Extended Factors for Linear Contingency Analysis

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Abstract

This paper presents preliminary results on three new tools to quickly assess the impact of line outages and reclosure on generators. The first deals with the estimation of the angle across the breaker of an opened line. The second deals with the estimation of the immediate currents that arise in generators in response to a line outage. The third deals with the estimation of the immediate currents that arise in generators in response to a line closure. The first would be used to determine if a reclosing relay might block a reclosure. The second would be used to determine if a line outage might damage a generator. The third would be used to determine if the override of a blocked reclosure might be allowable. The concepts are illustrated on test cases.

1. Introduction

Linear methods for contingency analysis have been in use for many years [1-6]. They remain the primary tool for estimating the impact of an outage in many security analysis programs. Despite their long history, their capabilities have not been fully exploited in several key applications.

In most cases, these linear methods approximate the full nonlinear power-flow solution to produce the "long-term" steady-state conditions after an outage. They are used primarily to verify that thermal line limits are not exceeded after an outage. This paper addresses several different issues associated with potential outages. It focuses on the impact of a line outage (or reclosure) on generator currents.

When a line is outaged, the currents throughout the network change virtually instantaneously to satisfy Kirchhoff's current and voltage laws before any generator or load dynamics begin. If these immediate current changes are too large, damage to the generator shafts can occur. When these currents appear in each generator, they will create power mismatches that will either accelerate or decelerate the shafts. If the system is stable, the dynamics should settle to new conditions that are only slightly different from the pre-outage case. Traditional power-flow methods (linear or nonlinear) will compute only this final system state – not the potentially dangerous instantaneous redistribution of current.

In addition, certain line reclosing relays contain blocking signals that will not allow reclosure when the angle difference across the open breaker exceeds a specified value (perhaps 45 degrees). The purpose of this blocking is to avoid possible damage to generator shafts as a result of large currents which may arise when reclosure is done when pre-closure angles are large.

The paper begins with a review of the theory and formulation of traditional distribution factors. These factors are then extended to include the estimation of line-outage angles as well as potential changes in generator currents due to line outages and reclosures.

2. Power transfer distribution factors (PTDF)

The basis for this analysis begins by considering linear circuits with voltage and current sources interconnected by impedances. Consider an n-bus plus ground network modeled with the admittance matrix referenced to ground. For an operating condition called case A, Kirchhoff's current law at each bus is:

$$\begin{bmatrix} I_1^A \\ \vdots \\ I_n^A \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \cdots & \vdots \\ Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1^A \\ \vdots \\ V_n^A \end{bmatrix} \tag{1}$$

with each bus injection current I_i^A coming from the ground through a path not included in Y. The Y matrix may include any line, transformer, load admittance connected between any two buses or between a bus and ground.

For a generator, this injection current is the generator current. For a load (not included in Y), this injection current is the negative of the load current. All quantities are in per unit.

For this analysis, let bus 1 be an ideal voltage source with voltage fixed for all cases as:

$$V_1 = V_1^0 \quad (2)$$

Eliminating the bus 1 current from the network model gives,

$$\begin{bmatrix} I_2^A \\ \vdots \\ I_n^A \end{bmatrix} = \begin{bmatrix} Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots \\ Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_2^A \\ \vdots \\ V_n^A \end{bmatrix} + \begin{bmatrix} Y_{21} \\ \vdots \\ Y_{n1} \end{bmatrix} V_1^0 \quad (3)$$

Solving for the case A voltages gives

$$\begin{bmatrix} V_2^A \\ \vdots \\ V_n^A \end{bmatrix} = \begin{bmatrix} Z_{22} & \cdots & Z_{2n} \\ \vdots & \cdots & \vdots \\ Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_2^A - Y_{21} V_1^0 \\ \vdots \\ I_n^A - Y_{n1} V_1^0 \end{bmatrix} \quad (4)$$

The line currents for case A are:

$$I_{ij}^A = \frac{V_i^A - V_j^A}{\bar{z}_{ij}} \quad (5)$$

where $\bar{z}_{ij}$ is the primitive line *ij* impedance. Now consider changes in injection currents from case A to case B. The case B network equations (for unchanged impedances) are:

$$\begin{bmatrix} V_2^B \\ \vdots \\ V_n^B \end{bmatrix} = \begin{bmatrix} Z_{22} & \cdots & Z_{2n} \\ \vdots & \cdots & \vdots \\ Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_2^B - Y_{21} V_1^0 \\ \vdots \\ I_n^B - Y_{n1} V_1^0 \end{bmatrix} \quad (6)$$

The line currents for case B are:

$$I_{ij}^B = \frac{V_i^B - V_j^B}{\bar{z}_{ij}} \quad (7)$$

From equations (4)-(7) the change in voltages and line current *ij* between cases A and B are:

$$\begin{bmatrix} \Delta V_2 \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{22} & \cdots & Z_{2n} \\ \vdots & \cdots & \vdots \\ Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \vdots \\ \Delta I_n \end{bmatrix} \quad (8)$$

$$\Delta I_{ij} = I_{ij}^B - I_{ij}^A = \sum_{k=2}^{n} \left[\frac{Z_{ik} - Z_{jk}}{\bar{z}_{ij}} \right] \Delta I_k \quad (9)$$

where

$$\Delta I_k = I_k^B - I_k^A \quad (10)$$

In cases where bus i or j equal 1, the entries of Z are defined to be zero.

This change can be written as

$$\Delta I_{ij} = \sum_{k=2}^{n} T_{ij,k} \Delta I_k \quad (11)$$

where

$$T_{ij,k} \triangleq \left(\frac{Z_{ik} - Z_{jk}}{\bar{z}_{ij}} \right) \quad (12)$$

is a “current” transfer distribution factor (CTDF). In power flow studies, it is customary to convert these to “power” transfer distribution factors by neglecting resistance, and assuming voltages to be near unity and define the *PTDF* as:

$$PTDF_{ij,k} \triangleq \left(\frac{X_{ik} - X_{jk}}{\bar{x}_{ij}} \right) \quad (13)$$

with the change in line real power flow in response to real power injection changes approximated as:

$$\Delta P_{ij} \approx \sum_{k=2}^{n} PTDF_{ij,k} \Delta P_k \quad (14)$$

For the derivations presented in the following sections, the change from case A to case B will always be done with constant impedances and constant topology. For example, the change due to a line outage is not the change from case A to case B. Fictitious changes in injections (i.e. from case A to case B) will be used to compute the impact of a line outage on flows and voltages. These injection changes are applied to networks either with the line "IN", or the line "OUT". As such, the factors will be labeled to reflect this change.

3. Line outage distribution factor (LODF)

The current transfer distribution factors above can be used to approximate the effects of a line outage as follows. Consider a change in the injection current at bus *i* equal to:

$$\Delta I_i = -\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \tag{15}$$

where I_{ij}^{IN} is the flow in line *ij* before the line outage, and $T_{ij,i}^{IN}$ is the current transfer distribution factor before the line outage. This injection will zero the flow in line *ij*, and cause the following change in all other lines.

$$\Delta I_{ab} = -T_{ab,i}^{IN}\left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}}\right) \tag{16}$$

With the line flow zeroed, the line *ij* may be opened without any resulting change in flows. The new network equations with line *ij* removed have new current transfer distribution factors denoted as $T_{ab,i}^{OUT}$. When the injection of (15) is removed (after the line is outaged) to restore all injections to their original values, the total change in line flow is:

$$\Delta I_{ab} = -T_{ab,i}^{IN}\left(\frac{I_{ik}^{IN}}{T_{ij,i}^{IN}}\right) + T_{ab,i}^{OUT}\left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}}\right) \tag{17}$$

or

$$\Delta I_{ab} = \left(\frac{T_{ab,i}^{OUT} - T_{ab,i}^{IN}}{T_{ij,i}^{IN}}\right) I_{ij}^{IN} \tag{18}$$

Neglecting resistance again and assuming voltages to be near unity, the *LODF* is defined as:

$$LODF_{ab,ij} \triangleq \frac{PTDF_{ab,i}^{OUT} - PTDF_{ab,i}^{IN}}{PTDF_{ij,i}^{IN}} \tag{19}$$

with the change in line real power flow in response to a line outage approximated as:

$$\Delta P_{ab} \approx LODF_{ab,ij} P_{ij}^{IN} \tag{20}$$

An equivalent derivation is given in [5] using "DC" load flow assumptions.

4. Line closure distribution factor (LCDF)

The current transfer distribution factors can also be used to approximate the effects of a line closure. For this case it is necessary to find an injection that makes the voltages equal at each end of the line which is to be closed. From the above equations for voltage, the injection needed at bus *i* to make the voltages at bus *i* and bus *j* equal is:

$$\Delta I_i = \left(\frac{V_j^{OUT} - V_i^{OUT}}{Z_{ii}^{OUT} - Z_{ji}^{OUT}}\right) \tag{21}$$

where Z_{ii}^{OUT} and Z_{ji}^{OUT} are from the network with the line out (before closure).

The change in line current I_{ab} due to this initial injection change is found by multiplying the injection current change times the "line out" current distribution factor. Since the voltages at each end of the line are now equal, the line may be closed without impacting the system. Then the initial injection change must be removed. The additional change in I_{ab} due to the removal of the original injection change is found by multiplying the injection current change times the negative of the "line in" current distribution factor giving:

$$\Delta I_{ab} = (T_{ab,i}^{OUT} - T_{ab,i}^{IN})\left(\frac{V_j^{OUT} - V_i^{OUT}}{Z_{ii}^{OUT} - Z_{ji}^{OUT}}\right) \tag{22}$$

or

$$\Delta I_{ab} = \left(\frac{T_{ab,i}^{IN} - T_{ab,i}^{OUT}}{T_{ij,i}^{OUT}}\right)\left(\frac{V_i^{OUT} - V_j^{OUT}}{\bar{z}_{ij}}\right) \tag{23}$$

Again neglecting resistance, the line closure distribution factor is defined as:

$$LCDF_{ab,ij} \triangleq \frac{PTDF_{ab,i}^{IN} - PTDF_{ab,i}^{OUT}}{PTDF_{ij,i}^{OUT}} \tag{24}$$

and can be used to approximate the change in real power line flow in response to a line closure as:

$$\Delta P_{ab} \approx LCDF_{ab,ij} P_{ij}^{\wedge} \tag{25}$$

where

$$P_{ij}^{\wedge} \triangleq \text{Real}\ \{V_i^{OUT} \left(\frac{V_i^{OUT} - V_j^{OUT}}{\bar{z}_{ij}} \right)^*\} \tag{26}$$

where * denotes complex conjugation.

5. Line outage angle factor (LOAF)

Looking at the change in bus voltage angles as a result of a line outage, we recall that the voltage changes at bus i and bus j in response to the injection current change of (15) are

$$\Delta V_i = -Z_{ii}^{IN} \frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \tag{27}$$

$$\Delta V_j = -Z_{ji}^{IN} \frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \tag{28}$$

When the injection change is removed in the outaged system, the total change in bus voltages is:

$$\begin{aligned} \Delta V_i &= -Z_{ii}^{IN} \left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \right) + Z_{ii}^{OUT} \left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \right) \\ \Delta V_j &= -Z_{ji}^{IN} \left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \right) + Z_{ji}^{OUT} \left(\frac{I_{ij}^{IN}}{T_{ij,i}^{IN}} \right) \end{aligned} \tag{29}$$

The change in voltage between buses i and j due to the outage of line ij is then:

$$\begin{aligned} \Delta V_i - \Delta V_j &= \left(\frac{Z_{ii}^{OUT} - Z_{ji}^{OUT}}{T_{ij,i}^{IN}} \right) I_{ij}^{IN} \\ &\quad - \left(\frac{Z_{ii}^{IN} - Z_{ji}^{IN}}{T_{ij,i}^{IN}} \right) I_{ij}^{IN} \end{aligned}$$

$$= \left(\frac{Z_{ii}^{OUT} - Z_{ji}^{OUT}}{Z_{ii}^{IN} - Z_{ji}^{IN}} \bar{z}_{ij} - \bar{z}_{ij} \right) I_{ij}^{IN} \tag{30}$$

Since the original network (with line ij in service) had

$$V_i^{IN} - V_j^{IN} = \bar{z}_{ij} I_{ij}^{IN} \tag{31}$$

the difference between voltages at buses i and j after line ij is removed is:

$$\begin{aligned} V_i^{OUT} - V_j^{OUT} &= \Delta V_i - \Delta V_j + V_i^{IN} - V_j^{IN} \\ &= \left(\frac{Z_{ii}^{OUT} - Z_{ji}^{OUT}}{Z_{ii}^{IN} - Z_{ji}^{IN}} \right) \bar{z}_{ij} I_{ij}^{IN} \end{aligned} \tag{32}$$

Using the approximations:

$$\cos \theta_i \approx 1 \tag{33}$$

$$\cos \theta_j \approx 1 \tag{34}$$

$$\sin \theta_i \approx \theta_i \tag{35}$$

$$\sin \theta_j \approx \theta_j \tag{36}$$

$$I_{ij}^{IN} \approx P_{ij}^{IN} \tag{37}$$

and neglecting resistance again, the angle difference across the opened line ij is approximately,

$$\theta_i - \theta_j \approx LOAF_{ij}\ P_{ij}^{IN} \tag{38}$$

where the line outage angle factor is defined as:

$$LOAF_{ij} \triangleq \left(\frac{X_{ii}^{OUT} - X_{ji}^{OUT}}{X_{ii}^{IN} - X_{ji}^{IN}} \right) \bar{x}_{ij} \tag{39}$$

$$= \left(\frac{PTDF_{ij,i}^{OUT}}{PTDF_{ij,i}^{IN}} \right) \bar{x}_{ij} \tag{40}$$

6. Line outage generation factor (LOGF)

For this analysis, we arbitrarily assume that buses numbered 1,…,m are generator terminal buses and m+1,…,n are load buses.

In the above LODF derivation, the assumption of constant current (power) at each generator reflects the new steady-state condition of constant scheduled output. This will normally not be the case for the small time between when the line is opened and when the controls react to preserve initial set points. During this time, dangerous currents can exist in some of the system generators, causing undesirable shaft torques. To compute these temporary, but potentially dangerous currents, the generators need to be modeled as constant voltages behind respective transient reactances. When the line is outaged, the new currents immediately change to match the constraint of constant internal generator voltage (magnitude and angle).

The above LODF derivation can be directly applied to this case where the objective is to predict generator currents. The addition of the transient reactance at each generator bus creates a branch flow for which we wish to compute the current after any line outage. To reflect the constraint of constant internal voltage, the analysis proceeds exactly as before with the exception that the CTDF's and changes in generator currents are computed as follows:

(a) Add $1/jX_d'$ (generator transient reactance) to the diagonal of the nxn Y_{BUS} (including bus 1 – a generator).
(b) Compute the CTDF's using the full nxn Z_{BUS} in (12).
(c) The generator current changes are estimated using (18) with $Z_{ij} = 0$ for subscripts i.j = n+1,…,n+m.

The subscripts n+1,…,n+m are the internal, constant-voltage buses of the m generators. Since these buses are constant voltage, they are not included in the Y_{BUS} (as was the case where bus 1 was not included in (3)).

7. Line closure generation factor (LCGF)

For the closure of a line, the immediate change in generator currents is computed in the same manner as in section 6. above, only using (23) to compute the change in currents. These currents will be the currents which appear in each generator immediately after the switch is closed (before generator control actions). The currents are however "steady'state" currents that result from the switching action – not the transient currents.

8. Illustrations

Preliminary results were obtained on the 3-machine, 9-bus system of Figure 1, with data from [7].

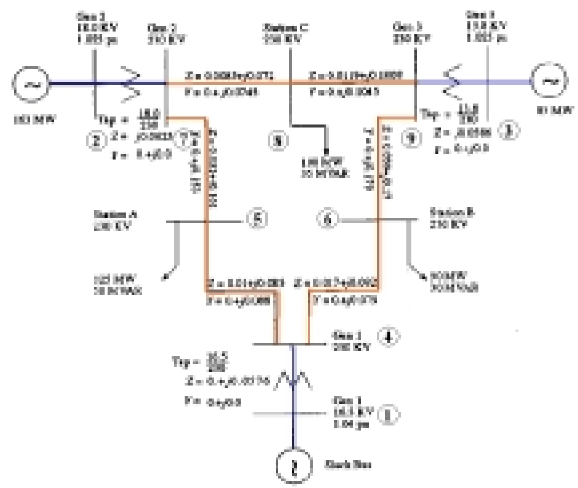

Figure 1. Test System

Figure 2. shows a comparison of the LOAF with a full power flow solution for the outage of six different lines. In each case, the error in the linear angle factors does not exceed about 15%.

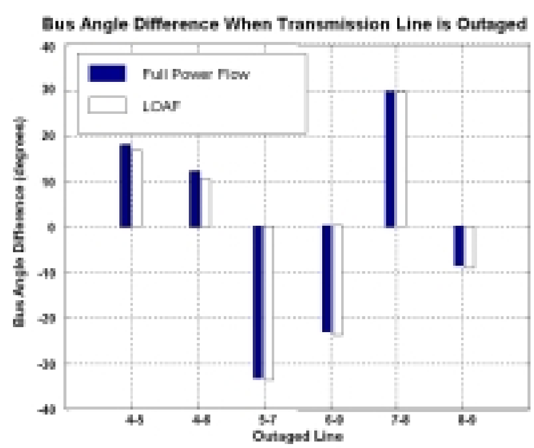

Figure 2. LOAF Comparisons

Figures 3-5 show the generator current predictions immediately following the outage of six lines using the LOGF. Comparisons with exact values using a dynamic simulation have not yet been made. In these cases, the load flow Ybus matrix was augmented to include the internal transient reactances X'_d as given in [7]. Additional analysis also needs to be performed to determine the importance of adding these reactances to the analysis. It is possible that ordinary load-flow data may be sufficient for reasonable estimations of generator currents.

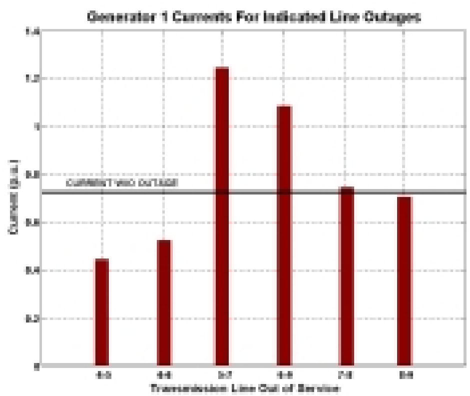

Figure 3. Generator 1 currents

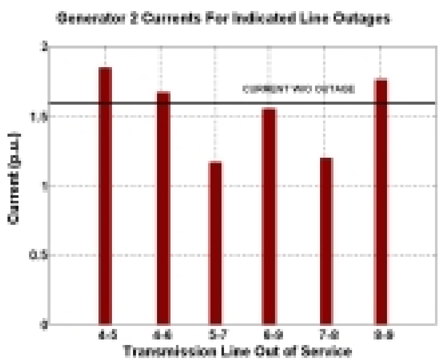

Figure 4. Generator 2 currents

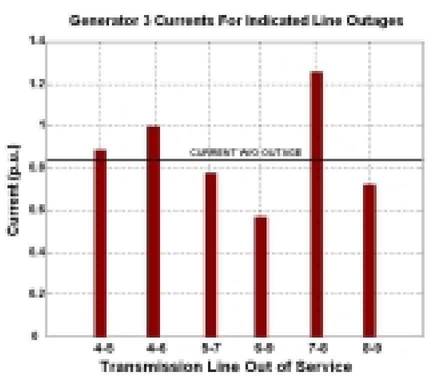

Figure 5. Generator 3 currents

The linear factors indicate that there can be significant immediate changes in generator currents following a line outage. These results need to be verified with full dynamic simulation. Similar results need to be computed for the line closure factors.

9. Conclusions

The extension of linear factors for contingency analysis to predict opened-line angles and immediate generator currents was presented as a straightforward utilization of traditional linear load flow techniques. These additional factors show promise as new tools for rapid contingency analysis that includes consideration of line outage impact on generators. More extensive comparisons with exact nonlinear solutions is needed to fully evaluate the usefulness of these tools.

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11. References

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