

## Optimization and Visualization of the North American Eastern Interconnect Power Market

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### Abstract

This paper uses a first generation OPF model of the Eastern Interconnect to gauge the potential benefits deriving from RTOs in an ideal world. So long as the lines and generators are operating as planned, there is sufficient low cost capacity to keep peak demand prices in the Eastern Interconnect below \$50 MWh. Under a reasonable approximation of ideal conditions a repeat of San Diego's experience is unlikely. Unfortunately a few downed lines or generator outages can greatly impact local prices even with unrestricted trade.

### 1. Introduction

The road to competitive electricity markets in the United States is becoming littered with roadkill. In June 1998 Midwest wholesale prices jumped from about \$30MWh to over \$5000MWh, bankrupting at least two suppliers [1]. In the Spring of 2000 capacity markets in New York and New England were destroyed by market manipulators [2]. During the summer of 2000, San Diego electricity consumers saw their bills double as demand approached the physical limits of generation [3]. Not surprisingly, politicians and regulators have imposed price caps and are calling for price fixing investigations and re-regulation [4]. Born again "competitive" utilities are demanding that legislatures and courts once again bail them out of their financial distress [5]. Restructuring wasn't supposed to progress that way.

The Federal Energy Regulatory Commission (FERC) has concluded that its "open access to transmission" policy, Orders 888 and 889, has not been sufficient to bring effective competition to electricity markets. Reflecting Adam Smith's telling insight (1776) that competition is limited by the size of the market, FERC decreed in December 1999 that all private and public transmission owners form and join Regional Transmission Organizations [6]. Large regional markets would replace balkanized utility systems. Presumably fierce price competition would result because RTOs would encompass

more generators vying for customers, all on a level playing field.

RTOs are to be operating by December 15, 2001 and are to address the Commission's finding that "...there remain important transmission-related impediments to a competitive wholesale market." [7]. These impediments include "...economic inefficiencies... in the current operation and expansion of the transmission grid, and (2) continuing opportunities for transmission owners to unduly discriminate...to favor their own or their affiliates' power marketing activities." [8]

This paper uses a first generation optimal power flow (OPF) derived model of the Eastern Interconnect to gauge the potential benefits deriving from RTOs in an ideal world [9]. Specifically we ask, if the Eastern Interconnect were managed as one huge RTO, would there be significant short term benefits from perfect competition and free trade? Since any real RTOs will be smaller than the Eastern Interconnect and would likely have significant departures from competitive pricing (price equals marginal cost) our estimate overstates the actual short-term benefit of RTOs. Next we show how sensitive these estimates are to relatively small changes in the network configuration.

A perfectly functioning RTO managing the Eastern Interconnect could bring significant benefits. So long as the lines and generators are operating as planned, there is sufficient low cost capacity to keep peak demand prices below \$50 MWh. Of course this result assumes purely competitive pricing and that the RTO ensures free trade. But, under a reasonable approximation of ideal conditions an Eastern Interconnect repeat of San Diego's experience is unlikely. Unfortunately a few downed lines or generator outages can greatly impact local prices even with unrestricted trade.

The risk is RTOs might not act in the general interest. If they do not, the sensitivity analyses suggest they could cause local price spikes and protect noncompetitive pricing. There is a growing literature suggesting that Independent System Operator governance and special

interest capture are problems that are not solved [10],[11] and[12]. Further in the foreseeable future there are likely to be several RTOs in the Eastern Interconnect. An individual RTO could act to maximize the welfare of its members at the expense of others. We intend to address these topics in the future with the aid of the Eastern Interconnect model.

The paper is organized as follows. Section 2 presents the electrical system model of the Eastern Interconnect that underlies our estimates. The electrical grid is a corrected version of the North American Electrical Reliability Council's (NERC) Summer 2000 Base Case; generator costs are taken from the FERC Form 1. Section 3 discusses the OPF solution algorithm as implemented in PowerWorld Simulator. Section 4 compares the electricity prices and the distribution of generator production under administered and free trade. Section 5 shows the sensitivity of these results to generator outages, line outages and line limit errors, while Section 6 is the conclusion.

## 2. Eastern Interconnect System Model

The starting point for this study was NERC's summer 2000 Base Case model of the Eastern Interconnect [13]. The Summer 2000 Base Case was built by NERC's Multiregional Modeling Working Group from regional power flow models provided by NERC's members. This model is intended to "...realistically simulate bulk electric system behavior." The Committee's guidance for component models ensures that the integrated model is realistic:

Of paramount importance in this effort is the detail in which the various systems are modeled. The detail included in each system model must be adequate for all inter and intraregional study activities but not necessarily as detailed as required for internal studies. This means that each system model should include sufficient detail to ensure that power transfers or contingencies can be realistically simulated.[14]

The Summer 2000 Base Case consists of 33,538 buses, 22,812 loads, 5312 generators, 2361 switched shunts, 45,421 ac lines/transformers (lines), 10 dc lines, and 107 control areas. Total load is 536.4 GW. Of the buses approximately 31,000 are located in the U.S. with the remainder in Canada. For this study optimization was only

performed on the U.S. buses. Figure 1 shows an interactive one-line diagram of a portion of the system; overall the one-line contains approximately 7300 of the higher voltage buses and 9300 lines.

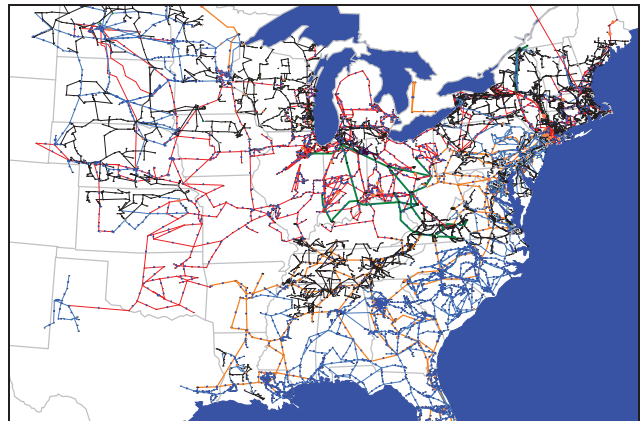


Figure 1: Eastern Interconnection One-line

In order to perform OPF studies on this model two major issues needed to be addressed. The first was the large number of initial line violations, while the second was obtaining estimates of generator costs. Using the default "A" limit set the case initially had 248 line limit violations, with 17 of the line flows at 150% or more of their limit (the highest was 321%). Thankfully many of these limit violations were in Canada and hence outside the scope of this study. Of the remainder, most were either limit violations on generator step-up transformers or violations on lines supplying radial load. These limits were disabled during the study, with the assumptions that generator step-up transformers are always sized to match the rated capacity of the unit and violations on radial loads can not be enforced by generator controls. The remaining limits were either 1) disabled if they could not be enforced by the available generator controls (such as in a radial load network), 2) disabled if they appeared to be adjacent to equivalenced portions of the network, or 3) enforced by the OPF. The NERC flowgates were also modeled in the case. However due to a lack of publicly available limits only those flowgates described on the NERC Market Redispatch website [15] were both monitored and enforced. The case also had low voltages at a handful of buses, with seven bus voltage magnitudes below 0.9 pu (the lowest was 0.803 pu) and 300 between 0.90 and 0.95 per unit. The bus voltages were monitored during the simulation but were not corrected.

The second issue was determination of generator operating costs. Unfortunately, the NERC 2000 list of generators does not report operating costs. Therefore, generators reported on the NERC 2000 must be identified and matched with generators reported on other survey Forms such as the FERC Form I [16] or the EIA-412 [17] where cost data is collected. Once matched, operating cost data for specific generators can be prepared for the model. Table 1 displays the technological character of the generating units modeled. The NERC 2000 case specifies a total of 5,312 generators, of which 4,693 are located in

were calculated from the EIA-759 (1999), "Monthly Power Plant Report," and FERC Form 423 (1999), "Monthly Report of Cost and Quality of Fuels for Electric Plants." Fossil cost data were developed from the FERC Form 423 (1999), "Monthly Report of Cost and Quality of Fuels for Electric Plants," 1999. Nuclear cost data were calculated from the FERC Form I, "Annual Report of Major Electric Utilities, Licensees and Others," 1998. 1999 data are used for fossil plants (including combined cycles and turbines), and 1998 data, converted to 1999 dollars are used for nuclear plants. A uniformly low operating cost was

Table 1. Number of Generators, Matched and Costed, by NERC Region

Technology Type	Data	NERC Region									Total
		ECAR	FRCC	MAIN	MAPP	NEPOOL	NYPP	PJM	SERC	SPP	
Fluidized Bed	Capacity (MW)	180	250		186	290	50	272	92	320	1,640
	Units (no.)	1	1		2	2	1	3	2	2	14
Combined Cycle	Capacity (MW)	1,620	4,291		359	2,818	3,472	2952	1,400	767	17,678
	Units (no.)	4	33		5	39	52	37	27	8	205
Gas Turbine	Capacity (MW)	4,783	5,523	3,831	4,056	2,225	2,556	8,467	11,947	2,919	46,307
	Units (no.)	98	122	81	106	53	61	152	248	52	973
Hydroelectricity	Capacity (MW)	774	47	529	2,749	1,339	3,815	992	12,055	2,050	24,349
	Units (no.)	30	4	58	68	111	54	23	343	64	755
Internal Combustion	Capacity (MW)	148	133	58	1,002	652	0	42		295	2,329
	Units (no.)	13	16	3	69	24	1	4		21	151
Nuclear: Boiling Water	Capacity (MW)	2,279		6,177	1,976		2,831	7,743	7,704		28,710
	Units (no.)	2		7	3		3	7	8		30
Nuclear: Pressurized	Capacity (MW)	4,503	3,954	7,128	1,604	5,386	980	2,998	23,359	1,164	51,076
	Units (no.)	5	5	10	3	6	1	3	24	1	58
Pump Storage	Capacity (MW)	2,127		440		1,647	1,300	1,349	7,716	438	15,017
	Units (no.)	8		2		7	16	11	37	12	93
Fossil Steam	Capacity (MW)	84,893	22,294	32,056	21,374	10,362	16,992	30,518	90,522	34,736	343,748
	Units (no.)	293	93	167	146	67	97	147	370	159	1,539
Wind Turbine	Capacity (MW)				265						265
	Units (no.)				5						5
unknown	Capacity (MW)	5,774	1,802	5,200	3,114	2,887	2,382	3,647	10,588	2,221	39,325
	Units (no.)	97	35	99	92	168	101	97	148	40	871
Total U.S. Capacity modeled (MW)		107,081	38,294	55,419	43,677	27,605	34,379	58,980	165,384	44,910	570,445
Total Units (no.)		551	309	427	506	477	387	484	1,207	359	4,693

the United States, with the remainder located in Canada. Total capacity of the U.S. units sums to over 570 GW. Units identifiable as fossil steam comprised 344 GW of the total capacity, with another 80 GW attributable to nuclear sources. Only 39 GW of capacity could not be identified to known generators.

Table 2 displays totals for generators where fuel costs were calculated at the plant level, using the average annual heat rate as an indicator of conversion efficiency, multiplied by the composite fuel cost as burned at the plant. Heat rates

assigned to all units identifiable as either hydroelectric or pumped storage. In this manner, costs were developed for 3,391 of the 4,693 U.S. generators, or 72%. Coverage in terms of capacity is much greater, as over 507 GW of the 570 GW (89%) are dispatched using these operating costs.

In the real world marginal costs increase as generation approaches design capacity. However for this study a constant cost model was used because no reliable means of estimating heat rate curves for the several types of generator was available. Rather, an average annual heat

Table 2: Costed Capacity Using FERC Form I Data, by NERC Region (MW)

Technology	Data	NERC Region									Grand
		ECAR	FRCC	MAIN	MAPP	NEPOOL	NYPP	PJM	SERC	SPP	
Combined Cycle	Capacity (MW)	270	3,633		284	2,230	3,090	2,599	1,100	528	13,734
	Units (No.)	1	26		2	35	42	29	24	6	165
Gas Turbine	Capacity (MW)	4,624	5,244	3,721	3,801	1,478	2,464	8,200	11,121	2,791	43,445
	Units (No.)	91	115	76	96	38	58	146	237	47	904
Hydroelectric	Capacity (MW)	774	47	529	2,749	1,339	3,815	992	12,055	2,050	24,349
	Units (No.)	30	4	58	68	111	54	23	343	64	755
Nuclear, Boiling Water	Capacity (MW)	2,279		6,177	1,976		2,831	7,743	7,704		28,710
	Units (No.)	2		7	3		3	7	8		30
Nuclear, Pressurized	Capacity (MW)	4,503	3,954	7,128	1,604	5,386	980	2,998	23,359	1,164	51,076
	Units (No.)	5	5	10	3	6	1	3	24	1	58
Pump Storage	Capacity (MW)	2,127		440		1,647	1,300	1,349	7,716	438	15,017
	Units (No.)	8		2		7	16	11	37	12	93
Fossil Steam	Capacity (MW)	82,782	21,380	31,620	20,199	9,256	15,612	27,240	88,556	34,073	330,719
	Units (No.)	289	75	157	112	41	85	138	343	146	1,386
Total Capacity (MW)		97,359	34,259	49,615	30,612	21,336	30,092	51,121	151,612	41,044	507,050
Total Units (No.)		426	225	310	284	238	259	357	1,016	276	3,391

rate at the plant level, as reported from FERC Form I data, was applied to the fuel cost incurred by the plant to arrive at an operating cost estimate which jointly reflects the cost of fuel input and the operating efficiency of the plant. The heat rate was calculated as the sum of the fuel input converted to MMBtu divided by the net output for the reporting year. Fuel cost (\$/MMBtu) was calculated as total cost of fuel at the plant divided by total heat input of all fuels burned. The resulting statistic, rendered in mills per kilowatt hour, was used as the estimate of marginal cost.

### 3. OPF Algorithm

Determination of the cheapest way of meeting demand is sensitive to numerous physical constraints imposed on the market by the transmission system. The solution of this problem requires the use of an OPF algorithm. The goal of the OPF is the minimization (or occasionally maximization) of some objective function, subject to a variety of equality and inequality constraints. Often the objective function consists of the total generation cost in some set of areas, while the equality and inequality constraints include the power flow equations, generation/load balance, generator Mvar limits, branch flow limits, and transmission interface limits.

For the case study presented here an LP based approach was used [18]. Overall the LP based methods iterate between solving the power flow to take into account system non-linearities and solving an LP to redispatch the control variables subject to certain equality and inequality

constraints. The basic steps in the LP algorithm employed here are

1. Solve the power flow equations.
2. Determine constraint violations; linearize the pertinent power flow constraint equations with respect to the control variables.
3. Solve the LP using the Revised Simplex Method with explicit bounds on individual variables in order to get the change in the control variables.
4. Update the control variables and then update the power system state using a linearized network model.
5. If the changes in the control variables are above a tolerance update the LP constraint equations and go to 3; otherwise resolve the power flow equations.
6. If any of the control variables changed during step 3 go to 2. Otherwise the solution has been reached; calculate the final solution cost and the bus/constraint marginal prices.

The key to making the LP OPF fast is to minimize the number of constraints explicitly included in the LP basis. Most of the system constraints are enforced during the power flow solution. These include the power flow equations, generator reactive power limits, LTC transformer limits and switched capacitor limits. The LP basis then only includes the power balance constraints for the areas on OPF control, and any binding network inequality constraints or any network inequality constraints that are likely to become binding during the iteration. Note that while voltage limits were not enforced by the LP, automatic reactive control devices including LTC transformers and switched shunts were included in the power flow solution. These devices helped substantially to maintain a reasonable system voltage profile.



Once an optimal solution has been determined, the marginal costs for enforcing the different constraints can be determined from the control costs and the final LP basis matrix:

$$\lambda^T = c_B^T B^{-1}$$

where

$$\lambda^T = \text{marginal costs of enforcing}$$

constraints

$$c_B^T = \text{control costs}$$

$$B = \text{LP basis matrix}$$

The bus MW marginal costs (also known as the locational marginal prices or LMPs) are then computed as

$$\lambda_{\text{buses}}^T = \lambda^T S$$

where

$$\lambda_{\text{buses}}^T = \text{bus MW marginal costs}$$

$$S = \text{matrix of sensitivity of bus MW injections to the set of constraints}$$

Similar to what is currently done in the LMP calculation for the PJM system [19], the incremental impact of system losses was not considered here (the actual system losses were, of course, included in the power flow solution). With this approximation in the absence of transmission system congestion, the bus marginal prices in an entire area would be identical. However, when congestion is present the marginal prices vary depending on the constraint locations the area no longer has a single marginal price.

#### 4. Administered vs Free Trade Results

The OPF was then used to compare two different operating philosophies, the administered trade model and the free trade model. The administered trade model supposes that the operating areas in the model import or export the amounts initially specified in the FERC file<sup>i</sup>. Thus in this scenario each area was self optimized; that is, it was optimally dispatched while simultaneously enforcing its area interchange constraint and the limit constraints on its lines. In total 74 areas were included in the optimization, with LMPs computed at slightly more than 30,600 buses. This resulted in a solution with a total operating cost of

\$8,148,700/hour. The LMPs ranged from a high of \$ 700MWh to a low of \$ -72MWh with less than 1% having values above \$ 100MWh and only three buses having negative values<sup>ii</sup>. The average LMP was \$ 35.79MWh while the standard deviation was \$20.60MWh. Because of the area interchange constraints the area average LMPs varied considerably, from a high of \$ 99.90MWh to a low of \$ 9.47MWh. At the solution the flows on 21 lines were being enforced as binding constraints; this represents only about 0.05% of the 40,000 lines modeled in OPF areas.

Figure 2 contours the regional variation in the LMPs at approximately 6700 of the 31,600 buses [20], [21] using a color range of between \$10MWh and \$50MWh. The patchwork appearance of the contour is due, for the most part, to the area constraints. These heterogeneous costs are what open the possibility of lowering system cost by shifting from high (marginal) cost generators to lower (marginal) cost generators.

Next the system was modeled using the free trade model in which the area interchange constraints between the U.S. operating areas were relaxed (those with Canadian areas were still enforced). Thus the entire U.S. portion of the Eastern Interconnect was treated as though it were a single operating area. The intent of this approach is to approximate a perfectly functioning RTO.

As expected, relaxing the area constraints resulted in a lower operating cost, with the value dropping by about 3.3% from \$8,148,700/hour down \$7,879,000/hour. With the area constraints relaxed power was free to flow from the low cost areas to those with higher costs. The largest changes occurred in the Virginia Power control area, which increased its exports by almost 1800 MWs, while Commonwealth Edison and the Florida Control area saw their imports increase by 1665 and 1445 MW respectively. This resulted in the average area LMPs converging as well, with the highest value now just \$ 41.57MWh and the lowest \$ 29.03MWh.

The most surprising result of this study was the degree to which it could be possible to operate the entire Eastern Interconnect as a single control. While the number of congested lines increased slightly from 21 to 30, the LMPs were surprising uniform across the region. Compared to the administered trade model the average LMP decreased slightly from \$35.79MWh to \$33.35,

<sup>i</sup> The FERC 715 files only specifies the total imports or exports for each area; bilateral transactions are NOT specified.

<sup>ii</sup> Negative LMPs simply indicate that buses on one side of a constraint should reduce generation or increase there load. Negative LMPs occurred on the PJM system during each of the three summer months in 1999, reaching \$-199.33MWh in July.

while the standard deviation decreased substantially from \$20.60MWh to \$6.27MWh. This change is even more dramatic when comparing Figure 2 to Figure 3, which contours the new LMPs using the Figure 2 color scale. Figure 4 presents a graphical comparison of the LMP variation between the two cases. Finally, even with a much narrower color scale of between \$28MWh and \$42 MWh Figure 5 shows there is still little regional variation in the LMPs.

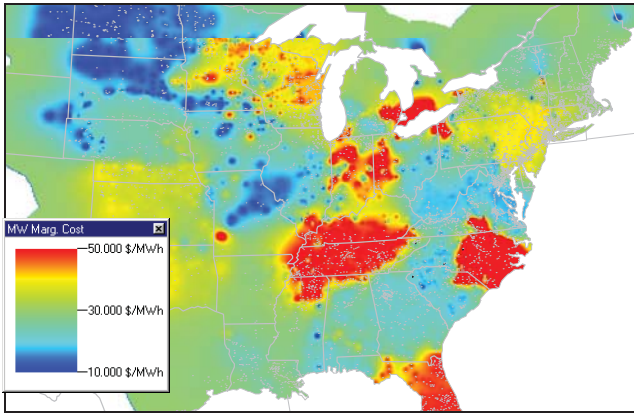


Figure 2: Administered Trade LMP Contour

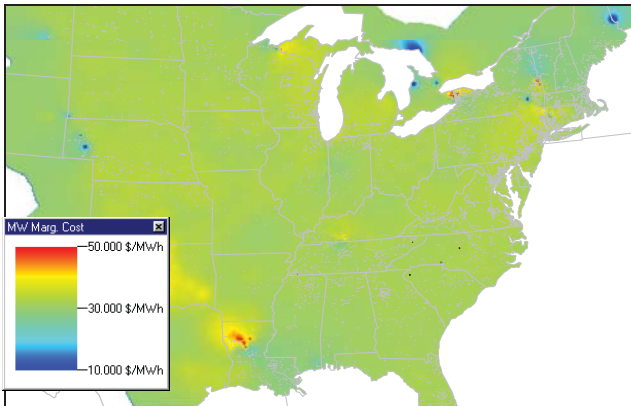


Figure 3: Free Trade LMP Contour

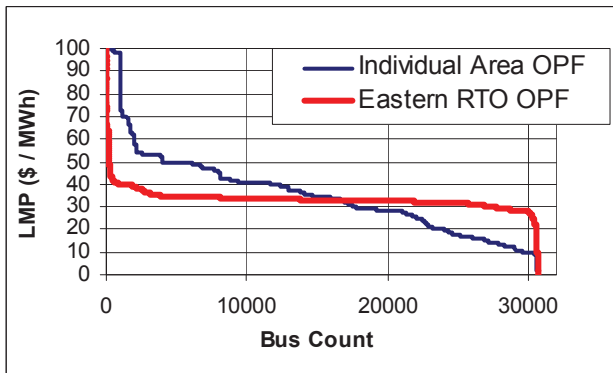


Figure 4: LMP Comparison

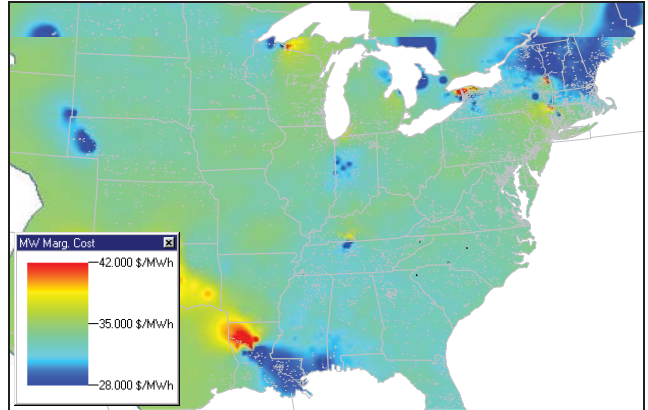


Figure 5: Figure 3 Contour with Narrower Color Key

### 5. Sensitivity of Results to Model Variation

Before concluding it is important to discuss the sensitivity of the results to model parameters. The results from any engineering study can, of course, only be as good as model from which they are derived. In probing the sensitivity of the previous results to model errors what we have found is in some areas the system is fairly robust, with outages of large generators or lines having relatively minor impacts on system prices, while in other areas such outages can have significant impacts on system prices. Overall, given that the case models a peak demand condition, the system is certainly stressed. But as indicated by the supply curve shown in Figure 6 (with the y-axis showing generation cost in \$ MWh and the vertical line showing the current load), generation is available on the grid. High regional prices can therefore be attributed to transmission system constraints.

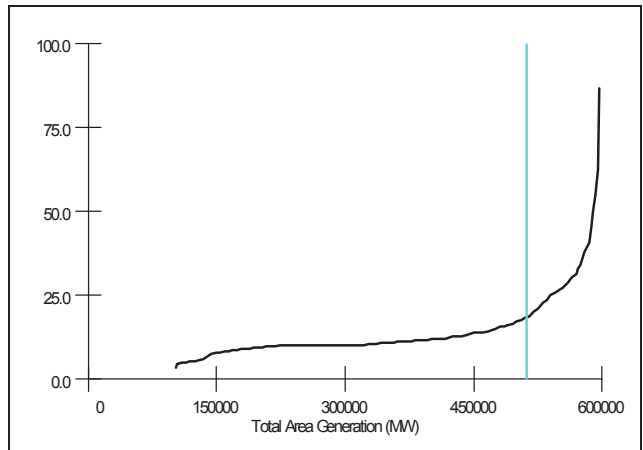


Figure 6: Eastern U.S. Generation Supply Curve

For example, the outage of a 1200 MW unit in the TVA area (at bus “N1 WBN”) had a relatively modest impact on the LMPs, increasing the average price in TVA by only

about \$0.80MWh. In contrast, the outage of a 890 MW unit in the Florida area (at bus “CR RV G3”) caused the prices in that area to soar, increasing from an average of \$32.27MWh to \$45.87MWh. Figure 7 contours the new LMPs using the same scale as Figure 5 of between \$28MWh and \$42 MWh, while Figure 8 presents a Florida close-up with a color scale of between \$30MWh and \$50MWh. Here the high price differences are due to congestion on several relatively low voltage lines (138 kV, 115 kV and 69 kV).

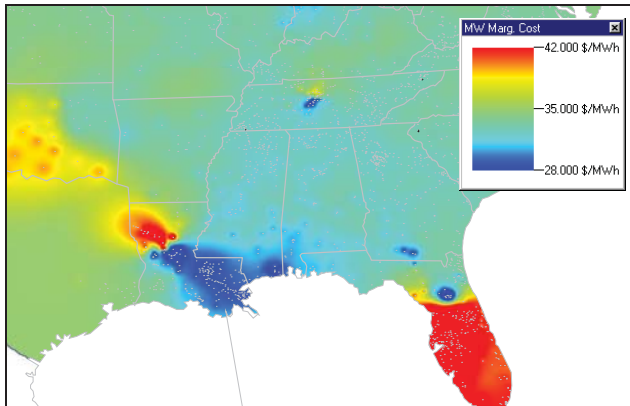


Figure 7: LMP Contours for Generator Outage in Florida

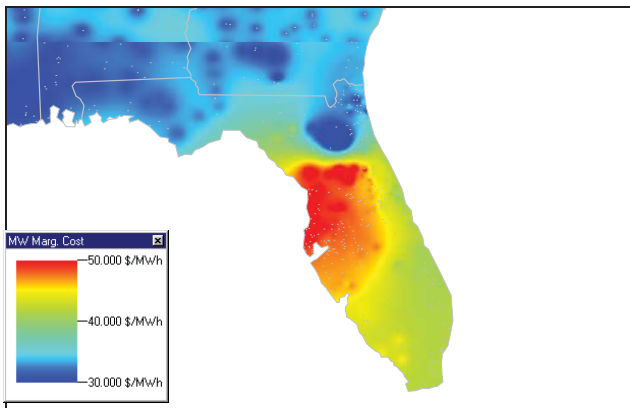


Figure 8: Florida Close-up

## 7. Conclusion

The results of this study support FERC’s contention that Regional Transmission Organizations acting in the public interest could bring significant benefits to electricity consumers. The model indicates that the Eastern Interconnect could be operated as a single system with only a minor increase in fully loaded lines under conditions of peak summer demand. That result holds without any increase in transmission investment to support the increase in inter-regional trade. Under peak demand, summer prices drop about 8% and, more importantly, the standard deviation of prices drops about 70%. Overall system costs

decline a bit over 3% which is considerable considering that most generators must be dispatched to meet peak demand.

There is no inherent reason for the Eastern Interconnect to experience San Diego’s fate if major equipment is available as planned. Under expected equipment availability, the Eastern Interconnect has sufficient generator capacity and transmission capability to meet peak demands at manageable prices. Given a chance, the existing transmission system is capable of flattening the price profile from the Rockies to the Atlantic.

If equipment fails or is withheld from the market, there would be high price areas that trade could not eliminate. For example, if several Florida generators were to become unavailable, either due to mechanical problems or there being withheld, prices through much of the state could increase substantially. Consequently there may be even more pressure for component reliability as the electrical system becomes more tightly integrated.

The critical question now is not whether RTOs have the potential for improving efficiency. The critical question is will they? Can RTOs be motivated to put system efficiency as their paramount goal? As the Florida example shows there remain opportunities for price increases even with free trade. A less-than –public spirited RTO would be capable through “technical” standards of furthering private gain at the expense of the general interest. The public policy problem is to identify the RTO’s interest with the public’s.

## 8. Acknowledgement and References

The generator cost data base was constructed by Mr. Thomas Leckey of the Energy Information Administration and Ms. Marilyn Walker of the Department of Justice. A table matching NERC Names and reference numbers to those used by the EIA and U.S. Environmental protection agency are available from the authors on request.

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- [2] See for example, Quint, Arnold and Young, W, Request of the New York Independent System Operator, Inc. for Suspension of Market Based Pricing for 10-minute Reserves and to Shorten Notice Period, March 27, 2000, file no: 55430.000001 who report to FERC “...substantial evidence that the levels of bids and quantities offered, and the resulting prices, have deviated substantially

from...[what].. would be expected if the 10-minute reserves markets were workably competitive.” pages 1-2

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[4] *ibid.*

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[6] FERC Docket No. RM99-2-000;Order No. 2000, December 20,1999

[7] *ibid.* page 32

[8] *ibid.* page 32

[9] The OPF model is intended as a research tool and is available from the authors upon request. The model requires PowerWorld Simulator ([www.powerworld.com](http://www.powerworld.com)) to run.

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