 0 \\ 0 & 0 & s_{ca,Li} \end{bmatrix}$$

- 2) constant current

$$I_{Li} = T^T I_{Lppi} \quad (6)$$

$$\text{where } I_{Lppi} = [I_{ab,Li} \quad I_{bc,Li} \quad I_{ca,Li}]^T$$

- 3) constant impedance

$$I_{Li} = T^T Y_{Lppi} V_{ppi} \quad (7)$$

$$\text{where } Y_{Lppi} = \begin{bmatrix} Y_{ab,Li} & 0 & 0 \\ 0 & Y_{bc,Li} & 0 \\ 0 & 0 & Y_{ca,Li} \end{bmatrix}, \quad V_{ppi} = \begin{bmatrix} V_{ab,i} \\ V_{bc,i} \\ V_{ca,i} \end{bmatrix}$$

2.3 Shunt Capacitors

Shunt capacitors can be modeled as Y connected or Δ connected constant impedance loads.

2.4 Dispersed Generators

Dispersed generators are classified as constant PQ or PV nodes. For PQ units, the models are identical with constant power load models, except that the current is injected into the bus. For PV units, the connected bus is modeled as a PV node. If the computed reactive power generation Q_{ig} is out of the reactive generation limits, then the reactive power generation is set to that limit and the unit acts as a PQ node. Some dispersed storage units may also act as a constant current node but for purposes of the load flow the PQ model is adequate.

2.5 Distribution Transformers

The general form of the three-phase transformer model is shown in Fig. 1, where shunt admittance Y_m represents core losses, series impedance Z_t represents leakage impedance, V_p represents primary voltage, and V_s represents secondary voltage.

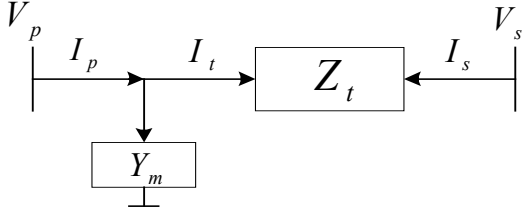


Fig. 1 General Form of 3-phase Transformer Model

All types of three-phase transformers, such as $Y0-Y0$ transformer, $\Delta - Y0$ transformer, $Y - \Delta$ transformer can be modeled in this framework [6].

III. Compensation-Based Method

The compensation-based method for three-phase power flow calculation was developed in [4]. The main procedure is the following:

1. Read network data and perform network indexing.
2. Construct breakpoint impedance matrix Z_B and PV node sensitivity matrix Z_V . The weakly meshed system is converted to a radial network by breaking all loops at breakpoints. In Z_B , the diagonal elements are the sum of the impedance of lines which form the loop connecting the two buses of a breakpoint, and the off-diagonal elements are the sum of the impedance in common lines for two breakpoint loops. Z_V can be formed from the PV node to the feeder node in a similar way with Z_B .
3. Perform backward current and forward voltage sweep iterations. If the maximum power mismatch in each node for all phases is less than the power convergence criterion, ε_p , then proceed to the next step.
4. Calculate breakpoint voltage mismatch ΔV_B . For a breakpoint j , which is separated into two end nodes, $j1$ and $j2$, we have

$$\Delta V_{B,j} = V_{j1} - V_{j2} \quad (8)$$

5. If the maximum breakpoint voltage mismatch is greater than the breakpoint voltage convergence criterion, ε_b , update breakpoint current injection J using (9)–(10) below, return to step 3. Otherwise, go to the next step.

$$Z_B \Delta J = \Delta V_B \quad (9)$$

$$J = J + \Delta J \quad (10)$$

For a breakpoint j , current compensation $-J_j$ is injected into end node $j1$, and current compensation J_j is injected into end node $j2$.

6. Calculate PV node voltage mismatch ΔV_V . For PV node i ,

$$\Delta V_{V,i} = V_{set,i} - V_{1,i} \quad (11)$$

where $V_{1,i}$ is the magnitude of positive sequence voltage in node i , and $V_{set,i}$ is the voltage set value in PV node i .

7. If the maximum PV node voltage mismatch is greater than the PV node voltage convergence criterion, ε_v , update PV node current injection I_q using (12)–(15) below, return to step 3. Otherwise, the final power flow has been obtained.

$$Z_V \Delta I_q = \Delta V_V \quad (12)$$

$$\begin{cases} \Delta i_{qa,i} = |\Delta i_{q,i}| e^{j(90^\circ + \delta_{va,i})} \\ \Delta i_{qb,i} = |\Delta i_{q,i}| e^{j(90^\circ + \delta_{vb,i})} \\ \Delta i_{qc,i} = |\Delta i_{q,i}| e^{j(90^\circ + \delta_{vc,i})} \end{cases} \quad (13)$$

This is 90° leading current, which will decrease reactive power generation. If $\Delta V_{V,i} < 0$ ($V_{1,i}$ is greater than $V_{set,i}$),

$$I_{q,i} = I_{q,i} + \Delta I_{q,i} \quad (14)$$

so that less reactive power generation will be injected into PV node i . If $\Delta V_{V,i} > 0$ ($V_{1,i}$ is less than $V_{set,i}$),

$$I_{q,i} = I_{q,i} - \Delta I_{q,i} \quad (15)$$

So that more reactive power generation will be injected into PV node i .

IV. CONVERGENCE RATE ANALYSIS

Good power flow algorithms should have fast and consistent convergence rate under diverse operating conditions. In this section, numerical simulations on the convergence properties of compensation-based power flow method are performed on the system in Fig. 2 based on a distribution feeder in the Kumamoto area of Japan. The network parameters can be found in [7]. G2 and G11 are constant PQ generators. For sections 4.4-4.6, the following convergence criteria are always chosen:

$$\varepsilon_p = 5 \times 10^{-4}, \varepsilon_b = 1 \times 10^{-4}, \varepsilon_v = 1 \times 10^{-4}$$

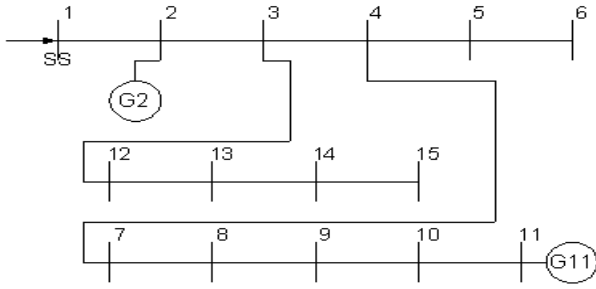


Fig. 2 Kumamoto 15-Bus Distribution System Diagram

4.1 Unbalanced Loads

Unbalanced loads are more common in a distribution system. To simulate the effect of three-phase unbalanced loads, first, three-phase balanced loads are assumed. Then while holding the total three-phase load constant at each node, the load in phase *A* is increased. *UF* represents the unbalanced factor.

Fig. 3 shows the maximum power mismatches after three iterations of backward/forward sweep calculation when *UF* is increased from 1.0 and 10.0. It can be seen that the maximum power mismatch is increasing nearly linearly as three-phase loads become more unbalanced.

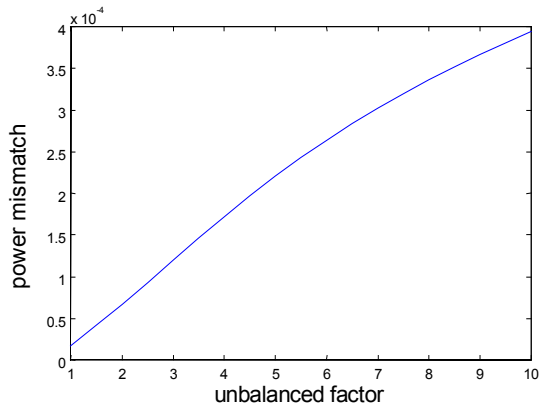


Fig. 3 Effect of Unbalanced Load

4.2 Large One-Phase Load

In the distribution system, there are one-phase loads that may be large. Here, we want to analyze the convergence property of the backward/forward method given a large load in a single phase. A scalar load factor *LF* is applied to phase-A loads while keeping phase-B and phase-C loads constant. Fig. 4 shows the maximum power mismatches after three iterations of backward/forward sweep when the load factor is increased from 1.0 to 2.5.

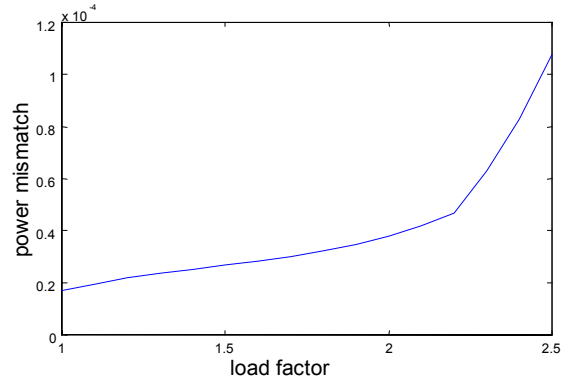


Fig. 4 Effect of Large One-Phase Load

In fig. 4, when the *LF* is less than 2, the maximum power mismatch is increasing nearly-linearly with *LF* increasing. This is mainly because of the unbalance loads. When *LF* is larger than 2, the maximum power mismatch becomes rapidly larger with phase-A loads increase. Table 1 further illustrates that the iteration number of backward/forward sweep increases rapidly with large *LF*. This is primarily a function of heavy loading as opposed to the impact of unbalance.

Table 1. Iteration Count for Increasing *LF* when $\varepsilon_p = 1e-4$

<i>LF</i>	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
iteration	3	3	4	6	7	10	17	diverge

4.3 Breakpoint Selections

Assume bus-15 is directly connected to bus-10 in fig. 2, so the radial Kumamoto distribution system becomes weakly meshed. In a weakly-meshed distribution system, every node in each single loop can be selected as a breakpoint to break the loop. In this system, the single loop is 10-9-8-7-4-3-12-13-14-15(10). Different breakpoints are selected and the compensation-based method is run to obtain the power flow solutions. Fig. 5 shows the effect on the required breakpoint compensation iterations with different breakpoint selections.

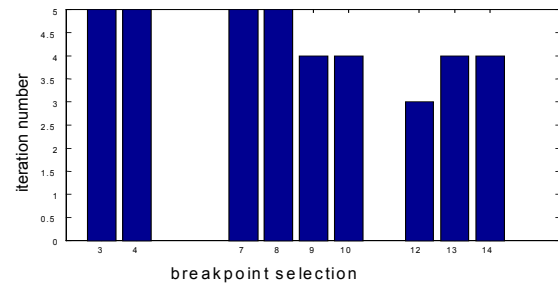


Fig. 5 Effect of Breakpoint Selection

4.4 PV Node Locations

In the original Kumamoto radial system, different *PV* node locations are chosen (voltage controlled dispersed generation units are installed at different buses) where $V_{set} = 0.99$. Fig. 6 shows the effect on the required *PV* compensation iterations with different *PV* node positions (reactive power limits are not considered).

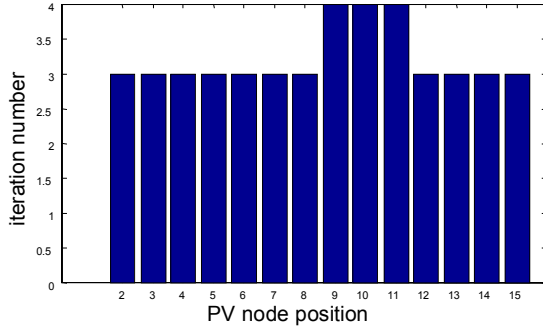


Fig. 6 Effect of *PV* Node Position

4.5 Number of Meshed Loops

Proceeding from the earlier modification to the radial Kumamoto system, case 1 will have a single loop if bus-10 is directly connected to bus-15. We allow for case 2, bus-6 to be connected to bus-8 to provide two loops, and then for case 3 to have three loops with bus-7 connected to bus-14. If bus-8, 10, and 14 are chosen as breakpoints, there are four breakpoint compensation iterations needed to obtain solutions in all these three cases. The breakpoint compensation iterations tend to be independent of the number of meshed loops.

4.6 Number of PV Nodes

In case 1, there is only one *PV* node, bus-6. For case 2, we allow bus-8 to be a *PV* node as well and then additionally in case 3 to have bus-4 as a *PV* node. Table 2 illustrates the number of *PV* compensation iterations in these three cases (n_V is the number of *PV* nodes).

Table 2. Iteration Number with n_V Effect

case	1	2	3
n_V	1	2	3
iteration	3	4	5

The *PV* compensation iterations tend to slowly increase with the number of *PV* nodes.

4.7 Summary of Analysis

Based on the above simulations, the convergence property of compensation-based power flow method is summarized:

- Compared with Fig. 3 and Fig. 4, it is clear that the convergence rate is much more sensitive to large single phase loads than load unbalance.
- Sections 4.3 to 4.5 show that the convergence rate is insensitive to breakpoint selection, dispersed generation installation position and number of meshed loops. The simulation in section 4.6 shows that increased number of *PV* nodes has a minor effect on the convergence rate.
- During a slow dynamic simulation, because the power flow calculation can use the power solution obtained in the previous time step as initialization, it will converge quickly, typically in one or two iterations.
- During the first iterations of the power flow calculation, the calculated breakpoint voltage mismatch ΔV_B or *PV* node voltage mismatch ΔV_V is usually relatively large (say, more than 10 times the corresponding convergence criteria). In these iterations, it is unnecessary to pursue inner-loop convergence criterions. A single iteration of the backward/forward sweep is sufficient if the previous breakpoint mismatch is more than 10 times its convergence criterion. Also, a single iteration of breakpoint current compensation is enough if the previous *PV* node voltage mismatch is more than 10 times its convergence criterion.

V. ADAPTIVE COMPENSATION-BASED METHOD

Based on the previous analysis, our adaptive compensation-based power flow method for use in dynamic simulations is the following:

1. Time step Δt_i is varied based on the previous number of iterations n_{i-1} of power flow calculation as follows:
 - If $n_{i-1} = 1$, then $\Delta t_i = 1.5\Delta t_{i-1}$.
 - If $n_{i-1} = 2$, then $\Delta t_i = \Delta t_{i-1} / 1.2$.
 - If $n_{i-1} \geq 3$, then $\Delta t_i = t_{\min}$.
 - Δt_i is restricted to $[t_{\min}, t_{\max}]$.
2. For backward/forward sweep iterations, if the previous calculated breakpoint voltage mismatch ΔV_B or *PV* node voltage mismatch ΔV_V is more than 10 times the corresponding convergence criteria, perform only one backward/forward iteration. Otherwise, continue the iteration until the maximum power mismatch is less than the power convergence criterion ϵ_p .
3. Calculate breakpoint voltage mismatch and compensation current. If the previous calculated *PV*

node voltage mismatch ΔV_V is more than 10 times the PV convergence criterion, perform only one iteration of the breakpoint compensation and proceed to next step. Otherwise, perform iterations between steps 2 and 3 until convergence is obtained.

4. Calculate PV node voltage mismatch and compensation current. Repeat steps 2, 3 and 4 iterations until convergence is obtained.

5.1 Computational Analysis

The original radial system is modified to be weakly meshed system with three voltage constant dispersed generations (bus-10 is directly connected with bus-15, bus-4, 6, and 8 are PV nodes). Comparison of convergence property between the original compensated-based method and the adaptive method has been made. Table 3 shows the comparison results when a scalar load factor LF is applied to all phase-A loads.

Table 3. Convergence Comparison

LF	Original			Adaptive		
	outer	middle	inner	outer	middle	inner
1.0	5	11	14	6	8	8
3.0	7	16	23	7	9	9
5.0	8	21	42	9	13	13
8.0	Diverges			11	23	27
9.0	Diverges			12	32	38

outer: number of outer-loop iterations which is PV compensation loop.

middle: number of middle-loop iterations which is breakpoint compensation loop.

inner: number of inner-loop iterations which is backward/forward sweep loop.

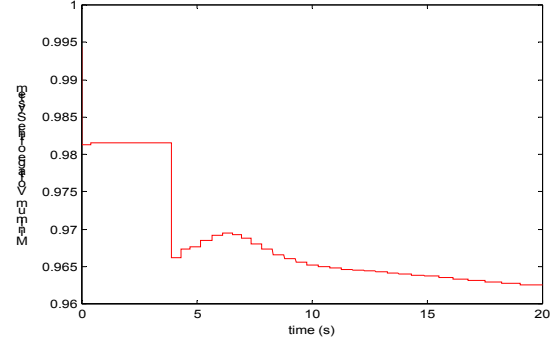
From the simulation results, note the following:

- During iterations, if the PV node voltage mismatch is large, there is no need to pursue small breakpoint voltage mismatch and power mismatch. Also, if the breakpoint voltage mismatch is large, there is no need to pursue small power mismatch. Thus, compared with the original method, although the outer iterations are roughly the same in the adaptive method, the middle and inner iterations are far fewer. Thus, the power flow calculation with the adaptive method is much faster than the original method.
- When LF is equal to 8.0 or 9.0, the power flow solution using the original method diverges. This is mainly because the backward/forward sweep algorithm will diverge with very heavy loads. In the adaptive method, before the breakpoint voltage mismatch ΔV_B or PV node voltage mismatch ΔV_V is less than 10 times the corresponding convergence criteria, only one backward/forward iteration needs to be performed. So the backward/forward sweep loop, breakpoint compensation loop, and PV compensation loop are running simultaneously and the compensation current is

adjusted continuously. Thus, our adaptive method is more robust.

5.2 Dynamic Analysis

When the power flow calculation is used in dynamic simulation, a variable time step allows more robust and faster computations. Fig. 7 shows the comparison of minimum voltage in the Kumamoto system (Fig. 2) when there is a sudden large increase of the loads.



(a) time step Δt fixed at 0.5 sec.

(b) time step Δt varies between [0.1, 2.5] sec.

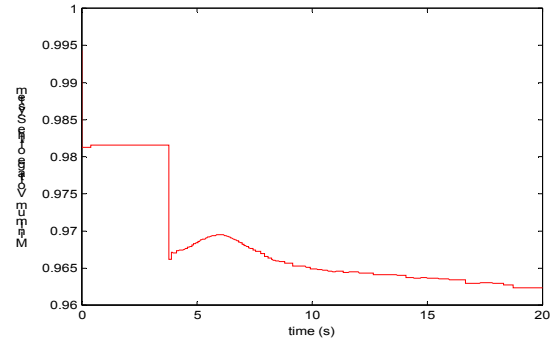


Fig. 7 Comparison of Dynamic Minimum Voltage

From the simulation results, note the following:

- Before the large increase of the loads ($t < 3.78$ sec), much larger time steps are taken for the variable step method than for the fixed step.
- After the large increase of the loads ($t > 3.78$ sec), much smaller time steps are taken than for the fixed step. With the system approaching the new steady state, time steps again increase.
- In dynamic simulations, using the adaptive step method, more accurate and smooth results can be obtained when a large change occurs in the system, and results can be obtained quickly without losing accuracy when the system is nearly steady.
- In terms of computations, although the adaptive method takes much smaller time steps than the fixed step method immediately after the large increase of the loads, the overall number of floating point operations is

fewer (19.88 Mops) than in the original method (22.33 Mops). This is simply due to large time steps once the disturbance has settled out. Note these variable time steps are independent of the integration time step.

- If the system undergoes no large change during simulation, the number of floating point operations is also less than for the original method as large time steps will be taken.

VI. CONCLUSIONS

An adaptive compensation-based method is developed for distribution power flow analysis based on exploitation of the convergence property of compensation-based method. Comprehensive models have been made for all elements in distributed systems. It is demonstrated that the adaptive compensation-based method is faster and more robust. The algorithm is particularly suited for modeling slow dynamics on the distribution system.

VII. ACKNOWLEDGEMENT

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BIOGRAPHIES

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