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The Efficiency of Multi-Unit Electricity Auctions

Wedad Elmaghraby* and Shmuel S. Oren**

Using a complete information game-theoretic model, we analyze the performance of different electricity auction structures in attaining efficiency (i.e., least-cost dispatch). We find that an auction structure where generators are allowed to bid for load "slices" outperforms an auction structure where generators submit bids for different hours in the day.

INTRODUCTION

The electric power industry around the world is undergoing a process of privatization, deregulation and restructuring. This transition is fueled by technological and social change that led to a fundamental re-examination of conventional wisdom concerning natural monopolies and economies of scale in this industry.

While the restructuring approaches implemented or proposed in various parts of the world and within the US are diverse in many aspects, they share several important elements which include competitive generation, spot energy markets and power auctions. The purpose of the auction is to provide a mechanism through which generators can submit bids to supply electricity. The most challenging aspect of designing an electricity auction is that daily demand which fluctuates from period to period, must be satisfied by a set of suppliers, with different costs, in a least-cost manner. Even in a centralized model with known generator costs, determining the optimal dispatch is a computationally difficult problem. It is an even greater challenge to design an auction where generators voluntarily chose to be efficiently dispatched.

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In any electricity auction, generators must submit bids, which indicate the minimum prices at which they are willing to generate electricity. In some auction designs (e.g., the in the UK system) bids can also include state transition costs such as start up and ramping costs as well as constraints on availability and dispatch.¹ The structure of the auction and the determination of prices paid to the winning bidders can vary. It is desirable that the designer of an electricity auction defines the structure and prices in such a way as to provide generators with the incentive to bid so as to minimize generation costs. We shall refer to the set of generators, which minimizes generation costs as the *efficient dispatch* and to an auction that induces an efficient dispatch as an *efficient auction*.

There are several aspects of an auction for electricity that separate it from the vast body of auction literature and make designing an efficient auction a challenging task. The most obvious is the structure of generation costs. Generators have many cost components (e.g., ramp-up costs, no-load costs, etc.) which must be recovered through their sales revenue. In addition generators are subject to intertemporal dispatch constraints that relate their output in different time periods. These characteristics create cost dependencies in intertemporal production so that the average cost of generating Q GW of electricity varies with the number of units generated and the dispatch schedule. Such dependencies complicate both the bidding strategies and the bid evaluation protocols.

The long standing tradition of vertical integration and centralized dispatch in the electric utility industry resulted in advanced computational tools for optimal dispatch of generating resources, which take as inputs all the costs and operational constraints for each available resource as well as demand data and reliability requirements. Such tools are designed based on the premise of perfect information about these inputs and produce as outputs two types of decision variables. The optimal commitment schedules specifying the periodic on/off state of each resource are typically produced for a range of the next 168 periods. These schedules and a rough estimate of the periodic output level of each resource are calculated by a unit commitment algorithm which is a mixed nonlinear and integer optimization program run every period on a rolling horizon basis for the next 168 hours. The unit commitment schedules are used as inputs to an optimal power flow calculation, which uses a nonlinear optimization algorithm run repeatedly at short time intervals to obtain the up-to-the-minute output levels of each generator on line. The optimal power flow employs a more realistic model of the power system that takes into consideration transmission and security constraints as well as various physical aspects associated with alternating current (AC) systems.

Some restructuring designs have attempted to preserve the central unit commitment protocols with competitive generation by employing a multipart

1. See Patrick and Wolak (1996) for an analysis of the United Kingdom auction design.

auction to elicit the inputs needed for the traditional unit commitment algorithms. In such auctions, bidders are required to submit for the day ahead, supply functions for energy as well as all the other cost components and dispatch constraints needed for central unit commitment procedures. Such an auction structure is employed in the UK where unit commitment is performed using the GOAL algorithm and it is part of the proposed restructuring plan for the New York Power Pool. Unfortunately, multi-part auctions are not well understood and have limited theoretical foundation that would enable a practical incentive compatible design of such auctions.

In a recent paper, Hobbs et al. (1998) studied an incentive compatible multi-part electricity auction based on the Vickery-Clarke-Grove mechanism. Unfortunately, as noted by the authors, this approach has limited practical value due to revenue deficiency (payments to generators will systematically exceed energy sales revenues). Furthermore, the discriminatory nature of that auction is likely to be controversial at best. In all current implementations of multipart electricity auctions, revenue adequacy is achieved through settlement systems that are based on uniform hourly (or half-hourly) energy prices and a possible capacity payment that are covered by "uplift charges" (e.g., NYPP, UK, Argentina and Spain have such capacity payments). The UK experience suggests that such auctions are susceptible to gaming and manipulation that could undermine the efficiency of central unit commitment.

Recent work by Johnson, Oren and Svoboda (1997) further suggests that even if bidders could be induced to reveal true costs and constraints, central unit commitment may still be inappropriate in a competitive generation environment. In particular, the authors have demonstrated that unit commitment algorithms designed for an environment with central generation ownership have multiple equally good solutions with varying resource schedules. When generation ownership is dispersed among many independent parties, such variations have diverse profit implications for the different parties. One fundamental source of potential inequalities is the fact that in the presence of nonconvex cost functions, marginally accepted bids make positive profits. In the absence of some compensation to the marginal losers, selecting winning bids in a multiple solution case has inequity consequences or will cause bidders to undercut each other by distorting information that is needed for efficiency central unit commitment. Consider, for example, a situation where there are two equal base load plants and a peaking plant where, efficiency dispatch to meet the load will commit a single base plant and the peaking plant. Since the base plant will receive the peaking plant's marginal cost during the peak and its own marginal cost during off peak (when the peaking plant does not operate), the dispatched base plant will make a positive profits while the rejected bid makes zero profit. This may induce the base plants to shave their bids by revealing false information. While such competition might be a desired outcome for long-

term efficiency, it undermines the short-term efficiency objectives of central unit commitment, which will be based on distorted information. It should be noted that the mechanism studied by Hobbs et al. (1998) pays generators their computed cost plus a lump sum payment that vanishes in case of a tie. Hence, under that scheme, accepted marginal bids in the event of multiple solutions do break even.

The resulting ambiguity in the bid selection protocols cannot be resolved by tie breaking procedures since it is not practical to compute all the optimal solutions (even one good solution is computationally challenging). Furthermore, the optimal unit commitment produced by a specific algorithm (out of the many possible) is affected by fine tuning of the program which consequently may be systematically biased in favor of some generators to the detriment of others. The repetitive nature of the auction could mitigate the adverse effects of such biases by “averaging out” these effects (see Rothkopf (1999)). It was also suggested by Rothkopf et al. (1998) that disputes associated with the imperfection of solutions to the central unit commitment problem (which is NP hard) could be resolved by offering bidders the opportunity to improve on the proposed schedule within a given time period.

An alternative approach to the multi-part auction that has been adopted in the California Power Exchange protocol and in the Victoria pool in New South Wales, Australia relies on self-commitment.² In other words, unit commitment decisions are left to the bidders while the auction structure is simplified to a single price per tender per plant.³ A tender consists of one or multiple blocks of energy defined in terms of their timing and capacity (e.g., 2 GW supplied for one period between 1 and 2 PM). All the production costs incurred by a generator (including fixed, intertemporal and dispatch constraints costs) are internalized in such an auction and reflected in the single bid price.

From an auction theory perspective power auctions with self-commitment may be interpreted as multi-unit auctions with complementarities. There are a few papers that address the issue of multi-unit auctions. Wilson (1979) began the study of “share” auctions, where bidders with a common

2. One of the prevailing approaches to electricity market design (e.g., California PX, Victoria Pool in Australia, Nordpool) has been to rely on self dispatch, i.e., unit commitment decisions are made by the generators before knowing the outcome of the auction, via hourly energy only auctions. In California generators of the investor owned utilities must bid all their generation through the Power Exchange (until the year 2001) and in Victoria all the generated power is cleared through an energy auction. In some energy markets, such as the one in Alberta, generators circumvent the energy auction and secure their dispatch and payments via bilateral contracts, thereby making the energy auction a residual market. In this paper, we will focus our attention to markets where generators secure their dispatch via an energy auction.

3. In reality, generators may own several units (gensets) and may submit a separate bid per unit. I shall refer to a unit as a plant. It is important to point out that the result in this paper also hold when generators are allowed to submit a (finite) step bid function for a plant's entire capacity.

valuation submit demand curves and are awarded a fraction of a unit's shares at a market-clearing price. He concludes that a seller's revenue is greatly reduced using a share auction when compared to selling the entire unit as an indivisible object. Maskin and Riley (1989) study the design of optimal multi-unit auctions with private valuations and show that neither a uniform price or discriminatory price auction is an optimal auction mechanism, i.e., neither maximize the seller's expected revenue. Hausch (1986) studies a two-object auction, where there are two bidders with common valuations who desire both objects. Hausch finds that the seller's revenue is greater when both objects are sold simultaneously versus sequentially. Rothkopf, Pekec, and Harstad (1998) identify a special class of multi-unit auctions in which bidders can submit a bid for different combinations of objects and the auction is computationally manageable.

Bikhchandani (1996) establishes the allocative efficiency properties of first⁴ price auctions when several heterogeneous objects are sold simultaneously (one auction for each type of object) and bidders may desire more than one object. The bidders' are assumed to have no budget constraints, private valuations for the objects that are common knowledge. Bikhchandani finds that, for a first price auction, an efficient pure-strategy Nash Equilibrium exists if and only if a Walrasian equilibrium exists. This result holds whether there is one or several of each type of object sold. In the case of several of each type of object, the identical objects are sold in the same auction and the bidders are allowed to submit a separate bid for each of the identical objects.

There has been a recent growth of auction theory literature in this area in response to the recent FCC spectrum auctions and the multitude of interesting questions they have raised. In the FCC auctions, bidders, comprised of US telecommunication companies, cellular telephone companies, and cable-television companies, competed to win various spectrum licenses for different geographical area. The synergies arising from owning licenses in adjoining geographical area create dependencies in (some) bidders' valuations for individual licenses (see McMillan (1994), Cramton (1995) and McMillan and McAfee (1996) for further discussion of the FCC spectrum auctions). Ausubel and Cramton (1996) question the superior allocative efficiency properties of uniform pricing rules using Wilson's (1979) "share" auction framework with private valuations. They find that the efficiency of 2nd price (uniform) auctions in a single-unit auction do not carry over to a multi-unit framework. They conclude that when bidders desire more than one object, or a large share of the total objects being auctioned, they have an incentive to underbid or "shade" their bids, resulting in an inefficient allocation. Milgrom (1998) explains the different auction formats that were candidates for the FCC spectrum auction and their relative strengths and

4. A first price auction represents a first-price sealed bid auction.

weaknesses. Isaac and James (1998) use an experiment setting to evaluate the performance of a Vickrey auction in a simple setting with multiple units, which exhibit synergies in value.

Von der Fehr and Harbord's (1993) analysis of the UK electricity industry is the only other study we know of that identifies an electricity auction as a multi-unit auction with private valuations and attempts to study the strategic bidding behavior of generators. Von der Fehr and Harbord assume a two-generator framework with uncertain demand and known marginal costs. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium.

Our objective in this paper is to address specific types of such auctions that are relevant to the context of electric power. In particular we focus on three alternative ways of structuring a multi-unit power auction and use the framework of games with complete information to examine the efficiency of their outcome. Specifically, we focus on the following question: Will the proposed auction structure induce the efficient (i.e., least social cost) dispatch in a non-cooperative⁵ setting where generators know each other's costs? While the analysis employs a very simplistic stylized model of demand and generation cost, we employ the insight of the analysis to outline the practical implementation of a new auction structure that promises efficient self-commitment and dispatch in the absence of collusion.

2. ELECTRICITY MARKET

2.1 Bidders

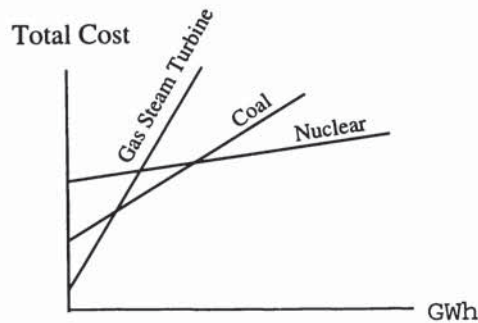
In an electricity auction, generators compete in price of generation to win dispatch. Generation technologies, for example, nuclear, combined-cycle gas turbines plants, and combustion gas turbines, are generally characterized by four traits. First, a generator's costs to generate fall into two general categories; there is a fixed "start-up" cost incurred each time a generating plant is turned on, and variable cost per GWh once the plant is up and running. Second, there exists an inverse relationship between the start-up cost associated with a technology and its variable cost. For example, a nuclear plant has a large start-up cost but relatively small variable cost per GWh, while a gas-steam turbine has a

5. Given the fact that deregulation is often being coupled with divestiture of generation plants to reduce market power (as evidenced in California) it is plausible to assume that with several companies participating in the market place, collusion will be difficult to sustain despite the repetitive (daily) nature of the auction. Rapidly changing conditions may also mitigate collusion but ultimately active monitoring and enforcement might be necessary.

relatively low start-up cost, but incurs a large variable cost per GWh. A third trait is that generation plants have a constraint on the maximum number of GW they generate at any point in time and are unable to store electricity,⁶ but have few restrictions on the duration for which they can generate.⁷ Finally, the size of a generation plant is generally decreasing in its variable cost, i.e., the capacity of a low variable cost nuclear plant is much larger than that of a high variable cost combustion gas turbine plant.

The goal of our paper is to establish the types of equilibrium dispatches that are supported by various auction mechanisms. In particular, we are interested in finding an auction structure that supports only efficient dispatches in equilibrium. Given a market with ample “efficient” resources, in the efficient dispatch, no inefficient plants will be used. Therefore, we will focus our attention on markets where all generation technologies are efficient, i.e., least-cost, for some output level. Low start-up, high variable cost technologies are the most efficient source over low output (total GWh) levels, while high start-up, low variable cost technologies are the most efficient source over higher output levels. Figure 1 below plots the total cost of generation associated with different technology types, assuming a generating plant is “switched-on” only once per day. The horizontal axis measures the total number of GWhs generated over time.

Figure 1. Costs for Different Generation Technologies



6. Some generating plants are able to store the *potential* for generating electricity, e.g., hydroelectric generators can store water, but generators are unable to store electricity. Therefore, there will always be a limit on the total MW a generating plant can generate.

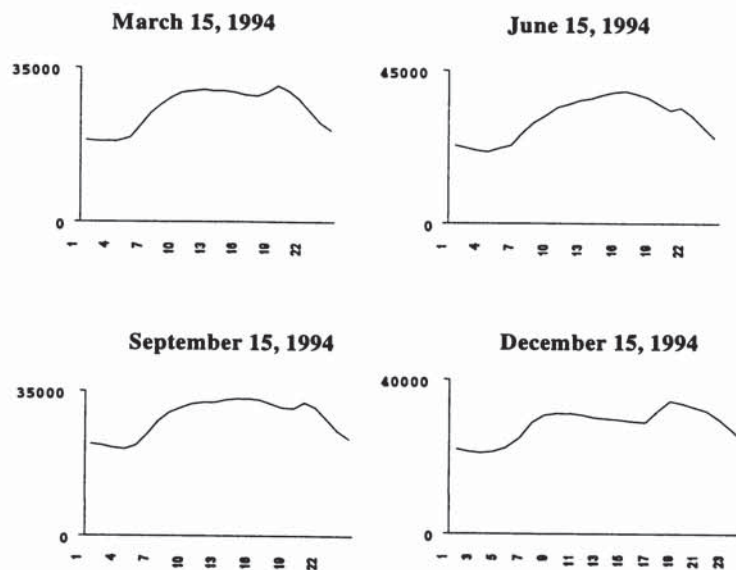
7. Generators do occasionally have to go off-line for maintenance, but this is not a relevant constraint over one day.

2.2 Market Structure

An auction for electricity must assure that daily electricity demand, which varies throughout the day, will be satisfied by a set of generators. Figure 2 below plots the total daily demand in California during the middle of March, June, September and December. Generally, demand is described to be one of three types: base, shoulder or peak load.

Several types of generation sources are needed in order to meet the varying consumption of demand throughout the day. Generation technologies can be classified by three types: base (*b*), shoulder (*s*) and peaking (*p*). Examples of base, shoulder and peaking plants are nuclear, coal and combustion gas turbine, respectively. Typically a generating plant's capacity, i.e., the maximum rate at which it can generate at any point in time, is a decreasing function of its variable cost and an increasing function of its start-up cost. Nuclear plants, which have high start-up costs, are classified as base plants and typically have a capacity several times larger than a shoulder plant such as a coal plant, which in turn has a larger capacity than a peaking plant such as a gas-steam plant. In the California market, the average (height of) base load, shoulder, and peak demand is approximately 21, 31, and 36 GW respectively, while the average size of a base load, shoulder, peak plant is 2, 1, and 0.5 GW respectively. This fact necessitates several generation plants of each type to be turned on in order to satisfy demand in a least-cost manner.

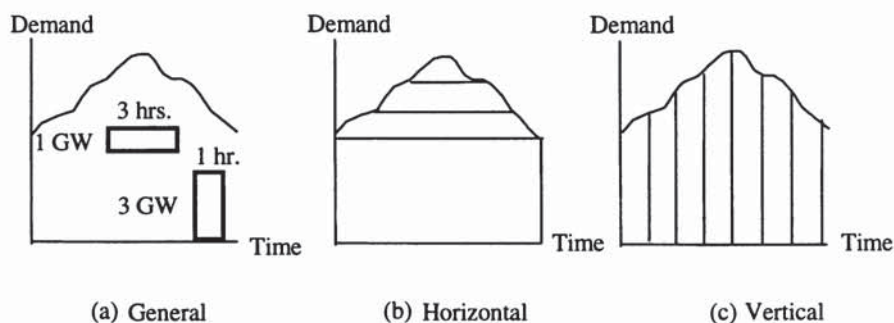
Figure 2. California Demand Load during the Months of March, June, September and December in 1994



2.3 Auction Structure

In this paper, we will ignore transmission constraints and assume that the power auction treats all the demand and supply as if it was at a single location. This simplification is consistent with the UK system, the California power exchange, the Victoria pool and other systems around the world where transmission constraints and congestion management are handled outside the power auction. We will further assume that there is no demand side bidding which is also consistent with most currently operating and proposed power auctions. Under this simplified structure the objective of the power auctioneer is to “fill” a forecasted load curve for a specified time period (say the next 24 periods) with tenders consisting of blocks of energy specified by capacity (GW) duration (periods) and timing, see graph (a) in Figure 3 below.

Figure 3. Various Partitioning Forms of Demand Load for Auctions

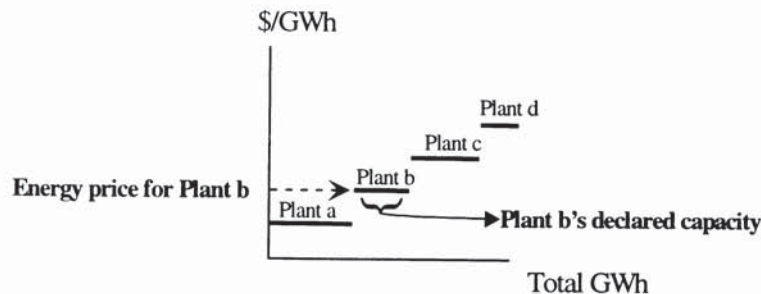


There are many possible ways to structure such an auction in terms of the tenders allowed, the bid evaluation process and the prices paid to winning tenders. In this paper, we will analyze the performance of three types of auction structures, a Daily Supply Curve-vertical (DSC-vertical), a Hourly Supply Curve-vertical (HSC-vertical)⁸ and a horizontal auction, and examine their ability to yield efficient dispatches. These auctions can be viewed as combinatorial auctions where the combinations are restricted to be one of two forms, either vertical or horizontal.

8. These auction structures are abstractions from the auctions currently in practice. A daily supply curve (DSC) is used in the England and Wales electricity auction. Likewise, the California Power Exchange accepts bids for each hour in the day. However, the auctions addressed in this paper are not exact characterization of the actual operating auction. For example, a DSC auction differs from the England and Wales auction in that a DSC auction has a uni-dimensional, price only bid while the bid structure in England and Wales is multi-dimensional (with a capacity availability component amongst other operating characteristics).

In all of these auctions, generators submit (convex) step supply curves. The supply curves indicate the minimum price that must be paid to generate, with each step corresponding to one generation plant (see Figure 4). From the submitted bids, the auctioneer constructs a cumulative supply curve, which is used to dispatch generators. In a *vertical* auction, daily demand is partitioned into hourly (or half-hourly in the case of the UK) markets. Generators submit supply curves that state the minimum price at which they are willing to generate. All generators chosen for dispatch are paid the highest accepted bid price.

Figure 4. Structure of Energy Bids; Separate Bid per Generating Plant



The electricity auctions in the United Kingdom and California can both be characterized as *vertical* auctions. A DSC-vertical and HSC-vertical auction differ in the number of supply curves the participating generators are allowed to submit. Using the characterization set forth in von der Fehr and Harbord (1993) and Green and Newbery (1992), in the UK electricity auction, generators are allowed to submit a single supply curve, which is valid for the 24-hours covered by the auction, indicating the minimum price at which they are willing to generate at different output levels.⁹ The auctioneer then constructs a single cumulative supply curve that is used to dispatch the generators for all periods in the day. In contrast, in the California electricity auction, generators are allowed to *simultaneously* submit a separate supply curve for each hour in the day. All generators chosen for dispatch in a period are paid the marginal price in that period, i.e., the highest accepted bid price. The decision to schedule a generator in any period t is determined solely on its bids for that period and independently of its schedule in any other period. We will show that there are inherent problems with trying to bid non-convex generation costs into a vertical auction, regardless of the number of bids allowed.

9. In von der Fehr and Harbord (1993) the supply curves are assumed to be step functions while Green and Newbery(1992) assume that generators submit smooth supply curves.

An alternative auction structure that, we will argue, lends itself more readily to the intertemporal cost dependencies in generation is a *horizontal* auction. In a *horizontal* auction, demand is divided into distinct horizontal demand sets that are auctioned *sequentially*. Demand sets are formed by grouping demand roughly according to its duration, i.e., a distinct set for each duration t . Hence, generators submit a supply curve for each set, indicating the price at which they are willing to generate k megawatts for a duration of t periods, where $-1 < k < K + 1$ for a fixed $t > 0$. The auctioneer constructs a least-cost supply curve from the submitted bids in each set and the winning bidders are *paid their bid price*. The sequencing of the auctions is such that the demand set with the longest duration is auctioned first, then the second longest, and so forth. The results of any previous auctions are made known before each auction.

One interpretation of a horizontal auction is that the auctioneer is segmenting demand by its types (base, shoulder or peak), thereby creating distinct auctions for the different types of demand. Generating electricity for the 24-period base load demand is a fundamentally different product than generating for just the few peak periods. By partitioning demand by its types, generators are able to submit separate bids for the different types of products provided.

The intuitive motivation for the alternative bid formats is articulated by Wilson (1998) who notes,

“The bid format is a key factor. For example, if the market is organized to provide hourly schedules and prices, then this tends to serve the interests of demanders for whom the time of power delivery is important, and suppliers with flexibility (e.g., hydro), whereas it tends to ignore the considerations of suppliers from thermal sources, who are mainly concerned with obtaining operating schedules over consecutive hours sufficient to recover the fixed costs of start-up and who are unconcerned about timing *per se*. Schemes have been devised that allow demanders to bid on a time-of-day basis while suppliers bid for operating runs of various duration; prices can then be stated equivalently in terms of hourly prices for demanders and duration prices for suppliers.”

An additional point worth mentioning here is that the frequency of the auctions for the different demand types need not be the same. For base load demand, the typical “up” time for a base load plant is several months. Therefore, while our model will examine a horizontal auction conducted on a daily basis, there is nothing to stop the baseload auction from being held once every few months; at that time plants would commit themselves to generate for the duration between auctions.

In the next section, we assume a simple model for demand and generation costs and demonstrate why neither a DSC-vertical or HSC-vertical auction can guarantee the efficient dispatch in equilibrium. We then prove that a horizontal auction *does* guarantee efficiency in our model. We conclude with the reasons for and the intuition behind these results.

3. MODEL

For simplicity, we assume that demand over the length of a day is represented by a step function, see Figure 5(a). Figure 5(b) represents the daily demand in terms of its load duration curve. When daily demand is single peaked, we can use the load duration curve in our analysis without loss of generality. Electricity auctions are conducted on a day ahead basis, where generators bid to supply the next day's forecasted demand. We model the daily demand as public information and known to the generators. In particular, we will assume that the load duration curve is as given in Figure 6 below, i.e., there is demand during only three periods of that day and demand is constant within each period.

Figure 5. Daily Demand Represented as a Load Duration Curve

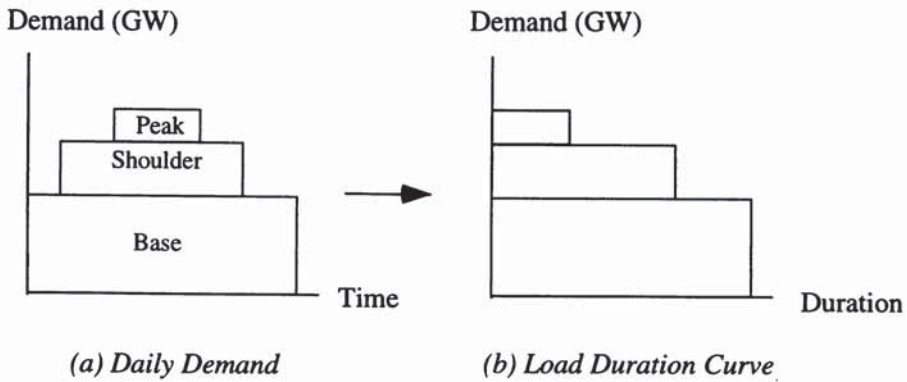
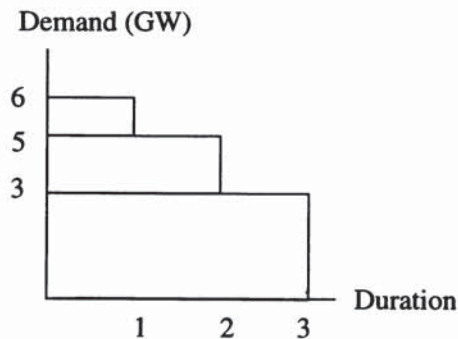


Figure 6. Assumed Demand Load



Throughout the paper we assume that generation plants have two costs, which are publicly known, associated with generation; a fixed “start-up” cost, f , to begin to generate and a variable cost per GWh, v , once the plant is up and running. Due to this cost structure, there exist cost dependencies in intertemporal production. In the following sections, we assume that there are three types of generation plants; base $\{b\}$, shoulder $\{s\}$ and peaking $\{p\}$ (see Figure 7). A generator can own several different types of plants; let G^i_x denote a plant of technology type x owned by generator i . The cost to generate q GWh from a plant of type x is denoted by $c_x(q)$. We make the following assumptions:

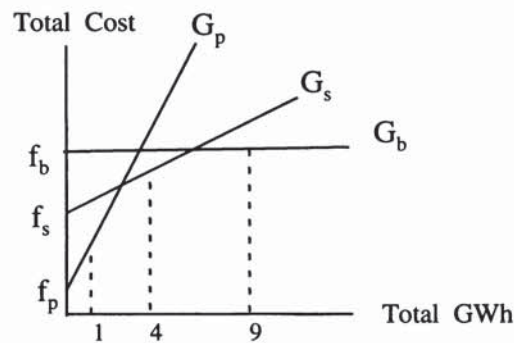
Assumption 1 For each technology type, there exists at least one extra plant present in the market, i.e., one more than is needed in the efficient dispatch.¹⁰

Assumption 2 Each technology type is owned by more than one generator (i.e., no one generator is a monopolist for a particular technology type).

Assumption 3 In the event of a tie for the lowest bid, the auctioneer dispatches plants in descending order of capacity.

Assumption 4 Plants that win in an auction are dispatched until the demand in that auction is satisfied or the plant has reached its capacity.

Figure 7. Assumed Generation Types and Respective Costs



10. Due to the presence of excess efficient technology types, the presence of inefficient plants in the market would not alter any of the paper's results.

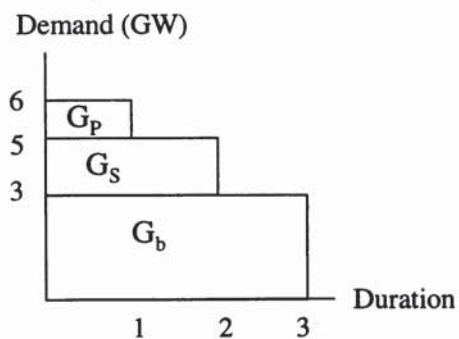
For expository purposes, we assume that there exist three generators, each of whom owns two plants, unless stated otherwise. In particular, assume that generator 1 owns a base and shoulder plant, generator 2 owns a base and peak plant and generator 3 owns a shoulder and peak plant. This assumption is not necessary, and the results of this paper hold for any market configuration as long as Assumptions (1) and (2) are satisfied. Table 1 lists the assumed capacity constraints for each type of plant. The capacity limitations imply that the maximum a base (shoulder, peak) load plant can generate at any point in time is 3 (2,1), but imposes no constraint on the duration for which a plant can generate.

Table 1. Capacities of Technology Types

Technology Type	Capacity
Base	3
Shoulder	2
Peak	1

Under these assumptions, we explore the simplest scenario where the size of the base (shoulder, peak) demand is equal to the capacity of one base (shoulder, peak) plant. The capacity plus demand assumptions imply that there is a unique efficient dispatch, as given in Figure 8. This simple example is rich enough to capture the failings and strengths of the three auctions structures presented here. In the next section we shall see that neither form of a vertical auction is able to guarantee the efficient dispatch in equilibrium.

Figure 8. Efficient Dispatch for Assumed Demand Load



3.1 Vertical Auctions

3.1.1 DSC-vertical Auction

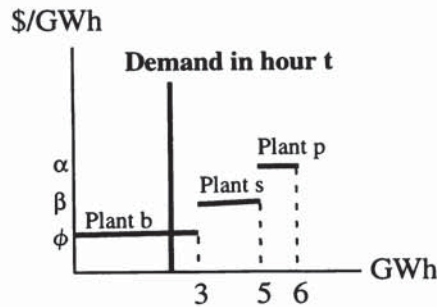
In a vertical auction where each generator owns two plants, each generator submits a supply curve, which consists of two bids, which reflect the

minimum energy price at which the generator is willing to generate up to its plants' capacity. The auctioneer then dispatches generators in increasing order of bids. We have found that even in our simple model of cost and demand, a DSC-vertical auction does not support an efficient dispatch in its set of Nash Equilibria.

Proposition 5 *In a complete information setting, a DSC-vertical auction cannot guarantee the efficient dispatch in equilibrium.*

Proof. Suppose, without loss of generality, that generator 1 owns a base and shoulder plant, generator 2 owns a base and peak plant which he bids into the auction and generator 3 owns a shoulder and peak plant. Given a daily demand as in Figure 8, for the efficient dispatch to result from the submitted bids requires that the lowest bid is submitted by exactly one base plant, the next lowest bid is submitted by exactly one shoulder plant, and the third lowest bid is submitted by a peak plant. Suppose this is true (which results in a cumulative supply curve whose first half is given in Figure 9: denote these three bids by ϕ base < β shoulder < α peak. Recall that the auctioneer uses the (same) cumulative supply curve to dispatch the plants in all three periods. The intersection of demand in any period with the cumulative supply curve determines the clearing price for that period. Given these bids, the same base plant would be dispatched all three periods at its capacity of 3 GW, the same shoulder plant would be dispatched in the first two periods at its capacity of 2 GW and a peak plant would be dispatched in the first period at his capacity of 1 GW. We will argue that such bids cannot occur in equilibrium as a result of two conflicting issues: The generators' desire to increase their bids (and hence their revenue) as much as possible while still ensuring dispatch and the discontinuity of their profits as a function of bids in combination with their non-convex generation costs.

Figure 9. Necessary Bids for Efficient Dispatch to Result in a DSC-Vertical Auction



If $\phi < \beta < \alpha$ constitutes an equilibrium set of bids, it must be that $\phi + 2\epsilon = \beta + \epsilon = \alpha$, where ϵ is the smallest bid increment ϵ .¹¹ If not, then at least one of the plants has an incentive to increase its bid (and hence its profit in the period) in which it is the price-setter, without altering its dispatch schedule. The clearing prices in periods 1, 2, and 3 would be α , $\alpha - \epsilon$, and $\alpha - 2\epsilon$ respectively. In addition, α must be equal to $f_p + v_p$ in equilibrium (due to the presence of an identical plant, which is not included in the dispatch and is owned by a different generator). Each generators' revenue in each period and total cost in the efficient dispatch are summarized in Table 2 below (Assume that all plants not listed here submit a bid higher than α , are not dispatched and receive a profit equal to zero).

Table 2. Revenues and Costs for Generators in Efficient Dispatch Under a UK-Vertical Auction

Plant	Revenue in period 1	Revenue in period 2	Revenue in period 3	Total Cost
G_b^i ($i=1$ or 2)	3α	$3(\alpha - \epsilon)$	$3(\alpha - 2\epsilon)$	$f_b + 9v_b$
G_s^j ($j=1$ or 3)	2α	$2(\alpha - \epsilon)$	0	$f_s + 4v_s$
G_p^k ($k=2$ or 3)	α	0	0	$f_p + v_p$

The bid ordering $\phi < \beta < \alpha$ is an equilibrium set of bids if and only if no generator has an incentive to deviate from its bid. However, G_p^k does have the incentive to change his bid given $\phi + 2\epsilon = \beta + \epsilon = \alpha$. Given these bids, G_p^k would prefer to bid $\alpha - 3\epsilon$ and undercut G_b^i as the base load generator as long as f_p is greater than two times the smallest bid increment (in this case, as long as $f_p > 2\epsilon$). By doing so, he foregoes ϵ in revenue in the first period, but wins a dispatch in the second period at G_s^j 's bid price of $\alpha - \epsilon$ and in the third period at G_b^i 's bid price of $\alpha - 2\epsilon$. To see why this is true, note that G_p^k has the incentive to deviate from a bid of α as long as

$$\alpha - (f_p + v_p) < [(\alpha - \epsilon) + (\alpha - \epsilon) + (\alpha - 2\epsilon)] - (f_p + 3v_p)$$

$$\alpha > v_p + 2\epsilon$$

11. Bids are integer multiples of ϵ , i.e., $0, \epsilon, 2\epsilon, 3\epsilon$, etc.

Given there exist two peaking plants in the market owned by different generators (Assumptions (1) and (2)), in an efficient equilibrium dispatch, α must be equal to $f_p + v_p$. Therefore, all that is needed in order for G_p^k to have the incentive to undercut G_b^i 's bid is for $f_p > 2\epsilon$. Start-up costs typically run several orders of magnitudes larger than the minimum bid increment (which is 1 cent in the California Power Exchange). Therefore the bid ordering $\phi < \beta < \alpha$ will not be supportable in equilibrium and we have shown that the efficient dispatch cannot be supported and hence cannot be guaranteed in equilibrium.

3.1.2 HSC-vertical Auction

The inability of a DSC-vertical auction to guarantee the efficient dispatch might be thought to be a result of the restriction of one bid per day. The auction in California, in contrast to that in the UK, allows generators to submit a separate supply function for each hour in the day. We shall show that despite the added flexibility of separate bids for each period, the HSC-vertical auction cannot guarantee efficiency in equilibrium. This is not to say, however, that it cannot support the efficient dispatch; both inefficient and the efficient dispatch are supportable in equilibrium.

Proposition 6 *In a complete information setting, a HSC-vertical auction cannot guarantee the efficient dispatch in equilibrium.*

Proof. As in section 3.1.1, suppose that generator 1 owns a base and shoulder plant, generator 2 owns a base and peak plant and generator 3 owns a shoulder and peak plant. In a HSC-vertical auction, a generator must submit a bid for each plant for each period $t=1, 2, 3$. Given the demand in Figure 6 and capacity assumptions, Table 3 defines a set of equilibrium bids for the generators which constitutes an *inefficient* dispatch in the HSC-vertical auction (see Figure 10 for the inefficient dispatch).

Figure 10. Inefficient Dispatch Supportable in a HSC-vertical Auction

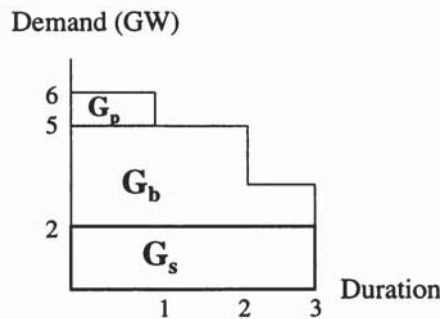


Table 3. Inefficient Equilibrium Bids

Generators	Bid in period 1	Bid in period 2	Bid in period 3
G_b^1	0	$\frac{f_s + 4v_s - 2(f_p + v_p - \epsilon)}{2}$	$f_p + v_p - \epsilon$
G_b^2	$f_b + v_p + \epsilon$	$\frac{f_s + 4v_s - 2(f_p + v_p - \epsilon)}{2} + \epsilon$	$f_p + v_p$
G_s^3	0	0	0
G_s^1	$f_s + 2v_s$	$f_s + 2v_s$	$f_s + v_s$
$G_p^i, i=2 \text{ and } 3$	$f_p + v_p$	$f_p + v_p$	$f_p + v_p$

Given their opponents' strategies in Table 3, no generator has an incentive to deviate from its bids. This profile of bids results in plant G_s^3 being dispatched at its capacity of 2 GWh in all three periods, G_b^1 being dispatched at its capacity of 3 GWh in periods 1 and 2 and at 1 GWh in period 3, and G_p^3 or G_p^2 being dispatched for 1 GWh in period 1. The clearing price received by all winning generators in period 1 is $f_p + v_p$ per GWh, $[f_s + 4v_s - 2(f_p + v_p - \epsilon)]/2$ per GWh in period 2 and $f_p + v_p - \epsilon$ per GWh in period 3. Since G_s^3 's payoff is not determined by its bid, generator 3 has every incentive to bid as low as possible to ensure dispatch for his shoulder plant. By submitting a bid of zero in all three periods, G_s^3 is able to win dispatch at a positive profit. Although G_b^1 is (one of) the least-cost producers of 9 GWh, it is unable to profitably undercut G_s^3 's bids of zero.

This simple example clearly illustrates why a HSC-vertical auction cannot guarantee efficiency in an environment with non-convex costs. It is quite likely that in any given period, not all the plants will be dispatched at the same output level. In such a scenario, there exists an opportunity for a relatively inefficient generator to accrue a positive profit by bidding zero and ensuring dispatch without fear of receiving its below-cost bid price. With the knowledge that in equilibrium the clearing price is guaranteed to be at least the cost of the marginal bidder in period t , a relatively inefficient generator can "sneak-in" to the dispatch schedule by submitting a zero bid, get dispatched at a higher level in period t than the marginal price-setting bidder and accrue a positive profit due to the non-convexity of its cost curve. The presence of excess generation capacity for each type reassures us that the HSC-vertical auction's failure to guarantee efficiency is not due to market power, but instead points to a more fundamental auction design flaw.

3.2 Horizontal Auction (HA)

We can learn from the failure of a vertical auction to guarantee efficiency in equilibrium that there are disadvantages to failing to account for cost dependencies in electricity generation when designing an auction. An auction structure that does exactly that, is a horizontal auction. Below we provide a simple example that illustrates the intuition behind the ability of a horizontal auction to support only the efficient dispatch in equilibrium. While the example presented here is designed such that there is only one winner per auction, the results carry over to a more general framework where there is more than one winner per auction.

Assume the same framework of demand, costs and generators (Figures 7 and 8) as in previous section, in addition to Assumptions (1) - (4). In a horizontal auction, base load demand is auctioned first. After the winner is made public knowledge, the shoulder load is auctioned and then the peak. As always, the lowest bid in an auction determines the winner.^{12,13} The bids for the base, shoulder and peak load, b_b , b_s and b_p , respectively, indicate the minimum price a plant will accept being paid to generate 1 GW for 3, 2, and 1 time period, respectively. Due to the sequential nature of the auctions, the appropriate equilibrium concept to focus our attention on is Subgame Perfect Nash Equilibrium (SPNE).

Proposition 7 *Given Assumptions (1)-(4), the unique SPNE outcome of the horizontal auction of demand load in Figure 8 is efficient.*

Proof. To prove the proposition, we will show that no other dispatch can be supported in equilibrium. Denote the total dispatch by the types of plants that win in each auction, i.e., (B,S,P) implies that a base plant won in the baseload auction, shoulder plant won in the shoulderload auction and peak plant won in the peakload auction. We will show that for an inefficient dispatch to be supported in equilibrium, the non-negative profit requirement combined with the Nash equilibrium bidding requirements (optimal response given all opponents' bids) will lead to a contradiction.

Given Assumptions (1)-(4), the following is an exhaustive list of possible inefficient dispatches (the dispatch denote only what types of plants are dispatched, it does not indicate the owner(s) of the plants or at what level the plants are dispatched):

12. I assume that if two different plants submit the same bid in an auction, the efficient one will be chosen for dispatch. (This is a standard assumption made in game theory when analyzing equilibria).

13. Recall that when there is greater than one winner per auction, each winner is paid his bid.

(B, S, B) (B, S, S) (B, PP, S) (SS, PP, B) (B, PP, B) (SP, S, S)
 (SP, S, B) (SS, SP, B) (SS, PP, S) (SP, S, P) (SS, SP, P)

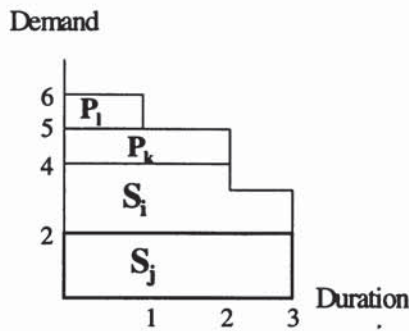
The first eight possible inefficient dispatches cannot be supported in equilibrium do the presence of an efficient plant that is not dispatched elsewhere, placing an upperbound on the winning bid below the current dispatched plant's generation costs. For example, in the case of (B,S,B), there are two peak plants who are not dispatched and therefore will create an upper bound on $b_p \leq c_p(1)$. A base plant's cost of generating 1 GWh is $c_B(1) > c_p(1)$ and therefore the dispatch (B,S,B) cannot be supported in equilibrium - either the peak plant would have incentive to undercut any $b_p > c_p(1)$ or the base plant would be made strictly better off by withdrawing his bid if $b_p \leq c_p(1)$. (Note: this argument does not require that the two plants in question be owned by different generators). Similar simple arguments can be made for the next seven dispatches listed. The remaining three dispatches ((SS,SP,P), (SS,PP,S), and, (SP,S,P)) use a similar logic, but require a few more steps in establishing the contradiction.

In the case of the inefficient dispatch (SS,SP,P), in equilibrium we would need that (i) the peak plant dispatched in the shoulder load would not be better off by undercutting b_p , (ii) neither baseload would want to undercut b_b , (iii) the peak plant in the peak load auction is operating at non-negative profit, and (iv) the shoulder plant dispatched at full capacity in the baseload would not be better off by undercutting b_s , i.e. (see Figure 11),

$$(i) b_s - c_p(2) \geq b_p - c_p(1), (ii) b_b \leq \frac{c_B(9)}{3}, (iii) b_p \geq c_p(1), \text{ and } (iv) 2b_b - c_s(6) \geq 2b_s - c_s(4) \\
 \rightarrow c_B(9) \geq 3b_b \geq 3(b_s + v_s) \geq 3(b_p + v_p + v_s) \geq 3(c_p(1) + v_p + v_s) > c_s(9)$$

which directly contradicts the efficient dispatch assumption (that $c_B(9) < c_j(9)$ for $j=S$ and P). Therefore (SS,SP,P) cannot be supported in equilibrium.

Figure 11. Inefficient Dispatch (SS, PP, S)



In the case of the inefficient dispatch (SS, PP, S) , in equilibrium we would need that (i) the shoulder plant dispatched at full capacity in the baseload would not be better off by undercutting b_s , (ii) neither baseload plant would want to undercut b_b and (iii) the peak plant in the shoulder load auction is operating at non-negative profit, i.e.,

$$(i) 2b_b - c_s(6) > 2b_s - c_s(4), \quad (ii) b_b \leq \frac{c_B(9)}{3}, \quad \text{and} \quad (iii) b_s \geq c_p(2)$$

$$\Rightarrow c_B(9) \geq 3b_b > 3(b_s + v_s) \geq 3(c_p(2) + v_s) > c_s(9)$$

which directly contradicts the efficient dispatch assumption (that $c_B(9) < c_j(9)$ for $j=S$ and P). Therefore (SS, PP, S) cannot be supported in equilibrium.

Finally, in the case of the inefficient dispatch (SP, S, P) , in equilibrium we would need that (i) the peak plant dispatched in the baseload would not be better off by undercutting b_p , (ii) neither baseload would want to undercut b_b , and (iii) the peak plant in the peak load auction is operating at non-negative profit, i.e.,

$$(i) b_b - c_p(3) \geq b_p - c_p(1), \quad (ii) b_b \leq \frac{c_B(9)}{3}, \quad \text{and} \quad (iii) b_p \geq c_p(1)$$

$$\Rightarrow c_B(9) \geq 3b_b \geq 3(b_p + 2v_p) \geq 3(c_p(1) + 2v_p) > c_p(9)$$

which directly contradicts the efficient dispatch assumption (that $c_B(9) < c_j(9)$ for $j=S$ and P). Therefore (SP, S, P) cannot be supported in equilibrium.

Therefore, none of the possible inefficient dispatches can be supported in equilibrium. In the efficient dispatch, (B, S, P) , b_b , b_s and b_p must satisfy, $b_b = [c_B(9)]/3 < b_s = [c_S(4)]/2 < b_p = c_p(1)$, since there exists one extra plant of each type (owned by a different generator than the current one in the dispatch) which will cause the plants to be bid in at cost. These bids are supportable in equilibrium, and hence the unique SPNE is the efficient dispatch.

The above analysis can be extended to a framework with n generators of each type where there are more than one winner per auction, i.e., the size of the base (shoulder, peak) load is greater than the capacity of a base (shoulder, peak) plant and therefore requires more than one plant to be used in order to satisfy demand. In order for the results to carry to a more general framework, the size of the plants must be small relative to the demand for which they are bidding—which can be reinterpreted as the capacity of a plant is exhausted within an auction if chosen for dispatch. Combined with Assumptions (1)-(4),

this implies that all the plants dispatched within each auction are of the same type and are dispatched equal amounts. This assumption is well justified in the California market, where the average height of base load, shoulder, and peak demand is approximately 21, 31, and 36 GW respectively, while the average capacity of a base load, shoulder, and peak plant is 2,1, and 0.5 GW respectively. Given that all the plants dispatched within an auction are of the same type and win identical dispatches, the above argument proves that all Subgame Perfect Nash Equilibria of a horizontal auction results in the unique efficient dispatch.

Our results for the horizontal auction can be viewed as an extension of Bikhchandani's (1996). He concludes that under complete information, when multiple units of heterogeneous objects are sold simultaneously, with one auction for each type of object, every pure-strategy Nash Equilibrium is efficient.¹⁴ By creating an auction for each type of demand load, we have effectively recreated Bikhchandani's environment for an electricity auction (in this case, the objects are the different types of demand), and extended upon his analysis by establishing the efficiency of the auction in a sequential setting with purchasing constraints and permitting only a single bid per auction.

4. IMPLEMENTING HORIZONTAL AUCTIONS

The above analysis, although based on highly stylized models of the demand, generating cost and auction structure exposes a basic shortcoming of vertical auctions and suggest that horizontal auctions may be more compatible with the notion of self-commitment. In this section, we will outline how a horizontal or a "Load Slice" auction may work in a real world environment. The description is intentionally vague recognizing the available flexibility in designing such auctions. We will first assume as in our stylized model a central market which ignores the locational characteristics of supply and demand and a fixed load curve which excludes demand side bidding. Later we will discuss how these assumptions could be relaxed. Under these restrictions the process may proceed as follows:

The auctioneer or exchange operator posts a load forecast for the bid period (say 24 periods) before the bidding process begins (few periods prior to the onset of the bid period). The auction is then done in several rounds filling up the load curve from the bottom up as illustrated in Figure 12. In each round,

14. Bikhchandani (1996) does not consider purchasing constraints (captured here by the presence of capacity constraints). In addition, Bikhchandani allows for a bidder to submit several bids within each auction; this would be equivalent to allowing a bidder to submit a (possibly different) bid for each 1 GW strip of demand in the baseload (shoulder, peak) auction. In our model, the bidder is constrained to submit only one bid per auction.

the auctioneer solicits bids for a load slice of a specified number of GW with a fixed schedule prescribed by the demand curve (the initial solicitation is for base load dispatched for the entire bid period then for shoulder load and finally for peak load.) In each round, bidders bid tenders consisting of capacity increments in GW that they are willing to commit to the specified schedule and a total price (or an average price per GWh). Since the schedule is known, bidders can easily calculate their total cost for serving a slice including all start-up costs and the costs associated with intertemporal constraints which they can factor into their bid price.

Winning bids in each round are selected based on lowest average price per GWh and winners are paid their bid price. It should be emphasized that the price per GWh will vary (most likely increase) as we move from lower base load slices to the upper peak load slices. The number of slices (and rounds) is a design parameter of the auction. The slices should be “thin” enough so that the dispatch schedule in each slice is approximately uniform.¹⁵ This approach also allows different load slices to be auctioned at different time intervals; for instance, base load could be auctioned weekly while peak load could be auctioned twice a day. The appropriate time intervals for each load slice would be another design consideration.

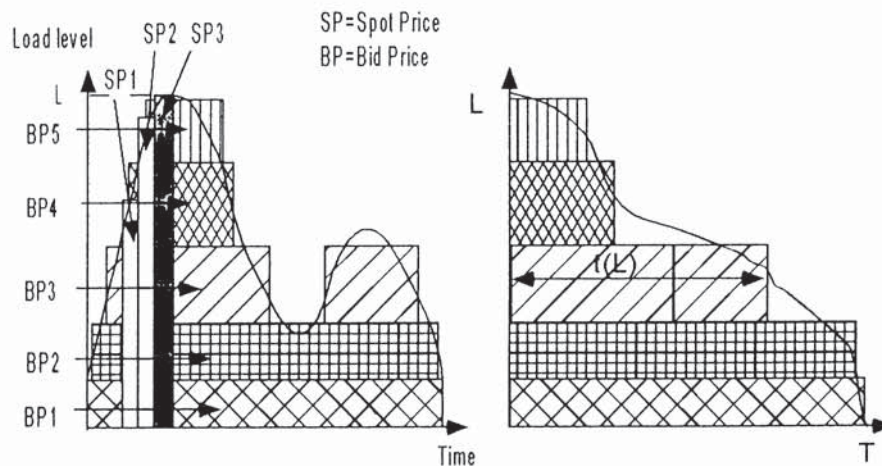
The above design may be refined by allowing bidders to specify also a marginal energy price for minor adjustments to their energy output within a specified range (that would not affect the intertemporal costs). Such information might be useful in defining spot energy prices. As much as it is natural for the suppliers to define their tenders as horizontal slices of loads with prescribed schedules, it is natural for consumers to think of electricity as a time differentiated commodity offered each period at a uniform spot price. Since energy consumed in a particular period cannot be traced to a specific supply source it should be sold at a uniform price. Economic efficiency dictates that the spot price at any period should be set at the marginal cost of the highest load slice active at that period.

Hence, the market can be organized so that on the supply side power is acquired through a load slice sequential auction while on the demand side the energy is sold in a spot market with vertical slices priced uniformly in each time period (see Figure 12). Figure 12 shows how the load can be partitioned into horizontal load slices, each of which represents the scheduled operation of an incremental unit of capacity. If the load slices are aggregated over a 24-hour period, the total operating duration of each slice can be captured by a load

15. An alternative implementation could take the form of a Dutch Auction where the auctioneer posts a price per MWh for a load slice with a specified schedule and raises the price until a dispatch commitment is made. The auctioneer then moves onto the next slice, using the last accepted price as his starting bid, and continues the process until the entire load curve is filled.

duration curve as shown. However, the load duration curve suppresses the number of start-ups that will affect a bidder's costs. Figure 12 also illustrates how on the selling side, the load can be partitioned into vertical slices that will be priced uniformly so that all MWhs consumed during a particular hour are priced the same, regardless of their generation source. It is possible to show that under some restrictive assumptions about the cost structure on the supply side and about the load pattern, such a scheme will break even in the sense that the spot market sales will generate just enough revenue to recover the payments to suppliers (see Appendix). In general, however, the spot market will run a deficit (due to the intertemporal costs rolled into the supply bids) which must be recovered through a fixed charge or an "uplift charge."

Figure 12. Load Slice Bidding with Spot Market Selling



The above scheme can be extended to include demand side bidding and locational factors. Both of these aspects will play a role during the peak load periods and need to be accounted for in the auction rounds for the upper slices. With regard to demand side bidding, such bids can be entered as a reservation price in the upper auction rounds. In other words, if bid prices exceed the demand side bids all bids are rejected in favor of demand curtailments. The locational factors, i.e., congestion affects, can be accounted for by running a power flow analysis as the load curve is being filled up. When congestion occurs, a locational penalty can be imposed as an adder to the bid price of upper slice bids originating at the constrained locations. This approach discriminates against peaking units in pricing congestion. Since the locational penalties are not imposed while the base load slices are being auctioned off, this amounts to giving priority to such units in congestion management. Intuitively, this seems

the right thing to do since the baseload units are less capable to respond to congestion signals. However, we have not yet analyzed the efficiency implications of such an approach.

5. CONCLUSIONS

Simulation studies and empirical evidence suggest that central unit commitment is inappropriate in a competitive electricity environment. Yet the traditional approach of auctioning power in an energy only hourly auction is incompatible with self-commitment due to intertemporal costs and nonconvexities in the production function for energy. One approach that has been adopted by the California Power Exchange proposal is a multi-round auction with activity rules which allow bidders to revise their bids so as to account for intertemporal costs resulting from their dispatch schedule. We propose a different approach that structures the auction so that the bid tenders allow bidders to account readily for these costs. Our proposed approach is supported by a game theoretic analysis of a stylized model that shows that, unlike the vertical slice auctions, a sequential horizontal slice auction will induce efficient dispatch.

APPENDIX

In this appendix, we demonstrate that under some idealized conditions, selling power at a uniform hourly marginal cost will generate just enough revenue to pay for competition acquisition of power through a load slice auction. The scheme mentioned in section 4 can be visualized as purchasing power to fill the load curve in uniformly priced “horizontal slices” where the price increases as the slice is higher in the stack. The power is then resold as uniformly priced vertical slices corresponding to different time periods, where the price for each slice is the spot price or the highest marginal price of the operating generation units.

We further assume that the load pattern is unimodal which implies at most one start-up in each load slice, and further assume that there is sufficient competition in each generation technology to make the optimal generation mix feasible. We ignore on/off switching aspects such as ramping-up and assume that a generator’s total generation costs at a given load level depend strictly on the dispatch duration (this includes as a special case a two-part cost structure consisting of a start-up and marginal operating cost). Define the cost to generator type i of generating at capacity for t periods to be $C_i(t)$. The marginal cost of a generator may depend on the dispatch duration. We also assume that generation capacity of each type can be procured in any quantity, i.e., is a continuous variable.

In an idealized competitive load slice bidding environment, a generator will commit its capacity at a price per GWh equal to its average cost for the posted dispatch duration. Thus, each load slice will be served by the generating unit with the lowest average cost. Figure 13 illustrates the cost functions of different technologies as functions of duration. An efficient load slice auction will result in the selection of the cheapest technology for each duration, so that the overall supply function is a function of slice duration, that corresponds to the optimal technology mix, is given by the lowest envelope of the different cost functions. Hence, if $t(L)$ denotes the duration corresponding to load level L , L is the maximum load level, T is the maximum duration, i.e., the base load duration, and the total cost of serving the load curve is given by

$$\begin{aligned} \text{Total Cost} &= \int_0^L \left(\min_i \frac{C_i(t(L))}{t(L)} \right) t(L) dL = \int_0^L \min_i C_i(t(L)) dL \\ &= \int_0^L \bar{C}(t(L)) dL = \bar{C}(T)L(T) + \int_{L(T)}^L \bar{C}(t(L)) dL \end{aligned}$$

where $\bar{C}(t) = \min C_i(t)$ and $L(T)$ is the base load level. Splitting the integral in the last expression accounts for the fact that the function $L(t)$ is discontinuous at $t=T$ $\{L(T^+) = 0\}$.

Integrating by parts gives us

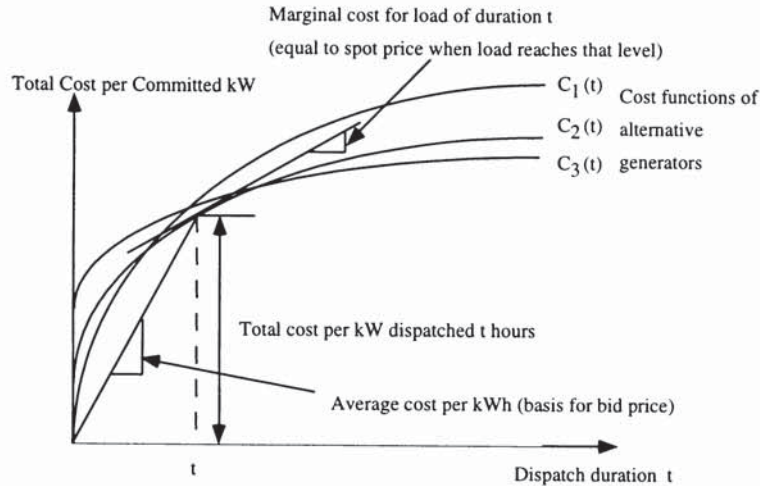
$$\text{Total Cost} = \bar{C}(T)L(T) = \bar{C}(t)L(t) \Big|_T^0 + \int_0^T \bar{C}'(t)L(t) dt = \bar{C}(0)L + \int_0^T \bar{C}'(t)L(t) dt$$

If we include curtailment as one of the supply technologies, then the function $C(0) = 0$, so the first term in the above expression vanishes.¹⁶ Also $C'(t)$ is the marginal cost of serving a load level whose duration is t . Since marginal cost is decreasing with duration, this is the highest marginal cost of generating units operating when the load level has duration t . Hence, $C'(t)$ is the spot price when the load level corresponds to duration t , and the integral of the spot price times the load over the entire time period equal the total cost. It thus

16. We assume that curtailment cost is a positive, finite cost that is equal to some measure of loss of load, (e.g., value of lost load (VOLL) per curtailed MWh). We assume, however, that curtailment has no start up cost and hence is continuous at $t=0$.

follows that charging the spot price for the entire load in each time slice recovers total acquisition costs.

Figure 13. Optimal Technology Mix (given by the lower envelope)



It should be noted that ramp-up costs and other costs associated with on/off switching cannot be recovered by spot prices that equal marginal costs. This is true in any system whether the acquisition is done through time slice bidding or load slice bidding, as described above. Recovery of switching costs requires some sort of “uplift” of the spot prices. In the above system, generators are aware of these costs at the time bids are made and are able to internalize these costs. However, the spot prices will not recover the entire acquisition costs and some sort of fixed charge or uplift of the spot prices will be needed to achieve revenue neutrality.

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