

# An OPF based Algorithm to Evaluate Load Curtailment Incorporating Voltage Stability Margin Criterion

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**Abstract**--This paper proposes a method to compute load curtailment evaluation, using optimal power flow (OPF) computation, incorporating the steady state voltage stability margin constraints. A steady state voltage stability indicator is first discussed for its applicability as a suitable indicator for representing stability margin from the collapse point. The load curtailment formulation is then evolved and described into the OPF's objective function. A criterion based on the voltage stability indicator is then incorporated as an additional constraint into the OPF. Examples are constructed to demonstrate the quantitative effects of the stability margin criterion in evaluating load curtailment.

**Index Terms**--Load curtailment, OPF, Voltage stability

## I. INTRODUCTION

In the recent years voltage stability issues have received considerable focus due to the several voltage stability crisis situations having occurred all around the globe. In the US these incidents have become more complicated with the ongoing process of deregulation. Economic competition, sometimes, results in paying less attention to security features of the overall system. Congestion management has become one of an important issue in the context of deregulation because of the increased demand and competition. In a deregulated environment, congestion alleviation could mean load curtailment in certain situations. The utilities would definitely prefer to curtail a load as lower as possible during a viability crisis situation. However, from the overall system viewpoint, any policy of load curtailment has definitely to incorporate voltage stability margin considerations. Thus, in the emerging deregulation market any control action has to incorporate security features to maintain an acceptable level of system reliability.

A power flow can have any number of operating limit violations. When such situations occur, the violations can be alleviated by appropriate or various corrective actions. The analytical process of evolving this procedure is known as

Optimal Power Flow[1]. The current practice is to use the constraints based on the operating limits imposed by bus An OPF based Algorithm to Evaluate Load Curtailment Incorporating Voltage Stability Margin Criterion (S-11)voltages, branch flows, power transfers over interfaces, etc. The system troubles in contingency analysis are violations of such constraints. Controls may include generator, real power phase shifter angle, bus load-curtailment or all the three. The objective of the corrective action algorithm is to observe all constraints while minimizing the weighted sum of the control movement. The Newton based approach to OPF was proposed in [2]. In [3] the authors have formulated the OPF extension to take into effect the contingencies that occur in power systems. The non-solvability of the Newton process due to the singularity of the Jacobian matrix is got over by modifying the OPF through load shedding or by relaxing some in-equalities [4].

Methods to understand the voltage instability phenomenon and quantify the stability indices have been reported in works [5,6,7]. In [8] a voltage stability indicator has been discussed whose value changes between zero (no load) and one (voltage collapse). The indicator incorporates the effect of all other loads in the system on the evaluation of index at individual load buses. The overall voltage stability of the system could be identified by the largest value of the index evaluated amongst all the load buses. This indicator can also be used as a normalized quantitative measure, for estimation of the voltage stability margin from the operating point. In this paper, the authors have used this capability of the indicator. Works in the direction of developing algorithms to incorporate stability issues into power system operational analysis are going on. The reported work [9] attempts to formulate the incorporation of the transient angle stability, into an OPF routine, as an additional constraint.

In this paper the authors have proposed and formulated an algorithm to include the voltage stability margin feature into the load curtailment objective function of an OPF.

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## II. VOLTAGE STABILITY MARGIN CONSTRAINT

The transmission system can be represented using a hybrid representation, by the following set of equations

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = H \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix}$$

$V_L, I_L$  are the voltage and current vectors at the load buses

$V_G, I_G$  are the voltage and current vectors at the generator buses

$Z_{LL}, F_{LG}, K_{GL}, Y_{GG}$  are the sub-matrices of the hybrid matrix H.

The H matrix can be evaluated from the Y bus matrix by a partial inversion, where the voltages at the load buses are exchanged against their currents. This representation can then be used to define a voltage stability indicator at the load bus, namely  $L_j$  which is given by,

$$L_j = \left| 1 + \frac{V_{0j}}{V_j} \right| \quad (1)$$

where,

$$V_{0j} = - \sum_{i \in G} F_{ji} V_i \quad (2)$$

The term  $V_{0j}$  is representative of an equivalent generator comprising the contribution from all generators.

The index  $L_j$  can also be derived and expressed in terms of the power terms as the following.

$$L_j = \left| \frac{S_{j+}^*}{Y_{jj+} V_j^2} \right| \quad (3)$$

where,

$$S_{j+} = S_j + S_{jcorr}$$

\* indicates the complex conjugate of the vector

$$S_{jcorr} = \left( \sum_{\substack{i \in \text{Loads} \\ i \neq j}} \frac{Z_{ji}^* S_i}{Z_{jj}^* V_i} \right) V_j \quad (4)$$

$$Y_{jj+} = \frac{1}{Z_{jj}} \quad (5)$$

The complex power term component  $S_{jcorr}$  represents the contributions of the other loads in the system to the index

evaluated at the node j.

It can be seen that when a load bus approaches a steady state voltage collapse situation, the index L approaches the numerical value 1.0. Hence for an overall system voltage stability condition, the index evaluated at any of the buses must be less than unity. Thus the index value L gives an indication of how far the system is from voltage collapse. This feature of this indicator has been exploited in our proposed algorithm to evolve a voltage collapse margin incorporated OPF routine.

In the conventional optimal power flow approach, the objective is to minimize the total amount of load curtailment considering the load flow system constraints like line flow, voltage magnitude, the maximum active and reactive power generation etc. The control variables for the OPF evaluation are the real and reactive power generation of each generation bus and the real and reactive load at each load bus.

## III. LOAD CURTAILMENT FORMULATION INCORPORATING VOLTAGE STABILITY MARGIN

The OPF problem formulation which we have used is presented herewith. In order to keep the load power factor as a constant, we assume that when a certain amount of real load has been shed at one bus, the corresponding reactive load will also be shed in the same proportions.

$$\text{Objective: } \min \sum_{i=1}^n \text{load\_curtail}_i$$

S.T.:

$$P_{gi} - P_{li} - \sum_{j=1}^n |V_i| |V_j| (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) = 0 \quad (7)$$

$$Q_{gi} - Q_{li} - \sum_{j=1}^n |V_i| |V_j| (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) = 0 \quad (8)$$

$$P_{li} / P_{lireq} = Q_{li} / Q_{lireq} \quad (9)$$

$$0 \leq P_{li} \leq P_{lireq} \quad (10)$$

$$0 \leq Q_{li} \leq Q_{lireq} \quad (11)$$

$$|V_i|_{\min} \leq |V_i| \leq |V_i|_{\max} \quad (12)$$

$$P_{gimin} \leq P_{gi} \leq P_{gimax} \quad (13)$$

$$Q_{gimin} \leq Q_{gi} \leq Q_{gimax} \quad (14)$$

$$P_{ij}^2 + Q_{ij}^2 \leq S_{ij\max}^2 \quad (15)$$

$$L_i \leq L_{crit} \quad (16)$$

Here,

$$\text{load\_curtail}_i = P_{lireq} - P_{li}$$

where,

$P_{lireq}$ : real load demand at bus i

$P_{li}$ : actual real load supply at bus i

$n$  : total number of load flow buses in the system

$P_{gi}$  : real power generation at bus  $i$

$Q_{gi}$  : reactive power generation at bus  $i$

$Q_{lreq}$  : reactive load demand at bus  $i$

$Q_{li}$  : actual reactive load supply at bus  $i$

$|V_i|$  : voltage magnitude at bus  $i$

$|V_j|$  : voltage magnitude at bus  $j$

$G_{ij}, B_{ij}$  : real/reactive part of the  $ij^{\text{th}}$  element of the bus admittance matrix

$\delta_{ij}$  : angle difference between the voltage phasor at bus  $i$  and bus  $j$

$P_{gimin}, P_{gimax}$  : minimum/maximum real power generation at generation bus  $i$

$Q_{gimin}, Q_{gimax}$  : minimum/maximum reactive power generation at generation bus  $i$

$|V_i|_{min}, |V_i|_{max}$  : minimum/maximum voltage magnitude at bus  $i$

$P_{ij}, Q_{ij}$  : real /reactive power flow through transmission line  $ij$

$S_{ijmax}$  : maximum apparent power flow allowable through the  $ij^{\text{th}}$  line

$L_i$  is the index  $L$  evaluated at the  $i^{\text{th}}$  bus other than the generation buses

$L_{crit}$  is the threshold value of the index acceptable for the system

It can be observed in the OPF formulation that it includes the power balance equations (7,8) generation limits (13,14), line loading limits (15), voltage magnitude limits (12). For the load curtailment policy which we have adopted, i.e constant power factor, an additional constraint (9) has been added. To incorporate the feature of voltage stability margin into the OPFs description the constraint (16) has been included.

#### IV. THE EFFECT OF INCORPORATING VOLTAGE STABILITY MARGIN ON LOAD CURTAILMENT EVALUATION

##### A. Case I

The WSCC 9 bus system is used as our test system (Fig 1) to illustrate how load curtailment evaluation is effected by our OPF algorithm which incorporates the steady state voltage stability margin criterion.

The network parameter for the WSCC 9 bus test system, on which we have chosen to implement our algorithm, is as shown in Table I.

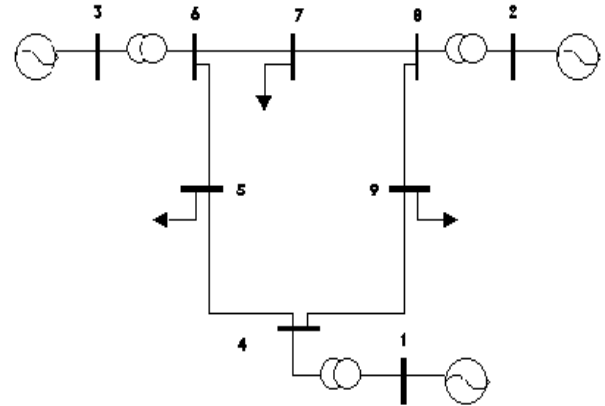


Fig. 1. WSCC Nine-bus test system

TABLE I  
LINE PARAMETERS FOR THE WSCC NINE-BUS SYSTEM

| Line | Resistance (p.u) | Reactance (p.u) | Susceptance (p.u) |
|------|------------------|-----------------|-------------------|
| 1-4  | 0.0000           | 0.0576          | 0.0000            |
| 4-5  | 0.0170           | 0.0920          | 0.1580            |
| 5-6  | 0.0390           | 0.1700          | 0.3580            |
| 3-6  | 0.0000           | 0.0586          | 0.0000            |
| 6-7  | 0.0119           | 0.1008          | 0.2090            |
| 7-8  | 0.0085           | 0.0720          | 0.1490            |
| 8-2  | 0.0000           | 0.0625          | 0.0000            |
| 8-9  | 0.0320           | 0.1610          | 0.3060            |
| 9-4  | 0.0100           | 0.0850          | 0.1760            |

For this case we chose the following loading before running the OPF based load curtailment algorithm. Load bus 5 was supposedly having a load demand of  $150 + j 120$  MVA, bus 7 a load demand of  $100 + j 35$  MVA and load bus 9 having demand of  $125 + j 50$  MVA. All the generator buses are taken to be PV buses with scheduled voltage at 1.0 p.u. The maximum and minimum acceptable voltage magnitudes at all load buses are taken as 1.1 and 0.9 p.u.

After running OPF, without incorporating the voltage stability margin criterion as a constraint, the load curtailment value got is  $26.51 + j 21.21$  MVA. However, by introducing the voltage stability index  $L$  as an additional constraint as proposed in the paper, the curtailment value is  $90.57 + j 72.45$  if  $L_{crit}$  is chosen as 0.1 The curtailment is  $36.45 + j 29.16$  for  $L_{crit}$  taken to be 0.2. This brings out the fact that if one intends to operate the system far away from a possible voltage collapse situation, i.e lower  $L_{crit}$  value, it would be at the cost of a large load curtailment. However, if  $L_{crit}$  is taken as anywhere above 0.25 the load curtailment value obtained is  $26.51 + j 21.21$  which is the same as obtained using the conventional OPF algorithm. In this situation the lower limit of the allowable minimum voltage

at bus 5 has hit its limit.

Thus, the proposed algorithm is able to incorporate both the features of voltage magnitude constraint and the voltage stability margin criteria. We summarize the results for the above case study in Table II.

TABLE II  
RESULTS FOR THE CASE I SCENARIO

| Constraints Imposed                   | Curtailment (MVA) at BUS 5 |
|---------------------------------------|----------------------------|
| Without Stability Index criteria      | 26.51 + j 21.21            |
| Index L $\leq$ 0.1 at all load buses  | 90.57 + j 72.45            |
| Index L $\leq$ 0.2 at all load buses  | 36.45 + j 29.16            |
| Index L $\leq$ 0.25 at all load buses | 26.51 + j 21.21            |

### B. Case II

The same WSCC system as described in Case I, is used in this scenario too. In this case we will demonstrate how the new proposed algorithm evaluates the load curtailment in situations where line contingencies for a steady operating system occur.

For this case, load bus 5 is supposedly having a load demand of  $120 + j 60$  MVA, bus 7 a load demand of  $100 + j 35$  MVA and load bus 9 having demand of  $125 + j 50$  MVA. All the generator buses are taken to be PV buses with scheduled voltage at 1.0 p.u as before and the maximum and minimum acceptable voltage magnitudes at all load buses are taken to be 1.1 and 0.9 p.u.

When the line 5-6 is down it is observed that there is no load curtailment for the conventional OPF algorithm that does not include the voltage stability margin criteria. However, the curtailment as evaluated by the proposed algorithm taking  $L_{crit}$  as 0.2 comes out to be  $5.8 + j 2.9$  MVA. Similarly if a line 4-5 contingency happens then the curtailment evaluated by conventional OPF is  $12.04 + j 6.02$  MVA while the load curtailment evaluated by the proposed algorithm comes out to be  $18.1 + j 9.05$  MVA.

Thus it is observable from the results that incorporating the voltage stability margin feature into the load curtailment policy is effective during contingency conditions too. The result for this case is summarized in Table III.

TABLE III  
RESULTS FOR THE CASE II SCENARIO

| Line Outage | Curtailment at BUS 5 (Without L Constraint) | Curtailment at BUS 5 (With L Constraint $\leq$ 0.2) |
|-------------|---------------------------------------------|-----------------------------------------------------|
| 5-6 out     | No curtailment                              | $5.8 + j 2.9$                                       |
| 4-5 out     | $12.04 + j 6.02$                            | $18.1 + j 9.05$                                     |

## V. CONCLUSIONS

The load curtailment evaluation is effected by the proposed incorporation of the voltage stability margin index into the OPF

algorithm. The amount of curtailment evaluated is observed to increase if more voltage stability margin, from a possible collapse, is required in a system. The algorithm discussed is effective during contingency evaluation also.

## VI. FUTURE WORK

This paper brings out the fact that incorporation of the voltage stability margin criteria into the load shedding procedure affects the amount of curtailment. We expect this to directly inference the indices like Expected Energy Not Served (EENS) in reliability studies of composite power systems.

## VII. REFERENCES

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## VIII. BIOGRAPHIES

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