Capturing Non-Convexities in a Multi-Unit Electricity Auction

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Abstract

As electricity auctions are being created around the world in newly deregulated electricity supply industries, several questions regarding the design of these auctions are being raised. This paper analyzes the ability of various electricity auction mechanisms to satisfy demand while attaining productive efficiency, i.e., minimizing total generation costs. Four possible auction mechanisms are considered and their performance is evaluated under three demand scenarios. Only a horizontal sequential auction is found to support an efficient equilibrium bidding strategy in all three demand scenarios.

1 Introduction

California's decision to deregulate its electricity supply industry, effective January 1, 1998, has caused an upheaval in a traditionally regulated industry. One of the many challenges facing the deregulated electricity supply industry is to create an appropriate medium through which electricity buyers and sellers can actively trade electricity. One of the goals of deregulation is to create a competitive decentralized market for generation, and hence much of the success of the deregulation rides upon the choice of the correct medium for this trade.

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Auction based mechanisms for electricity dispatch have been implemented in the United Kingdom, Norway and Australia. California is following in their steps and has created an electricity auction, the Power Exchange, whose role it is to facilitate the matching of electricity supply with demand on a daily basis. This will be done via an auction (similar to a commodity exchange), held one day before the day in question, where generators will be allowed to submit their supply and consumers[1] to submit their demand for energy the following day. Generators and consumers will submit their bids for day t's generation and consumption in an auction held on day $t-1$. The auctioneer collects the bids and, on their basis, determines the market clearing prices for electricity.

An electricity auction can be viewed as a multi-unit auction with private valuations where there may exist complementarities in cost over several units. This characterization of an electricity auction allows us to model the interactions of bidders (generators) in a game-theoretic framework and better understand the interplay between the auction (game) rules and the generators' optimal, equilibrium strategies.

There are several variable dimensions to an electricity auction which should be considered by the auction designers when designing the auction, e.g., pricing rules, how to bundle demand into lots for auction, and sequencing of auctions. Each resulting auction mechanism will affect the incentives bidders face and the strategies they will use when bidding in generation supply. This paper analyzes the ability of various auction mechanisms to result in the dispatch that minimizes total generation costs, referred to as the *efficient dispatch,* by identifying the generators' bidding behavior when the electricity auction is conducted only once[2]. Empirical evidence given by Wolak and Patrick (1997) indicates that generators in the England and Wales market took advantage of the capacity-declaration portion of their multi-dimensional bids by not bidding truthfully in order to extract exorbitant rents during some hours of the year. Therefore it is imperative to understand the effect a particular auction mechanism will have on generators' bidding strategies.

There are several electricity industry-specific characteristics that make designing an auction which induces the efficient dispatch a formidable challenge. For example, electricity cannot be stored; generation costs are nonconvex; there exist several different generation technologies; gener-

[1] Consumers may be themselves generation companies or distribution companies.

[2] In reality, the electrcity auction will be repeated daily.

ators have constraints on their generation (capacity constraints), and the shape of daily demand is constantly changing. Two previous papers, Elmaghraby and Oren (1997) and Elmaghraby(1997), analyze the productive efficiency of various auction mechanisms in a complete information framework. They found that while the California auction design can support inefficeincy in equilibrium and hence cannot guarantee efficiency, an alternative auction design, a horizontal auction[3], can guarantee efficiency in pure-strategy equilibria. This paper extends the analysis into an incomplete information framework. It can be argued that, while generators do have knowledge of their opponents' generation technologies[4], the exact make-up of the generation plants that will be bidding in any particular auction is not certain and is better modeled in an incomplete information framework. In addition, generators contract for a large portion of the fuels needed in the generation of electricity. The exact nature of the contracts and the exact price at which they are struck are private information of the contracting generator, and hence introduce an element of cost uncertainty. In this paper, I readdress the question of the ability (or lack thereof) of an auction mechanism to induce profit-maximizing generators to bid so as to result in the unique efficient dispatch in a *one-shot incomplete information framework.*

One of the main observations of this paper is that the more closely the shape of demand lots mimics the capacity structure of generators, the greater the ability of an auction to result in an efficient dispatch. This result is quite intuitive, for if we are able to bundle demand into equal lots whose size equals that of generators' capacities, all the non-convexities of generation costs are captured in the lot and the auction reduces to a multi-unit auction of homogeneous objects with independent valuations across different units(lots)[5] where generators wish to win at most one object.

However, the need to standardize the electricity auction, i.e., define pre-set demand lots that are not changed on a daily basis, combined with the changing shape of daily demand imply that there

[3]Referred to as a $1 - horizontal$ auction in Elmaghraby (1997).

[4]Investor-Owned Utilities, which account for 77% of the total GW generated in California (California Energy Commission Report, November 1995, Table 6-2), have to annually submit a public report declaring their different generation plants, its associated costs, generation capacity, etc.

[5]McAfee and Vincent (1993) found that if bidders exhibit NDARA, then the units will be allocated to the bidders who value them the most. In the case of a procurement auction, such as an electricity auction, this result can be interpreted as the least-cost generators will win and productive efficiency will be attained.

will exist times when interdependencies between demand lots arise. Hence, I am concerned with designing a standardized electricity auction that is robust "enough" to changes in daily demand to be able to provide generators with the right incentives so as to result in the efficient dispatch. Due to the added complexity of studying multi-unit auctions in an incomplete information framework, it is very difficult to identify an auction mechanism which guarantees productive efficiency in equilibrium for all demand, and hence complementarity, scenarios. Therefore my search is for an auction mechanism that will induce the efficient dispatch under *simple complementarity* (synergy) scenarios.

In section 2, I provide the reader with some background literature on multi-unit auctions. I then go on to characterize an electricity auction as a multi-unit auction with private valuations which are possibly dependent over several units, and outline the different auction mechanisms considered and the cost characteristics of generators. In section 3, I present the model of demand and costs used throughout the paper and present my results in section 4.

2 Background

2.1 Literature Review

In order to appreciate the new and interesting questions posed by an electricity auction, it is important to examine its place in the existing auction literature. The largest portion of auction literature looks at models where bidders desire at most one object (McAfee and McMillan (1987) provide an excellent survey of the auction literature). As I shall explain in more detail in the next section, an electricity auction is a multi-unit demand, private valuations auction where there may exist complemenataries across units[6]. As many researchers are pointing out, it is not possible to carry over the results from single unit auctions and apply them to an electricity auction.

There are a few papers that study multi-unit auctions with complementarites which were inspired by the recent FCC spectrum auction. In the FCC auctions, bidders, comprised of US

[6]Two objects are said to be complements (have superadditive value, or exhibit synergies) when their valuation independently is less then when combined, (i.e., if $v[\omega]$ is a bidder's value for object ω, and $v[\tau]$ is its value for object τ, then $v[\omega] + v[\tau] < v[\omega + \tau]$).

telecommunication companies, cellular telephone companies, and cable-television companies, competed to win various spectrum licenses for different geographical area. The synergies arising from owning licenses in adjoining geographical area create dependencies in (some) bidders' valuations for individual licenses (see McMillan (1994), Cramton (1995) and McMillan and McAfee (1996) for further discussion of the FCC spectrum auctions). Using the FCC spectrum auctions as their motivation, Krishna and Rosenthal (1996) study auctions where there are two types of bidders, global and local. Global bidders desire more than one object and their valuation for multiple objects is greater than the sum of each individual object's valuation, while local bidders desire at most one object. They are able to identify equlibrium bidding strategies when individual licenses are auctioned individually and simultaneously. They remark, however, that the equilibrium bidding strategies need not necessarily result in allocative efficiency. Ausubel and Cramton (1996) question the superior allocative efficiency properties of uniform pricing rules using Wilson's (1979) "share" auction framework with private valuations. They find that the efficiency of 2^{nd} price (uniform) auctions in a single-unit auction do not carry over to a multi-unit framework. They conclude that when bidders desire more than one object, or a large share of the total objects being aucitonned, they have an incentive to underbid or "shade" their bids, resulting in an inefficient allocation. Levin (1997) searches for optimal auction design, i.e., what is the optimal way to auction goods when there exist complementarities between the goods. He finds, in the case of two goods, that bundling the goods and auctionning them together increases the revenue of the seller, but does not necessarily lead to the efficient allocation.

Several other papers have addressed the issue of multi-unit auctions. Colwell and Yavas (1994) examine the relative revenues raised in the simultaneous and sequential auctions of adjacent land tracts. Hausch (1986) studies a two-object auction, where there are two bidders with common valuations who desire both objects. Hausch finds that the seller's revenue is greater when both objects are sold simultaneously versus sequentially. Gandal (1997), in an empirical paper, looks at the sequential auctioning of cable television licenses in Israel and concludes that there may have existed some interdependencies among licenses' valuation. Krishna(1993) examines the efficiency properties of a sequential auction of capacity to an incumbent and several potential new entrants. She finds that the sequential timing of the auctions leads to the benefits of aggregation not being realized.

Von der Fehr and Harbord's (1993) analysis of the United Kingdom's Electricity Industry is the only other study I know of that identifies an electricity auction as a multi-unit auction with private valuations and attempts to study the strategic bidding behavior of generators. von der Fehr and Harbord assume a framework with two generators who have (different) constant marginal costs of generation and whose costs are common knowledge. Demand for electricity is uncertain but its distribution is known. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium. Building on their analysis, I incorporate the presence of fixed "start-up" costs into generation costs and extend their study of bidding behavior to alternative auction mechanisms, relaxing their restrictive assumption of two generators.

2.2 Characterization of Electricity Auction

An electricity auction, such as the Power Exchange, is a double auction where consumers and suppliers actively trade. Ideally, an electricity auction would achieve allocative efficiency on the demand side and productive efficiency, i.e., minimize total generation costs, on the supply side. This paper takes a first step at evaluating the overall efficiency of an auction by focusing on its ability to provide generators with the correct incentives so as to result in a dispatch that satisfies demand and minimizes *total* generation costs. This is done by holding the demand-side of the auction to be fixed, i.e., assuming that demand is no longer bid into the auction but is forecasted, i.e., is inelastic and deterministic. In this situation, an electricity auction is a *procurement* auction, where bidding is done on the price of service (in the case, generation) and the lowest bid(s) win. The role of the auctioneer is to compile the submitted bids and determine the least-cost way to satisfy the daily demand, i.e., to determine which generators win dispatch (are chosen to generate), when and for how long. By assuming an inelastic and deterministic demand, I limit my criterion for judging an electricity auction to its ability to result in productive efficiency, i.e., the efficient use of resources[7].

What separates designing an auction for electricity from the vast body of auction literature is the mechanism of generation costs. Generators have different types of costs (e.g., ramp-up costs,

[7] I will assume, without loss of generality, that there is a unique dispatch that minimizes total generation costs.

no-load costs, etc.) which must be recovered through their sales revenues. Generation costs can be broadly classified into two groups: fixed "start-up" costs which are incurred when a generator is turned on to generate, and variable costs which are incurred with each additional MWh generated. Due to this cost mechanism, there exist cost *dependencies* in both time and quantity dimensions, i.e., the (average) cost to generate 1 MW during hour t depends upon the number of additional MW generated during hour t and other hours.

We can interpret the basic object being auctioned as a 1 MWh block of energy, where each individual MWh is indistinguishable from another except for the hour in which it occurs and its placement in the hour (i.e., the q^{th} MW of demand in hour t). Generators may wish to win several blocks of energy (by winning a block they win the right to supply that 1 MWh of demand at a price determined through the auction process) and hence bid for multiple objects. A generator's profit from being choosen to generate a MWh is the difference between the auction price it is paid and its own *private* cost for supplying the MWh. A generator may be constrained from supplying the entire demand the following day by the presence of capacity constraints on its generation level at any point in time (i.e., if a generator has a capacity of K, the maximum level of MW at which it can generate at any point in time is K MW). For most hours in the day, demand is greater than the capacity of any one generator; hence a multiple of existing generators must be chosen to supply demand. In summary, an electricity auction is a multi-unit auction with private valuations where there exists complementarites in generation across units.

2.3 Bidders

In an electricity auction, generators compete in price of generation to win dispatch. Generation technologies, for example, nuclear, coal and combined-cycle gas turbines plants, are generally characterized by four traits. First, a generator's costs to generate fall into two general categories; there is a fixed "start-up" cost incurred each time a generating plant is turned on, and variable cost per MWh once the plant is up and running. Second, there exists an inverse relationship between the start-up cost associated with a technology and its variable cost. For example, a nuclear plant has a large start-up cost but relatively small variable cost per MWh, while a gas combustion turbine (GCT) has a relatively low start-up cost, but incurs a large variable cost per MWh. A third trait of generation technologies is that each technology is least-cost for some output level. Low start-up, high variable cost technologies are the most efficient source over small output (total MWh) levels, while high start-up, low variable cost technologies are the most efficient source over higher output levels. Finally, generators have a constraint on the maximum number of MW they generate at any point in time and are unable to store electricity[8], but have few restrictions on the duration for which they can generate[9].

Figure 1 plots the total cost of generation associated with different technology types, assuming a generating plant is "switched-on" only once per day. The horizontal axis measures the total number of MWhs generated over time.

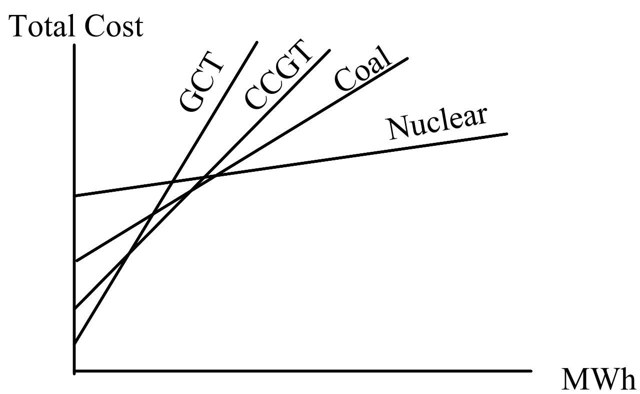

Figure 1: Costs for different generation technologies. (GCT = gas combustion turbine, CCGT = combined cycle gas turbine)

In this paper, I assume a setting with $n \geq 2$ generators. Each generator's technology is defined by its type, θ which is distributed on $[0, 1]$ with a cumulative density function $F(.)$. Assume that the density function $f(.)$ is positive and bounded everywhere on $[0, 1]$. Before the start of an auction, nature randomly assigns every generator a type from $[0, 1]$. Each generator knows its own type and the common distribution $F(.)$ from which its opponents' types is drawn.

[8]Some generating plants are able to store the *potential* for generating electricity, e.g., hydroelectric generators can store water, but generators are unable to store electricity. Therefore, there will always be a limit on the total MW a generating plant can generate.

[9]Generators do occasionally have to go off-line for maintenance, but this is not a relevant constraint over one day.

Assume the cost of a generator of technology type θ to generate q MWh is given by the cost function[10]

$$\begin{aligned} C(q,\theta) &= a(\theta) + b(\theta)q && (1)\\ a'(\theta) &< 0,\, b'(\theta) > 0 && (2)\\ \forall\theta\ \exists\ \ \hat{q}(\theta) \text{ such that } C(\hat{q}(\theta),\theta) &< C(\hat{q}(\theta),\widetilde{\theta}) \quad \forall\ \widetilde{\theta} \neq \theta && (3)\\ \hat{q}'(\theta) &< 0 && (4) \end{aligned}$$

$a(\theta)$ and $b(\theta)$ can be interpreted as the start-up and variable cost, respectively, of a generator of type θ. The form of $C(q,\theta)$ allows us to capture the inverse relationship between start-up and variable cost for different generation technologies. Figure 2 plots $C(q,\theta)$ for the case where $a(\theta) = (2-\theta)^2$ and $b(\theta) = 1.5\theta$. As assumed, each generation technology is the efficient technology for some output level.

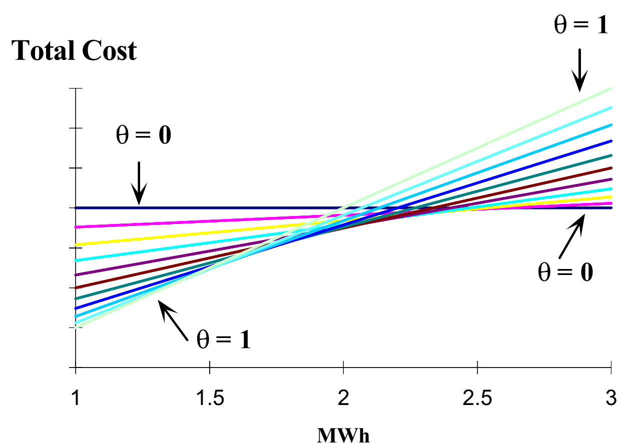

Figure 2: Total costs for different values of θ.

Assuming a total cost given by equations (1)-(4), the ranking of technologies in terms of efficiency for different output levels can be fully summarized by one of three shapes. Either the

[10]The results of this paper also hold under alternative assumptions on the form of the cost function, e.g., $a(\theta) + b(\theta)\log(q+1)$ and $a(\theta) + b(\theta)q^{\frac{1}{2}}$.

ordering of technologies in terms of efficiency is monotonically decreasing (as it is for $q = 1$ and shown in figure 3(a)), is a unimodal convex function (as it is for $q = 2$ and shown in figure 3(b)), or is monotonically increasing (as it is for $q = 3$ and shown in figure 3(c)). This characterization of efficiency proves to be quite useful later in the paper when identifying an efficient bid ordering.

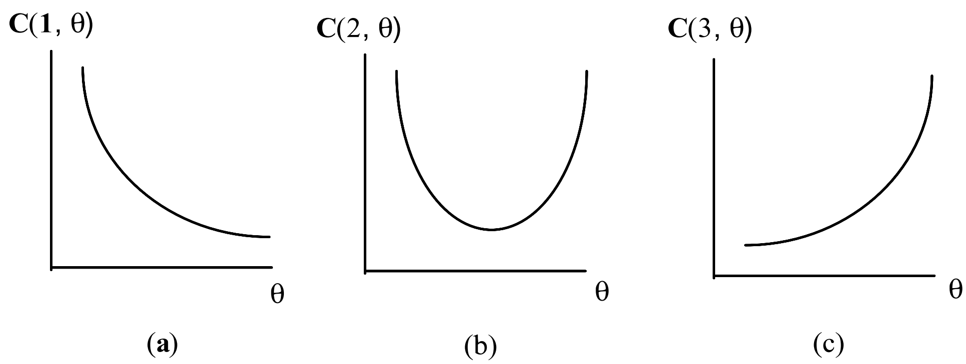

Figure 3: Cost functions for output level $q = 1, 2, 3$.

2.4 Auction mechanisms

In designing an electricity auction, the auctioneer must decide on auction characteristics such as 1) what type of bids generators will be asked to submit, 2) how to bundle demand lots, and 3) what the sequencing of the auctions shall be.

Currently in the United Kingdom, generators are asked to submit multi-dimensional bids, e.g., an energy price to generate, a capacity declaration, and ramping constraints. Instead of facilitating arriving at a central planner's efficient dispatch, the U.K.'s multi-dimensional bids have served as a vehicle for the creation of a nontransparent market and the exercise of market power, as evidenced in an empirical study of the market by Wolak and Patrick(1997). A simulation paper, by Raymond and Svoboda(1997) studies the relationship between a bid mechanism and the incentives to deviate from revealing true generation costs. They find that "perverse incentive effects are likely to be exacerbated when, in addition to perturbing market prices, a power exchange auction allows multi-part bids and non-price bid components in its bid formats."

In contrast, California has decided to limit the bids in its Power Exchange to be uni-dimensional, that is, generators will be asked to submit only an energy price reflecting the minimum price they must be paid to generate. Generators can submit one energy bid per generating plant[11] per demand lot, where the definition of the demand lot is dependent on the auction mechanism chosen by the auctioneer. Therefore, in my analysis, I restrict my attention to auction mechanisms with uni-dimensional bids.

While there are countless possible auction mechanisms, I identify four likely candidates: 1) a horizontal sequential auction, 2) a vertical sequential auction, 3) a horizontal simultaneous auction and 4) a vertical simultaneous auction.

Bundling of Demand If we interpret the basic object being auctioned as 1 MWh, then there are several possible ways to auction the objects. It is possible to allow generators to submit package bids (where each generator can bid on any bundle of demand). Under this scenario, the auctioneer would have to solve a combinatorial optimization problem in order to compute the least-cost dispatch.[12] Given the large number of individual bids under consideration, this approach would present a computational nightmare and hence the grouping of individual MWhs into predetermined demand lots must be considered.

There are two natural ways to bundle individual MWh into lots: horizontal and vertical bundling. When MWhs are vertically bundled, they are grouped according to the hour in which they occur, in which case daily demand can be viewed as being vertically partitioned and auctioned. Hence I refer to this mechanism as a *vertical auction* (see figure 4 for an example of a vertical auction). For each hour t, a generator bids the minimum price at which it is willing to generate 1 megawatt **during** hour t, where $t > 0$.

An alternative way to bundle demand is horizontally, by slice the demand load into strips where the height of the strips is small relative to generators' capacities. An example of a *horizontal auction* is illustrated in figure 5, where the height of each slice is set at 1 MW. Generators submit a bid for each lot indicating the minimum price at which they are willing to generate 1 megawatt for a **duration** of t hours, where $t > 0$ is the length of the strip .

[11] In reality, generators may submit a separate energy bid per genset. For simplification and without loss of generality, this paper assumes that a genset and plant are identical.

[12] See Rothkopf et al. (1995) for description of computationally manageable combinatorial auctions.

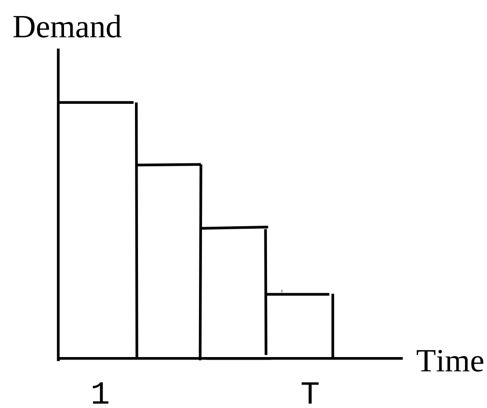

Figure 4: Vertical auction of demand.

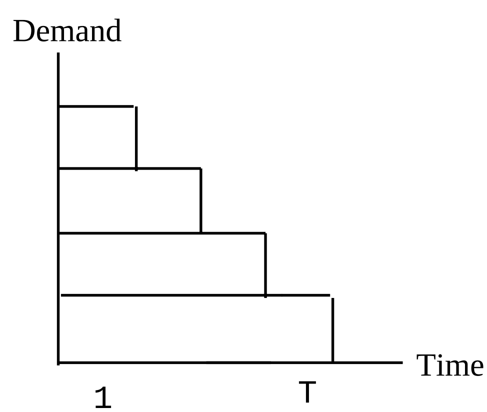

Figure 5: Horizontal auction of demand.

While there exist countless ways to bundle demand, the two bundling forms identified here are the most practical and logical to examine in an electricity auction setting. As stated earlier, the electricity auctions in operation in the United Kingdom and Australia and the proposed auction for California are vertical auctions, where generators must bid their generation into hourly markets. However, the decision to operate a plant is not made on an hourly basis. The physical characteristics of most generation plants require planning its scheduling for a duration of time. Therefore, allowing generators to bid for durations of operation is, I will argue, a more natural way to design an electricity auction.

Sequencing of Auctions When there is more than one demand lot to be auctioned, the auctioneer must decide how to sequence their sale: the auctioneer must decide whether to conduct the auctions sequentially or simultaneously. In a sequential auction, demand lots are auctioned sequentially; before each auction the results of any previous auctions are made known. I define the sequencing of the auctions such that in a sequential vertical auction, hour 1 is auctioned, followed by hours $2, 3, ...24$. In a sequential horizontal auction, the demand load is auctioned from bottom to top, i.e., the bottom slice is auctioned, then the slice directly above it, and so forth. In a simultaneous auction, the bids are submitted, and allocation decisions for all demand lots are made simultaneously.

Given these auction dimensions, there are four possible auction mechanisms: 1) a horizontal sequential auction, 2) a vertical sequential auction, 3) a horizontal simultaneous auction, and 4) a vertical simultaneous. It should be noted that the electricity auctions already operational in the United Kingdom and Australia can loosely be characterized as vertical simultaneous auctions.[13]

3 Model

Assume that the daily demand, which is inelastic and deterministic, is at two levels during the day and is given as in figure 6 (for simplicity, assume that there are only two hours in the day); demand during hour 1 is 2 MW and during hour 2 is 1 MW. While, albeit a simple demand setting,

[13]It is a loose characterization since the United Kingdom's auction allows for multi-part bids.

it is rich enough to capture the relative weaknesses and strengths of different auction mechanisms. By changing the number of plants each generator owns and each plant's capacity, it is possible to create three important demand scenarios which any electricity auction mechanism should be able to "handle" and yield the efficient dispatch. These scenarios are 1) *Dependent*: the entire demand can be supplied by one generator using the same generating plant, 2) *Independent*: the entire demand can be supplied by one generator using different generating plants, and 3) *Either-Or*: many generators must be used to satisfy demand. (The names given to each scenario characterize the demand scenario in a horizontal auction and will become clear in the next section.) An auction mechanism which is unable to yield the efficient dispatch in such a simple demand setting is highly unlikely to perform well when facing a more complex setting.

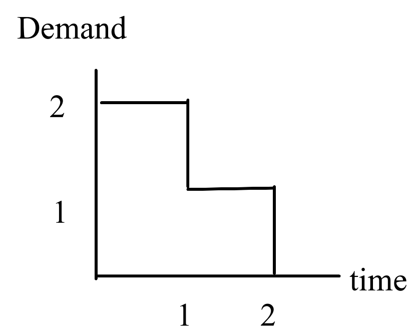

Figure 6: Daily forecasted demand.

Figures 7(a) and 7(b) illustrate the demand lots defined by a horizontal and vertical auction of demand, respectively. In both the sequential horizontal and vertical auctions, generators submit a bid for lot **2**, and after the winner is announced, submit a bid for lot **1**. In the simultaneous horizontal and vertical auctions, bids for both lots are submitted simultaneously. The bid for lot **2** (lot **1**) in a horizontal auction indicates the minimum price a generator must be paid to generate 1 MW for 2 (1) hours. The bid for lot **2** (lot **1**) in a vertical auction indicates the minimum price a generator must be paid to generate 2 (1) MW for 1 hour.

In a procurement auction, the bidder(s) submitting the lowest bid(s) wins. As stated earlier, an electricity auction where the next day's demand is forecasted is a procurement auction. Hence the generator(s) win dispatch in ascending order of their submitted bids. I assume that winning

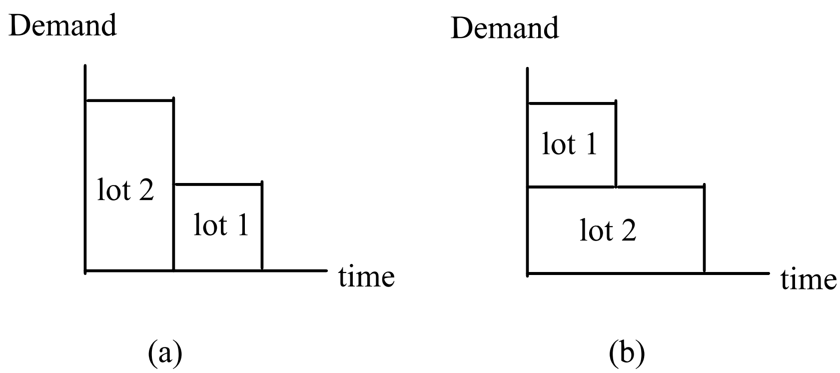

Figure 7: Vertical and horizontal auctions of demand.

generators are paid according to a 2^{nd} price rule, i.e., the winner(s) is(are) paid the lowest rejected bid in the demand lot. A generator that is chosen for dispatch is committed to generate the specified load it won or incur a large penalty.[14] Once chosen for dispatch, it may be called upon to generate at its capacity, i.e., a generator is unable to withhold generation. This assumption is manifested by the auctioneer's knowledge of each generator's capacity and the uni-dimensional structure of all bids[15].

Assume that all generators are risk-neutral and choose their bids so as to maximize their expected profits, where profits are defined as the difference between the total payments received on and the total cost incurred to meet its generation commitment.

3.1 Efficient Bidding

In any auction setting, a generator will adopt a strategy that maximizes its expected profit conditional on what strategies it believes its opponents are using. Given the symmetry in beliefs, it is natural to believe that two generators of the same type will bid the same way. Hence, for all

[14]The penalty is set high enough such that it is never in the interest of a generator to renege on its generation commitment.

[15]Admittedly, this simplifying assumption possibly omits interesting strategic behavior in quantity declarations on the part of generators. For example, Ausubel and Cramton (1996) find that bidders have an incentive to "shade" their quantity bids under a uniform-pricing rule.

auction mechanisms, I focus my attention on the symmetric equilibrium bidding functions[16] B_1, for lot 1, and B_2, for lot 2, which are functions of the type parameter θ[17]. As the definition of the demand lots changes from one auction mechanism to another, so will the generators' bidding strategies. The bidding strategies B_1 and B_2 map from a generators' type θ to a non-negative bid, for all $\theta \in (0, 1)$.

For the equilibrium bidding strategies to yield the efficient dispatch, they must each constitute an efficient bid ordering.

Definition 1 *An* efficient bid ordering *is an ordering of bids as a function of θ which, for all realizations of θ, yields the efficient dispatch.*

I define an equilibrium bidding strategy which is an efficient bid ordering (yields the efficient dispatch) to be an *efficient Bayesian equilibrium bidding strategy (EBEBS).* The process of finding the EBEBS of an auction mechanism comprises two steps. First, the existence of an efficient bid ordering must be established. Once established, the existence of an EBEBS must be checked. This is done by solving the expected profit maximization problem for one generator, assuming all others are bidding their true types and using an EBEBS. If either the efficient bid ordering or the EBEBS is found not to exist, then the auction mechanism under consideration does not have a symmetric pure-strategy EBEBS.

4 Robustness Results

In this section, we test the ability of the four identified auction mechanisms to achieve productive efficiency under three alternative demand scenarios. If, in a demand scenario, there exist EBEBS in an auction mechanism, then it achieves productive efficiency and the efficient dispatch. The goal is to identify which, if any, auction mechanisms can yield the efficient dispatch under all three demand scenarios, i.e., to identify an auction mechanism for which there exists a symmetric pure-strategy

[16]I focus only on symmetric pure-strategy equilbrium bidding strategies.

[17]The exception is B_1 in a sequential acution under a dependent demand scenario. See section 4.1 for further discussion.

EBEBS under all three demand scenarios. This is done to identify an auction mechanism which is robust to possible complementarities in costs.

While we assume the same demand shape in each of the following three scenarios, the participating generators' cost functions are varied yielding different efficiency orderings. This allows us to explore possible complemenarity scenarios without unneccessary mathematical complications.

4.1 Dependent

In the *Dependent* scenario, the entire demand can be supplied by one generator using the same generating plant. In this case generators are bidding for demand lots where the cost to supply one demand lot is dependent upon the outcome in the auction of another lot. This scenario can be effectively captured in a framework where demand is as in figure 6, F is uniform distribution over [0,1][18], there are $n + 1 > 2$ generators, each owning 1 plant with a capacity of 2 MW, generators' costs are given by equations (1)-(4), and the generator's efficiency orderings for $q = 1$ and 3 are as in figure 3. (No efficiency ordering assumption need be made for $q = 2$). Due to the capacity limits of 2 MW, it is possible for one generator to supply the entire demand.

Recall that the bid for lot **2** (lot **1**) in a horizontal auction indicates the minimum price a generator must be paid to generate 1 MW for 2 (1) hours. The bid for lot **2** (lot **1**) in a vertical auction indicates the minimum price a generator must be paid to generate 2(1) MW for 1 hour. When auctioned sequentially, lot **2** is auctioned before lot **1**. In the dependent framework, if the EBEBS for the sequential auctions should exist, $B_{\mathbf{2}}$ will be symmetric but $B_{\mathbf{1}}$ will not. This is because the winner of lot **2**'s true cost to supply lot **1** is no longer given by equation 1 but is only $b(\theta)$.

Proposition 2 *Under the dependent framework outlined above, there exist EBEBS for the horizontal sequential and vertical sequential auction of demand. These EBEBS are,*

$$B_{\mathbf{2}}(\theta) = C(2,\theta) + \frac{1}{n}((1-\theta)^{1-n}((1-n)n(\int_{\theta}^{1}(\alpha-\theta)^{n-2}(C(1,\alpha) - C_1(1,\theta))\,d\alpha))) \tag{5}$$

[18]Without loss of generality we may assume that θ is uniform over [0,1], since any cumulative distribution function F can be converted to a uniform distribution by redefining the index θ as a fractile.

$$B_{\mathbf{1}}(\theta) = \begin{Bmatrix} b(\theta) = C_1(1,\theta) \ \textit{if won lot } \mathbf{2} \\ C(1,\theta) \ \textit{otherwise} \end{Bmatrix} \tag{6}$$

Proof: See Appendix.

Remark: (The following remarks apply equally to the horizontal and vertical sequential auctions.) For the cost function and capacity limits chosen, it is in fact efficient for one generator, the generator with the lowest θ, to supply the entire demand. $C(q, \theta)$ is defined such that it is always less costly to increase the output of a generator already on (the generator who won in the first auction) than to turn-on a new generator[19]. A sequential auction of the demand allows generators to effectively reflect their relative cost advantages of generating 3 MWh and results in the efficient dispatch. It makes this possible by ensuring that whomever wins lot **2** will win lot **1**. This is because, as a 2^{nd} price auction, the generators have a dominant strategy to bid their true costs in the second auction. Hence, the winner of first auction is guaranteed to win in the second. Knowledge of the equilibrium behavior in the second auction induces generators to compete in their cost to supply 3 MWh (the entire demand) in the first auction, and results in the efficient dispatch in equilibrium.

Proposition 3 *Under the dependent framework outline above, there do not exist EBEBS for the horizontal simultaneous or vertical simultaneous auctions of demand.*

Proof: See Appendix,

Remark: (The following discussion applies both to horizontal simultaneous and vertical simultaneous auctions). In order for an auction to achieve the efficient dispatch, it must succeed in providing generators with the incentive to submit bids that are a monotonic transformation of their costs $C(q, \theta)$ for some output level q. A simultaneous auction of lots with dependent cost does not provide generators with these incentives and hence can not be expected to yield the efficient dispatch in equilibrium.

To the achieve the efficient dispatch in this dependent framework, the generators must bid for lots **2** and **1** on the basis of $C(3, \theta)$, i.e., B_1 and B_2 must be monotonically increasing. However the

[19]This is a direct result of $a(\theta) \geq b(\widehat{\theta}) - b(\theta)$, for $\forall \theta, \widehat{\theta} \in [0, 1]$, $\theta \neq \widehat{\theta}$.

mechanism of a simultaneous auction does not provide generators with the incentive to bid as such. Given that both bids are submitted simultaneously, it is not (optimal) profit maximizing behavior for say, a generator with a large θ, to bid on the basis of $C(3, \theta)$. Its comparative advantage is in the generation of 1 MWh (recall from figure 3(a) that $C(1, \theta)$ is monotonically decreasing) and hence it will bid aggressively for lot **1** on the basis of $C(1, \theta)$.[20]

In the light of proposition 2, it is possible to eliminate from consideration any form of simultaneous auction. While the nature of a sequential auction provides generators with the proper incentive to compete on their cost to supply the entire demand, a simultaneous auction does not and hence there does not exist a symmetric pure-strategy EBEBS.

4.2 Independent

The second scenario to be examined is one where the entire demand can be supplied by one generator using different generating plants. This situation is captured in an environment with demand given as in figure 6 and $n + 1 > 2$ generators, each of whom own 2 identical plants with a capacity of 1 MW each[21]. In this demand scenario the generators' costs over lots are independent in a horizontal auction. Assume that demand is as in figure 6, F is uniform distribution over [0,1], generators' costs are given by equations (1)-(4), and the generator's efficiency orderings for $q = 1$ and 2 are as in figure 8. (No efficiency ordering assumption need be made for $q = 3$).

In a horizontal sequential auction, the efficient dispatch consists of the lowest order statistic θ winning lot **2** and the highest order statistic θ winning lot **1**. In a vertical sequential auction, the efficient dispatch consists of the lowest and highest order statistic θs winning in lot **2** and the lowest order statistic θ winning lot **1**.

Proposition 4 *Under the independent scenario outlined above, there exist EBEBS in the horizontal sequential auction of demand. The EBEBS are,*

$$B_{\mathbf{2}}(\theta) = C(2, \theta) \tag{7}$$

[20]It is of interest to note that conditional bids (a generator's bid for *lot i* is dependent upon whether or not it wins *lot j*) would alleviate this problem.

[21]A generator of type θ has two plants of technology type θ.

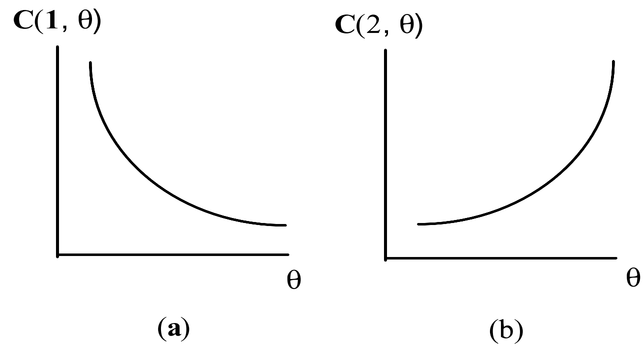

Figure 8: Cost function for 1 and 2 MWh.

$$B_{\mathbf{1}}(\theta) = C(1, \theta)$$

Proof: Since each generator owns two plants, each with a capacity of 1 MW, its costs to supply lots **2** and **1** are independent. This is because the lots' height (measured in MW) exactly matches each of the plant's capacity. Hence the two auctions can be analyzed independently as two separate and unrelated auctions. In a 2^{nd} price auction, it is a dominant strategy for generator to bid their true costs and hence the equilibrium bids are an efficient bid ordering. It is important to note that this result holds the most general of assumptions, i.e., when $\theta \in$[0,1] with cumulative distribution F and the generators' costs are given by equations (1)-(4) and there are no assumption made on the efficiency orderings for any quantity.

Proposition 5 *Under the independent framework outlined above, there do not exist EBEBS in a vertical sequential auction of demand.*

Proof: See Appendix.

Remark: The first step in establishing with existence of an EBEBS is to identify the efficient bid ordering for the auction mechanism. I found that there does not exist an efficient bid ordering in a vertical sequential auction under the independent framework. In order to achieve the efficient dispatch, the lowest and highest order statistics θs (the least-cost generators of 2 and 1 MWh, respectively) must submit the lowest bids and win dispatch in hour 1. Using a simple 3 generator example, it is easy to prove that, for different realizations of θ, the bid ordering necessary to induce the efficient dispatch in lot **2** are contradictory and hence an efficient bid ordering do not exist.

A vertical sequential auction fails to provide generators with the necessary incentive to submit bids that are a monotonic transformation of their relative efficiency with respect to some output level. Instead, it requires that, in lot **2**, generators differentiate themselves as the potential least-cost producer of 1 MWh or 2 MWhs and bid on the basis of two different costs, $C(1, \theta)$ and $C(2, \theta)$, simultaneously. On the other hand, a horizontal sequential auction succeeds in providing the correct incentives by shaping the demand lots to the generators' capacity mechanism. In the auction for lot **2**, all generators compete on their relative efficiency to supply 2 MWh. Likewise, in the auction of lot **1**, they complete on their relative efficiency to supply 1 MWh.

From propositions 3 and 4 we are able to narrow our list of successful auction mechanisms to a horizontal sequential auction. We find that in fact, a horizontal sequential auction is able to provide generators with the correct incentives so as to guarantee the efficient dispatch in any "independent" scenario, that is for any values of $a(\theta)$ and $b(\theta)$ satisfying equations (1)-(4).

4.3 Either-or

The lack of existence of EBEBS for a vertical sequential auction leaves us only with a horizontal sequential auction to consider in the final scenario. Generators will often face a scenario where many generators must be used to satisfy demand. In a horizontal auction, this reduces down to the choice of either winning one lot or another (recall that in the independent scenario, a generator faced no capacity constraints and could win both lots). A generator must often decide whether to bid aggressively for lots auctioned early on and, should it win, be unable to participate in further auctions due to capacity constraints, or wait for later auctions in which to bid aggressively.

This scenario can be effectively captured in a framework with demand given as in figure 6 and $n+1 > 2$ generators, each of whom owns 1 plant with a capacity of 1 MW. Assume that $\theta \in$[0,1] with cumulative distribution F, the generators' costs are given by equations (1)-(4), and the generator's efficiency orderings for $q = 1$ and 2 are as in figure 8. (No efficiency ordering assumption need be made for $q = 3$). Given the assumed cost mechanism, the efficient dispatch requires the lowest order statistic θ to win lot **2** and the highest order statistic θ to win lot **1**.

Proposition 6 *In the either-or framework outlined above, there exist EBEBS for the horizontal sequential auction of demand. The EBEBS are,*

$$B_{\mathbf{2}}(\theta) = C(2, \theta) \tag{8}$$

$$B_{\mathbf{1}}(\theta) = C(1, \theta)$$

Proof: See Appendix.

Remark: Not only does a horizontal sequential auction of demand yield the efficient dispatch, but generators bid their true costs in both auctions. This result is quite intuitive: A generator who owns 1 plant with a capacity of 1 MW, can win in at most one auction. Since there do not exist dependent valuations across lots, generators compete in each lot on the basis of their costs, $C(1, \theta)$ for lot **1** and $C(2, \theta)$ for lot **2**. Due to the strict monotonicity, in opposite directions, of $C(1, \theta)$ and $C(2, \theta)$ a generator can win in at most one auction (it can either the be highest order statistic θ, or the lowest order statistic θ, but not both). Given a generator can hope to win in at most one 2nd price auction, its dominant strategy is true-cost bidding in both auctions. It must be emphasized that true-cost bidding is a direct result of $C(1, \theta)$ being strictly monotonically decreasing in θ and $C(2, \theta)$ being strictly monotonically increasing in θ and it is not generalizable to a general cost function framework. However, for any forms of $a(.)$ and $b(.)$ satisfying figure 8, there do exist EBEBS in the horizontal sequential auction.

5 Conclusion

This paper has addressed the question of whether various electricity auction mechanism can guarantee productive efficiency under different complementarity scenarios. In particular, I looked at four likely candidate mechanisms and evaluated their performance under three scenarios. Only a horizontal sequential auction was found to guarantee productive efficiency in equilibrium under all three complementarity scenarios. The key to its success lies in its ability to either 1) shape demand lots to mimic the capacity mechanism of generators (as in the independent and either-or scenarios) or 2) provide generators with the correct incentive to submit bids that are a monotonic transformation of their cost $C(q^*, \theta)$ for some output level $q^* > 0$ (as in the dependent scenario).

These results speak in favor of adopting a horizontal auction for electricity. However, there is still much research left to be done before we can guarantee productive efficiency at all times. The efficiency of horizontal sequential auctions must be tested under a richer variety of demand and cost scenarios. In addition, it would be of great interest to relax the uni-dimensionality assumption on bids and to study the strategic behavior of generators when quantity as well as energy bids are allowed. Finally, the efficiency properties of auction mechanisms in a model which allows for demand as well as supply-side bidding should be studied.

Appendix

Recall that the cost functions assumed here satisfy the following four conditions:

Condition 1) $C(q,\theta) = a(\theta) + b(\theta)q$

Condition 2) $a'(\theta) < 0,\ b'(\theta) > 0$

Condition 3) $\forall\theta\ \exists\ \ \hat{q}(\theta)$ such that $C(\hat{q}(\theta),\theta) < C(\hat{q}(\theta),\widetilde{\theta})\quad \forall\ \widetilde{\theta} \neq \theta$

Condition 4) $\hat{q}'(\theta) < 0$

Conditions 1 & 2 imply that any pair of total cost lines cross only once. Conditions 1-3 imply that *all* lines must cross. Conditions 1-4 imply that, for any $\hat{\theta}$ and corresponding $\hat{q}(\hat{\theta})$, $C(q,\theta)$ is monotinically increasing for all $\theta < \hat{\theta}$ and is monotonically increasing for all $\theta > \hat{\theta}$. This last observation is what allows us to model the efficiency ordering for any quantity as either strictly decreasing, increasing, or unimodal (as in figure 3).

The unimodularity of efficiency ordering can easily be seen using a simply example. Suppose that there exist only four generator types, θ_i for $i = 1,..,4$, whose total cost curves are given as in figure X. Define q^*_{ij} to be the quantity at which $C(q,\theta_i) = C(q,\theta_j)$. We know that for any $q < q^*_{12}$, $C(q,\theta_1) < C(q,\theta_2)$ and for any $q > q^*_{12}$, $C(q,\theta_1) > C(q,\theta_2)$. Likewise we know that for any $q < q^*_{13}$, $C(q,\theta_1) < C(q,\theta_3)$ and that for any $q > q^*_{13}$, $C(q,\theta_1) > C(q,\theta_3)$. Repeating this exercise for all possible pairs gives us :

- For $\hat{q}(\theta_1) < q^*_{12}$, $C(q,\theta_1) < C(q,\theta_2) < C(q,\theta_3) < C(q,\theta_4)$
- For $q^*_{12} < \hat{q}(\theta_2) < q^*_{23}$, $C(q,\theta_1) > C(q,\theta_2) < C(q,\theta_3) < C(q,\theta_4)$
- For $q^*_{23} < \hat{q}(\theta_3) < q^*_{34}$, $C(q,\theta_1) > C(q,\theta_2) > C(q,\theta_3) < C(q,\theta_4)$
- For $q^*_{34} < \hat{q}(\theta_4)$, $C(q,\theta_1) > C(q,\theta_2) > C(q,\theta_3) > C(q,\theta_4)$

In the proofs to all the propositions, I consider the expected profit maximization problem of a generator of type θ (referred to as generator θ), assuming that all other generators bids truthfully, i.e., a generators of type $\omega \in [0,1]$ bids $[B_1(\omega), B_2(\omega)]$ for lots **2** and **1**, respectively. Define $\theta_{(1)} \leq \theta_{(2)} \leq ... \leq \theta_{(n)}$ to be the order statistics of the other generators' types.

Proof to Proposition 1: The proofs for the horizontal and vertical sequential auctions are identical. Recall that the generators' costs are given by equations (1)-(4) and the generator's efficiency orderings for $q = 1$ and 3 are as in figure 3. In order to identify, should they exist, the EBEBS B_1 and B_2 for the sequential auction, it is appropriate to employ backwards induction. After lot **2** has been auctioned and its winner made public knowledge, the generators bid for lot **1**. The auction for lot **1** is a 2^{nd} price single object auction (the single object being lot **1**) and hence the generators have a dominant strategy to bid their true costs to supply lot **1**. For the assumed cost function, $a(\theta) + b(\theta) \geq b(\widehat{\theta})$, for $\forall \theta, \widehat{\theta} \in [0,1]$, $\theta \neq \widehat{\theta}$, implying that the cost of supplying lot **1** for the generator who won lot **2** is less than for all other generators. These two facts combined imply that the winner of lot **2** will also win lot **1**.

Stepping back to the first auction, in order for the efficient dispatch to occur in equilibrium, the generators must bid for lot **2** in increasing order of technology type, i.e., the efficient bid ordering implies that B_2 must be strictly monotonically increasing. This is evident from the fact that the winner of lot **2** will also win lot **1** and that $C(3, \theta)$ is strictly monotonically increasing (see figure 3(c)).

Suppose that B_2 is increasing and consider the decision of generator θ. Generator θ's expected profit from both auctions given it bids as type r when its true type is θ is

$$V(r, \theta) = E\{B_2(\theta_{(1)}) + C(1, \theta_{(n)}) - C(3, \theta) | r < \theta_{(1)}\} \ \Pr(r < \theta_{(1)}) \tag{9}$$

$$\begin{aligned} &= n \int_r^1 [(C(1, \alpha) - C_1(q, \theta))(F(\alpha) - F(r))^{n-1} \\ &+(B_2(\alpha) - C(2, \theta))(1 - F(\alpha))^{n-1}] f(\alpha) d\alpha \end{aligned} \tag{10}$$

If generator θ submits the lowest bid and wins lot **2** it receives $B_2(\theta_{(1)})$ for lot **2** and $C(1, \theta_{(n)})$ for lot **1** (recall that $C(1, \theta)$ is monotonically decreasing in θ).

Differentiating equation (10) and setting $V_1(\theta,\theta)=0$ (under the assumption that $\theta \tilde{} U(0,1)$) yields equation (5), the necessary condition for truthful telling to be optimal for generator θ. As shown below, B_2 is (as was assumed) monotonically increasing for all $n \geq 2$.

Lemma 7 *$B_2(\theta)$ is monotonically increasing.*

Proof. $B_2'(\theta) = C_2(2,\theta) + \frac{(1-n)}{(1-\theta)^{n-1}}\left[\frac{(n-1)}{(1-\theta)}\int_\theta^1 (\alpha-\theta)^{n-2}(C(1,a)+C_1(q,\theta))d\alpha\right]$

$+\frac{(1-n)}{(1-\theta)^{n-1}}\left[\int_\theta^1 (2-n)(\alpha-\theta)^{n-3}(C(1,a)+C_1(q,\theta)) - C_{12}(q,\theta)(\alpha-\theta)^{n-2}d\alpha\right]$

For B_2 to be monotonically increasing, the right hand side of the equation above must be positive. For the right hand side to be positive, we need (this expression is derived by integrating the term in the integrand expression and noting that

$$\begin{aligned} C_2(2,\theta) + C_{12}(1,\theta) &= C_2(3,\theta) \\ &\geq \left(\frac{(n-1)}{(1-\theta)^{n-1}}\right)\left[\frac{(n-1)}{(1-\theta)}\int_\theta^1 (\alpha-\theta)^{n-2}(C(1,a)+C_1(q,\theta))d\alpha\right. \\ &\quad \left. - \int_\theta^1 (n-2)(\alpha-\theta)^{n-3}(C(1,a)+C_1(q,\theta))\right] \end{aligned}$$

which can be rewritten as,

$$C_2(3,\theta) \geq \frac{(n-1)}{(1-\theta)^{n-1}}\int_\theta^1 [\underbrace{C(1,a)+C_1(q,\theta)}_{\eta}][\underbrace{\frac{(n-1)}{(1-\theta)}(\alpha-\theta)^{n-2}-(n-3)(\alpha-\theta)^{n-3}}_{\gamma}$$

$]d\alpha$

To ensure that $B_2(\theta)$ is monotonically increasing, it is sufficient to show that the integral is negative. η is by definition decreasing in θ while γ is both positive and negative for over $\theta \in (0,1)$. The value of θ, defined to be θ^*, at which γ changes signs is

$$\theta^* = \frac{n-2+\theta}{n-1}$$

On the unit interval, the range over which γ is negative is larger than that over which it is positive, i.e., $(\theta, \frac{n-2+\theta}{n-1}) > (\frac{n-2+\theta}{n-1}, 1)$. Since η is decreasing in θ, this implies that $B_2(\theta)$ is monotonically increasing. (See figure below).

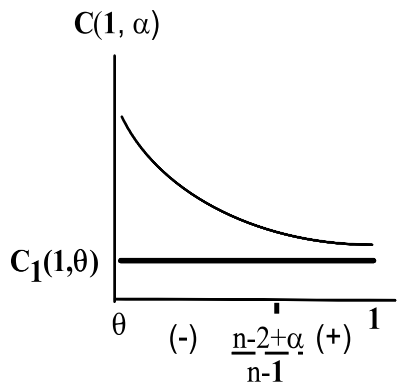

Figure 9:

For $B_2(\theta)$ to be an optimal response for generator θ, it is sufficient for $V_{12}(r,\theta) \geq 0$.[22]

Indeed, $V_{12}(r,\theta) = n(1-r)^{n-1}(C_2(2,\theta) + C_{12}(q,\theta) = n(1-r)^{n-1}C_2(3,\theta) \geq 0 \quad \forall r, \theta \in [0,1]$ and $n > 1$. Therefore, there exist EBEBS for the horizontal and vertical sequential auctions under the assumed dependent framework, which are as in equations (5) and (6). ♦

Proof to Proposition 2: The proofs for the horizontal and vertical sequential auctions are identical. For the efficient dispatch to occur in equilibrium, B_1 and B_2 must both be strictly monotonically increasing in θ, i.e., the lowest technology parameter generator wins in both auctions. Suppose that B_1 and B_2 are strictly monotonically increasing and consider the decision of generator θ. The expected profit of generator θ who bids as type r for lot **1** and type s for lot **2** is given by $V(r,s,\theta)$,

$$V(r,s,\theta) = E\{B_1(\theta_{(1)}) - C(1,\theta) | r < \theta_{(1)} < s\} \Pr(r < \theta_{(1)} < s) \tag{11}$$

$$+E\{B_2(\theta_{(1)}) - C(2,\theta) | s < \theta_{(1)} < r\} \Pr(s < \theta_{(1)} < r)$$

$$+E\{B_1(\theta_{(1)}) + B_2(\theta_{(1)}) - C(3,\theta) | r \text{ and } s < \theta_{(1)}\} \Pr(r \text{ and } s < \theta_{(1)})$$

$V(r,s,\theta)$ takes on different forms for $r < s$ and $r > s$. In the case of $r < s$, generator θ can either win only lot **1** or both lots **1** and **2**. In the case of $r > s$, generator θ can either win only lot

[22]See Guesnerie and Laffont (1984) for a proof of the sufficiency condition.

2 or both lots **1** and **2.**

For the case of $r < s$, $V(r, s, \theta)$ reduces to,

$$V(r, s, \theta) = \int_{s}^{r} (B_1(\omega) - C(1, \theta))d[1 - F(\omega)]^n \tag{12}$$

$$+ \int_{s}^{r} (B_1(\omega) + B_2(\omega) - C(3, \theta))d[1 - F(\omega)]^n$$

A necessary condition for truthful-bidding to be an optimal for generator θ, is given by $V_1(\theta, s, \theta)$ $= 0$, which implies

$$B_1(\theta) = C(1, \theta) \tag{13}$$

i.e., generator θ must bid its true cost to generate only lot **1.** But $C(1, \theta)$ is monotonically decreasing, and hence contradicts our initial assumption on the shape of B_1. As a result, there does not exist an EBEBS for lot **1** in an horizontal or vertical simultaneous auction. A similar argument, in the case of $r > s$, demonstrates that there does not exist an EBEBS for lot **2**. However, the nonexistence of an EBEBS for lot **1** is sufficient to refute the ability of a horizontal or vertical simultaneous auction to yield the efficient dispatch in equilibrium.♦

Proof to Proposition 4: The first step in establishing with existence of an EBEBS is to identify the efficient bid ordering for the auction mechanism. I found that there does not exist an efficient bid ordering in a vertical sequential auction under the independent framework. In order to achieve the efficient dispatch, the lowest and highest order statistics θs (the least-cost generators of 2 and 1 MWh, respectively) must submit the lowest bids and win dispatch in hour 1. Using a simple 3 generator example, it is easy to prove that, for different realizations of θ, the bid ordering necessary to induce the efficient dispatch in lot **2** are contradictory and hence an efficient bid ordering does not exist. The results generalize to other cost functional forms and n generators, $n > 2$.

Suppose there exist 3 generators, of technology types (0.1, 0.3, 0.5) whose costs are given by equations (1)-(4) where $a(\theta) = (2 - \theta)^2$ and $b(\theta) = 2\theta$. In order to result in the efficient dispatch, generators 0.5 and 0.1 must win in lot **2** and generator 0.1 must win lot **1,** i.e., $B_2(0.5) < B_2(0.3)$.

Suppose, instead, that the three generators where of types (0.3, 0.5, 0.7). In this scenario, for the efficient dispatch, generators 0.3 and 0.7 must win in lot **2** and generator 0.3 must win lot **1**, i.e., $B_2(0.3) < B_2(0.5)$. This directly contradicts the bid ordering that would be necessary in the first scenario considered. In fact, there does not exist an efficient bid ordering for lot **2** which yields the efficient dispatch for all realizations of θ.⧫

Proof of Proposition 5: Recall that the generators' costs are given by equations (1)-(4) and the generators' efficiency ordering for $q = 1, 2$ is as in figure 8. In this either-or framework, a generator may either win lot **1** or **2**, but not both. For the efficient dispatch to occur, bids for lot **1** must be strictly monotonically decreasing in technology parameter θ and bids for lot **2** must be strictly monotonically increasing in θ. These can easily be seen to be the efficient bid ordering from figures 8(a) and 8(b).

Employing backwards induction, it is a dominant strategy for generators to bid their true costs for lot **1** in the second auction, i.e., $B_1(\theta) = C(1, \theta)$ for $\theta \in [0, 1]$. Stepping back to the first auction, for lot **2**, consider the decision of generator θ. Suppose that B_2 is a monotonically increasing bidding function. Generator θ wishes to bid for lot **2** so as to maximize its total expected profits, i.e., to bid as type r to maximize

$$V(r, \theta) = E\{B_2(\theta_{(1)}) - C(2, \theta) | r < \theta_{(1)}\} \Pr(r < \theta_{(1)}) \tag{14}$$

$$+E\{B_1(\theta_{(n)}) - C(1, \theta) | r < \theta_{(1)} \text{ and } \theta_{(n)} < \theta\} \Pr(r < \theta_{(1)} \text{ and } \theta_{(n)} < \theta)$$

For $r \leq \theta$

$$V(r, \theta) = \int_1^r (B_2(\omega) - C(2, \theta)) d[1 - F(\omega)]^n + \int_0^r (C(1, \omega) - C(1, \theta)) d[F(\omega)]^n \tag{15}$$

$$+ \int_r^\theta (C(1, \omega) - C(1, \theta)) d\widetilde{F}_{1n}(r, \omega)$$

where $\widetilde{F}_{1n}(r, \omega) = \Pr\{\text{at least 1 generator is of type} \leq r, \text{ at least } n \text{ generators are of types} \leq \omega\}$.

For $r \geq \theta$

$$V(r, \theta) = \int_1^r (B_2(\omega) - C(2, \theta)) d[1 - F(\omega)]^n + \int_0^\theta (C(1, \omega) - C(1, \theta)) d[F(\omega)]^n \tag{16}$$

Differentiating equations (15) or (16) and setting $V_1(\theta, \theta) = 0$ yields the necessary condition for truthful bidding,

$$B_2(\theta) = C(2, \theta) \tag{17}$$

B_2 is monotonically increasing, as was assumed. To check for sufficiency of truthful bidding as generator θ's optimal response, $V_{12}(r, \theta)$ must be non-negative.

$$V_{12}(r, \theta) = n(1 - F(\theta))^{n-1} f(\theta) \geq 0 \text{ for all } \theta. \tag{18}$$

Truthful bidding is in fact optimal and, in equilibrium, lots **2** and **1** will be awarded to the least-cost producers of 2 and 1 MWh, respectively. ⧫

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