

# Power System State Estimation: Modeling Error Effects and Impact on System Operation

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**Abstract.** State estimation has been introduced to power systems and implemented in the 60s, using a single frequency, balanced and symmetric power system model under steady state conditions. This implementation is still prevalent today. The single frequency, balanced and symmetric system assumptions have simplified the implementation but have generated practical problems. This paper examines these simplified assumptions and their impact on the state estimation performance. It provides a theoretical basis for the well known fact that the reliability of the state estimator algorithms has been below expectations. Specifically, sensitivity analysis methods are used to quantify the impact of modeling simplifications and measurement schemes on the performance of state estimation. The results clearly illustrate that the traditional state estimation algorithm is biased. These biases affect the accuracy of state estimation and its convergence characteristics. The paper also reviews the traditional state estimation approach against recent technological advances that have enabled synchronized measurements. The implications and possibilities of this new technology are discussed in this paper. Specifically, an example application of the new technology for a Three Phase State Estimator is described. A power system state estimation based on a) multiphase model, b) voltage and current waveform measurements, and c) synchronized measurements is formulated. The paper focuses on the following: a) modeling, b) implementation, c) observability and d) performance. The overall performance of the system is described in terms of confidence level versus error. These concepts are illustrated with simple systems. In addition, we demonstrate the performance of the proposed methods on an actual system (New York Power Authority system) using actual synchronized measurements. The paper concludes with a commentary on the implications of improved state estimation methods on the security/reliability monitoring and control of an electric power system.

## 1. Introduction

State estimation was introduced by Gauss and Legendre (around 1800). The basic idea was to "fine-tune" state variables by minimizing the sum of the residual squares. This is the well-known least squares (LS) method, which has become the cornerstone of classical statistics. The reasons

for its popularity are easy to understand: At the time of its invention there was no computers, and the fact that the LS estimator could be computed explicitly from the data (by means of some matrix algebra) made it the only feasible approach. Even now, most statistical packages still use the same technique because of tradition and computational speed. Also, for one-dimensional problems, the LS criterion yields the arithmetic mean of the observations, which at that time seemed to be the most reasonable estimator. Afterwards, Gauss introduced the normal (or Gaussian) distribution as the error distribution for which LS is optimal. Since then, the combination of Gaussian assumptions and LS has become a standard mechanism for the generation of statistical techniques.

In a real time environment, state estimation was applied to power systems by Schweppe and Wildes in the late 1960's [1]. Over the past twenty five years, the basic structure of power system state estimation has remained practically the same:

- Single phase model
- P, Q, V measurement set
- Non-simultaneousness of measurements
- Single frequency model

The above basic structure of the power system state estimation implies the following assumptions (which in turn result in a biased state estimator):

1. all current and voltage waveforms are pure sinusoids with constant frequency and magnitude
2. the system operates under balanced three phase conditions
3. the power system is a symmetric three phase system which is fully described by its positive sequence network

These assumptions introduce a discrepancy between the physical system and the mathematical model (bias) and have resulted in practical difficulties manifested by poor numerical reliability of the iterative state estimation algorithm. Substantial efforts to fine tune the mathematical models in actual field implementations are required.

To alleviate the sources of error, new measurement systems and estimation methods are needed. For example, by utilizing synchronized measurements [3], the problem of

time skewness can be alleviated. Synchronization is achieved via a GPS (Global Positioning System) which provides the synchronizing signal with accuracy of better than 1 sec. By utilizing three phase measurements the system imbalance can be accounted for. Finally, by using full three phase model for the power system, the system asymmetry can be accounted for. Under ESEERCO (Empire State Electric Energy Research Corporation)/NYPA sponsorship, a multisite phasor measurement system has been implemented based on: (a) Synchronized measurements of voltage and current waveforms, (b) Three phase measurements, and (c) Use of full three phase models.

The state estimation based on this system is not subject to the mentioned biases of the traditional state estimation. This state estimation is formulated in its general form that allows estimation of waveform distortion as well resulting in the Harmonic State Estimation. This paper focuses on the application of this system as a three phase state estimation, free of time skewness, imbalance errors and asymmetry errors. The paper presents quantitative descriptions of the errors resulting from these biases and therefore provides a quantitative evaluation of the merits of the proposed three phase state estimator.

The paper begins with a brief review of the LS state estimation algorithm. Subsequently, expressions are derived of the biases from several sources of error. Then, the state estimator based on multiphase synchronized measurements is introduced. The general case of this estimator is the Harmonic Measurement System (HMS) and the three phase state estimator is a special case. Results of this state estimator and its impact on accuracy are presented.

## 2. Review of LS State Estimation

A present day implementation of state estimation includes additional functions as it is illustrated in Figure 1. Note that the basic state estimation algorithm is supplemented with a number of supporting functions, such as topology processor, data preprocessing, observability analysis, bad data rejection, parameter estimation and possibly remote calibration. Here we will focus on the basic state estimation algorithm. The state estimation is a mathematical procedure by which the state of an electric power system is extracted from a set of measurements. Traditionally, the measurements are P, Q and V (real power, reactive power and voltage magnitude). In general, any measurement can be expressed as a function of the system state. Let  $z_i$  denote a measured quantity:

$$z_i = h_i(x) \quad (1)$$

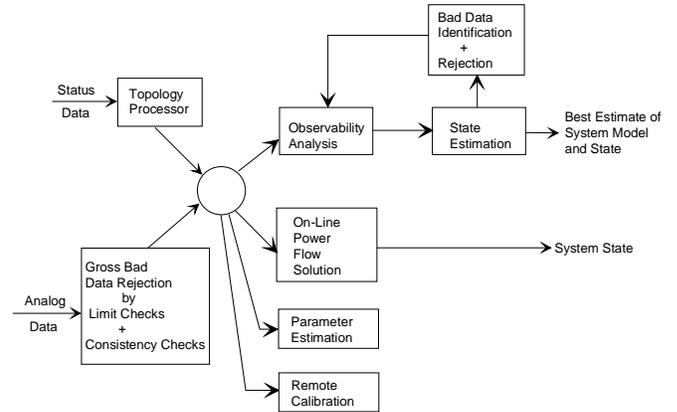
where  $x$  is the system state and  $h_i$  is a function specific to the measured quantity  $z_i$ . Assume that  $m$  measurements are

taken. Then, all measurements can be written in compact form:

$$z = h(x) \quad (2)$$

where

- $x$  is the system state - an  $n \times 1$  vector
- $z$  is a vector of measured quantities - an  $m \times 1$  vector
- $h$  is a vector function - an  $m \times 1$  vector function.



**Figure 1. Conceptual View of Real Time Power System Modeling and State Estimation**

Typically more measurements are taken than the number of state variables to be determined, i.e.  $m > n$ . In this case, the set of Equations (2) represents an overdetermined set of nonlinear equations in real variables.

The least squares solution of the overdetermined system (2) is the vector  $x$  which minimizes the sum of the squares of the components of the residual vector  $r$ ,  $r = z - h(x)$ . Mathematically, this is expressed as follows:

$$\text{Minimize } J = \sum_{i=1}^m r_i^2 = r^T r \quad (3)$$

A variation of this method is the weighted least squares method which minimizes the sum of the weighted squares of the components of the residual vector  $r$ . Mathematically, this is expressed as follows:

$$\text{Minimize } J = \sum_{i=1}^m w_i r_i^2 = r^T W r \quad (4)$$

where:  $w_i$  : the weight for the residual  $r_i$ ,  $W$  : a diagonal matrix, the diagonal elements being the weights  $w_i$ .

The unknown vector  $x$  is obtained from the solution of the necessary conditions, which in matrix notation are expressed

as follows:

$$\frac{dJ}{dx} = 0 \quad (5)$$

Note that:

$$\frac{dJ}{dx} = \frac{d}{dx} [(Hx - b)^T W (Hx - b)] = 2H^T W (Hx - b) = 0 \quad (6)$$

Upon solution of last equation for the state vector  $x$

$$x = (H^T W H)^{-1} H^T W b \quad (7)$$

Equation (7) provides the solution to the linear estimation problem (4).

To obtain the solution to the nonlinear estimation problem (4), assume that an initial guess of the vector  $x^0$  is known. The system (2) is linearized around the point  $x^0$  yielding:

$$b = h(x^0) + \left. \frac{\partial h(x)}{\partial x} \right|_{x=x^0} (x - x^0) + \text{h.o.t.}$$

Where h.o.t. denotes higher order terms. Assuming that the vector  $x^0$  is very close to the solution, then the higher order terms (h.o.t.) are negligibly small and are omitted from above equation.

Let

$$\left. \frac{\partial h(x)}{\partial x} \right|_{x=x^0} = H$$

Then

$$b = h(x^0) + H(x - x^0)$$

and

$$r = Hx - Hx^0 + h(x^0) - b$$

Observe that the vector  $-Hx^0 + h(x^0) - b$  is known. Let

$$-Hx^0 + h(x^0) - b = -b'$$

Now:

$$r = Hx - b'$$

Now the problem is identical to the linear estimation problem. Thus, the solution is:

$$\begin{aligned} x &= (H^T W H)^{-1} H^T W b' \\ &= (H^T W H)^{-1} H^T W [Hx^0 - h(x^0) + b] \\ &= (H^T W H)^{-1} (H^T W H)x^0 - (H^T W H)^{-1} H^T W [h(x^0) - b] \\ &= x^0 - (H^T W H)^{-1} H^T W [h(x^0) - b] \end{aligned}$$

The last equation is generalized into the following iterative equation:

$$x^{+1} = x^0 - (H^T W H)^{-1} H^T W [h(x^0) - b] \quad (8)$$

where  $H$  is the matrix  $\partial h(x)/\partial x$  computed at  $x = x^0$ . This is the Jacobian of the vector function  $h(x)$ .

In summary, the least squares solution of the linear estimation problem is given by Eq. (7) and the least squares solution of the nonlinear estimation problem can be obtained with the iterative algorithm (8).

### 3. Sources of Bias

The LS state estimation procedure is an **unbiased estimator if and only if the model is accurate (exact) and the measurement error is statistically distributed**. Both of these conditions may not exist in a practical system. In this section we concentrate on the bias resulting from model inaccuracies and we discuss the effect of measurement errors. In particular model inaccuracies result from: (a) unbalanced operating conditions and (b) asymmetries of power system models.

#### 3.1 Balanced Operation

An actual power transmission system operates near balanced conditions. The imbalance may be small or large depending on the design of the system. As an example, Figure 2 illustrates the three phase voltages and currents on an actual system. Note for example a 10% difference in the currents of Phases A and B of transmission line to GILBOA. The voltage in this case has only a 0.2% difference between two phases.

Because of imbalance, the measurements may have an error. We represent this as follows:

$$z = z_t + \Delta z$$

where  $z_t$  is the true measurement (assuming a balance system),  $\Delta z$  is the measurement error due to imbalance, and  $z$  is the actual measurement.

Application of the LS state estimation procedure, assuming no other error sources, yields:

$$x = x_t + (H^T W H)^{-1} H^T W \Delta z \quad (9)$$

where  $x_t$  is the true state of the system or the unbiased state estimate, and the second term is the bias resulting from the imbalance measurement error. Note that the bias from unbalanced operation depends on the level of imbalance as well as the system parameters (matrix  $H$ ).

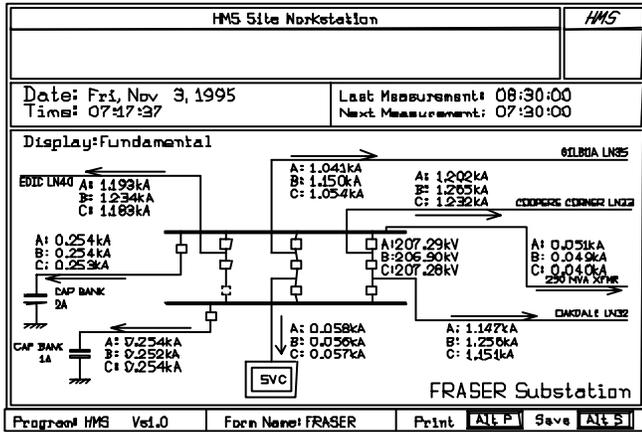


Figure 2. Actual Three Phase Voltages and Currents in FRASER Substation

### 3.2 System Symmetry

An actual power transmission system is never symmetric. While some power system elements are designed to be near symmetric, transmission lines are never symmetric. The impedance of any phase is different than the impedance of any other phase. In many cases, this imbalance can be corrected with transposition. Because of cost many lines are not transposed.

The asymmetry may be small or large depending on the design of the system. One power system component that contributes to the asymmetry is the three phase untransposed line. As an example, Figure 3 illustrates an actual three phase line. For the purpose of quantifying the asymmetry of this line, two asymmetry metrics are defined:

$$S_1 = \frac{1}{2} \frac{|z_{\max} - z_{\min}|}{|z_1|}$$

$$S_2 = \frac{1}{2} \frac{|y_{\max} - y_{\min}|}{|y_1|}$$

where  $z_1$  is the positive sequence series impedance of the line,  $z_{\max}$  and  $z_{\min}$  are the max and min series impedances of the individual phases,  $y_1$  is the positive sequence shunt admittance of the line,  $y_{\max}$  and  $y_{\min}$  are the max and min shunt admittances of the individual phases.

The above indices provide in a quantitative manner the level of asymmetry among phases of a transmission line. As a numerical example, these metrics have been computed for the line of Figure 3 and are presented in Figure 3. Note that the asymmetry is in the order of 5 to 6%.

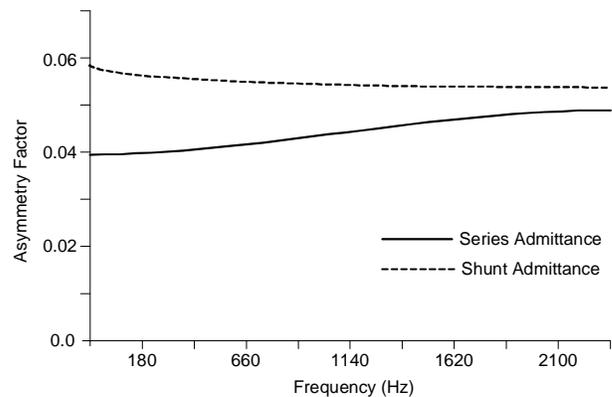
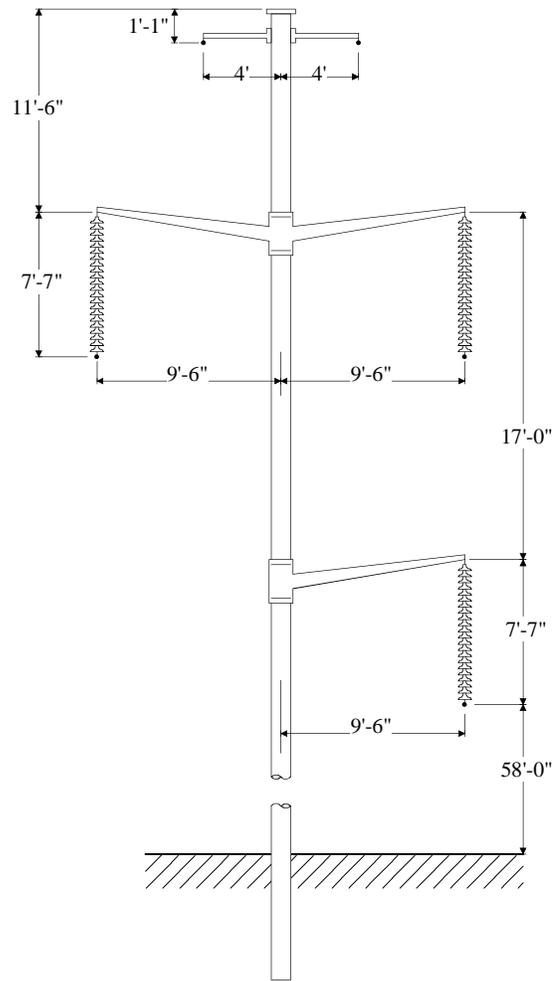


Figure 3. Line Asymmetry Indices

Because of the presence of non-symmetric components, the state estimate using the positive sequence model of the power system is biased. An estimate of the bias can be computed as follows. First observe that because of power system component asymmetry, the relationship of a measurement to the system model will have an error. Specifically:

$$z = h(x) + \Delta h(x)$$

where  $h(x)$  is the function relating the measurement to the state vector assuming symmetric power system components,  $\Delta h(x)$  is the difference between the symmetric model (positive sequence model) and the asymmetric model.

Now the jacobian matrix of the measurements becomes:

$$H = H_s + \Delta H$$

where  $H_s$  is the jacobian matrix assuming symmetric power system elements.

Application of the LS state estimation procedure, assuming no other error sources, yields:

$$x = (x_t + (H^TWH)^{-1}H^TW\Delta z)(\Delta H^TWH)^{-1} (I + 2(\Delta H^TWH)(H^TWH)^{-1})^{-1}(\Delta H^TWH) \quad (10)$$

where  $x_t$  is the state of the system assuming a symmetric model, and the other terms represent the bias resulting from the system asymmetry.

### 3.3 Measurement Errors

State estimators are based on the assumption that measurement errors are statistically distributed with zero mean. The traditional implementation of state estimation uses sensors of V, P and Q. When the sensors are properly calibrated, the measurement error is very close to meeting the requirements of state estimation. However, recent trends resulted in the use of sensorless technology for power system measurements. Sensorless technology refers to the use of A/D converter technology to sample the voltage and current waveforms. Once the sampled waveforms are available, the required measurements can be retrieved with numerical computations.

Independently of the technology used for measurements, it is important to examine whether there is bias in the measurements. This can be best achieved by examining the entire measurement channel of a typical power system instrumentation [14]. The major sources of error (see Figure 4) are (a) the instrument transformers, (b) the cables connecting the instrument transformers to the sensors or A/D converters and (c) the sensors or A/D converters. Figure 5 illustrates the transfer functions of a typical instrument transformer. It can be observed that the characteristics of instrument transformers near the power frequency are flat. One can conclude that for power frequency measurements, there is no appreciable measurement bias from instrument transformers. However for measurements at harmonic frequencies, a substantial measurement bias can occur. Another source of measurement bias may result from A/D converters. Figure 6 illustrates the transfer function of a

specific A/D converter. Note the magnitude and phase bias even at power frequency. It is important to note that the measurement bias is dependent upon the design of the A/D converter. The measurement bias resulting from control cables is variable depending on the total length of the cables.

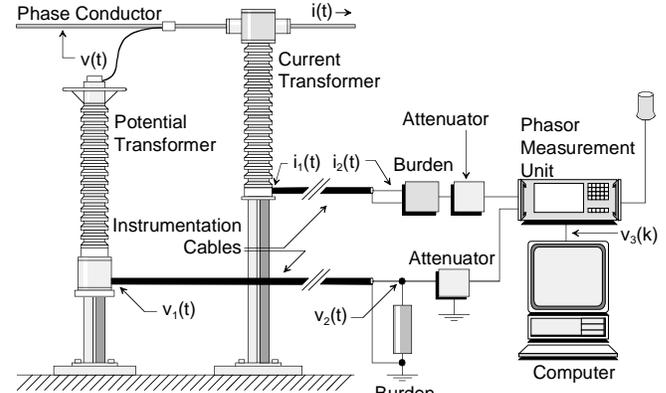


Figure 4. Components of Typical Voltage and Current Instrumentation Channel

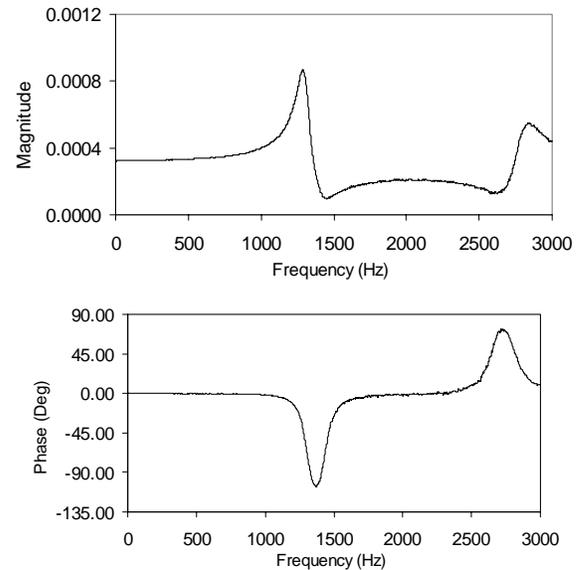


Figure 5. Magnitude and Phase of Frequency Response of a 200 kV/115 Potential Transformer

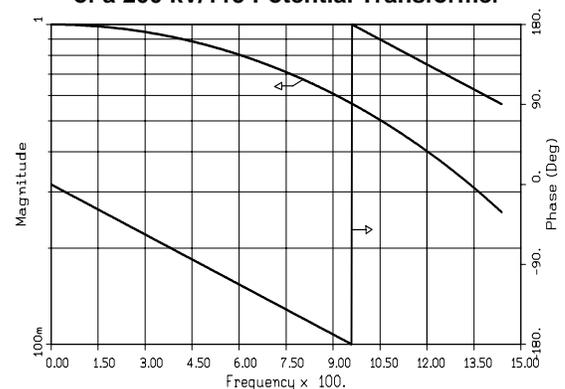


Figure 6. Magnitude and Phase of Frequency Response of the PMU-1620 Unit

The measurement bias can be corrected with software. Such methods have been developed [18], but their use in state estimation is very limited. It is important to note that the above sources of error cannot be corrected with better (more accurate) instrumentation. To avoid these sources of error, three phase measurements and a three phase system model is required. Such a system has been developed and it is described next.

#### 4. The Harmonic Measurement System (HMS)

The HMS consists of the Phasor Measurement Unit and a Personal Computer (local system) installed in substations and a centrally located master workstation as illustrated in Figure 7. Every local system has a Global Positioning System (GPS) receiver to synchronize harmonic phase measurements (with accuracy of 1  $\mu$ sec). This time reference allows the measurement of the phase angle of the fundamental with accuracy 0.02 degrees. The local system uses input signals from existing instrument transformers. The captured voltage and current waveforms are processed by a local site computer with error correction algorithms which compensate the error introduced during the measurement process from instrument transformers, cables, and effects of burdens. The error correction algorithms use the characteristic transfer functions of instrument transformers which have been measured and stored in a data bank [4]. Subsequently, the corrected waveforms are processed locally by the site computer to obtain the harmonic information.

The harmonic information obtained at every local system is sent to the master station for global data processing, including harmonic state estimation. Note that the harmonic state estimation here is system wide estimation instead of local estimation referred to in the literature. If the harmonics are limited to the fundamental, then the state estimator is a three phase estimator. The subject of this paper is the use of this system as a three phase state estimator.

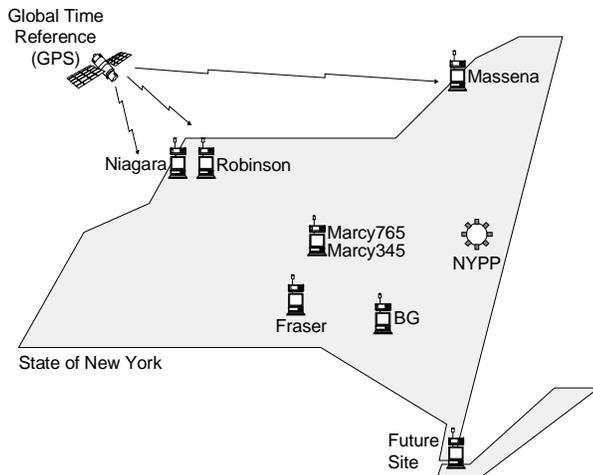


Figure 7. The present Harmonic Measurement System

#### 5. Formulation of the Three-Phase State Estimation

The formulation is presented with the following postulated model:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\eta} \quad (11)$$

where  $\mathbf{z}$  is a vector of fundamental frequency measurements;  $\mathbf{x}$  is a vector of fundamental voltages (state);  $\boldsymbol{\eta}$  is a vector of error;  $\mathbf{h}$  is a vector function depending on the system modeling. The three phase state estimator is formulated by selecting the system state in terms of the voltages in all three phases, the three phase measurements and the three phase system model. The three phase state and the three phase measurements are described here. The three phase system model is addressed in section 6.

##### 5.1 Three Phase System State

Similar to the conventional state estimation, the voltages are defined as system state. The difference is that we use node (phase) voltages versus bus voltages in the conventional state estimation. Specifically, the node (phase) voltage is expressed as:

$$v_i(t) = \text{Re}\{e^{j\omega t}(v_{real,i} + jv_{imag,i})\} \quad (12)$$

The set of variables  $v_{real,i}$  and  $v_{imag,i}$  are state variables, one set for each phase. Here the rectangular coordinate system is used for convenience. The number of state variables for a bus are  $3 \times 2 = 6$ .

##### 5.2 Three-Phase Measurements

The measurement set consists of synchronized sampled waveforms. The synchronization ensures the exact time of the sample with accuracy of 1  $\mu$ sec. From the sampled waveforms, the quantities  $z_{real,i}$  and  $z_{imag,i}$  are computed which constitute the measurements in accordance to the following postulated model:

$$z_i(t) = \text{Re}\{e^{j\omega t}(z_{real,i} + jz_{imag,i})\} \quad (13)$$

Real and reactive power measurements are not used in the proposed three phase state estimator for the following reason: since voltage and current is measured, and since the real power and reactive power is derived from these measurements, all the information needed is included in the V and I waveform measurements. It certainly does not mean that the real and reactive power measurements can not be processed in the HSE, but they do not provide additional information.

The measurements,  $\mathbf{z}$ , are related to the state variables with the equations (2) and (3) respectively.

$$\tilde{\mathbf{z}}_{\text{current}} = \tilde{\mathbf{Y}}\tilde{\mathbf{x}} + \text{error} \quad (14)$$

$$\tilde{\mathbf{z}}_{\text{voltage}} = \tilde{\mathbf{T}}\tilde{\mathbf{x}} + \text{error} \quad (15)$$

where overstrike  $\sim$  means complex value. *Error* is also a complex value. Matrix  $\tilde{Y}$  is an admittance matrix of proper dimensions.  $T$  is a matrix whose entries are either 1 or 0. If the measured state variables are ordered first in  $\tilde{x}$  in the same order as in  $\tilde{z}$ , then matrix  $T$  has the form  $[I \mid 0]$  with identity matrix  $I$  and zero matrix  $0$  having proper dimensions. Equations (2) and (3) can be lumped into one equation below:

$$\tilde{z} = \tilde{H}\tilde{x} + \tilde{r} \quad (16)$$

Equation (16) is linear. The least square estimation requires only one iteration (direct solution). This advantage comes from the use of the rectangular coordinate system.

### 5.3 Least Squares Estimation

The least square estimation is formed as an optimization problem:

$$\begin{aligned} \text{Minimize: } J &= \tilde{r}^H W \tilde{r} \\ \text{Subject to: } \tilde{r} &= \tilde{z} - \tilde{H}\tilde{x} \end{aligned} \quad (17)$$

where superscript  $H$  means Hermitian transpose. By separating the complex variables into real and imaginary parts, problem (5) is transformed to :

$$\begin{aligned} \text{Min: } & r_{\text{real}}^T W r_{\text{real}} + r_{\text{imag}}^T W r_{\text{imag}} \\ \text{S.t. } & r_{\text{real}} = z_{\text{real}} - (H_{\text{real}} x_{\text{real}} - H_{\text{imag}} x_{\text{imag}}) \\ & r_{\text{imag}} = z_{\text{imag}} - (H_{\text{real}} x_{\text{imag}} + H_{\text{imag}} x_{\text{real}}) \end{aligned} \quad (18)$$

The solution is obtained from the following equation via a Cholesky factorization and a forward and back substitution:

$$\begin{bmatrix} A & B \\ -B & A \end{bmatrix} \begin{bmatrix} x_{\text{real}} \\ x_{\text{imag}} \end{bmatrix} = \begin{bmatrix} H_{\text{real}}^T W z_{\text{real}} + H_{\text{imag}}^T W z_{\text{imag}} \\ H_{\text{real}}^T W z_{\text{imag}} - H_{\text{imag}}^T W z_{\text{real}} \end{bmatrix} \quad (19)$$

### 6. Three-Phase Power System Model

For any estimation problem, a model must be hypothesized which relates the measurements to the state of the system. This task can be achieved by considering individual system components. The major system components are transmission lines, transformers, generators, and AC/DC converters. All components are classified into linear and nonlinear. If current waveform meters are placed at all interfaces with nonlinear devices, the model relating measurements to the state of the system is linear[3]. For this reason, only linear devices need to be modeled in the present implementation of the Three-Phase State Estimation. The postulated model for each linear device is:

$$\tilde{I}_d = \tilde{Y}_d \tilde{V}_d + \tilde{I}_{s,d} \quad (20)$$

where

$\tilde{I}_d$  are the currents at the terminals of the device

$\tilde{V}_d$  are the voltages at the terminals of the device

$\tilde{I}_{s,d}$  are equivalent current sources

$\tilde{Y}_d$  is the admittance matrix of the device

Note that for passive devices (i.e. a line),  $\tilde{I}_{s,d}$  is zero.

The admittance matrix  $\tilde{Y}_d$  contains the modeling detail for a device. Consider for example a transmission line. Rigorous modeling of a line yields a matrix  $\tilde{Y}_d$  which corresponds to a nonsymmetric system, i.e. each phase exhibits different admittance. Yet in conventional state estimation, a line is represented with its positive sequence admittance, i.e. we assume the line to be symmetric. In addition, the currents  $\tilde{I}_d$  and voltages  $\tilde{V}_d$  may not be balanced. The importance of asymmetry and imbalance has been quantified earlier.

### 7. Quality Evaluation of the Three-Phase SE

The overall accuracy and performance of the Three-Phase State estimator is evaluated by using the concept of the confidence level. Typically, the confidence level is used to measure the goodness of fit of the measurements to the system model. Reference [7] describes methods to quantify the quality of measurements and resulting errors versus confidence level. Reversely, the confidence level can be used to quantify the goodness of fit of the system model to system measurements. The confidence level is computed by arguing that the objective function of problem (5) is chi-squared distributed with  $m-n$  degrees of freedom if  $r_i$ 's are normalized independent Gaussian variables with zero mean. In this case, the chi-square distribution provides a quantitative measure of quality of the measurement system as follows: Let the value of the objective function  $J$ , computed at the estimated  $x^*$ , be  $z_1$ . Obviously, since  $x^*$  minimizes the objective  $J$ , any other state vector  $x$  will yield a larger value of  $J$ , i.e.,  $J(x) \geq z_1$ . The probability  $\text{Pr}[J \geq z_1]$  is provided by the chi-square distribution and expresses a measure of how well the measurements fit the model. This probability is called confidence level. In summary, the confidence level is computed with the following steps:

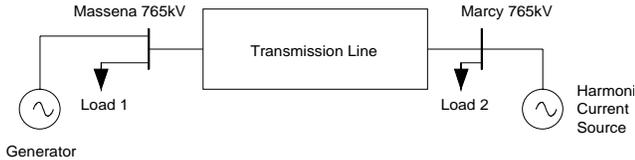
- Step 1: Compute the harmonic state estimate  $x^*$
- Step 2: Evaluate the function  $J^* = [h(x^*) - z]^H W [h(x^*) - z] = a$
- Step 3: Compute  $p = 1.0 - P(a, m-n)$ , where  $P(a, u)$  is the probability that  $J \leq a$ ,  $u$  is the degree of freedom

Quite often, the deviation of the estimated state is used to measured the accuracy of estimation. It should be emphasized that this is meaningful only when the confidence level is high.

### 8. Test Results

The HMS has been used to evaluate the effects of system asymmetry, system imbalance, and quality of estimate. For clarity of presentation, we use a subsystem of the HMS, namely the New York Power Authority 765kV line between MARCY and MASSENA to demonstrate some of our

findings. Both ends of this line are instrumented with a site workstation of the HMS, thus providing complete measurements at the interface of this subsystem with the rest of the system. Figure 8 illustrates a single line diagram of the test system. The transmission line data can be found in [12]. The purpose of this test is to evaluate the effect of different transmission line models on the three phase state estimation.



**Figure 8. Test System Single Line Diagram**

The effect of the transmission line models and measurement schemes are assessed by computing the confidence level, and the sensitivity indexes. Specific analyses are:

1. Perform the three-phase state estimation using three phase unsymmetric transmission line model and three phase measurements. Obtain the confidence level.
2. The three-phase state estimation is executed again using symmetric transmission line model and three phase measurements. The new confidence level is computed.
3. The three-phase state estimation is executed again using three phase unsymmetric transmission line model and single phase measurements. The new confidence level is computed.

Table 1 reports the computed confidence level. The confidence level could be low if bad measurements exist or system components are modeled poorly or both. Since all other parameters were kept constant except the line model, the results indicate the effect of the line model. Note that when using the three phase asymmetric line model the confidence level is very high, 100% as shown in first row of Table 1. We can see that the confidence level drops dramatically if a symmetric transmission line model is used (second row of Table 1). Finally if an asymmetric model with single phase measurements are used (phase A only) the confidence level drops to zero (third row, Table 1). Next we computed the bias resulting from the line asymmetry and the bias resulting from the imbalance in the measurements (see equations ( ) and ( )). The results are illustrated in Table 2. Note that the biases are substantial compared to the expected precision of state estimates.

**Table 1. Confidence Level of Using Different Transmission Line Model**

	Confidence Level (%)
Three Phase Unsymmetric Model, Three Phase Measurements	100.0
Three Phase Symmetric Model, Three Phase Measurements	13.02
Three Phase Unsymmetric Model, Single Phase Measurements	0.0

**Table 2. Computed SE Biases Due to Model Asymmetry and Measurement Unbalance**

	Magnitude	Phase
<b>Computed Bias Due to Line Asymmetry</b>	0.06%	0.028 deg.
<b>Computed Bias Due to Measurement Imbalance</b>	0.14%	0.095 deg.

## 9. Conclusions

The conventional state estimation has inherent biases resulting from system operational imbalance and system model asymmetries. We presented equations for quantifying the biases in conventional state estimation. In addition, we presented a proposed three phase state estimation that is based on synchronized measurements, three phase instrumentation and asymmetric three phase power system model. This state estimator does not exhibit the biases discussed earlier and it is direct, i.e. it does not require an iterative algorithm to obtain the solution. Using this system, we have presented numerical results that quantify the performance of this estimator. The system has been partially implemented in the New York Power Pool system. As technology advances, this approach becomes more feasible for practical systems. In the meanwhile, we have seen hybrid state estimators, i.e. estimators that are partially based on the conventional approach and partially on synchronized and three phase measurements. Hybrid approaches are not discussed in this paper.

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