O.R. Applications

# Multi-unit auctions with complementarities: Issues of efficiency in electricity auctions

Wedad J. Elmaghraby *

*Industrial and Systems Engineering, Georgia Institute of Technology, Altlanta, GA 30305, USA*

## Abstract

In this paper, we analyze the ability of different auction structures to induce the efficient dispatch in a one-shot framework where generators know their own and competitors' costs with certainty. In particular, we are interested in identifying which, if any, rules in an auction structure yield only the efficient dispatch in equilibrium. We find that a critical component to a successful auction design is the way in which demand is bundled and hence the way bids are defined. While an auction mechanism which allows for more than one winner in an auction may support inefficient dispatches in equilibrium, we find that an auction where there is exactly one winner per lot, where the lots are formed to capture the cost structure of generation plants, and all lots are auctioned simultaneously, supports only efficient dispatches in equilibrium.
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## 1. Introduction

Many governments believe that it is in their and their constituents' best interest to restructure their electricity supply industries. The goal of the regulators is to introduce competition into their electricity supply industries and create the appropriate medium through which electricity buyers and sellers can actively trade electricity, in the hope that such a competitive market will promote efficiency. Auction-based mechanisms for electricity dispatch have already been implemented in various countries around the world (e.g., the United States, United Kingdom and Australia). The design of an electricity auction which induces an efficient use of generation resources is complicated by the fact that electricity demand, which fluctuates from hour to hour, must be satisfied by many suppliers with different costs, and that the generators lack the ability to store electricity in inventory. It is proving to be a great challenge to design an efficient auction where the generators' schedules can no longer be dictated by a central planner, but must result as a consequence of their submitted bids.

* Tel.: +1-404-894-2359.
*E-mail address:* wedad@isye.gatech.edu (W.J. Elmaghraby).

The road to deregulation has been very bumpy; for example, California, the first state in the US to create an electricity auction, was forced to shutdown operation of its electricity auction early in 2001 due to exorbitant electricity prices and the resulting threat of bankruptcy of its load serving entities (see Borenstein (2002) for an excellent discussion of the failed California electricity market). The exercise of market power as well as inefficient utilization of generation resources were both cited as problems plaguing the California design. In their original auction design, generators were allowed to submit (energy-only) bids for each hour of the day; each hourly market was then cleared independently of the bids in all other hours. We refer to an auction where each (sub-)auction is cleared independently as a *simple* auction. In their redesigned auction, California will abandon the simple auction format and will join the ranks of the Northeast power pools (e.g., Pennsylvania–New Jersey–Maryland and the New York Independent System Operator) and solve a unit commitment problem over the entire day to establish a dispatch schedule.

The question that this paper poses is: Is it possible to design a simple (sealed bid) auction for electricity that supports only the efficient dispatch (in equilibrium)? That is, was there is a way that the CA auction design could have been altered while maintaining its simple format so as to result in productive efficiency?

There are two forms of inefficiency which may arise from an ill-designed market; productive inefficiency and allocative inefficiency. Productive inefficiency implies an inefficient use of resources in the production of goods, while allocative inefficiency occurs when the goods are not consumed by those who value it the most. Currently, most retail consumers do not see the real-time price of electricity and hence have limited ability to respond to changes in electricity prices. Rather, their demand is represented in the wholesale electricity auction by a (forecasted) inelastic demand curve. While there has been much discussion as to the importance of allowing consumers to respond to electricity prices in real-time, Stoft (2002) calculates the expected savings from doing so, and

hence achieving allocative efficiency, to be small. "Fully implemented real-time pricing...would reduce the cost of supply by approximately 2.25% of retail costs" (Stoft, 2002, p. 14). As a result, the most promising efficiency gains to be had from the proper design and use of an electricity auction lies within the efficient use of generation resources. The purpose of this paper is to study the impact of various auction rules on yielding productive efficiency, referred to as the efficient dispatch. We are interested in identifying if there exists a simple auction structure that yields only the efficient dispatch in equilibrium.

In order to gain a better understanding of the incentives created by different auction structures and the ability of each auction structure to induce the efficient dispatch, we focus only on the generation (supply) side of the market and we assume that electricity demand is deterministic, inelastic, common knowledge and that the auction is conducted only once. While in reality, the auction will be repeated daily (and hence the set of equilibrium bidding strategies may be larger), we can gain insight into some of the equilibrium strategies for the repeated game by examining a one-shot version (for the equilibrium strategies to the one-shot game will always constitute an equilibrium in the repeated game).

In addition, we examine the performance of auctions in a complete information setting. Complete information, may appear at first to be quite restrictive and unrealistic. However, in an industry such as the electricity supply industry, which has a long history of regulation (often with all the generation sources under government management as was the case in the United Kingdom before 1990), it is reasonable to assume that generators are aware of what types of plants their competitors own and what the costs associated with generation are. (For example, investor-owned utilities, which accounted for 77% of the total power generated in California in 1995 (Table 6-2, CEC Report), had to annually submit a public report declaring their different generation plants, their associated costs, generation capacity, etc.). Furthermore, the cost of primary inputs to electricity generation, e.g., fuel and emission permits, are generally known, as well as

generic heat rates for generator types (and in some cases, the actual heat rates for particular units).

We explore the performance of a simple hourly auction (as was adopted in California) under various auction formats (simultaneous/sequential, single-part/multi-part bids, discriminatory/uniform price). We find that a simple hourly auction format, where there can be more than one winner in an hour, inherently opens the door for inefficiency. The most significant result of the paper is that we find that for the generation market modeled in this paper, when the auction is re-specified so as to allow for only one winner per lot, and all lots are auctioned simultaneously, the efficient dispatch, and only the efficient dispatch, can be supported in equilibrium.

In Section 2, we provide the reader with some background literature on multi-unit auctions. We then go on to characterize an electricity auction as a multi-unit auction with private valuations which are possibly dependent over several units, and outline the different auction mechanisms considered and the cost characteristics of generators. In Section 3, we argue that bundling demand into vertical lots, which allow for more than one winner in an auction, precludes guaranteeing the efficient dispatch in equilibrium. In Section 4, we present an auction mechanism that supports only efficient dispatches in equilibrium.

## 2. Electricity auctions

### 2.1. Multi-unit auction with complementarities

What separates designing an auction for electricity from the vast body of auction literature is the structure of generation costs. Generators have different types of costs (e.g., ramp-up costs, no-load costs, etc.) which must be recovered through their sales revenues. Generation costs can be broadly classified into two groups: fixed "start-up" costs which are incurred each time a generation plant is turned-on, and variable costs which are incurred with each additional MW hour generated. (Other fixed costs such as construction costs are ignored since we address the issue of relative effi-

ciency given that a certain mix of generation plants is in place.) Due to this cost structure, there exist economies of scale in generation or cost *complementarities* in both time and quantity dimensions, i.e., the (average) cost to generate 1 MW during hour $t$ depends upon the number of additional MW generated during hour $t$ and other hours.

A realistic depiction of generation costs must also take into account that different generators are efficient at different output levels. For example, nuclear plants, which have a large start-up cost but relatively small variable cost, are more cost-effective at high output levels while oil-fired plants, which have a relatively low start-up cost, but a large variable cost, are more cost-effective at lower output levels. To the best of our knowledge, with the exception of Elmaghraby and Oren (1999), this is the only paper that incorporates this characteristic (i.e., varying cost/valuation rankings) in the study of multi-unit auctions.

If the object being auctioned is defined as 1 MW hour of demand for energy, then demand can be interpreted as a collection of identical objects, distinguished only by the time at which they occur. Generators are sellers of electricity who may wish to win many objects (by winning an object they win the right to supply that 1 MW hour of demand at a price determined through the auction process) and hence have *multi-unit demand*.

A generator's (operating) profit from "winning" a MW hour is the difference between the auction price he is paid and his own *private* (operating) cost for supplying the MW hour. A generator is constrained from winning all objects (and hence supplying the entire demand the following day) by the presence of capacity constraints on his generation level at any point in time (i.e., if a generator has a capacity of $K$, the maximum level of MW at which he can generate at any point in time is $K$ MW). Demand throughout the day is greater than the capacity of any one generator; hence several generators must be chosen to supply demand. Hence, an electricity auction is a multi-unit auction with private valuations, complementarities and "purchasing" capacity constraints.

### 2.1.1. Relevant auction literature

In order to appreciate the new and interesting questions posed by an electricity auction, it is important to examine its place in the existing auction literature. As explained above, an electricity auction is a multi-unit demand, private valuations auction where there may exist complementarities across units. The bulk of research in auction theory has traditionally focused on the use of single-unit auctions; unfortunately, most of the results for single unit auctions do not generalize to multi-unit settings (see Klemperer (1999) for an excellent survey of the literature). Hence, studying auctions for electricity forces us to leave the well-studied and understood world of single-unit auctions and explore the performance of auctions in a more complicated setting.

In contrast to simple auctions, one way to deal with the presence of complementarities across objects is to allow for package or combinational bid. Motivated by the Federal Communication Commission's upcoming combinational auction for spectrum licenses, a number of papers have examined the appropriate design of such auctions (e.g., Ausubel and Milgrom, 2002; Rothkopf et al., 1998). Hobbs et al. (2000) explore the use of a combinational auction for electricity under a Vickrey–Clark–Groves (VCG) payment scheme. VCG auctions, the multi-unit extension of a second price auction, have the desirable property that they are efficient. However, a problem with VCG auctions is that they require bidders to submit their valuations over all possible bundles and, more importantly, they are frequently revenue inadequate, i.e., the auctioneer must pay out more than (s)he collects. One way that bidders can exacerbate this revenue problem is by submitting bids under someone else's name (false-name bidding), thereby increasing his own payments, and the auctioneer's total outlay (see Rothkopf et al. (1990) for further discussion). As a result, VCG auctions are rarely used in practice and are impractical for electricity auctions.

Within the study of simple multi-unit auctions, a number of papers have examined presence of "bid shading" in equilibrium (e.g., Engelbrecht-Wiggans and Kahn, 1998; Noussair, 1995; Katzman, 1990; Ausubel and Cramton, 1996). These papers conclude that when bidders desire more than one object and they are allowed to submit demand curves (a separate bid for each unit), bidders have an incentive to underbid or "shade" their bids, in an effort to reduce the price they will have to pay. Hence, bid shading results in an inefficient allocation. This type of strategic behavior can be found under uniform and discriminatory pricing (although the effect is less under discriminatory auctions), as well as common and private valuation settings. A standard assumption in all of these papers is that bidders do not experience complementarities across multiple objects. Therefore, this paper extends the work above to incorporate the presence of complementarities. Furthermore, we identify another type of strategic behavior on the part of bidders under uniform price auctions. The manifestation of "bid shading" in a procurement auction is that sellers bid above their costs so as to increase the price that they are paid. We identify equilibria in which seller actually bid below their costs so as to guarantee that they are allocated units. This "zero" bid strategy creates a very similar effect to one identified by Back and Zender (1993) for the uniform auction of Treasury bills. Analyzing a continuous bid auction, where bidders have common valuations, Back and Zender show that bidders are able to costlessly deter competitors from bidding more aggressively by submitting extremely steep demand curves. We demonstrate that this type of strategy carries over in equilibrium to a private valuations with complementarities setting, with discrete bids.

Ausubel (2002), also under the assumption that bidders do not experience complementarities, proposes an ingenious ascending auction format that overcomes the problem of bid shading and supports only the efficient allocation in equilibrium. Ausubel proposes an open ascending auction, whereby bidders are charged the price at which they "clinched" an object, thereby reproducing the prices under a VCG auction. While Ausubel's auction has nice efficiency properties, it is unclear how his mechanism would extend to a setting with complementarities. In addition, all of the electricity markets currently in operation or under design are sealed bid; it is believed that a

sealed bid auction is the more appropriate format for the time-sensitive electricity auction. [1]

Gale (1990) considers a multi-object auction where bidders(producers) have constant marginal costs and superadditive profit functions. He finds that bundling all of the objects together maximizes the auctioneer's expected revenue. One of the few other papers that does consider synergies across multiple units and ''lumpy'' bids in a simple auction setting is Tenorio (1999). He examines a setting where there are two bidders, each with a demand for 2 or 3 units, and a total of 3 units to be sold. Bidders submit a bid indicating the number of units $q \geqslant 2$ they wish to purchase, and a corresponding per-unit price. He allows for complementarities by modeling bidder $i$'s valuation for $q$ units to be $v(t_i, q) = t_i \sum_{k=1}^{q} \phi^{k-1}$, where $t_i$ is a bidder's private parameter and $\phi$ is the same for all bidders. Since $\phi$ is common to all bidders, the bidders' (valuation) orderings are symmetric for each $q$, i.e., if $t_i > t_j$, then $v(t_i, q) > v(t_j, q)$ for all $q > 0$. Via numerical examples, Tenorio illustrates that both uniform and discriminatory auctions can result in inefficient outcomes. As stated in the previous section, we examine a more general setting where bidders' valuations (costs) rankings are not the same at different output levels; and allow for $n > 2$ bidders and $q > 3$ units to be auctioned. Furthermore, we demonstrate that allowing for multi-part bidding (a step supply curve, versus requiring a flat supply curve) does not alleviate the inefficiencies found in equilibrium.

There is a small, but growing, number of papers that theoretically investigate the performance of various electricity auction formats. There have been two modeling approaches in studying electricity auction: (i) a continuous supply curve approach first adopted by Green and Newbery (1997) in their analysis of the UK electricity market and (ii) the discrete multi-unit auction approach first adopted by Von der Fehr and Harbord (1993). Fabra et al. (2002) illustrate how each modeling approach can lead to very different predictions of market outcomes. Given that most electricity markets require generators to submit discrete step supply function, we adopt the multi-unit auction approach.

Von der Fehr and Harbord (1993) assume a one-period framework with two generators who have (different) constant marginal costs of generation and whose costs are common knowledge. The level of demand for electricity is uncertain but its distribution is known. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium. Adopting a similar setting, Fabra et al. (2002) study the welfare and revenue performance of discriminatory, uniform and Vickrey auctions. Elmaghraby and Oren (1999) extend their analysis to incorporate multiple generators, who have a fixed ''start-up'' costs with generation and identify conditions under which bundling demand according to its duration and auctioning the bundles sequentially is efficient. This paper builds on their analysis by examining markets where, although no one generator is pivotal in the market, there may not be a large surplus of each type of generation. We find that a sequential auction where ownership of generation technologies is not sufficiently dispersed may result in inefficient equilibria.

Finally, empirical studies of strategic bidding in the electricity markets have been address by Patrick and Wolak (1997), Wolfram (1998, 1999), and Borenstein et al. (2002).

### 2.2. Model

In order to gain a better understanding of the incentives provided by different auction structures and the ability of each auction structure to induce the efficient dispatch, we assume that electricity demand is both deterministic and completely inelastic and that the auction is conducted only once. While in reality, the auction will be repeated daily, we can gain some insight into the equilibrium strategies of the repeated game by examining

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[1] It is interesting to note that in his design of the original California Power Exchange, Robert Wilson initially designed the auction to be iterative/open, i.e., bidders would have 6 rounds in which to submit bids. However, difficulties in the software and time-constraints on the bidders forced the market operators to close the auction after one round, i.e., run a sealed bid auction.

a one-shot version. For simplicity, demand is always assumed to be a step function which is constant during each hour (any daily demand can be approximated, to a first order, by a step-function) and is public information and hence known to all generators.

A generation company (referred to as a generator) can own several generating plants. A generating plant can be one of $n$ technology types. Each plant type $i$ has two costs associated with generation: a fixed "start-up" cost, $f_i$, $f_i > 0$, and a constant variable cost per MW hour, $v_i$, $v_i > 0$, once the plant is up and running. [2] Each plant is characterized by a capacity $K$, the maximum rate at which he can generate electricity. (Without loss of generality, we will scale demand and all of the plants' capacities such that $K \geqslant 1$.) For simplicity, we assume that $K$ is always an integer. The cost of generating a total of $Q$ MW hour in $T$ h from a type $i$ plant is $C_i(Q) = f_i + Qv_i$ for $Q > 0$ where $Q = \sum_{t=1}^{T} q_t$ where $q_t$ is the number of MW generated during hour $t$ and $q_t \leqslant K$ and $C_i(0) = 0$. The capacity constraint implies that a plant cannot supply more than $K$ MW at any point in time, but places no restrictions on the duration for which he can generate. We assume that generation costs, the market structure and the efficient dispatch satisfy the following properties:

**Assumption 1.** All cost and capacity information is publicly known.

**Assumption 2.** All plants can generate instantaneously and for any duration of time.

**Assumption 3.** $v$ is strictly decreasing in $f$ and $K$ is increasing in $f$.

**Assumption 4.** The generation capacity in the market is such that all the plants owned by any one generator of size $K$ or higher can be removed and there is sufficient capacity from plants of size $K$ or higher to meet demand, i.e., no one generator is a "pivotal" supplier and must be dispatched in order to meet demand.

**Assumption 5.** Suppose plant type $p$ with capacity $K$ is dispatched for $q$ MW hours in the efficient dispatch and incurs a cost of $C_p(q)$. Then $C_p(q) \leqslant C_j(q)$ for all $j \neq p$.

Assumption 1 implies that the efficient dispatch is known by all (the auctioneer and generators alike). However, the decentralized nature of an electricity auction implies that the auctioneer must rely on the generators' bid in order to determine the final dispatch. Assumption 2 implies that we are abstracting away from any of the operational constraints on generation plants, e.g., ramp-up time, minimum-up times, etc. Assumption 3 implies that the technology types cost curves cross at most once and in a systematic order. Assumption 4 is an assumption on the presence of market power; it rules out any markets where extreme market power is present, by eliminating the existence of a "captured" market for any one generator. Assumption 5 states that we are examining a generation market where (i) an inefficient plant is never used in the least-cost dispatch (a plant is efficient if it defines the efficiency frontier for some output level) and (ii) in the efficient dispatch, the dispatched plants are always operating on the efficiency frontier. While Assumptions 4 and 5 are strong assumptions on the cost structure and number of plants in the market and implies that we are examining a particular type of efficient dispatch, they allows us to evaluate the performance of various auction designs in an abundant supply market environment. Based on these assumptions on the generation market, we have that there is a unique efficient dispatch [3] and that all plants win continuous blocks of MW hours, starting from the base of demand going up to the peak in the efficient dispatch.

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[2] Assuming a constant variable cost is a simplifying assumption of this paper. Variable generation costs are typically non-decreasing in the quantity produced at any point in time.

[3] That is, the technology types (not owners) and their dispatched quantities are unique.

### 2.3. Auction structures

In designing an electricity auction, the auctioneer must decide on multiple auction dimensions, such as: the bid format, what the pricing rule will be, the sequencing of the auction and how to bundle demand into lots. We assume the following auction rules apply to all of the auction structures considered,

*Auction rule 1:* Each (demand) lot is auctioned independently.

*Auction rule 2:* Generators submit a (binding) [4] energy-only price bid per demand lot per plant via a sealed bid and are dispatched in increasing order of bids for a lot.

*Auction rule 3:* In the event of a tie in an auction, the auctioneer selects the cheaper (in terms of real costs, not bids) plant as the winner.

Auction rule 1 implies that we are studying only simple auctions. Auction rule 2 implies that we allow only energy bids, i.e., generators cannot separately indicate their start-up costs, but must incorporate them into their energy bids.

California initially decided to bundle demand into vertical lots (hourly auctions), to pay the same uniform price to every winning generator in a lot, and to have generators submit their bids for all lots simultaneously. As will be shown in Section 3.1, this auction structure can support inefficient dispatches in equilibrium. In order to understand why this is true and how the auction might be modified to remedy this problem, we identify the different auction dimensions and the possible alternatives within each dimension.

*Bid format.* While we are analyzing auctions where generators submit energy-only bids, there is still a design question as to the maximum number steps to allow in a supply function. On one extreme, generators could be restricted to submitting

a single price per lot per plant, i.e., a flat supply curve. Conversely, assuming that output is an integer variable, generators could be allowed to submit a multiple step function supply curve, with a distinct step form each output level.

*Sequencing of auctions.* When there is more than one demand lot to be auctioned, the auctioneer must decide how to sequence their sale. In a simultaneous auction, the bids are submitted, and allocation decisions for all demand lots are made simultaneously. Alternatively, in a sequential auction, demand lots are auctioned sequentially; before each new auction, the results of any previous auctions are made known.

*Pricing rule.* Before the bidders submit their bids, they must know how the prices at which the transactions take place are to be determined. If there is more than one winner per lot, each with different bids, in a lot, do the winners get paid the same price, or different prices? The former pricing rule is called a uniform pricing rule and the latter a discriminatory pricing rule. Presently, all of the electricity auctions in operation adopt a *uniform pricing rule*, where the winners are paid the highest accepted bid price. (This is because an electricity auction is a procurement auction, and hence the goal is to solicit the lowest bids.) [5] Under a *discriminatory pricing rule*, each winner is paid his own bid price.

*Bundling of demand into lots.* In the case of electricity, the basic object to be auctioned is 1 MW hour of the forecasted daily demand. When there are several objects to be auctioned, the auctioneer must decide how to bundle the objects into lots for auction. While there exist countless ways to bundle demand, most electricity auctions par-

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[4] A plant that submits a winning bid for a lot will be expected to generate the specified load it won or incur a large penalty.

[5] An alternative pricing format that is possible but is not being used or under consideration by any of the electricity markets, would be to pay all the bidders the lowest rejected bid price, i.e., a second-price rule. Unfortunately, given the multi-unit nature of the auction, a second-price rule fails to have the nice incentive properties that are generally associated with the single-unit second-price auction. The multi-unit generalization of a second-price auction is referred to as a Vickrey–Clark–Groves (VCG) auction. This auction, while theoretically elegant, is rarely used in practice, as was discussed in Section 2.1.1.
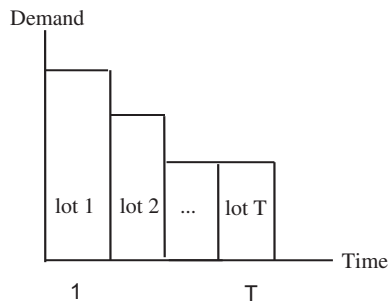
Fig. 1. Vertical auction.

tition demand into distinct auctions according to the hour in which they occur, referred to as vertical auctions.

**Definition 1.** In a vertical auction (see Fig. 1), daily demand is divided into $T$ hourly demand lots, where each demand lot contains all the demand in hour $t$, $t = 1 \ldots T$. For each hour $t$, generators submit a bid (per plant) that reflects the minimum price they must be paid per megawatt generated in hour $t$.

The question of interest to us is: In a complete information setting, can a vertical auction, in any of its multiple manifestations (simultaneous/sequential, single/multiple step bid, uniform/discriminatory), induce non-cooperative profit-maximizing generators to bid in a way that always results in an efficient dispatch in Nash equilibrium (or a Subgame perfect Nash equilibrium in the case of a sequential auction), i.e., are all equilibria efficient for all demand scenarios? We find that, even in our simple setting, the answer is no.

## 3. Demand lots with multiple winners: Vertical auctions

In this section, we will demonstrate that, for each form of a vertical auction (simultaneous, sequential, discriminatory or uniform), we are able to find an instance of demand for which either (1) there exists an inefficient equilibrium or (2) the efficient dispatch is not supported in equilibrium. Hence, a vertical auction cannot guarantee efficiency in equilibrium. It is important to point out

that the existence of an inefficient dispatch in equilibrium does not imply the absence of an equilibrium supporting the efficient dispatch. Given the presence of both efficient and inefficient equilibrium dispatches, game theory does not allow us to conclude which one will occur. However, the focus of this paper, i.e., to find an auction format that supports only the efficient dispatch in equilibrium, implies that we can immediately dismiss from consideration any auction with an inefficient dispatch in equilibrium.

Assume throughout Section 3 that there are four technology types and that each generator owns only one plant. There are two generators of technology type $n = 1$ ($G_{11}$ and $G_{12}$), one generator of type $n = 2$ ($G_2$), two generators of type $n = 3$ ($G_{31}$ and $G_{32}$), and one generator of type $n = 4$ ($G_4$). The capacity of the plant technology type $n = 1$ is $K = 1$ MW and the remaining technology types ($n = 2, 3, 4$) have a capacity of $K = 2$ MW. In addition, suppose that forecasted demand is as in Fig. 3. This demand model, albeit a simple one, is rich enough to illustrate the failings of a vertical auction. Fig. 2 plots the total costs of generation associated with different types of plant technology, assuming a generating plant is "switched on" only once (the horizontal axis measures the total number of MW hour generated over time).

Given the assumed demand, cost, and capacity functions, the unique efficient dispatch is given in Fig. 3. Note that the efficient dispatch is always the same, regardless of the auction mechanism. The efficient dispatch consists of the type 4 generator ($G_4$) supplying 2 MW hour in both time periods,
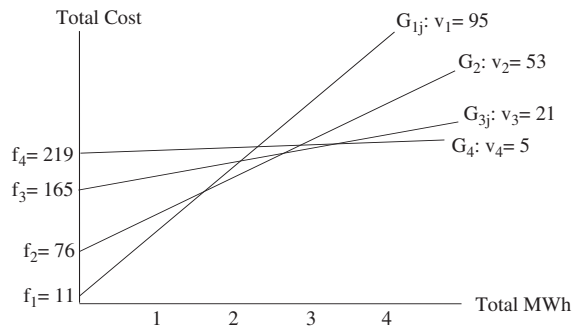


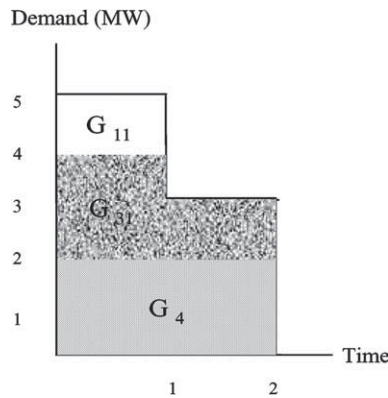Fig. 2. Total cost for plant technology types $n = 1, \ldots, 4$.

Fig. 3. The efficient dispatch.

the same type 3 generator $(G_{3j})$ supplying 2 MW hour in the first period and 1 MW hour in the second, and a type 1 generator $(G_{1j})$ supplying the top 1 MW. For expositional ease and without loss of generality, we will assume that the winning generators in the efficient dispatch are $G_{11}$, $G_{31}$, and $G_4$.

A vertical auction of the demand in Fig. 3 consists of the auction of two lots, as in Fig. 4. Each generator $G_i$ submits two bids, one for each lot $k$, defined to be $b_i^k$ (recall that each generator owns only one plant and therefore submits only one bid per lot). A bid of $b_i^k$ is the minimum amount generator $G_{ij}$ must be paid per MW hour generated in lot $k$. In order for the efficient dispatch to result from the submitted bids, $G_{11}$, $G_{31}$, and $G_4$ must submit the lowest bids in *lot 1* and $G_{31}$
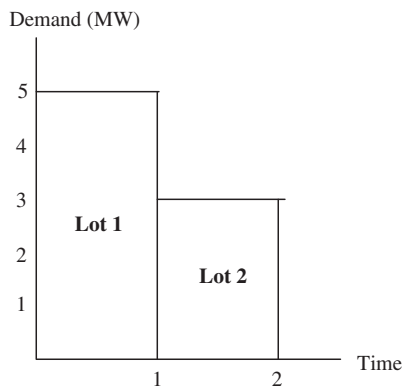


Fig. 4. Vertical auction of demand.

and $G_4$ must submit the lowest bids in *lot 2*, regardless of the pricing rule or auction sequencing. In *lots 1* and *2*, the winning bids must be ordered as follows: $b_4^1 < b_{11}^1$, $b_{31}^1 < b_1^1 1$, and $b_4^2 < b_{31}^2$.

### 3.1. Vertical uniform auction

{*Note*: These results can be found in Elmaghraby and Oren (1999) and are restated here for the sake of completeness of the proof. In their paper, Elmaghraby and Oren refer to a vertical uniform auction as an hourly supply curve-vertical auction}.

In a uniform price auction, all generators who win and are dispatched in a given lot are paid a uniform price equal to the highest accepted bid. We show in this section that when there is more than one winner per demand lot, a uniform pricing is unable to guarantee an efficient dispatch in equilibrium because: (1) The bid price is separated from the received price for all except the marginally dispatched generators. (2) The same MW hour price is paid for each MW hour generated in a lot. (3) The total cost curve is strictly concave and (4) generators may be dispatched at different output levels within an hour.

Given the demand in Fig. 3 and capacity assumptions, Table 1 defines a set of equilibrium bids for all the generators, which constitute an *inefficient* dispatch for a vertical, uniform, simultaneous auction (see Fig. 5 for the inefficient dispatch). (The same inefficient dispatch can be supported in a vertical, uniform, sequential auction. The inefficient equilibrium bidding strategies are provided in Appendix A.) The winning bids are followed by an asterisk. (*Note*: Neither these bids nor dispatch constitute the unique equilibrium for either auction. In particular, the efficient dispatch

Table 1
Inefficient dispatch in vertical uniform auction

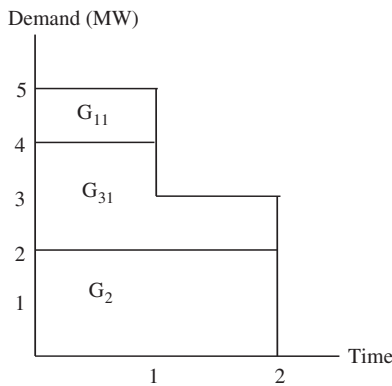| Generator | Bid for *lot 1* $(b_i^1)$ | Bid for *lot 2* $(b_i^2)$ |
|---|---|---|
| $G_{11}$ | 106* | 95 |
| $G_{12}$ | 106 | 106 |
| $G_2$ | 0* | 0* |
| $G_{31}$ | 0* | 94* |
| $G_{32}$ | 186 | 186 |
| $G_4$ | 224 | 224 |

Fig. 5. Inefficient equilibrium dispatch.

can also be supported in equilibrium.) Assume that the smallest bid increment $\delta$ is equal to one.

Given their competitors' strategies in Table 1, no generator has an incentive to deviate from his bids. This profile of bids results in $G_2$ being dispatched for 2 MW hour in *lots 1* and *2*, $G_{31}$ being dispatched for 2 MW hour in *lot 1* and 1 MW hour in *lot 2*, and $G_{11}$ being dispatched for 1 MW hour in *lot 1*. The clearing price received by all winning generators in *lot 1* is 106 per MW hour and in *lot 2* is 94 per MW hour. Since $G_2$'s payoff is not determined by her bid, she has every incentive to bid as low as possible to ensure dispatch. By submitting a bid of zero in *lots 1* and *2*, $G_2$ is able to win dispatch at a positive profit $(2 * 106 + 2 * 94 - 288 = 112)$. [6] Although $G_4$ is (one of) the least-cost producers of 4 MW hour, he is unable to profitably undercut $G_2$'s bids of $b_2^1 = b_2^2 = 0$.

This simple example clearly illustrates why a vertical, uniform auction cannot guarantee efficiency in a multi-unit environment with fixed plus variable (strictly concave) costs. If the demand in any hour $t$ is not an integer multiple of $K$, then not all the generators will be dispatched at the same output level within that hour. In this scenario, there exists an opportunity for a relatively inefficient generator to accrue a positive profit by bidding zero and ensuring dispatch without fear of receiving his below-cost bid price. With the

knowledge that in equilibrium the clearing price is guaranteed to be at least the cost of the marginal bidder in hour $t$, a relatively inefficient generator can "sneak-in" to the dispatch schedule by submitting a zero bid, get dispatched at a higher level in hour $t$ than the marginal price-setting bidder and accrue a positive profit due to the concavity of his cost curve. It is important to point out that this result also holds when each generator owns several (not necessarily identical) generation plants.

This "zero" bid strategy creates a very similar effect to one identified by Back and Zender (1993), under common valuations, for the uniform auction of Treasury bills. Bidders are able to costlessly deter competitors from bidding more aggressively by submitting extremely steep demand curves. The low bids on inframarginal quantities have no chance of determining the clearing price, but act as a deterrent to competitors from bidding more aggressively.

### 3.1.1. Multiple step bids in a uniform, vertical, simultaneous auction

It might be thought that the restricted bid structure, i.e., one in which generators are restricted to submitting a flat supply curve, is the culprit behind this "zero" bid strategy and resulting inefficiency in uniform-price auctions. As a fact, almost all electricity auctions allow generators to submit multiple steps per production plant; the original UK design allowed for 3 steps while the Spanish design allowed for 25 steps (Fabra et al., 2002). Therefore, allowing generators to submit a multiple step bid which is contingent on quantity should help reduce the existence of inefficient equilibria. We find that this is not true; inefficient equilibria continue to be supported in equilibrium despite the added flexibility of multi-step/quantity-specific bids.

Assume the same framework of generators and demand as in Section 3.1, and that the generators costs are given by Fig. 2. The bid format now allows generators to submit (up-to) a two-part bid in each hour. Since each generator can generate only in increments of 1 MW and has a capacity of $K = 2$, a 2-part bid can sufficiently capture and reflect its cost structure. Generators submit a two-part bid which is of the form

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[6] In addition, $G_{31}$ accrues a profit of $(2 * 106 + 94 - 228 = 78)$ and $G_{11}$ accrues a profit of zero.

$$\left\{\begin{array}{c} \text{minimum price to generate} \\ \text{1MW during hour } t \\ \text{minimum price to generate} \\ \text{2MW during hour } t \end{array}\right\}$$

for each hour $t = 1, 2$.

The bids in Table 2 support the same inefficient dispatch found in Section 3.1 and constitute a Nash equilibrium bidding strategy (asterisks follow winning bids). Under these bids, the least-cost dispatch is to accept $G_2$'s bids for 2 MW during both hours, $G_{31}$'s bid for 2 MW hour in hour 1 and 1 MW hour in hour 2, and $G_{1y}$'s bid for 1 MW during hour 1, for $y = 1$ or 2. This sets the clearing prices paid per MW in hours 1 and 2 to be $f_1 + v_1$ and $v_1 - \delta$, respectively. At these clearing prices, $G_2$ and $G_{31}$ earns a positive profit and $G_{1y}$ earn a zero profit. $G_2$ and $G_{31}$ have successfully submitted sufficiently low bids so as to make it impossible for any other generator to profitably undercut them in either time hour for either quantity level.

Not only does a richer bid structure not pre-empt inefficient dispatching in equilibrium, in addition it poses the combinatorial optimization problem for the auctioneer of identifying the least-cost manner of satisfying demand in each hour. In these simple examples, a plant can generate only at two possible output levels, while in reality the possible output levels are much larger.

Table 2
Inefficient equilibrium 2-part bidding strategies

| Generator | Bids for hour 1 | Bids for hour 2 |
|---|---|---|
| $G_{11}$ | $\begin{pmatrix} f_1 + v_1^* \\ - \end{pmatrix}$ | $\begin{pmatrix} v_1 \\ - \end{pmatrix}$ |
| $G_{12}$ | $\begin{pmatrix} f_1 + v_1^* \\ - \end{pmatrix}$ | $\begin{pmatrix} v_1 \\ - \end{pmatrix}$ |
| $G_2$ | $\begin{pmatrix} 0 \\ 0^* \end{pmatrix}$ | $\begin{pmatrix} 0 \\ 0^* \end{pmatrix}$ |
| $G_{31}$ | $\begin{pmatrix} 0 \\ 0^* \end{pmatrix}$ | $\begin{pmatrix} v_1 - \delta^* \\ 2v_1 - \delta \end{pmatrix}$ |
| $G_{32}$ | $\begin{pmatrix} f_3 + v_3 \\ f_3 + 2v_3 \end{pmatrix}$ | $\begin{pmatrix} f_3 + v_3 \\ f_3 + 2v_3 \end{pmatrix}$ |
| $G_4$ | $\begin{pmatrix} f_4 + v_4 \\ f_4 + 2v_4 \end{pmatrix}$ | $\begin{pmatrix} f_4 + v_4 \\ f_4 + 2v_4 \end{pmatrix}$ |

### 3.2. Vertical discriminatory auctions

Prior to their FERC-ordered shutdown, CA hired a Blue Ribbon panel to explore if switching to a discriminatory auction from a uniform-price auction would help alleviate the exercise of market power in the electricity auction (Kahn et al., 2001). The panel's conclusion was that a move to discriminatory pricing may create more problems than it solves. One of the potential problems the panel pointed to was that, under incomplete information, a discriminatory pricing rule creates a "Guess the Clearing Price" game; this game may give larger suppliers an informational advantage and hence encourage industry consolidation (pushing the market in the opposite direction from competitive). The panel also pointed out that the "Guess the Clearing Price" game could result in inefficient dispatches due to errors in guessing.

While the panel's focus was on the problems created by the use of a discriminatory pricing rule in a marketplace where generators have incomplete information about each others' costs, the exposition below demonstrates that a discriminatory price auction also creates instability in the marketplace *with* complete information. That is, while a uniform-price vertical auction of the demand in Fig. 3 fails to guarantee efficiency in equilibrium because of the possible existence of inefficient dispatches in equilibrium, a discriminatory-price vertical auction fails because it cannot support the efficient dispatch in equilibrium. The following exposition is true for both a vertical, discriminatory, sequential and a vertical, discriminatory, simultaneous auction with single or two-part bids.

In a discriminatory-price auction, each generator chosen for dispatch receives his own bid. For the efficient dispatch to occur in equilibrium in a vertical auction (as stated earlier) the three lowest bids in *lot 1* must be from a type 4, 3, and 1 generator (with the type 1 generator having the third lowest bid) and the two lowest bids in *lot 2* must be from a type 4 and 3 generator (with the type 3 generator having the second lowest bid). As there are two identical generators of type 1, in an efficient equilibrium, $G_{11}$ must be earning zero profits. If $G_{11}$ were earning a positive profit, then genera-

tor $G_{12}$, would have the incentive to undercut $b_{11}^1$ and replace $G_{11}$ as the third lowest bidder in *lot 1*. Therefore we know that, in an efficient equilibrium, $G_{11}$ will bid $b_{11}^1 = f_1 + v_1 = 106$. Since each generator is paid what he bids, and each generator wishes to maximize his profits, in an efficient equilibrium, $G_{31}$ and $G_4$ will bid $b_{31}^1 = b_4^1 = f_1 + v_1 - \delta = 105$, [7] and $G_{32}$ must submit a bid for *lot 1* greater than 106. At a bid of $b_{31}^1 = 105$, $G_{31}$ is dispatched at 2 MW hour in *lot 1* and earns a positive profit of $210 - 207 = 3$, while $G_{32}$ is not dispatched and earns zero profits. Therefore, $G_{32}$ has the incentive to undercut $b_{31}^1$ and replace $G_{31}$'s position in the bid ordering. But then we have just shown that the bids (and bid ordering) necessary to support an efficient dispatch are not equilibrium bidding strategies. Therefore, the efficient dispatch cannot be supported in a discriminatory-price, vertical, simultaneous or sequential auction.

The structure of vertical demand lots creates the need for more than one winner per lot, which in turn creates a barrier to guaranteeing efficiency. A generator's dispatch depends upon his bids' placement within in a lot. In this example, the efficient dispatch requires that different types of generators, with different cost structures, win within a lot. This fact, coupled with the winning generators' desire to maximize their profits, creates a situation where $G_{31}$ bids above its costs. But in the presence of another identical generators, it cannot be an equilibrium for $G_{31}$ to bid above its cost without other generators undercutting its bid.

## 4. Demand lots with only one winner: Horizontal auctions

In Section 3, we established the inability of vertical auctions to guarantee the existence of, and only of, efficient equilibria in a market that satisfies Assumptions 1–5. Therefore, it is appropriate to search for an alternative bundling form which assures only one winner per lot. Elmaghraby and Oren (1999) study an alternative bundling form,

they coin horizontal bundling, where electricity demand is partitioned according to its duration. They show that when (i) when the efficient dispatch requires identical plants to win within the same horizontal strip and all plants exhaust their entire capacity within a horizontal strip and (ii) no one generator is the sole provider of a plant type used in the efficient dispatch, a sequential horizontal auction supports only the efficient dispatch in equilibrium. If assumption (i) is relaxed, then a horizontal auction suffers from the same drawbacks as a vertical auction. That is, it is possible for there to be more than one winner per lot and winners are awarded different production quantities. In an electricity auction, where demand is larger than the capacity of any one generator, it is not possible to achieve only one winner per lot when demand is partitioned vertically. However, it is possible if we slightly redefine horizontal partitioning.

**Definition 2.** In a horizontal auction, demand lots are formed by partitioning the daily demand into horizontal strips that mimic the efficient dispatch. Generators submit a bid (per plant) that reflects the minimum price they must be paid to generate and supply all of the MW hours in a lot, i.e., only one winner per lot is allowed.

In a horizontal auction, the auctioneer must decide on the shape of the horizontal strips. Given her knowledge of the efficient dispatch, the auctioneer should partition demand into strips that mimic the efficient dispatch (see Fig. 6).

A horizontal auction is, by definition, a discriminatory, one-part bid auction since there is only one winner per lot and generators must supply all or none of the demand in a lot. There is still the possibility of conducting the auction of the lots simultaneously or sequentially. In this section, we show that when assumption (ii) of Elmaghraby and Oren (1999) is relaxed, i.e., all plants of a particular technology type are owned by a single generator, a sequential horizontal auction can support inefficient dispatches in equilibrium. However, when all the bids are submitted simultaneously, only the efficient dispatch is supported in a Nash equilibrium.

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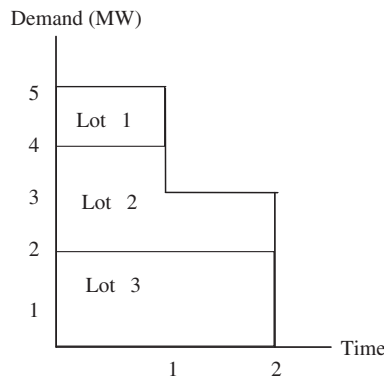[7] This would be their bid for 2 MW hours under a two-part bid.

Fig. 6. Demand lots in the horizontal auction.

Section 4.1 contains an example of a sequential auction of demand in which the set of equilibrium dispatches contains an inefficient dispatch. Section 4.2 concludes with a proof stating that the efficient dispatch is supportable in a pure-strategy Nash equilibrium and that it is the unique equilibrium dispatch of a horizontal simultaneous auction.

### 4.1. Horizontal sequential auction

A horizontal sequential auction allows inefficient dispatches to be supported in equilibrium if the ownership of a particular technology type is concentrated in the hands of a few generators. In particular, if each of a small number of technology types has a single owner, a relatively inefficient generator is able to bid strategically so as to squeeze out an efficient competitor.

**Theorem 1.** *In a market setting satisfying Assumptions 1–5, a horizontal sequential auction cannot guarantee efficiency in equilibrium if there is a concentrated distribution of generation resources.*

**Proof.** Theorem 1 is proven by counterexample. Suppose that there are only three generators in the market, $G_1$, $G_2$, and $G_3$ who each own two identical generating plants, denoted by $g_i$, with $K = 1$ each (for convenience, we have dropped the second subscript since $M = 1$). Furthermore, assume that the generation costs associated with plant $g_i$ are as follows: $(f_1, v_1) = (11, 95)$, $(f_2, v_2) = (71, 53)$ and $(f_1, v_1) = (165, 21)$.

Assume that there exists a daily demand given by Fig. 7, which is to be auctioned via a horizontal, sequential auction. The longest duration lot, *lot* 3, is auctioned first, followed by *lot* 2 and then *lot* 1. Fig. 7 also depicts the unique efficient dispatch. Despite the simple structure of demand, it is possible to support an inefficient dispatch in equilibrium, in particular the dispatch given in Fig. 8.

The equilibrium strategies supporting this inefficient dispatch are given in Appendix A. We briefly summarize here the strategies used and incentives behind them. Recall that each generator owns two identical plants with $K = 1$. The dispatch in Fig. 8 is a result of equilibrium bids where $G_2$ submits a bid of $f_3 + 3v_3 - \delta = 227$ for *lot* 3 for one of his $g_2$ plants. Such a bid is below both her and $G_3$'s cost to supply 3 MW hour and allows her to win *lot* 3. $G_2$ knows that by winning *lot* 3, she is committing herself to participate in only one of the two remaining auctions (due to capacity con-
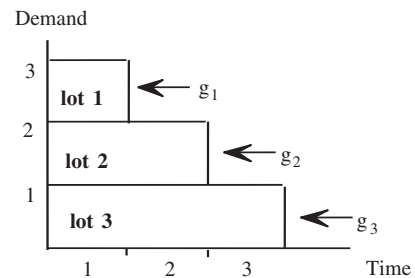


Fig. 7. The unique efficient dispatch for a sequential *horizontal* auction.
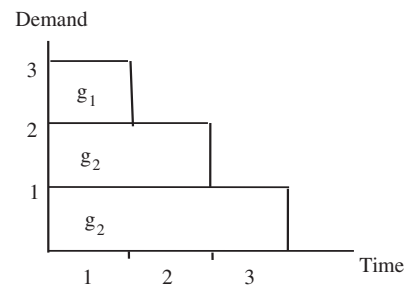


Fig. 8. Inefficient dispatch for a sequential *horizontal* auction.

straints). If all three generators were to participate in every auction, the upper bound on $G_2$'s winning bid for *lot* 2 is $f_1 + 2v_1 = 201$, and the upper bound on $G_1$'s winning bid for *lot* 1 is $f_2 + v_2 = 124$. Given $G_2$ has won *lot* 3, it is to $G_1$'s advantage that $G_2$ win *lot* 2 and hence be removed from participating in the auction for *lot* 1. With $G_2$ no longer participating in the auction, $G_1$ is able to win *lot* 1 at a bid of $f_3 + v_3 = 186$. Therefore, $G_1$'s optimal response is to not undercut $G_2$'s bid for *lot* 2 and allow $G_2$ to win *lot* 2 with a bid of $f_3 + 2v_3 = 207$. Hence, $G_2$ is able to undercut $G_3$'s lower costs for *lot* 3 with the knowledge that it is in $G_1$'s best interest to allow her to win *lot* 2 with a large profit margin. □

Bundling demand so that there is one winner per lot did not remove the incentives for relatively inefficient generators to bid below cost nor prevent the resulting inefficient dispatch. It is the ability to change the upper bounds on winning bids via strategic interactions that allows an inefficient dispatch to be supported in equilibrium. As a consequence of the sequential nature of the auction and the lack of competition from identical generators, $G_1$ knows that, given $G_2$ wins both *lot 3* and *lot 2*, the upper bound on a winning bid for *lot 1* is raised from 124 to 186. Similarly, the sole reason winning *lot 3* is part of an overall profitable strategy for $G_2$ is that $G_2$ is able to change the upper bound on her winning bid for *lot 2* from 201 to 207.

The inefficient dispatch given in Fig. 8 was made possible by the changing set of active bidders across the lots: In the final auction, no technology type 2 plant was bidding. In order to avoid such situations and to ensure the set of active bidder types is the same for each lot, there must be a "extreme" surplus of each technology type in the market (e.g., each technology type has sufficient capacity to supply entire demand). However, such an extreme market structure is not viable and certainly is not found in many electricity supply industries. Fortunately, there exists an alternative auction form which precludes inefficient dispatches in equilibrium and supports the efficient dispatch in equilibrium. It is to this auction that we turn our attention to next.

## 4.2. Horizontal simultaneous auctions

This section concludes our search for an auction which supports the efficient dispatch and *only* the efficient dispatch in a one-shot, pure strategy Nash equilibrium. A simultaneous *horizontal* (discriminatory) auction has an additional attribute necessary to eliminate inefficient dispatches and support the efficient dispatch: the simultaneity of bids. In a simultaneous auction, a bidder is unable to change the set of potential bidders for each lot via strategic bidding. A simultaneous auction with discriminatory pricing creates a strict upper bound on winning bids. When a generator is paid his bid and there is no possibility of changing the upper bound on a winning bid through strategic interaction, he will never operate one of his plants at a negative profit: A generator whose total bids for a plant are less than the plant's generation costs will always be better off by withdrawing his bid(s) and earning zero profit. In Theorem 2 we prove that the efficient dispatch is a Nash equilibrium for a market setting satisfying Assumptions 1–5. Theorem 3 demonstrates that inefficient dispatches cannot be a Nash equilibrium in these markets. Hence, the efficient dispatch, and only the efficient dispatch, can occur in equilibrium.

Defined below are bids that result in the efficient dispatch.

*Efficient bids.* Suppose that there are $\Omega$ lots for auction in a *horizontal* auction. Let $D^* = (D_1^*, \ldots, D_\Omega^*)$ denote the efficient dispatch, where $D_\omega^*$ indicates the plant that wins lot $\omega$ in the efficient dispatch. A set of bidding strategies that would result in the efficient dispatch in a simultaneous *horizontal* auction is:

For lots $l = 1, \ldots, \Omega$           (1)

plant $D_l^*$

    bids $\Pi(D_l^*) + C_{D_l^*}(\mathrm{MW}(D_l^*))$ for lot $l$,     (2)

all plants in $P_{-D_l^*}$ and $\overline{P}_{-D_l^*}$

    bid $\Pi(D_l^*) + C_{D_l^*}(\mathrm{MW}(D_l^*)) + \delta$ for lot $l$,     (3)

all plants (except $D_l^*$) in $P_{D_l^*}$ and $\overline{P}_{D_l^*}$

    bid $Z$ for lot $l$,                        (4)

where $MW(p)$ is the total number of MW hours generated by plant $p$ in $D^*$, $C_p(q)$ is plant $p$'s total cost to generate $q$ MW hours, $P$ is the set of plants that are in $D^*$, $P_i$ is the set of plants in $D^*$ that are owned by the same generator as plant $i$, $P_{-i}$ is the set of plants in $D^*$ that are owned by competitors of plant $i$, $\overline{P}$ is the set of plants that are not in $D^*$, $\overline{P}_i$ is the set of plants that are not in $D^*$ and that are owned by the same generator as plant $i$, and $\overline{P}_{-i}$ is the set of plants that are not in $D^*$ and that are owned by competitors of plant $i$, $\Pi(p)$ is plant $p$'s profit in the efficient dispatch $D^*$ and $Z$ is some very large number (that is greater than all winning bids).

A simple algorithm can be used to determine, $\Pi(\cdot)$, the maximum profit any dispatched plant can earn. Let $X(K)$ be the set of plants in set $X$ with a capacity of at least $K$. For each plant $p$ in $P$, $\Pi(p)$, can be determined as follows:

*Step 1* Set $t = 0$. $\forall p$ in $P$
Set $\pi_p^0 = +\infty$,
If $\overline{P}_{-p}(K) \neq \emptyset$ set $\pi_p^t = \min_{i \in \overline{P}_{-p}(K)} [C_i(MW(p)) - C_p(MW(p))]$.

*Step 2* Set $t \to t+1$. $\forall p$ in $P$,
If $P_{-p}(K) \neq \emptyset$ set $\pi_p^t = \min[\min_{i \in P_{-p}(K)} [C_i(MW(p)) + \pi_i^{t-1} - C_p(MW(p))], \pi_p^{t-1}]$;
otherwise set $\pi_p^t = \pi_p^{t-1}$.

*Step 3* If $\pi_p^t = \pi_p^{t-1}$ $\forall p$ in $P$; set $\Pi(p) = \pi_p^t$; otherwise, repeat step 2.

By Assumption 4 we know that either set $P_{-i}(K)$ or $\overline{P}_{-i}(K)$ is non-empty for all $i$. If generators bid according to the outlined strategy, the resulting dispatch will be efficient. However, whether or not these bids constitute an equilibrium will be addressed next.

**Theorem 2.** *Consider a market setting satisfying Assumptions 1–5. In this market setting, the set of pure-strategy Nash equilibria in a simultaneous horizontal auction is non-empty and contains an efficient dispatch.*

**Proof.** In order for the efficient bid strategies (defined in lines (1)–(4)) to constitute a Nash equilibrium, it must be true that (a) no plant has an incentive to deviate given all of its competitors'

bids and (b) no plant is paid less than its generation costs. Without loss of generality, we will focus on the proposed bids for plant $r$ who is owned by generator $G^r$ and who generates $q$ MW hours in $D^*$.

Step 1 ensures that it is unprofitable for any non-dispatched plant owned by another generator to undercut plant $r$'s bids, while step 2 ensures that it is unprofitable for any dispatched plant owned by another generator to undercut plant $r$'s bids. Some dispatched plants may be operating at a profit and therefore would require a bid (significantly) above their costs to induce them to undercut an opponent. This profit is incorporated in the calculation of $\Pi$ in step 2. Therefore, plant $r$'s strategy, as defined in lines (1)–(4) imply that no opponent can profitably undercut $r$'s bids.

In addition, we need to show that it is unprofitable for a generator to change any of his winning plants' bids. Obviously, it would not be profitable to decrease the winning bids. Hence, it remains to show that no generator has the incentive increase his bid on any of his plants in $D^*$. Assumption 4 implies that no one generator is able to simultaneously increase his bids on all of or any one of his plants, and still guarantee to have those plants be dispatched.

Finally, it remains to show that $\Pi(r) \geqslant 0$ (otherwise plant $r$ would be operating at a loss, which cannot be supported in equilibrium). This follows directly from Assumption 5 (which establishes plant $r$ as the least cost produce of $q$ MW hours), Assumption 3 and the construction of the bids via steps 1 and 2: Since plant $r$ is (one of) the least cost producer of $q$ MW hours, and its total bid is bound above by its competitors' smallest generation costs for $q$ MW hours plus that competitor's profit margin, $r$'s total bid will be greater than or equal to its own total generation costs. If $\Pi(r) < 0$, that would imply that one its competitor plants has a generation cost for $q$ MW hours that is below $r$'s, which directly contradicts Assumption 5. Therefore, we have satisfied the conditions that (a) no plant has an incentive to deviate given all of its competitors bids and (b) no generator is dispatching a plant below its generation costs, and have shown that

the bids given by lines (1)–(4) constitute an efficient Nash equilibrium.  □

In addition to supporting the efficient dispatch in equilibrium, we need to argue that a horizontal simultaneous auction does not support any inefficient dispatches in equilibrium: Theorem 3 does just that.

**Theorem 3.** *In any market setting satisfying Assumptions 1–5, the set of pure-strategy Nash equilibria in a simultaneous horizontal auction does not contain an inefficient dispatch.*

**Proof.** This proof is done by contradiction. We will demonstrate that there cannot exist an inefficient dispatch $\widehat{D}$ in equilibrium. The necessary conditions for $\widehat{D}$ to be an equilibrium implies that no single generator would be made strictly better off by switching its bid so as to win its dispatch in $D^*$. We will demonstrate that this condition cannot be simultaneously feasible for all generators. Suppose, as in Theorem 2, that there are $\Omega$ lots for auction in a horizontal auction and $P$ plants are participating in the auction. Let $D^* = (D1, \ldots, D_\Omega)$ denote a particular efficient dispatch, the components of which indicate which plant wins lot $\omega$ in an efficient dispatch. The total cost associated with the efficient dispatch is denoted by $C(D^*)$. Suppose that there *does* exist an inefficient dispatch, $\widehat{D}$, in equilibrium, i.e., the types of plants dispatched and their schedules in $\widehat{D}$ are not the same as in $\widehat{D}$. Let $\widehat{B} = (\widehat{B}_1, \ldots, \widehat{B}_\Omega)$ denote the winning bids and $C(\widehat{D})$ to be the total generation costs associated with the inefficient dispatch $\widehat{D}$.

Given that each generator is paid his bid and that all bids are submitted simultaneously, we know that no generator has an incentive to bid a plant below its cost. In equilibrium, each generator must be making a non-negative profit and no generator must have an incentive to change his bid given his competitors' bids. For example, for a generator that would have plants i and j dispatched for lots $\omega$ and $\gamma$, respectively, in $D^*$ but has nothing dispatched at all in $\widehat{D}$, the equilibrium bid $\widehat{B}_\omega$ must satisfy

$$\underbrace{\widehat{B}_\omega + \widehat{B}_\gamma - C_i(\omega) - C_i(\gamma)}_{\text{profit in dispatch } D^*} \leqslant \underbrace{0}_{\text{profit in dispatch } \widehat{D}}$$

In addition, for a generator $j$ who had his plants $j$ and $k$ dispatched for lots $x$ and $z$, respectively, in $D^*$, but only has plant $j$ dispatched for lot $y$ in $\widehat{D}$, the equilibrium bids $\widehat{B}_x$, $\widehat{B}_y$ and $\widehat{B}_z$ must satisfy

$$\underbrace{\widehat{B}_x + \widehat{B}_z - C_j(x) + C_j(z)}_{\text{profit in dispatch } D^*} \leqslant \underbrace{\widehat{B}_y - C_j(y)}_{\text{profit in dispatch } \widehat{D}}$$

Finally, if a generator did not have any of his plants dispatched in $D^*$ but is dispatched for lots $x$ and $z$ with his plant $k$ must satisfy

$$0 \leqslant \widehat{B}_x + \widehat{B}_z - C_k(x + z).$$

Summing up these incentive compatibility constraints over all generators (generators that are not dispatched earn a profit of zero) implies

$$\widehat{B} - C(D^*) \leqslant \widehat{B} - C(\widehat{D}) \Rightarrow C(\widehat{D}) \leqslant C(D^*)$$

which contradicts the assumption that $D^*$ is the efficient (least-cost) dispatch.  □

## 5. Conclusion

As auction based mechanisms for electricity dispatch are emerging in previously regulated electricity supply industries, it is imperative to understand the effect of auction rules and structure on efficiency in multi-unit auctions with possible complementarities. This paper addresses exactly this relationship by asking which simple auction structures provide bidders with the correct incentives so as to support only the efficient (least-cost) dispatch in equilibrium.

Under a complete information and one-shot framework, we have studied the performance of various auctions formats. We have found that a vertical auction, which allows for more than one winner per demand lot, is unable to guarantee efficiency in equilibrium in an environment where the bidders have a fixed plus variable production costs. We explored alternative auction designs that might better lend themselves to the particular cost

structure of electricity generation. Theorems 2 and 3 state that an alternative auction structure, a *horizontal* simultaneous auction, succeeds in supporting the efficient dispatch, and *only* the efficient dispatch, in equilibrium. It does this by restricting the numbers of winners per lot to be one, designing the lots to capture the synergies in generation costs, and having all bidders submit their bids on various lots simultaneously.

The results of this paper have some bearing on the auction mechanisms originally chosen in the UK, Australia and California (all three of which have now become defunct or undergone serious restructuring). All three auctions were designed so as to have generators submit hourly (or half-hourly) bids: The winners in each time period were paid the highest accepted price; this is the structure of a vertical, uniform auction. The results in Section 3 lead us to conclude that, even under abundant supply conditions, we should not expect these auctions to provide generators with the correct incentives so as to result in the efficient dispatch. These theorems also provide us with an alternative auction format that the desirable property of being efficient under complete information.

This paper has made some restrictive assumptions which must be relaxed before bidding behavior in an electricity auction can be accurately depicted. We examine the bidding behavior of generators assuming that the auction is held only once, while in reality the auction will be held on a daily basis. In addition, the both the auctioneer and the generators are assumed to have complete information about costs. While the analysis of complete information has been viewed by some researchers as a reasonable precursor to the incomplete information games (Bikhchandani, 1999), other authors have shown that equilibrium results obtained under complete information do not always carry over to an incomplete information framework (Katzman, 1999). An important next step would be to extend this analysis into a framework where generators know only their own costs with certainty and to test if the efficiency of simultaneous, horizontal auctions carries over to an incomplete information framework. Finally, this paper has analyzed the performance of the auctions under abundant, efficient supply conditions (Assumptions 4 and 5). It is important to explore which auction mechanisms will yield efficient dispatches in a market where generators have market power and/or inefficient plants are called upon to generate. It is our hope that the results of this paper provide us with guidance as to auction formats which may render themselves efficient in a more general setting.

## Appendix A

### A.1. Equilibrium bids for discriminatory, horizontal, sequential auction

The following Subgame perfect Nash equilibrium result in an inefficient dispatch in equilibrium for a discriminatory, horizontal, sequential auction. Assume that all generators submit the same bids for both of their plants unless otherwise specified. Once a particular plant has won a lot, he can no longer participate in later auctions.

$G_1$'s *strategy*:
Bid $f_3 + 3v_3 + \delta = 229$ for *lot 3*,
   If I win *lot 3*, bid $f_3 + 2v_3 - \delta = 206$ for *lot 2*
     If I win *lot 2*, bid $\infty$ for *lot 1*
     If 2 wins *lot 2*, bid $f_2 + v_2 = 124$ for *lot 1*
     If 3 wins *lot 2*, bid 124 for *lot 1*

$G_2$'s *strategy*:
Bid $f_3 + 3v_3 - \delta = 227$ for *lot 3*
   If 1 wins *lot 3*, bid 207 for *lot 2*
     If 1 wins *lot 2*, bid 185 for *lot 1*
     If I win *lot 2*, bid 124 for *lot 1*
     If 3 wins *lot 2*, bid 124 for *lot 1*

**Appendix A** (*continued*)

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If 2 wins *lot 3*, bid $f_3 + 2v_3 + \delta = 208$ for *lot 2*
  If I win *lot 2*, bid 124 for *lot 1*
  If 2 wins *lot 2*, bid $f_3 + v_3 = 186$ for *lot 1*
  If 3 wins *lot 2*, bid 124 for *lot 1*
If 3 wins *lot 3*, bid $f_1 + 2v_1 = 201$ for *lot 2*
  If I win *lot 2*, bid 124 for *lot 1*
  If 2 wins *lot 2*, bid 124 for *lot 1*
  If 3 wins *lot 2*, bid 124 for *lot 1*

If I win *lot 3*, bid 207 for *lot 2*
  If 1 wins *lot 2*, bid 124 for *lot 1*
  If I win *lot 2*, bid $\infty$ for *lot 1*
  If 3 wins *lot 2*, bid 124 for *lot 1*
If 3 wins *lot 3*, bid 207 for *lot 2*
  If 1 wins *lot 2*, bid 124 for *lot 1*
  If I win *lot 2*, bid 124 for *lot 1*
  If 3 wins *lot 2*, bid 124 for *lot 1*

$G_3$'s *strategy*:
Bid 228 for *lot 3*
  If 1 wins *lot 3*, bid 207 for *lot 2*
    If 1 wins *lot 2*, bid 186 for *lot 1*
    If 2 wins *lot 2*, bid 186 for *lot 1*
    If I win *lot 2*, bid 186 for *lot 1*
  If 2 wins *lot 3*, bid 207 for *lot 2*
    If 1 wins *lot 2*, bid 186 for *lot 1*
    If 2 wins *lot 2*, bid 186 for *lot 1*
    If I win *lot 2*, bid 186 for *lot 1*
  If I win *lot 3*, bid 207 for *lot 2*
    If 1 wins *lot 2*, bid 186 for *lot 1*
    If 2 wins *lot 2*, bid 186 for *lot 1*
    If I win *lot 2*, bid 186 for *lot 1*

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The result of these strategies is that $G_2$ wins *lot 3* and *lot 2*, and $G_1$ wins *lot 1*.

*A.2. Equilibrium bids for uniform, vertical, sequential auction*

  $G_{1y}$'s *strategy* ($y = 1, 2$)
Bid 106 for *lot 1*
  If I win at $t = 1$, bid 95 for *lot 2*
  Otherwise bid 106 for *lot 2*

  $G_2$'s *strategy*
Bid 0 for *lot 1*
  If I lose at $t = 1$, bid 129 for *lot 2*
  If I win at $t = 1$ and so do $G_{31}$ and $G_{11}$, bid 0 for *lot 2*
  If I win at $t = 1$, but neither $G_{11}$ nor $G_{12}$ do, bid 53 for *lot 2*
  Otherwise bid 94 for *lot 2*

  $G_{31}$'s *strategy*
Bid 0 for *lot 1*

  If I lose at $t = 1$, bid 186 for *lot 2*
  If I win at $t = 1$ and so do either ($G_{32}$ and $G_{11}$), ($G_{32}$ and $G_{12}$), ($G_2$ and $G_{11}$) or ($G_4$, $G_{11}$ and $G_{12}$), bid 94 for *lot 2*
  If I win at $t = 1$ and so do both $G_4$ and $G_2$, bid 52 for *lot 2*
  Otherwise bid 21 for *lot 2*

  $G_{32}$'s *strategy*
Bid 186 for *lot 1*
  If I lose at $t = 1$, bid 186 for *lot 2*
  If I win and either both $G_4$ and $G_{31}$ win or neither $G_4$ nor $G_{31}$ win at $t = 1$, bid 21 for *lot 2*
  If I win at $t = 1$ and either both ($G_4$ and $G_2$) or ($G_2$ and $G_{32}$) win at $t = 1$, bid 52 for *lot 2*
  Otherwise bid 94 for *lot 2*

  $G_{41}$'s *strategy*
Bid 224 for *lot 1*
  If I win in *lot 1*, bid 219 for *lot 2*
  Otherwise bid 224 for *lot 2*.

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