A Transient Stability Constrained Optimal Power Flow

Deqiang Gan (M) deqiang@ee.cornell.edu
Robert J. Thomas (F) rjt1@cornell.edu
Ray D. Zimmerman (M) rz10@cornell.edu

School of Electrical Engineering
Cornell University
Ithaca, NY 14853

Abstract

Stability is an important constraint in power system operation. Often trial and error heuristics are used that can be costly and imprecise. A new methodology that eliminates the need for repeated simulation to determine a transiently secure operating point is presented. The methodology involves a stability constrained Optimal Power Flow (OPF). The theoretical development is straightforward: swing equations are converted to numerically equivalent algebraic equations and then integrated into a standard OPF formulation. In this way standard nonlinear programming techniques can be applied to the problem.

Introduction

The cost of losing synchronous operation through a transient instability is extremely high in modern power systems. Consequently, utility engineers often perform a large number of stability studies in order to avoid the problem. Mathematically, the transient stability problem is described by solutions of a set of differential-algebraic equations [1,2,3]. The simplest form of the equations are the so-called swing equations. The current industry standard is to solve these equations via step-by-step integration (SBSI) methods. Since different operating points of a power system have different stability characteristics, transient stability can be maintained by searching for one that respects appropriate stability limits. Such a search using conventional methods has to be done by trial-and-error incorporating heuristics based on engineering experience and judgement. Recently, significant improvements in computer technology have encouraged the successful implementation of on-line dynamic security assessment programs [4,5,6]. While these new programs greatly improve the ability to monitor system stability, they also reveal that trial-and-error methods are not suitable for automated computation.

The disadvantage of SBSI methods has been recognized since the early stages of computer application in power systems. This encouraged extensive investigations into energy function methods [8,9,10,11]. These methods have their roots in Lyapunov stability theory and they provide a quantitative stability margin based on an assessment of the change in direction of the operating point [12,13,14]. Possibly for the same reason, research on pattern recognition and its variant, artificial neural networks, has also been rather active in the past two decades. Although these methods do not contain an explicit stability margin, they do provide for a simple mapping between controllable generation dispatch and indices such as an energy margin, rotor angles, etc. The simple mapping information can in turn be used in a preventive control formulation [15]. Other attempts to solve this preventive control problem can be found in, for example, references [4,16,17,18].

The unique feature of an OPF is that certain costs can be minimized while functional constraints such as line-flow and voltage limits are respected. Significant progress has been made in this area in recent years [19,20,21]. Given state-of-the-art OPF software, power engineers can perform studies for large systems with *n-1* steady-state constraints in a reasonable amount of time. It is relatively straightforward to include *n-1* contingency constraints since these constraints can be modeled via *algebraic* equations or inequalities. It is, however, an open question as to how to include stability constraints since stability is a dynamic concept and differential equations are involved. Recently, OPF practitioners began to discuss the possibility of including stability constraints in standard OPF formulations [19,21,22]. A few attempts based on either energy function methods or pattern recognition techniques have been pursued [12,13,14,15]. The importance of maintaining stability in power systems operation however calls for fundamentally strict, precise, yet flexible methodologies.

It is also worth mentioning that the emergence of competitive power markets also creates the need for a stability-constrained OPF because the traditional trial-and-error method could produce a discrimination among market players in stressed power systems [23]. As reported in [24], "the past practice of maintaining reliability by following operating guidelines based on off-line stability studies is not satisfactory in a deregulated environment".

In this paper, we develop a method for handling transient stability constraints. We demonstrate our idea by applying it to a stability-constrained OPF problem. The methodology is built upon a state-of-the-art OPF and SBSI techniques. By converting the differential equations into numerically equivalent algebraic equations, standard nonlinear programming techniques can be applied to the problem. We demonstrate via simulation results that stability constraints such as rotor angle limits and/or tie-line stability limits can be conveniently controlled in the *same* way thermal limits are controlled in the context of an OPF solution.

A Stability-Constrained Optimal Power Flow Formulation

A standard OPF problem can be formulated as follows [19]:

$$\text{Min} \quad f(P_g) \tag{1}$$

$$\text{S.T.} \quad P_g - P_L - P(V,q) = 0 \tag{2}$$

$$Q_g - Q_L - Q(V,q) = 0 \tag{3}$$

$$S(V,q) - S^M \le 0 \tag{4}$$

$$V^m \le V \le V^M \tag{5}$$

$$P_g^m \le P_g \le P_g^M \tag{6}$$

$$Q_g^m \le Q_g \le Q_g^M \tag{7}$$

Where $f(\cdot)$ is a cost function; (2) and (3) are the active and reactive power flow equations, respectively; P_g is the vector of generator active power output with upper bound P_g^M and lower bound P_g^m; Q_g is the vector of reactive power output with upper bound Q_g^M and lower bound Q_g^m; P_L and Q_L are vectors of real and reactive power demand; $P(V,q)$ and $Q(V,q)$ are vectors of real and imaginary network injections, respectively; $S(V,q)$ is a vector of apparent power across the transmission lines and S^M contains the thermal limits for those lines; V and q are vectors of bus voltage magnitudes and angles with lower and upper limits V^m and V^M, respectively. Note that P_g, Q_g, V, and q are the free variables in the problem.

Now, assume that the dynamics are governed by the so-called classical model in which the synchronous machine is characterized by a constant voltage E behind a transient reactance X'_d. For the sake of illustration the load is modeled by constant impedance. Note that more complicated models could be used without loss of generality. We have the following "swing" equation [1]:

$$\frac{dd_i}{dt} = w_i \tag{8}$$

$$\begin{aligned}\frac{dd_i}{dt} &= \frac{pf_0}{2H_i}\left[P_{gi} - \frac{1}{X'_{di}}\left(E_i W_{xi}\sin d_i - E_i W_{yi}\cos d_i\right)\right] \\ &= D_i\left(P_{gi}, E_i, W_{xi}, W_{yi}, d_i, w_i\right)\end{aligned} \tag{9}$$

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix} \cdot \begin{bmatrix} W_x \\ W_y \end{bmatrix} = \begin{bmatrix} I_x \\ I_y \end{bmatrix} \tag{10}$$

Where G and B contain the real and reactive part of the bus admittance matrix, respectively; W_x and W_y are vectors containing the real and imaginary part of the network (bus) voltages; f_0 is the nominal system frequency; H_i is the inertia of *ith* generator; w_i and d_i are the rotor speed and angle of *ith* generator. The i*th* entry of I_x and I_y is given by:

$$I_{xi} = \frac{E_i \sin d_i}{X'_d}, \quad I_{yi} = -\frac{E_i \cos d_i}{X'_d} \qquad \text{(generator buses)}$$

$$I_{xi} = 0, \quad I_{yi} = 0 \qquad \text{(non-generator buses)}$$

We require that a solution of the stability-constrained OPF respect the following constraint for each i:

$$\bar{d}_i = d_i - \frac{\sum_{k=1}^{ng} H_k d_k}{\sum_{k=1}^{ng} H_k} \le 100^\circ \tag{11}$$

Where *ng* is the number of generators and $\bar{d}_i$ is the *rotor angle with respect to a center of inertia* reference frame. We use rotor angle to indicate whether or not the system is stable. Note that other constraints such as voltage dip can also be included here.

A solution to a stability-constrained OPF would be a set of generator set-points that satisfy equations (1)-(11) for a set of credible contingencies. Unfortunately, this hard nonlinear programming problem contains both algebraic and differential equation constraints. Existing optimization methods cannot deal with this kind of problem directly. In the next section, we propose a method to attack the problem.

Outline of the Idea

As mentioned in preceding text, it is relatively straightforward to include *n-1* contingency constraints into OPF since these constraints can be modeled via *algebraic* equations or inequalities. It is, however, an open question about how to include stability constraints. Obviously the key to solving the problem is in handling the differential equations. Here we convert the differential-algebraic equations to numerically equivalent algebraic equations using trapezoidal rule. This yields:

$$d_i^{n+1} - d_i^n - \frac{h}{2}\left(w_i^{n+1} + w_i^n\right) = 0 \tag{12}$$

$$w_i^{n+1} - w_i^n - \frac{h}{2}\left(D^{n+1} + D^n\right) = 0 \tag{13}$$

$$GV_x^{n+1} - BV_y^{n+1} - I_x^{n+1} = 0 \tag{14}$$

$$BV_x^{n+1} + GV_y^{n+1} - I_y^{n+1} = 0 \tag{15}$$

$$\left(n = 1,2,\ldots,nend; \quad i = 1,2,\ldots,ng\right)$$

where h is the integration step length, n is the integration step counter, and *nend* is the number of integration steps [28]. The stability constraints can thus be expressed as follows:

$$d_i^n - \frac{\sum_{k=1}^{nq} H_k d_k^n}{\sum_{k=1}^{nq} H_k} \le 100^\circ \tag{16}$$

$$\left(n = 1,2,\ldots,nend; \quad i = 1,2,\ldots,ng\right)$$

Note that we still have to set up the equations for computing initial values of rotor angle and the equations for computing parameters of the swing equations. It is straightforward to show that:

$$w_i^1 = 0 \tag{17}$$

$$E_i V_i \sin\left(d_i^1 - q_i\right) + P_{gi} X'_{di} = 0 \tag{18}$$

$$E_i^2 - E_i V_i \cos\left(d_i^1 - q_i\right) - X'_{di} Q_{gi} = 0 \tag{19}$$

$$\left(i = 1,2,\ldots,ng\right)$$

$$G_{load,i} = \frac{P_{Li}}{V_i^2} \tag{20}$$

$$B_{load,i} = -\frac{Q_{Li}}{V_i^2} \tag{21}$$

$$\left(i = 1,2,\ldots,nb\right)$$

Where $G_{load,i}$ and $B_{load,i}$ represent the real and imaginary part of load impedance, and *nb* is the number of buses. In summary, we obtain the following *algebraic nonlinear program (NP) problem*:

$$\text{Min} \quad f(P_g) \tag{22}$$

S.T. (2) – (7)
(12) – (22)

This standard nonlinear programming problem can be solved using existing numerical methods. Indeed, the idea described in this section is surprisingly simple. In subsequent sections, we will develop a linear programming (LP) based computational procedure to solve this algebraic NP problem.

Computational Issues

In this section, we outline the overall procedure of our method and discuss computational complexities associated with stability constrained OPF problem.

A. An Algorithm

A model algorithm that has been tested on small power systems is outlined in Fig. 1. We constructed the model algorithms from direct extension of the successive linear programming method with constraint relaxation [22].

In what follows we explain the procedure described in Fig. 1. Since individual stability constraints are typically not binding, it is only prudent to begin by solving a standard OPF to start and to check to see if the solution of

the standard OPF respects stability constraints. If the solution does, then this solution is also the final solution of stability constrained OPF. If the solution does not respect stability constraints, then a complete stability constrained OPF must be solved.

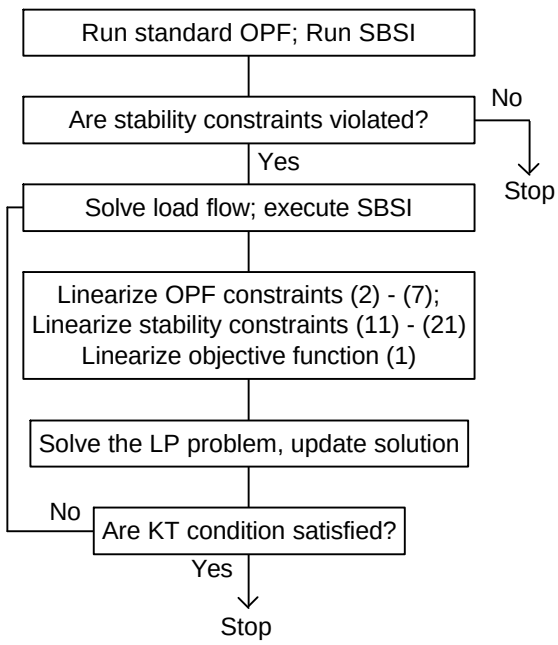

Fig 1: A procedure for the stability constrained OPF

The KT or Kuhn-Tucker condition alluded to in Fig. 1 is the optimality condition for *the algebraic NP problem*. Inside the main loop, load flow and swing equations are solved simultaneously. Based on our computational experience, this seems to be overly cautious. So in our prototype code, we solve load flow and swing equations sequentially. Our experience also indicates that the integration format used in SBSI and that in the algebraic NP problem should be consistent. Otherwise, the algorithm may not converge.

Linearizing the objective function and constraints is trivial. The only thing we would point out is that the number of stability constraints is very large.

B. Computational Complexity

The algebraic NP problem (22) contains a very large number of constraints. At this point, we are not able to validate whether or not the LP-based method is efficient for this problem. Rather, we offer some observations that could lead to a practical solution to this problem. We start our discussion by making a comparison between steady-state security constrained OPF and dynamic-security constrained OPF. As an example assume

- There 10 contingency constraint equations
- The integration step size is 0.1 second
- The integration period is 2 second
- There are 2 network switches (the point in time where the fault is applied and cleared)

Note that each integration step imposes one set of constraints (equations 12-16), so each contingency imposes a set of 22 constraints (2/0.1 + 2 constraints). Thus for this stability-constrained OPF problem, 220 constraints need to be appended to standard OPF. For steady-state security constrained OPF, 10 constraints would need to be appended to the standard OPF. This analysis is however overly simplistic for the following reasons:

First, for many occasions one is only interested in transient stability constrained problems in which only one contingency is involved at a time.

Second, we notice that the number of binding constraints for dynamic security is typically smaller than that for steady-state security. In perhaps any power system, the number of binding stability constraints is normally very small, say in the order of 5 or less.

Third, for most stability studies, we can apply the constraint relaxation technique explained below. Suppose the *maximum rotor angle at each integration step*, that is $\max(\bar{d}_i, i = 1,...,ng)$, reaches its maximum point at 0.8 second, then the constraints associated with those integration steps after, say, 1.0 second can be excluded from the LP problem (see Fig. 2).

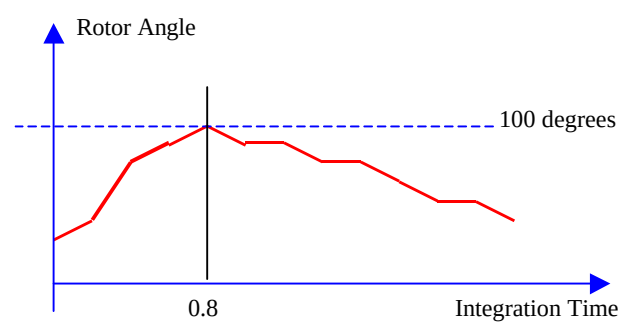

Fig. 2. Constraint Relaxation for the Stability Constrained OPF

The above technique, which is conceptually different from that described in [22], can reduce the size of the LP problem significantly (note that a full SBSI should always be performed to make sure that no stability limit is violated).

An Extension

The integration-based method described in the preceding sections also offers the basis of an analytical tool for other stability-related problems. We give some examples in this section.

Similar to standard OPF or steady-state security constrained OPF, the objective function of the stability constrained OPF can be defined as operating cost, transmission loss, as well as special objectives like the one given below:

$$\text{Min} \sum_i \left(P_{gi} - P_{gi}^0\right)^2$$

S.T. (2) – (7)
(11) – (20)

Where P_g^0 represents the desired operating point (typically the previous one). The objective of this OPF is to find a secure operating point that is close to the desired operating point. Such a problem is known as preventive control or generation rescheduling.

Another example is to estimate the loadability of power systems subject to stability constraint [25]. The objective function and load flow constraints in this problem should be defined as:

Min λ
S.T.

$$P_g - l\,P_L - P(V,q) = 0$$
$$Q_g - l\,Q_L - Q(V,q) = 0$$

(4) – (7), (11) – (20)

Where scalar λ denotes a parameter associated with load increases.

Now, the Total Transfer Capability (TTC) or stability limit of a tie line can be computed by solving:

Max *Interface Flow*
S.T. (2) – (7)
(11) – (20)

Note that once the TTC is obtained, it is trivial to compute Available Transfer Capability (ATC) [24]. The interface flow can be either of the point-to-point or the area-to-area type.

We would also like to mention that, the method could be modified to compute Critical Clearing Time (CCT), a measure of stability margin, for simple contingencies. The objective function in this case becomes:

Max *CCT*

More sophisticated implementation has to be formulated to accommodate CCT computation.

One of the advantages of our method is that it has less limitation on component modeling. Load can be expressed as any combination of constant impedance, constant current, and constant power. Generators can be modeled with a single-axis model, a two-axis model, or even a more detailed model [1]. Interesting network changes such as three-phase-ground faults or the removal of transmission lines can be modeled in a straightforward way.

Numerical Examples

The integration-based method was implemented using the MATPOWER package [26], a MATLAB-based power system analysis toolbox that is freely available for download from the site at http://www.pserc.cornell.edu/matpower/. The prototype code has been tested on the WSCC 3-machine 9-bus system and the system New England 10-machine 39-bus System. The results of New England system are presented here.

The operating point is given by a standard OPF. A three-phase-to-ground fault is applied to bus 29, the fault is cleared 0.1 second later coupled with the removal of line 29-28. The integration step size is set to 0.1 seconds and the integration is executed for 1.5 seconds. We note that the operating point did not respect the stability constraint (the relative rotor angle of generator at bus 29 is about 700 degrees at time 1.5 second (See Fig. 3).

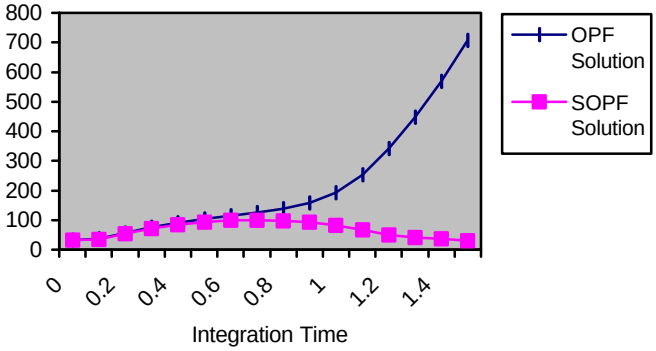

Fig 3: Maximum Angle (at each integration step)

The stability constrained OPF program was then run providing an operating point that respects stability constraints, as illustrated in Fig 3. The operating cost of the system was slightly increased. The iteration process in the stability constrained OPF is illustrated in Fig. 4.

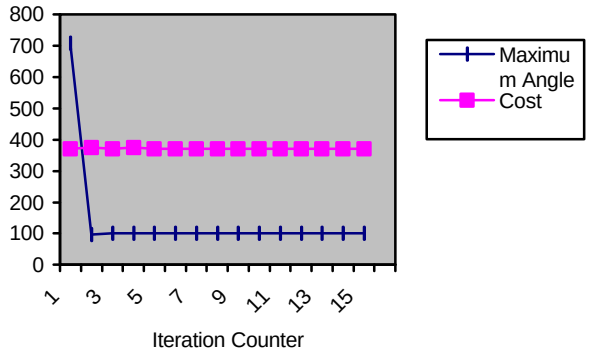

Fig 4. Iteration Process of Stability Constrained OPF

Conclusions

In the recent past tremendous effort has been spent on system stability issues. The objectives are to monitor and ultimately control the stability during power system operation. While the technology for stability simulation is rather stable now, little analytical development has been done for computing stability limits precisely. This is perhaps because computing the stability limits precisely has been thought to be impossible [27].

There is, however, an increasing need for solutions for this challenging problem. In this paper, we have developed a basis for one approach to this problem. The method naturally inherits the advantages of SBSI-based methods such as, it has little limitations on component modeling, it is robust, and it provides all relevant system swing information. We demonstrated that, using this general methodology, for the first time the stability limits of power systems can be precisely and automatically estimated. We are hoping that the methodology can be developed into a practical tool but this requires that it be efficiently implemented.

Acknowlegdement

We would like to thank Z. Yan and C. Murillo-Sanchez for their valuable help during the course of the study reported in the paper.

References

1. P.W. Sauer, M.A. Pai, *Power System Dynamics and Stability*, Prentice-Hall, Upper Saddle River, New Jersey, 1998
2. P. Kundur, *Power System Stability and Control*, McGraw-Hill, 1994
3. B. Stott, "Power System Dynamic Response Calculations", *Proceedings of the IEEE*, vol. 67, no. 2, 1979, pp 219-241
4. E. Vaahedi, Y. Mansour, et al, "A General Purpose Method for On-line Dynamic Security Assessment", *IEEE Trans. on Power Systems*, vol. 13, no. 1, 1998, pp. 243-249
5. K.W. Chan, R.W. Dunn, et al., "On-line Dynamic-security Contingency Screening and Ranking", *IEE Proceedings-Generation Transmission and Distribution*, vol. 144, no. 2, 1997, pp. 132-138
6. K. Demaree, T. Athay, et al., "An On-line Dynamic Security Analysis System Implementation", *IEEE Trans. on Power Systems*, vol. 9, no. 4, 1994, pp 1716-1722
7. C. Pottle, R.J. Thomas, et al., "Rapid Analysis of Transient Stability: Hardware Solutions", in Rapid Analysis of Transient Stability (a Tutorial), IEEE Publication No. 87TH0169-3-PWR, 1987
8. H.D. Chiang, "Direct Stability Analysis of Electric Power Systems Using Energy Functions: Theory, Application, and Perspective", *Proceedings of the IEEE*, vol. 83, no. 11, Nov. 1995, pp 1497-1529
9. M. Pavella, P.G. Murthy, *Transient Stability of Power Systems: Theory and Practice*, John Wiley & Sons, Inc., New York, 1994
10. A.A. Fouad, V. Vittal, *Power System Transient Stability Analysis Using the Transient Energy Function Method*, Prentice-Hall, Englewood Cliffs, 1991
11. M.A. Pai, *Energy Function Analysis for Power System Stability*, Kluwer Academic Publishers, 1989
12. A.A. Fouad, J. Tong, "Stability Constrained Optimal Rescheduling of Generation", *IEEE Trans. On Power Systems*, vol. 8, no. 1, 1993, pp. 105-112
13. J. Sterling, M.A. Pai, et al., "A Method of Secure and Optimal Operation of a Power System for Dynamic Contingencies", *Electric Machine and Power Systems*, 19-1991, pp. 639-655

14. K.S. Chandrashekhar, D.J. Hill, "Dynamic Security Dispatch: Basic Formulation", *IEEE Trans. On Power Apparatus and Systems*, vol. 102, no. 7, July 1983, pp 2145-2154.
15. V. Miranda, J.N. Fidalgo, et al., "Real Time Preventive Actions for Transient Stability Enhancement with a Hybrid Neural-optimization Approach", *IEEE Trans. on Power System*, vol. 10, no. 2, 1995, pp. 1029-1035
16. W. Li, A. Bose, "A Coherency Based Rescheduling Method for Dynamic Security", *Proceedings of PICA*, 1997, pp. 254-259
17. D. Gan, Z. Qu, et al., "Methodology and Computer Package for Generation Rescheduling", *IEE Proceedings-Generation Transmission and Distribution*, vol. 144, no. 3, May 1997, pp. 301-307
18. R.J. Kaye, F.F. Wu, "Dynamic Security Regions of Power Systems", *IEEE Trans. On Circuits and Systems*, vol. 29, Sept. 1982
19. J. Carpentier, "Towards a Secure and Optimal Automatic Operation of Power Systems", *IEEE PICA Conference Proceedings*, Montreal, Canada, 1987, pp. 2-37
20. B. Stott, O. Alsac, et al., "Security Analysis and Optimization", *Proceedings of IEEE*, vol. 75, no. 12, 1987, pp. 1623-1644
21. J.A. Momoh, R.J. Koessler, et al., "Challenges to Optimal Power Flow", *IEEE Trans on Power Systems*, vol. 12, no. 1, 1997, pp. 444-455
22. O. Alsac, J. Bright, et al., "Further Developments in LP-based Optimal Power Flow", *IEEE Trans. On Power Systems*, vol. 5, no. 3, 1990, pp. 697-711
23. R.J. Thomas, R.D. Zimmerman, et al., "An Internet-Based Platform for Testing Generation Scheduling Auctions", *Hawaii International Conference on System Sciences*, Hawaii, January 1998
24. S.C. Savulescu, L.G. Leffler, "Computation of Parallel Flows and the Total and Available Transfer Capability", 1997 PICA Tutorial.
25. D. Gan, Z. Qu, et al., "Loadability of Generation-Transmission Systems with Unified Steady-state and Dynamic Security Constraints", *14th IEEE/PES Transmission and Distribution Conference and Exposition*, Los Angles, California, September, 1996, pp. 537-542
26. R. Zimmerman, D. Gan, "MATPOWER: A MATLAB Power System Simulation Package", http://www.pserc.cornell.edu/matpower/
27. M. Ilic, F. Galiana, et al., "Transmission Capacity in Power Networks", *International Journal of Electrical Power & Energy Systems*, vol. 20, no. 2, 1998, pp. 99-110.
28. F. Alvarado, *IEEE Transactions on Power Apparatus & Systems*,1979