IEEE/PES Summer Meeting, Vancouver, BC. July 16, 2001.

STATE ESTIMATION FOR THE DETECTION OF MARKET PARAMETERS

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Abstract

Deregulation of electric utilities has led to a new competitive regime for utilities. In traditional systems, the main objective is to estimate voltages and flows. Estimators that include the ability to determine system parameters along with system conditions have been developed. In the evolving deregulated environment there are a host of new estimation needs. The various costs of the participants, estimation of the degree of market power, estimation of the price elasticities of the participants, and estimation of the volatilities of prices. This paper addresses two estimation needs. It defines the problem of estimation system status based on the knowledge of published PTDFs (Power Transfer Distribution Factors) by describing a new procedure for estimating parameters It also reformulates a previously presented but not much publicized method for estimating the cost elasticities of generators. Keywords: State estimation, spot pricing, ISO, PTDF, price elasticity.

1 Traditional State Estimation

Having an accurate picture of the state of a system is an important aspect of system operations. While a simple SCADA system is capable of providing operators with "raw" information about system operating conditions, only a state estimator is capable of "filtering" the information to provide a more accurate picture of the status of the system.

The traditional objective of state estimation is to reduce measurement errors by utilizing the redundancy available in most measurement systems. In particular, the objective is to reduce the variance of the estimates and improve their overall accuracy. There are other major objectives of traditional state estimation:

- Detection of erroneous measurements and bad data.
- Detection of erroneous assumptions about the system, particularly the status of switches and breakers.
- Ability to provide information for unmetered or unmonitored parts of the system.

Concepts from this paper were originally presented the ICIAM conference in Scotland, July 1999.

- Use of redundancy in order to improve the parameters for the electrical models of the system.

A more complete an accurate picture of the system has value in several ways. The traditional reason for an accurate picture of the system state is to assure system security. With an accurate picture of the system operating conditions, it is possible for security application software to do a better job assessing the impact of "events." However, a more accurate set of measurements also has significant value in terms of providing better numbers for "billable" quantities.

The traditional State Estimation problem is:

- Given a set of measurements $\mathbf{z}$, and a model of the network $\mathbf{h}(\mathbf{x}) = \mathbf{z}$
- Determine the unknowns $\mathbf{x}$ that minimize the residuals $\mathbf{r} = \mathbf{z} - \mathbf{h}(\mathbf{x})$.

The most common type of estimation minimizes the weighted least squares sums of the residuals. This objective has the advantage that, for Gaussian noise, it leads to the maximum likelihood estimate. It is also (comparatively) easy to implement, even when sparsity must be preserved. For additional references on least squares estimation, refer to [1].

The types of measurements available to state estimators depend on the nature of the available transducer. The most common measurements types include:

- Power flows on a line or transformer.
- Current flows. If only current magnitudes are metered, the problem gets complicated as a result of multiplicity of solutions. Refer to [2, 3, 4, 5] for additional information.
- Voltage magnitude measurements.
- Voltage phase angle measurements. This is a relatively new addition to the measurement repertoire. For additional information see [6, 7, 8].

It is also possible to extend the estimation problem to include both equality and inequality constraints into the formulation [9, 10, 11].

Evolution into a deregulated environment brings about new measurement needs. Measurements may include observed prices or temperature readings. There are also new parameters to estimate.

2 Impact of deregulation on state estimation

One of the important needs of a deregulated environment that can be met by state estimators is the precise determination of actual physical flows of power. While compliance with the terms a bilateral contract may be ascertained by the simple device of metering the power injected by the supplier at the supply point (presumably a specific system location) and the consumer at the delivery point (another specific system location), in general things are more complex:

- It may be necessary to determine the marginal losses within the system at both the supply and delivery points, since any efficient system for trading power will have to make allowances for loss compensation.
- Any metered quantity may need "backup metering" in case of failure of the main metering system. If a particular measurement of a specific power injection becomes unavailable, a state estimator may be able to supply the missing information based on other measurements that remain available.
- Many contracts (even bilateral contracts) do not involve single specific locations, but rather involve trading energy at several injection points for delivery at more than one location. Contracts may call for specific patterns of supply and delivery. An example of this type of contract is the so-called "sellers choice" contract, where the delivery point is not precisely specified [12]. There is a great deal of uncertainty associated with this type of contract. The objective of state estimation is to reduce this uncertainty.
- The ability to separate out the impact of any one business activity from any other one. Transmission Loading Relief procedures work because of beliefs that certain activities increase certain flows. An estimate of the accuracy of these effects (characterized by matrix sensitivities, also called PTDFs[1]) is most pertinent. This is relevant to the ability to accurately determine the marginal impact of flows on congested lines and/or corridors (*flowgates*). Improper detection of which transactions are causing which flows can lead to serious operational problems, since it may lead to the curtailment of an ineffective or perhaps even a helpful transaction.

It is not sufficient to estimate the value of system quantities. It is necessary to estimate the variances and expected errors associated with the various quantities. While estimation of variances and covariances has always been an integral part of the state estimation process, the methods for accuracy estimation come into greater focus as a result of deregulation. There is a great deal of economic "value" associated with a reduction in uncertainty.

Rather than continue with a long list of new estimation problems, we consider two specific examples.

3 Estimation of network status from PTDFs

This problem is defined thus:

- The user is given an observed or measured PTDF matrix, defining the relationship between nodal injections (relative to some reference location) and flows on relevant lines or corridors (so-called flowgates).
- The user also has a general knowledge of the topology of the grid for which the PTDFs have been given, but has no precise knowledge of the specific network conditions that led to these PTDFs. That is, the user knows the nodal Jacobian matrix J_n and the flow Jacobian matrix J_f, but J_n could be in error.

The presumed PTDF matrix S_0 is determined from:

$$S_0 = J_f J_n^{-1} \tag{1}$$

where J_f is the Jacobian of the flowgate flows with respect to all voltage and angle variables (of course, all lines can be designated as flowgates if desired, but this is uncommon). J_n is the conventional nodal injection Jacobian of all nodal injections. Both J_n and J_f exclude the slack location angle variable, as well as any columns (also rows, in the case of J_n) corresponding to fixed voltage magnitude locations. The column of S_0 corresponding to this reference location is all zeros.

An approximate version of the above can be obtained from the susceptance matrices, as follows:

$$S_0 \approx B_f B_n^{-1} \tag{2}$$

where B_n is the (real) nodal susceptance matrix, and B_f is the matrix of branch susceptances for all flowgates. Computationally, a "backsolve" version of equation (2) is preferable:

$$S_0' = B_n' \backslash B_f' \tag{3}$$

The problem of interest is to detect any incorrect status of lines or breakers. That is, given:

- An observed PTDF matrix S_1, containing n rows (the number of nodes in the network) and m columns ($m \ll n$), where m is the number of flowgates.
- A presumed topology and approximate parameters for the network, sufficient to permit the construction of "intact network" B_n and B_f matrices.
- A list of suspected branch status conditions (that is, a list of possible changes ΔB_k), zero or more of which could be valid.

Estimate, from an inspection of this suspected status list, which (if any) of the suspected lines was out of service when the PTDF matrix S_1 was obtained.

Two methods to determine the status of the system from PTDFs are considered:

Method 1: Create PTDF matrices S_k^0 for every presumed condition k by using the modifications ΔB_k to B_n to recalculate S. Make sure that each S_k^0 uses the same reference location as S_1. Compare each S_k^0 with the given matrix S_1. Select the case k that corresponds to a minimum norm discrepancy.

Method 2: Estimate the missing line using a reduced matrix approach in which the given matrix S_1 is compared only against the base case PTDF matrix S_0.

Method 1 is self-explanatory and requires no further description. It is also slow, since numerous PTDF matrices must be constructed and tested.

To describe method 2, let B_0 be the approximate nodal Jacobian matrix of dimension n by n, and B_f be the approximate flow Jacobian for the m flowgates[2], of

[1] A PTDF, or Power Transfer Distribution Factor, is the change in flow in a line as a result of an injection. The PTDFs depend on the choice of a reference location, but the effect of reference location always cancels out when the PTDFs are applied.

[2] To avoid unnecessary notational complexities, we do not explicitly distinguish between uses of the reduced matrices B_n and B_f when all rows and columns are considered and the case when the reference node rows and columns are excluded. This should be apparent from the context.

dimension n by m. Let S_0 be the sensitivity (PTDF) matrix for the flowgates of interest (a n by m matrix) under the assumption of all lines in service. Let S_1 be the observed or posted PTDFs. By the definition of PTDFs, we should have:

$$S_0' = B_0^{-1} B_f \tag{4}$$
$$S_1' = B_1^{-1} B_f \tag{5}$$

The challenge is to determine the structure of B_1 from a knowledge of B_0, S_0, S_1 and B_f. From the equations above eliminate B_f and establish that:

$$B_0 S_0' = B_1 S_1' \tag{6}$$

Here $B_1 = B_0 + \Delta B$ is unknown, and ΔB is to be determined. Substitution into the equation above and simplification leads to:

$$\Delta B S_1' = B_0 \Delta S' \tag{7}$$

where $\Delta S = S_1 - S_0$.

The objective is *not* to determine ΔB but rather to select from among a set of candidate ΔB_k matrices. The ΔB matrix corresponding to the omission of line i to j has the structure $(e_i - e_j) b_{ij} (e_i - e_j)'$, where e_i is a zero vector with a 1 in position i and b_{ij} is the direct susceptance from i to j.

Because the product $\Delta B S_1'$ should have nonzeros only in positions i and j, we calculate a new matrix $\Lambda = B_0 \Delta S$. In theory, this matrix should have the same structure as $\Delta B S_1'$. That is, is should have zeros in all rows other than i and j. In practice, it will only have an such structure as an approximation. To estimate the underlying topology, we seek the two rows of Λ with the largest norm. We identify the likely missing element as connecting i to j[3].

Several comments are in order:

- It is impossible (or nearly impossible) to distinguish the status of series branches.
- Multiple incorrect status conditions require either a sequential application of this method, or the development of a simultaneous removal method.
- More formal analysis would also consider the possible variance in the PTDF values.

3.1 Numerical example

Consider the 16-node 3-area network illustrated in Figure 1. The appendix lists all the branch susceptances, which is the only additional item needed to be able to replicate the results in this paper. The following are features of this system:

- The flowgates for which PTDFs are computed only for flowgates 7–14, 9–12 and 1–4.
- The "corridors" where uncertainty with respect to status exists are 4–5, 5–6, 15–13, 9–11 and 7–8.

[3] Alternatively, we can limit such selection to the subsets of pairs of rows that are defined by the given list of lines of uncertain status

Based on the nodal and flow Jacobians, the PTDFs corresponding to the flowgates are (node 1 is reference):

$$S_0 = \begin{bmatrix} 0.000 & 0.000 & 0.000 \\ -0.186 & -0.091 & -0.352 \\ -0.222 & -0.203 & -0.395 \\ -0.142 & 0.039 & -0.538 \\ -0.223 & -0.114 & -0.432 \\ -0.239 & -0.256 & -0.414 \\ 0.222 & -0.008 & -0.265 \\ -0.103 & 0.037 & -0.473 \\ -0.226 & 0.091 & -0.454 \\ -0.248 & -0.290 & -0.416 \\ -0.456 & -0.016 & -0.384 \\ -0.269 & -0.413 & -0.427 \\ -0.473 & -0.049 & -0.378 \\ -0.603 & -0.036 & -0.338 \\ -0.375 & -0.124 & -0.404 \\ -0.321 & -0.251 & -0.414 \end{bmatrix} \tag{8}$$

This *computed* matrix assumes no outages. When measurements are taken, the system behaves according to the following *observed* PTDFs:

$$S_1 = \begin{bmatrix} 0.000 & 0.000 & 0.000 \\ -0.188 & -0.109 & -0.335 \\ -0.223 & -0.223 & -0.376 \\ -0.136 & 0.046 & -0.542 \\ -0.231 & -0.145 & -0.407 \\ -0.242 & -0.277 & -0.395 \\ 0.226 & -0.009 & -0.262 \\ -0.096 & 0.040 & -0.472 \\ -0.225 & 0.084 & -0.446 \\ -0.253 & -0.309 & -0.400 \\ -0.451 & -0.022 & -0.375 \\ -0.270 & -0.428 & -0.412 \\ -0.468 & -0.055 & -0.369 \\ -0.599 & -0.039 & -0.332 \\ -0.373 & -0.132 & -0.394 \\ -0.323 & -0.263 & -0.402 \end{bmatrix} \tag{9}$$

These PTDFs differ from the ones above, but the nature of the difference is not evident. It is up to us to determine whether these PTDFs correspond to the intact system, and if not, to detect the most likely topology error(s) from among the set of suspected errors. The matrix above has one such error, but what is it? The two methods described above are:

- Method 1 compares observed PTDFs with PTDFs obtained from the systematic omission of every suspect line in turn. The PTDF that come closest in a norm sense to the observed PTDF is declared as the most likely correct status.
- Method 2 compares the new PTDFs against the base case PTDFs only and estimates the status from an observation of a Λ matrix.

A systematic comparison of the observed PTDFs with PTDFs calculated for each and every alternative outage condition under consideration gives the following norms for the error between the PTDFs for a each assumed outage condition and the given PTDFs S_1:

No outage	4–5	5–6	15–13	9–11	7–8
0.040	0.012	0.045	0.162	0.480	0.331

From these values, we infer that the correct status is the outage of line 4–5 (it has the smallest norm). This was, indeed, the case.

Fig. 1: ***The 16 bus test system. Flowgates and lines of uncertain status are indicated.***

Application of method 2 results in:

$$\Lambda = \begin{bmatrix} -0.55 & -0.02 & -0.30 \\ 0.04 & 0.01 & 0.03 \\ 0.18 & -0.01 & 0.10 \\ 0.80 & 3.42 & -3.00 \\ -1.52 & -3.64 & 2.56 \\ 0.16 & -0.02 & 0.09 \\ 0.20 & 0.02 & 0.10 \\ 0.70 & 0.01 & 0.40 \\ -0.05 & 0.20 & -0.01 \\ -0.27 & 0.03 & -0.15 \\ 0.07 & 0.00 & 0.04 \\ 0.14 & -0.03 & 0.08 \\ 0.08 & 0.00 & 0.05 \\ -0.04 & -0.00 & -0.02 \\ 0.13 & 0.03 & 0.07 \\ -0.09 & 0.01 & -0.05 \end{bmatrix} \tag{10}$$

The norm of the absolute values of the rows of this matrix is:

$$[.5, 0, .2. 3.4, 3.6, .2, .2, .7, .2, .3, .1, .1, .1, 0, .1, .1]'$$

The outaged line (corresponding to the two largest entries above) is correctly identified as (4–5).

Although both methods are successful in the detection of errors for this simple example, their main limitation their inability to distinguish between the status of two lines that are otherwise in series (or roughly in series) Also, detectability suffers as errors are introduced into the observed PTDFs.

4 Estimation of generator cost elasticities

Price is an important signal in a deregulated environment. A spot pricing system for transmission achieves optimal congestion management. However, using prices to *anticipate* the likely effect of prices on congestion relief requires the knowledge of price elasticities [13].

Assume that every generator has a marginal cost structure that is quadratic, with an increasing quadratic cost coefficient. The form of every generator cost function is:

$$f(P_g) = a + bP_g + \frac{1}{2}cP_g^2$$

Thus, the marginal cost of each generator is linear. Assume further that $c > 0$ for all generators (increasing marginal costs). As described in [13], by observing the behavior of generators under a variety of conditions, any passive party privy to the behavior of generators (e.g., what is the generator quantity bid or delivered when the price at a location is known) can "almost" estimate the quadratic coefficient values for all generators. To make the procedure complete, it is necessary to also use the observed responses of generators to transmission congestion charges. This paper revisits these results and discusses those aspects of the problem that are in the domain of estimation. The lossless case is the only case considered here.

Assume that in an uncongested market where no market power exists every generator's operates according to their own marginal cost. Equilibrium is attained when all marginal costs are the same:

$$\mathbf{b} + \mathbf{diag}(\mathbf{c}) \cdot \mathbf{P}^0 = \mathbf{ones}(n_g, 1) \cdot \lambda_0$$

λ represents the system marginal cost. In the absence of congestion and losses, is the same everywhere.

As conditions change, the generation pattern changes, and so does λ. Assume that the cost coefficients for every generator remain, however, constant. A second uncongested operating condition leads to a second operating point:

$$\mathbf{b} + \mathbf{diag}(\mathbf{c}) \cdot \mathbf{P}^1 = \mathbf{ones}(n_g, 1) \cdot \lambda_1$$

Subtracting these last two sets of equations from each other leads to

$$\mathbf{diag}(\mathbf{c}) \cdot \Delta\mathbf{P} = \Delta\lambda \cdot \mathbf{ones}(n_g, 1)$$

According to our assumption of marginal cost bidding, changes in generation pattern $\Delta\mathbf{P}$ must be according to the ratios of values of $\mathbf{c}$. Thus, anyone observing the market can estimate $\mathbf{c}$ values based on an observation of $\Delta\mathbf{P}$. Denote these estimates as $\tilde{\mathbf{c}}$. One $\tilde{c}_i$ coefficient must be guessed, since it is only the ratio among values that can be observed directly. Without loss of generality, assume that this is the coefficient for generator 1. The remaining coefficients are estimated from:

$$\tilde{c}_2 = \frac{\Delta P_1}{\Delta P_2}\tilde{c}_1 \,, \quad \ldots \quad , \tilde{c}_{n_g} = \frac{\Delta P_1}{\Delta P_{n_g}}\tilde{c}_1$$

A potential problem occurs when $\Delta P_i = 0$. However, under these circumstances it must be assumed that the particular generator is either unable or unwilling to participate in redispatch, and it must be thus excluded from the list of participating generators and treated as a constant output unit, therefore reducing the dimensionality of the problem. It can be seen further that, if $\Delta\lambda$ were known, the values for $\tilde{c}$ would be known accurately. As it is, these values become known up to a constant. That is:

$$\tilde{\mathbf{c}} \quad = \quad K\mathbf{c}$$

where K is a scalar, and $\mathbf{c}$ and $\tilde{\mathbf{c}}$ are vectors.

This process of "almost" estimating the quadratic coefficients by a passive observation of market behavior can be repeated as often as desired, leading to *redundant* solutions for the estimated values of $\tilde{\mathbf{c}}$. The determination of $\tilde{\mathbf{c}}$ under these circumstances becomes a least squares problem.

A market observer is likely to also want an estimate for the linear cost coefficients $\mathbf{b}$. Denote these estimates by $\tilde{\mathbf{b}}$. These coefficients have an arbitrary adder component, as described above. By assuming an arbitrary λ and inserting the observed P_i's in the optimality condition of the unconstrained problem, a market observer is able to estimate a value for $\tilde{\mathbf{b}}$ consistent with $\tilde{\mathbf{c}}$.

Assume that the market observer detects a congestion condition under $\mathbf{P}^3$. An OPF solution using a best estimate for the values of $\mathbf{c}$ leads to a prediction of the optimum operating point $\widetilde{\mathbf{P}}$, which differs from $\mathbf{P}^3$.

$$\begin{aligned} K\cdot\mathbf{c}\cdot\widetilde{\mathbf{P}} - \mathbf{ones}(n_g,1)\cdot\tilde{\lambda} + S^T\cdot\mu &= -\tilde{\mathbf{b}} \\ \mathbf{ones}(1,n_g)\cdot\widetilde{\mathbf{P}} &= P_D \\ \mu\geq 0 \qquad\qquad \mathbf{S}\cdot\widetilde{\mathbf{P}} &\leq \mathbf{p}^{\max} \end{aligned}$$

It is possible to attain economically optimal operation by means of congestion charges. Let β be the vector of proposed congestion charges. If congestion prices are sent to the market, the market settles into a new operating point P^4:

$$\begin{aligned} \mathbf{c}\cdot\mathbf{P}^4 - \mathbf{ones}(n_g,1)\cdot\lambda_3 &= -(\tilde{\mathbf{b}}+\beta) \\ \mathbf{ones}(1,n_g)\cdot\mathbf{P}^4 &= P_D \end{aligned}$$

Because the cost coefficients may not be correct, however, the solution $\mathbf{P}^4$ to which the market settles may be different from the predicted solution $\widetilde{\mathbf{P}}$. The difference is due to the unknown factor K. It is possible to determine K from an observation of the difference between the presumed solution change $\Delta\widetilde{\mathbf{P}} = \widetilde{\mathbf{P}} - \mathbf{P}^3$ to the observed change $\Delta\mathbf{P} = \mathbf{P}^4 - \mathbf{P}^3$ from:

$$K \quad = \quad \frac{\Delta P_i}{\Delta\widetilde{P}_i}$$

This solution for K is redundant (all n_g generators should give the same result). Redundancy is desirable to account for odd producer behavior, response errors and other practical estimation difficulties. Thus, K is estimated from the solution of a least squares problem.

Once K has been estimated, it is possible to induce the expected optimal congestion-relieving behavior by issuing a second and final round of additive price signals β^* obtained from a solution of the new OPF problem:

$$\begin{aligned} K\cdot\mathbf{diag}(\tilde{\mathbf{c}})\cdot\mathbf{P}^5 - \mathbf{ones}(n_g,1)\cdot\lambda + \mathbf{S}^T\cdot\mu &= -\tilde{\mathbf{b}} \\ \mathbf{ones}(1,n_g)\cdot\mathbf{P}^5 &= P_D \\ \mu\geq 0 \qquad\qquad \mathbf{S}\cdot\mathbf{P}^5 &\leq \mathbf{p}^{\max} \end{aligned}$$

Both the solution $\mathbf{P}^5$ and the eventual market response to the adders β^* are identical with the optimal operating point solution $\mathbf{P}^*$.

5 Conclusions

A general discussion of new challenges of state estimation brought about by deregulation has been followed by the description of two "new" estimation problems of unique importance in a deregulated environment. These and similar estimation problems are likely to prove quite useful for those operating and trading in deregulated power environments. More significantly, these new problems are indicators of the kind of new mathematical thinking required in all problems in state estimation in power systems. The two questions addressed in this paper have been: (a) can we determine the cost sensitivity of every generator in the system by simple market observation and (b) can we estimate the status of lines from a knowledge of the PTDFs. The answer in both cases is yes, subject to some caveats.

Many challenges remain. The procedure presented for topology error detection is somewhat ad-hoc. A more formal mathematical understanding of the conditions of topology error detectability is warranted. Likewise, the method for estimation of quadratic cost coefficients of generators is predicated on two premises: bidders for power will always bid exactly according to their marginal cost, and their cost has a quadratic structure. A more general analysis would relax one or both of these requirements: it would permit bidders that bid only an approximation to their marginal cost, and would also permit the inference of non-quadratic cost structures from multiple observations. It can also lead to an estimation of market power conditions and to an estimation of startup and shutdown costs for the various plants in the system.

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6 Data for example

From	To	Susceptance
1	2	17
1	4	50
1	7	67
2	3	33
2	5	33
2	9	17
3	6	67
4	8	200
4	5	18
4	9	100
5	6	33
5	9	67
5	12	40
6	10	67
6	12	33
7	8	50
7	14	100
8	9	50
8	14	13
9	11	20
9	12	67
9	15	50
10	12	33
11	13	50
11	14	25
12	15	50
12	16	20
13	14	25
13	15	25
14	15	50
15	16	20