MARKET POWER AND PRICE VOLATILITY IN RESTRUCTURED MARKETS FOR ELECTRICITY

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Abstract

The restructured market for electricity in the UK has experienced a systematic pattern of price spikes associated with the use of market power by the two dominant generators. Partly in response to this problem, the share of capacity owned by any individual generator after restructuring was limited in Victoria, Australia. As a result, a much more competitive market resulted with prices substantially lower than they were under regulation. Nevertheless, an erratic pattern of price spikes exists and the price volatility is a potential problem for customers. This paper argues that the use of a uniform price auction for electricity markets exacerbates price volatility. A discriminatory price auction is proposed as a better alternative that would reduce the responsiveness of price to errors in forecasting total load.

1. Introduction

A number of countries have restructured their markets for electricity for a variety of different reasons. They share, however, the objective of making the new market for generation more competitive with lower average prices. In the UK, high prices caused by the use of market power by the two largest generators have been a persistent problem (see von der Fehr and Harbord (1993), Newbury (1995), Wolak and Patrick (1997), and Littlechild (1998)). The electricity markets in Scandinavia and New Zealand also have dominant generators, but they are owned by the state and they practice some form of self-imposed restraint on the use of market power (see Wolak (1997) and Read (1998)). Consequently, prices are relatively stable but are probably higher than competitive levels.

Given the experience in the UK market, the restructuring of generation in Victoria, Australia required that each major power plant should be sold to a different buyer, effectively limiting the share of capacity owned by any individual company (see Outhred (1997) and Wolak (1997)). Hence, the foundation for a relatively competitive market with six competing generators was established as an improvement over the skewed pattern of ownership in the UK. The subsequent merging of the Victorian market with the state-owned generators in New South Wales in May 1997 did not change the situation appreciably. In fact, prices fell further after the merger.

Although lower prices for electricity in the Australian market are an encouraging sign, there are also erratic patterns of price spikes which lead to high price volatility. This type of price behavior is illustrated in Figures 1 and 2. In Figure 1, average daily prices are shown for the

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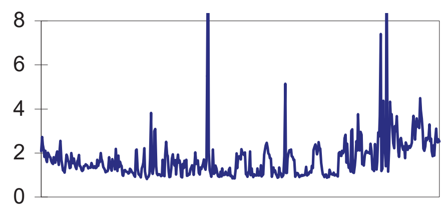
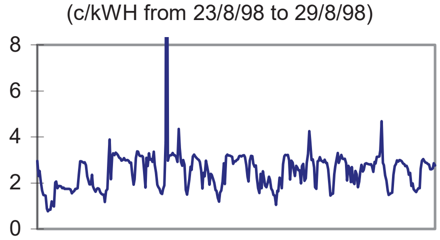
past year (9/97 to 8/98), and there are many spikes and no obvious seasonal patterns.

Figure 1: Average Daily Prices for Electricity in New South Wales (c/kWh from 9/97 to 8/98)



Actual half hourly spot prices for a recent week (23/8/98 to 29/8/98) are shown in Figure 2. Once again, the price spikes do not follow a regular daily pattern as they do during the winter months in the UK. Price volatility appears to be an intrinsic problem with this particular competitive market.

Figure 2: Spot Market Prices for Electricity in New South Wales (c/kWH from 23/8/98 to 29/8/98)



Most of the theoretical research on auctions has been directed to markets to sell items and the behavior of buyers. Assuming that the logic for setting offers in markets to buy items is equivalent to the logic for setting bids discussed in the literature, the results of Ausubel and Cramton (1997) suggest that the offer curves submitted by sellers in a multiple units auction will be higher and steeper than the true marginal cost curves if a uniform price auction is used (all successful sellers are paid the same price). Since the difference between the offer and the marginal cost increases as the number of units for sale increases, this behavior is an example of how market power can be used to increase the final price. Wolak and Patrick (1997) have shown that the two largest generators in the UK have used their market power successfully to raise prices this way. Backerman et.al. (1997) have used experimental economics to show that generators can capture congestion rents and make excess profits. In addition, Bernard et.al. (1998) have used POWERWEB (a simulation model of an electricity market used to test alternative types of auction at Cornell University) to show that participants can exploit opportunities for market power in load pockets. Such behavior is not surprising to most economists.

One of the implications of having steeply sloped offer curves is that the aggregated supply curve will be relatively price inelastic. Consequently, uncertainty in the load due to forecasting errors will be amplified into high price volatility. Furthermore, price spikes are more likely to occur when the expected load is high and the level of market power is at its greatest. Price spikes can also occur after unexpected outages of generators or transmission lines. In general, market power will make prices more volatile when a uniform price auction is used, and all restructured markets for electricity have adopted this type of auction.

The main objective of this paper is to demonstrate that a discriminatory price auction, in which generators are paid what the offer, may be a better form of auction for electricity markets. The reason is that the offer curves will be flatter and the aggregated supply curve more price elastic. Consequently, uncertainty about load will be dampened, and price volatility will be smaller than it is using a uniform price auction even if there is no appreciable market power.

Many economists believe that discriminatory price auctions are less efficient than uniform price auctions because Vickrey (1961) showed that buyers would submit honest bids if they paid the highest rejected bid in an auction to sell items. However, these results do not generalize, as Vickrey recognized, to situations in which some individuals want to buy more than one item. Swinkels (1997a and 1997b) has shown that both a uniform price and a discriminatory price auction approach the perfectly competitive market solution if the number of participants is sufficiently large. Furthermore, over 90% of the auctions to sell treasury bills in a sample of 42 countries use a discriminatory price auction (Bartolini and Cottarelli (1997)). Hence, there is no convincing reason to dismiss the consideration of a discriminatory price auction for electricity markets on theoretical or empirical grounds.

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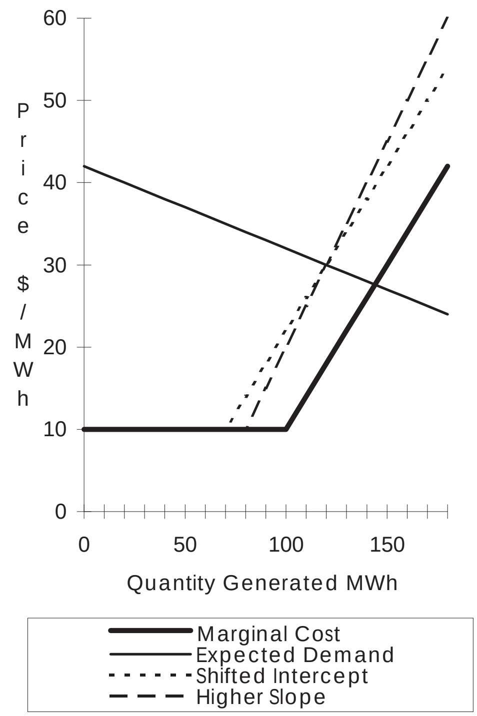
The objective of this paper is to compare the effects of using a uniform price and a discriminatory price auction for an electricity market. The following section of the paper defines the conditions faced by a single generator using a modified quadratic cost function. The optimum offer curve is derived in Section 3 for a uniform price auction, and the implications for the aggregated supply curve and price volatility are determined. The same steps are repeated in Section 4 for a discriminatory price auction. In the final section, the two supply curves for the two different auctions are compared. The results imply that both auctions are adversely affected by market power, but the price volatility is much lower using a discriminatory price auction.

2. The Specification for a Single Generator

Consider a spot market for electricity with N generators participating. Each generator submits offers to an Independent System Operator (ISO) and tries to maximize expected profits (short-run net revenue) subject to a known cost function. Using a uniform price auction, the same price is paid to the generators who submit the lowest offers to meet an expected load $E[Q_{tot}]$ (i.e. demand is perfectly inelastic). The price paid to generators is set at the intersection of the load and the combined offer curve for all generators. Since discontinuities in the offer curves are ruled out by the specified form of the cost functions, there is no need to distinguish between a Last Accepted Offer and a First Rejected Offer auction. They are identical. Using a discriminatory price auction, generators who submit the lowest offers to meet $E[Q_{tot}]$ are selected, but the prices paid correspond to the actual offers.

The form of the short-run cost curve for generation is specified as a displaced quadratic, implying that the marginal cost curve is a displaced linear function. This form is chosen to approximate the actual cost functions derived by Wolak and Patrick (W&P) (1997) for the UK. Using this functional form makes it possible to distinguish between offer curves that alter the slope of the marginal cost curve and offer curves that shift the location of the marginal cost curve (e.g. reduce the degree of displacement from the origin). The latter behavior was found by W&P to be a close approximation to the offers submitted by the two dominant generators in the UK. Examples of the two types of offer curve and the true marginal cost curve are shown in Figure 3.

Figure 3: Alternative Forms of Offer Curve



For the j^{th} generator, the short-run cost curve is specified as follows:

Total cost for generator j

$$C_j(Q_j) = c_{1j} + c_{2j} Q_j + c_{3j} \Delta Q_j^2 \qquad (1)$$

where $c_{ij} > 0$, $i = 1,2,3$ and $q_{0j} \geq 0$ are known parameters (q_{0j} is the degree of displacement from the origin), Q_j is the level of generation

$$\Delta Q_j = (Q_j - q_{0j}) \text{ for } Q_j > q_{0j}$$
$$= 0 \text{ otherwise.}$$

Given the form of the total cost in (1), the marginal cost curve can be written:

Marginal cost for generator j

$$\begin{aligned} MC_j(Q_j) &= c_{2j} + 2c_{3j}\Delta Q_j \qquad (2) \\ &= (c_{2j} - 2c_{3j}q_{0j}) + 2c_{3j}Q_j \\ &\quad \text{if } Q_j > q_{0j} \\ &= c_{2j} \text{ otherwise.} \end{aligned}$$

For simplicity, the offer curves are restricted to having the same linear form as the marginal

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cost curves, and the offer curve for generator j is defined as follows:

Offer curve for generator j

$$P_j(Q_j) = v_{1j} + v_{2j} Q_j \tag{3}$$

where v_{1j} and $v_{2j} > 0$ are constants specified by generator j, and Q_j is the level of generation supplied at price P_j. The quantity supplied can be written as a function of the price received to give:

Supply offered by generator j

$$S_j(P_j) = (P_j - v_{1j})/v_{2j} \tag{4}$$

Since the ISO dispatches generators using the offer curves submitted by the generators, the total payment for meeting load is minimized by paying the same price P to all generators so that $Q_{tot} = \Sigma_j S_j(P)$. (For this illustration, the costs of transmission losses and constraints are ignored.)

The supply curve for the other $(N - 1)$ generators is the sum of their supply curves, and the corresponding demand faced by generator j is the difference between the total load and this sum.

Demand faced by generator j

$$\begin{aligned} D_j(P_j) &= Q_{tot} - \Sigma_{i \neq j} S_j(P) \\ &= [Q_{tot} + \Sigma_{i \neq j} (v_{1i}/v_{2i})] - [\Sigma_{i \neq j} (1/v_{2i})]P \end{aligned} \tag{5}$$

Since generator j does not know the parameter values chosen by other generators for their offer curves (or the exact value of Q_{tot} that will occur), it is assumed that generator j forms the following subjective expectation of the linear relationship in (5):

Subjective expectation of demand by generator j

$$Q_j = A_{1j} - A_{2j} P \tag{6}$$

where $A_{1j} > 0$ and $A_{2j} > 0$ are constants determined by generator j. The expected demand faced by generator j is shown in Figure 3, and it is specified to go through the point where the two offer curves cross.

3. The Optimum Offer Curve Using a Uniform Price Auction

The profit function faced by generator j combines the expected demand relationship (6) with the true cost function (1). This is the standard problem faced by a producer with market power, and the solution determines the optimum level of generation Q_j and the market price P.

Maximize with respect to Q_j

$$R_j(Q_j) = PQ_j - C_j(Q_j) \tag{7}$$

subject to (6), where $C_j(Q_j)$ is the total cost defined in (1). The first order condition for maximizing (7) can be written:

$$P^* - Q_j^*/A_{2j} - MC_j(Q_j) = 0 \tag{8}$$

where $MC_j(Q_j)$ is the marginal cost defined in (2). Rearranging (7) gives the following expression for the optimum offer curve:

Optimum offer curve for generator j

$$\begin{aligned} P^* &= MC_j(Q_j^*) + (1/A_{2j})Q_j^* \\ &= (c_{2j} - 2c_{3j}\, q_{0j}) + (2c_{3j} + 1/A_{2j})\, Q_j^* \quad \text{if } Q_j^* > q_{0j} \\ &= c_{2j} + (1/A_{2j})Q_j^* \quad \text{otherwise.} \end{aligned} \tag{9}$$

In a competitive market, $1/A_{2j} = 0$ and $P = A_{1j}/A_{2j} =$ constant. Consequently, the optimum output for generator j is to set Q_j^* so that the corresponding marginal cost equals the price. In our example, $1/A_{2j} > 0$ in (9), and as a result, the optimum offer curve is more steeply sloped than the true marginal cost curve. In addition, the optimum offer curve has the same intercept as the true marginal cost curve, implying that the degree of displacement q_{0j} is unchanged. The kink in the marginal cost curve and the optimum offer curve occur at the same value $Q_j = q_{0j}$. An example is shown in Figure 4.

The form of offer curve in Figure 4 is consistent with the form of theoretical bid function derived by Ausubel and Cramton for an auction to buy more than one item. The difference between the offer and the true marginal cost increases as Q_j gets larger. Using the results in (9), the optimum parameter values for the offer curve (3) can be written in terms of the parameters of the marginal cost curve (2) and the expected demand curve (6) as follows:

For $Q_j^* > q_{0j}$

$$\begin{aligned} v_{1j}^* &= (c_{2j} - 2c_{3j}q_{0j}) \text{ and} \\ v_{2j}^* &= (2c_{3j} + 1/A_{2j}) \end{aligned} \tag{10}$$

For $Q_j^* \leq q_{0j}$

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$$v_{1j}^* = c_{2j} \text{ and } v_{2j}^* = 1/A_{2j} \quad (11)$$

The optimum solution to the maximization in (7) gives a single price P^* and a single quantity Q_j^*, and the expressions are:

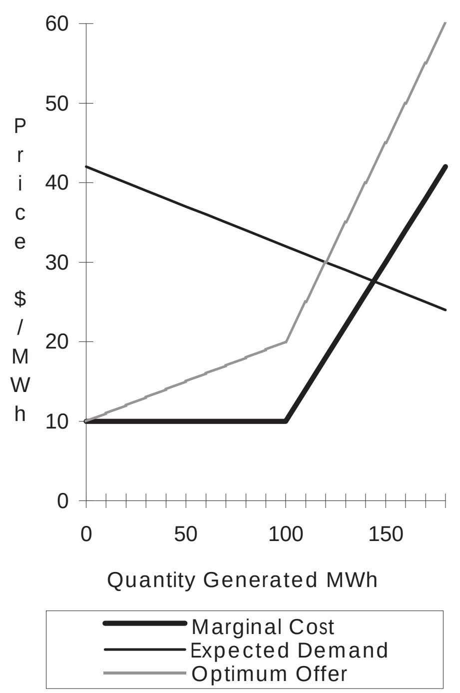
Optimum quantity and price

$$Q_j^* = A_{1j} - A_{2j}P^* \quad \text{from (6)} \quad (12)$$

$$P^* = (c_{2j} - 2c_{3j}q_{0j} + 2c_{3j}A_{1j} + A_{1j}/A_{2j}) \,/\, (2 + 2\,c_{3j}A_{2j}) \quad \text{from (9).}$$

These two values could be determined on an offer curve by increasing the slope of the marginal cost curve using (9) or by shifting the marginal cost curve by an appropriate amount to the left. For the example in Figure 3, this special case corresponds to the intersection of the two offer curves with expected demand at $P^* = 30$ and $Q_j^* = 120$. (The parameter values for the marginal cost curve (2) are $c_{2j} = 10$, $c_{3j} = 0.2$ and $q_{0j} = 100$, and for the expected demand curve (6), they are $A_{1j} = 420$ and $A_{2j} = 10$. The shifted intercept corresponds to resetting $q_{0j} = 70$.) The primary reason for submitting the offer curve in (9) rather than shifting the marginal cost curve to the left is that it gives the locus of optimum prices and quantities for any value of A_{1j}. Even though the expected demand faced by generator j in (6) is conditional on the expected behavior of generators, there is still uncertainty in the actual load Q_{tot}, and consequently, in the value of $[Q_{tot} + \Sigma_{i \neq j}(v_{1i}/v_{2i})]$ in (5) which is represented by A_{1j} in (6).

Figure 4: Optimum Offer Curve for One Generator



The offer curves of the two dominant generators in the UK were shown by W&P to correspond to withholding inexpensive capacity from the market (i.e. making q_{0j} smaller and keeping the slope $2c_{3j}$ unchanged). This is not consistent with the optimum behavior implied in (9). The explanation given by W&P is that reducing the capacity offered from low cost generators is less likely to incur government intervention from the oversight committee than raising prices. Since a typical company controls a number of different power plants of different types, the cost curve in (1) represents all plants controlled by generator j. Hence, it is inevitable that the quantities of capacity available for individual plants change due to maintenance schedules and other factors. Frequent changes in the price offered for generation from any specific plant would be harder to justify. A problem with the observed behavior in the UK, however, is that it leads to market inefficiencies because capacity from the low cost plants is held back from the market, and the true cost of generation is higher than it would be under both perfect competition and the optimum offer curve. This is a case where the threat of regulation may have a perverse effect on efficiency but may still lower the spot price. In contrast, the optimum offer curve in Figure 4 implies that the ranking of the true marginal costs of generating units is identical to the ranking of offers.

3.1 The Special Case of Identical Generators

In the numerical example, the true marginal cost at $Q_j = 120$ is only $MC_j = 18$, which corresponds to 60 percent of the optimum offer 30. An obvious question is whether this example is realistic. In the simplest case in which all N generators have the same cost curve and behave identically, the N offer curves will also be identical. The slope of the offer curve is $v_{2j}^* = (2c_{3j} + 1/A_{2j})$ for $Q_j^* > q_{0j}$ in (10), where A_{2j}, defined in (6), is the subjective value of the slope $[\Sigma_{i \neq j}(1/v_{2i}^*)]$ in (5). When $v_{2i}^* = v_2^*$ and $c_{3i} = c_3$ for all i, the following relationship holds:

$$v_2^* = 2c_3 + v_2^*/(N - 1) = 2c_3(N - 1)/(N - 2) \quad \text{for } N > 2 \qquad (13)$$

For the values $v_2^* = 0.5$ and $2c_3 = 0.4$ in the example, the number of generators is $N = 6$. Hence, the relatively large difference between the slopes of the offer curve and the marginal cost curve (the slope of the offer curve is 25 percent higher than the efficient value) corresponds to a relatively large number of competing generators by the standards of the electric utility industry. The maximum markup of the slope for $N > 2$ is 100 percent when $N = 3$.

It is interesting to note that the expression for v_2^* in (13) does not include the case of two identical duopolists. The reason is that the load faced by the duopolists in this example is completely inelastic, and there is no stable Nash equilibrium when $N = 2$. In (13), the only situation that is valid for $N = 2$ is when $c_3 = 0$ (i.e. the marginal cost curve is flat), but even in this situation, the value of v_2^* is still indeterminate. In general, the same problem exists when the duopolists are not identical. If a known maximum price is set for a market by the ISO, one would expect that each duopolist would submit a flat offer curve at the maximum price (as long as the procedure for breaking ties gives a fair share of the load to each participant). However, this solution would still not be a stable equilibrium. The threat of retaliation in an auction that is repeated many times, like an hourly market for electricity, is one possible reason for the duopolists to keep the price at the maximum.

3.2 The Observed Supply Curve for Identical Generators

Given the results for the optimum offer curve, it is possible to determine the implications for the aggregate supply curve for the N generators. From (4), the aggregate supply curve can be written (assuming $Q_j^* > q_{0j}$ for all j):

$$Q_{tot} = \sum_{j=1}^{N} S_j(P) = [\Sigma_j 1/v_{2j}^*]\, P - [\Sigma_j v_{1j}^*/v_{2j}^*] \qquad (14)$$

where v_{1j}^* and v_{2j}^* are the optimum values defined in (13). Since (14) is a linear function of P, it can be rewritten as an explicit function of P as follows:

$$P = (Q_{tot} + B_1)/B_2 \qquad (15)$$

where B_1 and B_2 are the intercept and the slope in (14), respectively.

The slopes of the optimum offer curves v_{2j}^* in (10) are larger than the efficient values (slopes of the marginal cost curves). Consequently, the value of the slope in (15) $1/B_2$ is also larger than the efficient value, and the supply curve based on offers is more price inelastic than the efficient supply curve based on marginal costs.

In the special case of N identical generators, the slope of all $N > 2$ offer curves is $v_2^* = 2c_3(N - 1)/(N - 2)$. As a result, the slope of the supply curve in (15) can be written:

$$1/B_2 = (2c_3/N)\,((N - 1)/(N - 2)) \qquad (16)$$

where $(2c_3/N)$ is the slope of the efficient aggregate supply curve based on marginal costs and $((N - 1)/(N - 2)) > 1$ for $N > 2$ represents the effect of market power.

Consider the total cost curve for a single generating unit:

$$c(Q_*) = c_{1*} + c_{2*}Q_* + c_{3*}(Q_* - q_{0*})^2 \qquad (17)$$

If generator j controls k_j units, then the aggregate cost curve (assuming all k_j units operate at the same level of output) can be written:

$$c_j(Q_j) = c_{1*}k_j + c_{2c}Q_j + (c_{3*}/k_j)(Q_j - k_j q_{0*})^2 \qquad (18)$$

where $Q_j = k_j Q_*$. Under this specification the average costs for (17) at Q_* and for (18) at Q_j are identical, and so are the two corresponding marginal costs. If there is a total of $k_{tot} = \Sigma_j k_j$ units of generating capacity, the least cost solution for meeting any level of load Q_{tot} is not affected by the pattern of ownership of capacity among the N generators. Furthermore, if the aggregate cost curve in (18) is assumed to hold for any $k_j > 0$, and not just for integer values, then each generator controls (k_{tot}/N) units of capacity in the case of N identical generators. In this special case, the parameters defining the cost for each generator are:

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$c_1 = c_{1*}k_{tot}/N$, $c_2 = c_{2c*}$, (19)
$c_3 = c_{3*}N/k_{tot}$ and $q_0 = q_{0*}k_{tot}/N$

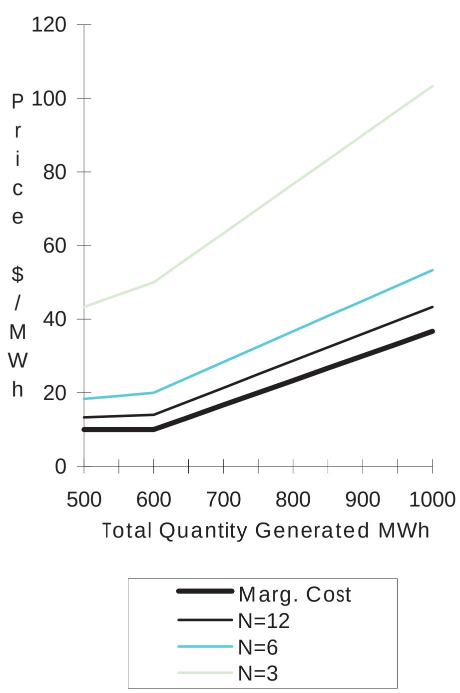
Substituting (19) into (16), the slope of the supply curve in (15) can be written:

$$1/B_2 = (2c_{3*}/k_{tot})((N - 1)/(N - 2)) \quad (20)$$

Consequently, the slope of the efficient supply curve based on marginal costs $(2c_{3*}/ k_{tot})$, unlike the slope in (16), is the same for all values of N . The number of generators N affects only the difference between the slope of the offer curve and the slope of the marginal cost curve. This places a clear focus on the role that market power plays in increasing the spot price P above the efficient level.

For the numerical example used in Figure 4, the offer curve corresponds to $N = 6$ identical generators with a total load of $Q_{tot} = 720$. Using $k_{tot} = 12$ (equivalent to a standard generating unit of size 60), the cost parameters for the marginal cost of the standard generating unit in (19) are $c_{2*} = 10$, $c_{3*} = 0.4$ and $q_{0*} =$ 50.

Figure 5: Total Supply Curves for N Identical Generators



The aggregate supply curves for $N = 3$, 6 and 12 identical generators are shown in Figure 5 together with the aggregated marginal cost curve (corresponding to $N = \infty$) . All three supply curves and the positively sloped part of the marginal cost curve have the same negative intercept (-30) . (The differences in the slopes among the supply curves would be more obvious if there was no displacement of the cost curve (i.e. $q_{0*} = 0$) because in that case the common intercept would be zero.)

The supply curves in Figure 5 are defined for values of the total load above $k_{tot}q_{0*} = 600$, and the expected load in the example is $Q_{tot} =$ 720 . For values of $Q_{tot} < 600$, the result in (20) implies that all offer curves would be flat because the slope of the marginal cost curve is zero. Hence, all supply curves would be the same as the marginal cost curve at the constant level $c_{2*} = 10$. The supply curves in Figure 5 are shown with the same form as the optimum offer curve in Figure 4 to give one possible choice of the form, but the values for $Q < 600$ are not strictly optimum.

3.3 The Implications for Price Volatility

When offers are submitted to the ISO , the values of B_1 and B_2 in (15) are fixed, and the market solution for the spot price P is determined by the realized value of the total load Q_{tot} . Consequently, the uncertainty about Q_{tot} is translated into uncertainty about P by the slope $1/B_2$ in (15). Using the results for $N > 2$ identical generators in (20), the variance of price can be written:

$$\begin{aligned} Var[P] &= (1/B_2)^2\, Var[Q_{tot}] \quad (21)\\ &= ((N-1)/(N-2))^2(2c_{3*}/k_{tot})^2\, Var[Q_{tot}] \end{aligned}$$

The important conclusion from (21) is that the spot price will be more volatile than the price in an efficient market because $((N-1)/(N - 2)) > 1$. The additional volatility due to market power gets smaller as the number of generators increases because $((N - 1)/(N - 2)) \rightarrow 1$ as $N \rightarrow \infty$.

The situation in actual spot markets for electricity is more complicated. In the UK, for example, W&P show that the kinked offer curves in Figure 3 are reasonable approximations to actual offer curves. During periods of low load, market solutions generally occur on the flat part of the marginal cost curve. When load is high, the spot price is determined by the steeper part of

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the offer curve. Hence, there is a mixture of two regimes setting the spot price. In addition, there is increasing evidence that the two dominant generators exploit situations when the total load is high and the expected demand in (6) is most inelastic (A_{2j} is small), and they submit offers that deviate even more from the marginal cost curve at these times (e.g. during the late afternoon on weekdays in the winter). Exploiting "bad" situations after an unexpected outage of a generator or a failure on the system, for example, is exactly the type of behavior that is likely to unleash the wrath of government regulators. In the UK, the blatant use of market power by the two dominant generators during periods of high load has resulted in punitive reactions from the government in the form of special taxes on profits and pressure to sell generating capacity.

4. The Optimum Offer Curve Using a Discriminatory Price Auction

In a discriminatory auction, the prices received by a generator correspond to the offer curve submitted to the ISO. Even though there is no direct link between the market clearing price P^* paid for the last unit accepted from generator j and the prices paid for the other (Q_j^* - 1) units accepted, the optimization problem is very similar to the situation using a uniform price auction.

If the subjective expected demand faced by generator j is given by (6), the objective is to maximize profits as before. A discriminatory monopolist would be able to extract the full surplus between the demand curve (6) and the marginal cost curve (2). However, there are limits on the ability of a generator to charge discriminatory prices. The most important one is that the ISO will rank the offers for individual units from the least expensive to the most expensive. Hence, the market clearing price P^* for the marginal unit accepted from generator j must correspond to the highest offer accepted from generator j. In other words, the offer curve must be monotonically non-decreasing (i.e. $v_{2j} \geq 0$ in (3)). Under this restriction, revenue is maximized for any optimum Q_j^* by specifying a flat offer curve (i.e. $v_{1j} = P^*$ and $v_{2j} = 0$ in (3)). The problem with this strategy is that it is not robust to uncertainty about the demand curve in (6) due to uncertainty about total load, for example. If the actual demand was lower than expected and the ISO accepts $Q_j^0 < Q_j^*$, there is no way to guarantee that the units accepted will be the ones with the lowest costs. To the ISO, all units from generator j cost the same if the offer curve is flat. Hence, it is reasonable to specify a minimum positive slope for the offer curve ($v_{2j} = v_{2m} > 0$ in (3)) to ensure that the ISO ranks units correctly. (This assumption also avoids the problem of indeterminancy that exists if all offer curves are flat for N identical generators.)

With the slope of the offer curve set at v_{2m}, the optimization problem for generator j can be written as a modification to (7) as follows:

Maximize with respect to Q_j.

$$R_j(Q_j) = PQ_j - v_{2m}Q_j^{\,2}/2 - C_j(Q_j) \qquad (22)$$

subject to (6), where $C_j(Q_j)$ is the total cost defined in (1). The first order conditions for maximizing (22) can be written:

$$P^* - Q_j^*/A_{2j} - v_{2m}\, Q_j^* - MC_j(Q_j^*) = 0 \qquad (23)$$

where $MC_j(Q_j^*)$ is the marginal cost in (2). Rearranging (23) gives the following expression for the optimum price and level of generation:

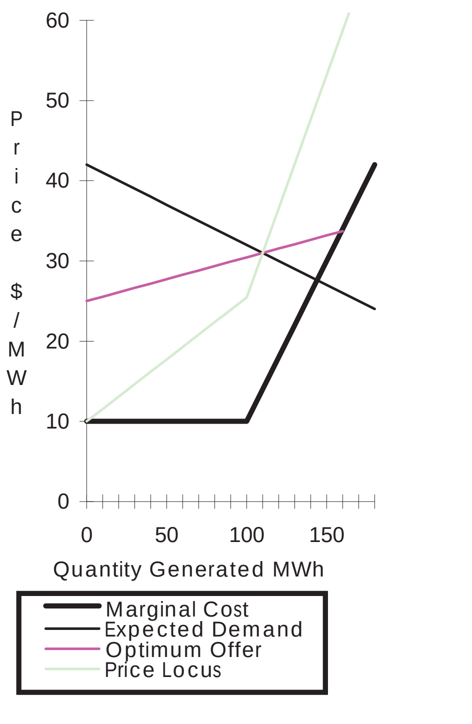
Optimum price locus

$$\begin{aligned} P^* &= MC_j(Q_j^*) + (1/A_{2j} + v_{2m})Q_j^* \qquad (24)\\ &= (c_{2j} - 2c_{3j}q_{0j}) + (2c_{3j} + 1/A_{2j} + v_{2m})Q_j^* \text{ if } Q_j^* > q_{0j}\\ &= c_{2j} + (1/A_{2j} + v_{2m})Q_j^* \text{ otherwise.} \end{aligned}$$

This result is almost identical to the corresponding expression for the uniform price auction in (9). The slope of the solution for the optimum price for a given value of A_{2j} is steeper in (24) due to the minimum slope v_{2m} that is required for the offer curve. The optimum price locus and the optimum offer curve are shown in Figure 6 using the same marginal cost and expected demand curves used in Figure 4 for a uniform price auction.

Assuming that the minimum slope for the offer curve is v_{2m} = 6/110 = 0.054545 (to give integer solutions for P^* and Q_j^*), the resulting optimum price is P^* = 31 (compared to 30 in Figure 4) and the optimum quantity of Q_j^* = 110 (compared to 120 in Figure 4). This illustrates the effect of the slightly higher slope of the locus of optimum prices in Figure 6 compared to the slope of the optimum offer curve in Figure 4. Although these differences in the optimum price

Figure 6: Optimum Offer Curve for One Generator



and quantity are relatively small, there is also a major difference because the slope of the optimum offer curve for the discriminatory price auction in Figure 6 is much lower (0.054545 versus 0.5 in Figure 4). This has important implications for reducing both the inflation of the spot price above the true marginal cost and the level of price volatility.

For $Q_j \leq Q_j^*$, the optimum offer curve is defined by the slope v_{2m} and the solution to (24) for P^* and Q_j^* ($v_{1j}^* = P^* - v_{2m}Q_j^*$ and $v_{2j}^* = v_{2m}$). There is still a question about the optimum offers for $Q_j > Q_j^*$. This is no longer determined automatically as it was using a uniform price auction because the optimum offer curve is not the same as the optimum locus for P^* and Q_j^* in (24). If $Q_j = Q_j^* + \Delta Q_j > Q_j^* > q_{0j}$, because the demand curve faced by generator j has shifted up and to the right, the optimization would be to maximize the profit from the additional generation ΔQ_j (because the revenue from Q_j^* is already determined). The objective function would be:

Maximize with respect to Q_j

$$\Delta R\,(\Delta Q_j) = \Delta Q_j P - v_{2m}\Delta Q^2 / 2 - [C_j(Q_j) - C_j(Q_j^*)] \qquad (25)$$

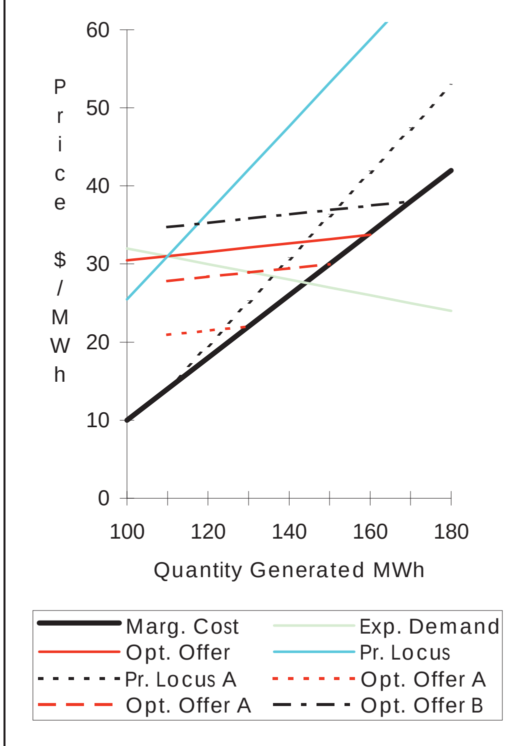
subject to a modified (6) with a larger intercept than before (the simplest way to shift the demand curve). The solution for the optimum price can be written as follows:

$$\begin{aligned} P^{**} &= MC_j\,(Q_j^{**}) + (1 / A_{2j} + v_{2m})\Delta Q^{**} \qquad (26) \\ &= (c_{2j} - 2c_{3j}\, q_{0j} + 2c_{3j}\, Q_j^*) + (2c_{3j} + 1 / A_{2j} + v_{2m})\Delta Q^{**} \end{aligned}$$

where $Q_j^{**} = Q_j^* + \Delta Q_j^{**}$ is the new optimum level of generation. The solution in (26) has the same slope as the solution in (24) but the intercept is shifted upwards and corresponds to $MC_j(Q_j^*)$. The solutions for the optimum prices in (24) and (26) are shown in Figure 7.

The marginal cost (Marg. Cost), price locus (Pr. Locus) optimum offer (Opt. Offer) and expected demand (Exp. Demand) are identical to the corresponding relationships in Figure 6 (the optimum solution is at $P^* = 31$ and $Q_j^* = 110$). The new optimum price locus is Pr. Locus A&B, and two possible optimum incremental offer

Figure 7: Optimum Offer Curves for Additional Demand



curves are shown in Figure 7. These correspond to two different levels of higher expected demand (Exp. Demand A and Exp. Demand B), and the optimum solutions for Q_j^{**} are 139 and 155, respectively (the intersections with Pr. Locus A&B).

For Exp. Demand A, the optimum price $P^{**} = 30$ is lower than the original solution $P^* = 31$, and Opt. Offer A is below the original Opt. Offer. The reason is that the choice of P^* affects the prices paid for Q_j^* but the choice of P^{**} only affects the prices paid for ΔQ_j^{**}. Hence, it is not optimum to inflate the offer P^{**} above the true marginal cost by such a large amount. In these situations, however, the basic rule of having a non-decreasing offer curve would be violated. If the increase of expected load is large enough ($Q_j^{**} > 144$), the optimum offer curve is above the original Opt. Offer. Opt. Offer B is one example.

Treating the problem as a series of incremental steps for $Q_j > Q_j^*$, a reasonable strategy is to extend the optimum offer curve derived for $Q_j < Q_j^*$ until it reaches the marginal cost curve. Beyond that point, the offers would be equal to the true marginal cost. The complete optimum offer curve for a discriminatory price auction is shown in Figure 6. However, it would be preferable, as a subject for further research, to consider the effects of uncertainty about the total load explicitly in deriving the optimum offer curve.

4.1 The Implications for Total Supply

For a discriminatory price auction, the slopes of the optimum offer curves are set at the minimum value v_{2m} for all N generators. Consequently, the sum of the N individual offer curves, corresponding to (14), can be written:

$$Q_{tot} = [\textstyle\sum_j v_{1j}^* / v_{2m}] + [N/v_{2m}]P \qquad (27)$$

where v_{1j}^* and v_{2m} are the parameters of the optimum offer curve for generator j. The intercept in (27) is bigger than it would be if the spot price fell on the true marginal cost curve. However, in most realistic situations the slope of the total supply curve (v_{2m} / N) will be substantially smaller than the slopes of both the total supply curve using a uniform price auction and the competitive supply curve based on the true marginal cost curves. Furthermore, total supply will be increasingly price elastic as the number of generators N gets larger.

If the N generators are identical and have cost curves defined by the standard parameters in (19), then the competitive supply curve is independent of N with a slope of $2c_{3*}/k_{tot}$ for $Q_{tot} > q_{0*}k_{tot}$ and a level of c_{2*} for $Q_{tot} \leq q_{0j}k_{tot}$. The slope of the optimum locus of prices in (23) can be simplified because the slopes of all optimum offer curves are always v_{2m} regardless of the size of N. Consequently, the slope of the expected demand curve in (6) can be written:

$$A_{2j} = (N - 1) / v_{2m} \qquad (28)$$

The optimum price locus in (23) is:

$$\begin{aligned} P^* &= MC_j(Q_j^*) + (v_{2m}N / (N - 1))Q_j^* \qquad (29) \\ &= (c_{2*} - 2c_{3*}q_{0*}) + (2c_{3*}N / k_{tot} + v_{2m}N/(N - 1))Q_j^* \end{aligned}$$

for N identical generators. It follows that the expected spot price in the market is determined by substituting $Q_j^* = E[Q_{tot}]/ N$ into (29). It is interesting to note that, unlike the uniform price auction, the conditions in (29) include the case of duopolists. It is only a monopolist who would be able to drive the spot price to the maximum allowed by the ISO.

Using the same parameter values for the cost function given below (20) and the minimum slope $v_{2m} = 6/110$, the optimum price loci and the supply curves for N = 2,3,6 and 12 identical generators are shown in Figures 8 and 9. The price loci are similar in form to the supply curves for a uniform price auction in Figure 5, but the supply curves in Figure 9 are much more price elastic than the supply curves in Figure 5. In addition, the market clearing prices for the same number of generators as Figure 5 are lower for the discriminatory price auction in Figure 9. The relative flatness of the supply curves in Figure 9 will always hold as long as the slope of the marginal cost curve for each generator is greater than v_{2m} (the slope of the offer curve using a uniform price auction is greater than or equal to the slope of the marginal cost curve). The ranking of the prices between the two auctions, however, is dependent on the values of the parameters.

The relatively small slopes of the supply curves in Figure 9 imply that market prices will be relatively unaffected by uncertainly about Q_{tot}. In other words, price volatility will be much lower for the supply curves in Figure 9 compared to the supply curves in Figure 5. The equivalent

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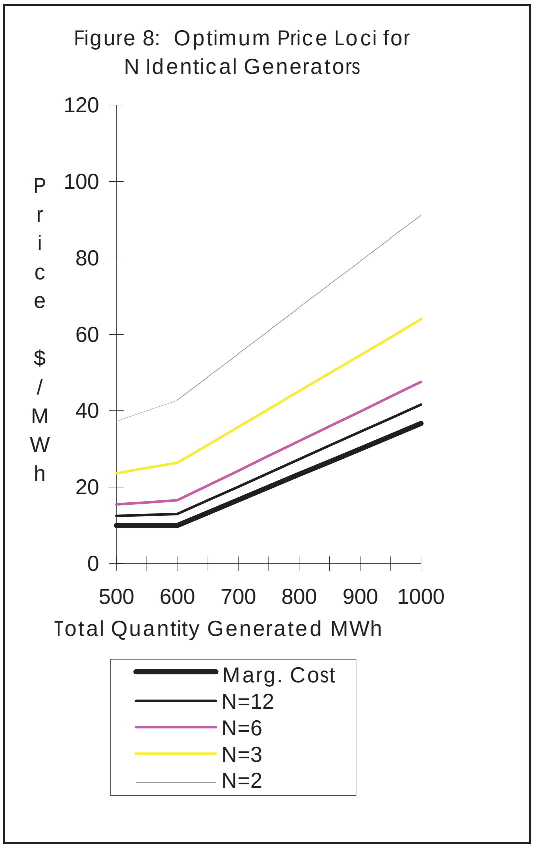


expression to (21) for a discriminatory price auction is:

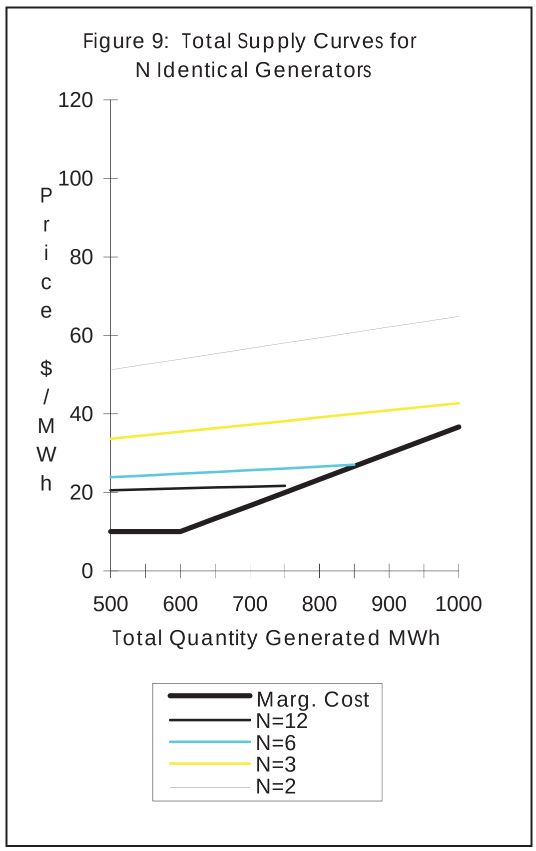
$$\mathrm{Var}[P] = (v_{2m} / N)^2 \mathrm{Var}[Q_{tot}] \qquad (30)$$

Consequently, $\mathrm{Var}[P] \rightarrow 0$ as $N \rightarrow \infty$ in (30) but using a uniform price auction in (21) Var[P] $\rightarrow$ the variance in a competitive market $((2c_{3*}/k_{tot})^2\mathrm{Var}[Q_{tot}])$. It should be noted that the low price volatility is the response to forecasting errors about Q_{tot} (i.e. the difference between $E[Q_{tot}]$ used to determine the offers and the actual Q_{tot} that occurs). Market prices will vary due to changes in the expected load (e.g. the daily load cycle), but the price elasticity of the supply curves imply that price spikes are less likely to occur using a discriminatory price auction than a uniform price auction. These issues are discussed further in the concluding section of the paper.

5. Summary and Conclusions

The main objective of this paper is to show that adopting a Discriminatory Price Auction (DPA) for electricity markets may be preferable to the current practice of using a Uniform Price Auction (UPA) to determine spot prices. Even though it is difficult to rank these two auctions consistently on the basis of economic efficiency or the level of the spot price, the supply curves will typically be more price elastic using a DPA. Consequently, price volatility caused by errors in forecasting the total load on the system will be lower, and the phenomenon of unexpected price spikes observed in the Australian market, for example, could be alleviated. Since price volatility is generally not a desirable feature of a market for customers or for new generators considering entry into the industry, less price volatility should be beneficial. Existing generators do benefit from the existence of price spikes, but there is no basis to judge whether average prices will be higher or lower using a DPA. Hence, profits for existing generators could be higher or lower if the type of auction is changed from a UPA to a DPA.

Using a UPA, the results derived in Section 3 show that offers submitted to the market will reflect the degree of market power held by a generator. The offer curves (and the aggregated supply curve) will be more price inelastic than the true marginal cost curves (and the



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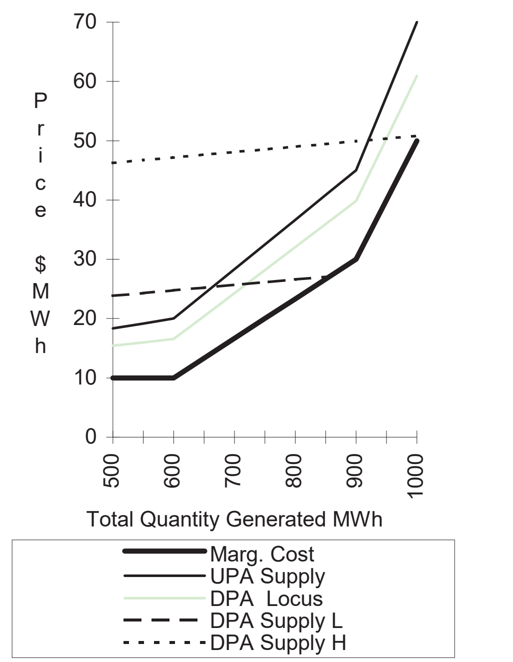
competitive supply curve). These characteristics are consistent with the type of optimum behavior derived by Ausubel and Cramton (1996) for bids in a UPA to sell multiple units, and to the actual behavior described by Wolak and Patrick (1997) for generators in the UK electricity market. Evidence shows that the two dominant generators in the UK have been willing to lose some market share in order to get higher prices and higher profits.

The results for a DPA discussed in Section 4 demonstrate that market power will increase market prices. In general, the relative size of the increase compared to a UPA is an empirical issue. Even though Swinkels (1997a and 1997b) has shown that both types of auction will approach the efficient competitive solution as the number of participants increases (in an auction to sell multiple units), there is no theoretical basis for ranking the auctions when the number of participants is small. The examples in Sections 3 and 4 are consistent with Swinkels results, and the market price approaches the competitive price as the number of generators increases using either auction.

The main distinguishing feature of the supply curve using a DPA is that it is relatively price elastic. As the number of generators increases, the optimum price locus in (29) gets closer to the true marginal cost curve but the slope of the offer curve does not change. Using a UPA, the offer curve and the optimum price locus are identical. Consequently, the offer curve for a UPA stays the same when the expected load changes, but the offer curve shifts up and down in response to these changes using a DPA. These features are illustrated in Figure 10 for a low load (L) and a high load (H). The results in Figure 10 represent the aggregated supply for N = 6 identical generators using the same parameter values as Figure 5 for a UPA and Figure 9 for a DPA. In addition, the slope of the marginal cost curve for $Q_{tot} > 900$ is increased from $2c_{3*}/k_{tot}$ to $6c_{3*}/k_{tot}$. Consequently, the slope of the supply curve for a UPA also increases for $Q_{tot} > 900$. In contrast, the slopes of the supply curves for a DPA have the same slopes but different intercepts.

The implications of the different supply curves in Figure 10 are summarized in Table 1. The market prices show how market power increases the prices above the competitive level for both auctions. Defining profits as the difference between total revenue and total costs

Figure 10: Total Supply Curves for N = 6 Generators



(with $c_{1*} = 0$ in (18)), profits are substantially higher than the competitive levels for both auctions. For example, prices are about 50% higher using a UPA and profits are roughly double the competitive levels. The lower profits for a DPA reflect both the lower prices and the effect of the slope of the supply curves (revenue is the area under the supply curve for a DPA). Defining the scalar multiplier of the variance of load as the Volatility Factor ($[((N - 1)/(N - 2))^2(2c_{3*}/k_{tot})^2]$ in (21) and $[(v_{2m}/N)^2]$ in (30)), the implications for price volatility are very different. The Volatility Factors for a UPA are higher than the competitive levels but they are close to zero for a DPA.

The low Volatility Factors are an attractive feature of a DPA because the effects of uncertainty in the load will be dampened compared to the competitive supply curve. The opposite is the case using a UPA and for this type of auction additional market power would increase the Volatility Factors even more. Consequently, price volatility and price spikes are likely to be much more of a problem for a UPA than they are for a DPA. It is not clear why the UPA has been chosen for electricity markets (prices may differ among generators because of transmission losses, but a uniform price is paid for all generation from a given location). In

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contrast, most markets for selling Treasury Bonds in different countries use a DPA.

Table 1: Characteristics of Supply for N = 6 Identical Generators

Load (MWh)	Competitive	UPA	DPA
	Market Price ($/MWh)		
720	18.0	30.0	25.9
950	40.0	57.5	50.4
	Profits (‘000$)		
720	5.9	13.9	8.6
950	22.3	43.6	28.0
	Volatility Factor x 1000		
720	4.4	6.9	0.08
950	40.0	62.5	0.08

Given the importance of price volatility in competitive markets for electricity, like the Australian case, there is a clear need for further research on discriminatory price auctions. This would provide an excellent opportunity to use an experimental setting, such as PowerWeb, to identify the relative merits of a discriminatory price auction and the conventional uniform price auctions for electricity markets. Finding out more about how these auctions perform under uncertainty is an essential step for understanding the causes of price volatility in electricity markets and for identifying ways to reduce it.

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