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# INITIAL CONCEPTS FOR APPLYING SENSITIVITY TO TRANSFER CAPABILITY

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## 1 Introduction

The available transfer capability indicates how much interarea power transfers can be increased without compromising system security. That is, the available transfer capability is the total transfer capability minus the base case transfer together with adjustments to allow some margin of safety. Much of the experience in computing transfer capabilities concerns line flow limits and uses DC load flow power system models. However, in some cases the total transfer capability is limited by voltage magnitude or voltage collapse and this paper mainly addresses these voltage limits. Accurate determination of these voltage limits requires nonlinear power system models and the continuation computation described below.

Once the total transfer capability is computed under one set of operating conditions or assumptions, it is useful to determine the effect on the total transfer capability of varying the operating conditions or assumptions. We suggest sensitivity methods for quickly estimating the effect on the transfer capability of varying the operating conditions, assumptions or data.

### 1.1 Transfer with no contingencies

We first consider the simple case of a transfer between two areas which neglects contingencies. We assume a base case with a loading and transfer schedule at which the power system is secure. The base transfer schedule includes existing transmission commitments. There are many ways to dispatch generators to transfer a given total power between the two areas. Therefore it is necessary to know or assume the generator dispatch policies in both areas to define how the transfer is to take place. As the total power transfer is increased, the power system state evolves and eventually some security criterion is violated.

For example, a line power flow could exceed its limit, a bus voltage magnitude could drop below the normal limit, or an operating point could disappear in a voltage collapse. The real power transfer at the first encountered security violation is the total transfer capability.

The computation in which the transfer is increased in steps and the power system operating point is computed at each step is often called continuation. Continuation amounts to ingenious ways of solving successive load flows (augmented with suitable equations describing dispatch and the amount of transfer) as the transfer is increased.

Determining the transfer capability under one set of conditions is rarely sufficient. The key next step in exploiting the results of the continuation is to compute the sensitivity of the transfer capability to a wide range of parameters. This transfer capability sensitivity computation is very fast and can be used to estimate the transfer capability for different parameters without rerunning the continuation. There are several useful ways to choose the parameters to be varied and some examples follow:

- **simultaneous transfers**: The sensitivity of the transfer capability to the amount of another transfer allows the effect of the other transfer on the transfer capability to be estimated. This could be used to quickly update the transfer capability when the other transfer is executed.
- **generator dispatch**: It is useful to know how the transfer capability depends on the assumed generator dispatch. The sensitivity of the transfer capability to the dispatch yields information on how to best change the generator dispatch to increase the transfer capability.
- **load**: The effect on transfer capability of varying the assumptions about the power system load can be estimated.
- **uncertain or inaccurate data**: If some power system data are uncertain or inaccurate, then the impact of this uncertainty on the transfer capability can be assessed by using the sensitivity of the transfer capability to the uncertain parameters. This would be particularly helpful in determining an appropriate transmission reliability margin. For example, the effect on transfer capability of inaccurate line impedances can be estimated.

### 1.2 Contingencies

In practice, contingencies are taken into account in defining and computing transfer capability. The simple security criterion considered in subsection 1.1 is made more stringent by additionally requiring that any contingency from a given list not cause the operating point to violate emergency limits. In principle, a continuation is run for each of the contingencies to find the transfers yielding the first voltage magnitude limit violation or voltage collapse. Thus each contingency and the base case yields a transfer. The transfer capability associated with voltage limits is then the smallest of all these transfers. The sensitivity computation has to be applied to the results of the continuation corresponding to the transfer capability. If the sensitivity is used to estimate the effect of a large change in a parameter, it is possible that the the limit and contingency defining the transfer capability could change. In principle, this could be taken into account by computing the sensitivity of the transfer capability for each of the contingencies.

There are well known and standard ways to estimate the effects of contingencies on line power or current limits using DC load flow models and large signal sensitivities obtained from network theorems [27, 14] or the matrix inversion lemma [4]. Therefore we would expect to handle the effect of contingencies on line power or current limits using a standard approach. DC load flow models are not effective for voltage magnitude and voltage collapse limits because system voltages vary in a nonlinear fashion with increased loading. Thus accounting for voltage limits requires the additional work of computing a continuation.

### 1.3 Simultaneous transfers

It is often the case that each area will simultaneously transfer power to several other areas. The amount of transfer from one area affects the allowable transfers with the other areas. In this case, the transfer capability depends upon assuming a specific schedule of transfers. Just as the transfer capability depends upon the assumed dispatch for a nonsimultaneous transfer, for a simultaneous transfer the transfer capability depends upon the assumed dispatch and the assumed *schedule* of transfers. In general, the transfer schedule can specify either some combination of interarea transfers occurring together or a sequence of interarea transfers occurring one after another. For simplicity, this paper assumes that the transfer schedule is some fixed combination of transfers occurring together in some fixed proportion (but see further discussion in the section 4). The effect on the transfer capability of varying the assumed schedule of transfers is of interest and can be estimated from the sensitivity of the transfer capability with respect to the assumed schedule of transfers.

## 2 Previous Work

There has been interest in quantifying the transmission transfer capabilities of power systems for many years. When systems were isolated and largely radial, these capabilities were fairly easy to determine and consisted mainly of a combination of thermal ratings and voltage drop limitations. In most cases, these two limitations were easily combined into a single power limitation (either MW, MVA, or SIL). As such, available transfer capability for a given transmission line at a given time could be interpreted as the difference between the power limitation and the existing power flow. NERC has been careful to distinguish the word "capacity" from the word "capability". Capacity is normally a specific device rating (i.e. thermal), whereas capability refers to a limitation which is highly dependent on system conditions.

As isolated systems became interconnected for economic and reliability reasons, looped networks introduced technical issues with the definition and calculation of available transfer capability. In addition, the differences between contract path and actual power-flow path introduced additional complexity to the quantification of available transfer capability. System stability became an important constraint for some areas of the interconnected network and this required the introduction of a third limiting phenomena. The St. Clair curves were one of the first attempts to include thermal, voltage, and stability constraints into a single transmission line loading limitation [23]. These results were later verified and extended from a more theoretical basis in [8]. This "single rating" concept is extremely valuable from a computational point of view. Linear load flow and linear programming solutions made transmission transfer capability determination relatively fast and easy [15, 16, 24, 10]. They focused on both the "Simultaneous Interchange Capability (SIC)" and the "Non-Simultaneous Interchange Capability (NSIC)".

The margin sensitivity approach used in this paper arose in the context of sensitivity of loading margins to voltage collapse [5, 12]. These methods were extended to quickly rank the effect of contingencies on the loading margin to voltage collapse in [28, 13]. Sensitivity formulas for the loading margin to voltage and flow limits were derived in [11, 20]. This paper continues a research direction in available transfer capability indicated in [21] by applying these margin sensitivity ideas to transfer capability.

## 3 Determining transfer capability

### 3.1 Definition of transfer capability and ATC

For this paper we use the main features of the NERC 1995 and 1996 definitions [17, 18]: The power system is judged to be secure for the purpose of interarea transfer if "all facility loadings are within normal ratings and all voltages are within normal limits", the system "remains stable following a disturbance that results in the loss of any single element", "the post-contingency system ... has all facility loadings within emergency ratings and all voltages within emergency limits" [21]. Some of the finer points of the transfer capability definition are not addressed in this paper.

The power system is partitioned into areas, each of which is defined by a set of buses. The transfer between two areas is the sum of the real powers flowing on all the lines which directly connect one area to the other area. (The point on the lines at which to measure the power must be specified.) A list of contingencies is chosen and a nominal transfer schedule is chosen. A secure base case is chosen. A base case transfer (existing transmission commitments) is determined. The transfer is then gradually increased starting at the base case transfer until the first security violation is encountered. The real power transfer at the first security violation is the total transfer capability.

The available transfer capability is then defined as

$$\begin{aligned}
&\text{Available Transfer Capability (ATC)} = \\
&\qquad \text{Total Transfer Capability (TTC)} \\
&\qquad -\text{Existing Transmission Commitments (ETC)} \\
&\qquad -\text{Transmission Reliability Margin (TRM)}
\end{aligned}$$

−Capacity Benefit Margin (CBM)

The following limits can be accounted for in the framework of this paper:

- power flow or current limits (normal and emergency)
- voltage magnitude upper and lower limits (normal and emergency)
- voltage collapse limit

The framework of this paper accounts directly only for limits which can be deduced from static model equations. Oscillation and transient stability limits are assumed to be studied offline and converted to power flow limits. The paper focuses on the voltage magnitude and voltage collapse limits. We do not recommend using voltage magnitude limits exclusively in place of the voltage collapse limit.

### 3.2 System modeling

This section details the modeling required to compute the transfer capability corresponding to voltage magnitude and voltage collapse limits. The continuation program used to establish the nominal transfer capability traces equilibrium solutions, not transient trajectories of the state. Thus, detailed equations that model the dynamic response of the system yield the same results as steady state equations provided that they have the same equilibrium solutions [6].

The model must include the following:

- Static equilibrium equations. The equilibrium equations can be standard or elaborate power balance equations, or the the right hand side of the system differential, or differential algebraic equations.
- Equilibrium equations that model the interarea transfers. For example, the simple approach of appending to the power balance equations equation (1) for each interarea transfer is sufficient:

  $$\mathrm{Transfer}(\mathrm{area}_1 \text{ to } \mathrm{area}_2) = \Sigma(\text{tie line flows}) \quad (1)$$

  A more detailed representation of the frequency based control system is conceivable, but may not be worthwhile.
- Static generator dispatch equations. As the transfers are altered, the dispatch equations should distribute area slack to the area generators proportionally according to the generator participation factors. For example, if the generator participation factor of the $i$th generator in the area is $\alpha_i$, the dispatch equation can take the form

  $$\mathrm{P}_{\text{area slack}} = \sum_i \alpha_i \,(\text{generator } i \text{ output}) \quad (2)$$

  A more detailed representation of the control system and governor equations is not necessary because the continuation only requires equilibrium solutions.

### 3.3 Continuation

Continuation methods are a basic tool for accurately computing transfer capabilities. The problem of computing a security margin to a security violation involves finding an equilibrium that satisfies a specific condition such as a variable reaching a limit. The continuation method works by tracing equilibrium solutions from a known operating point as the transfer is increased until the desired security violations are found.

Continuation methods are well explained in the texts [22, 9]. Continuation methods for determining the long term voltage stability of power systems have been presented in [1, 2, 3] and used in [12, 13, 19, 25]. The use of steady state continuation programs in power systems is now well established.

Continuation methods can be implemented with any set of power system equilibrium equations, although common descriptions of the programs often assume the standard power flow equations. [26] uses a continuation program with elaborate equilibrium equations similar to that envisioned for the method described in this paper. The continuation program used to establish the transfer capabilities must take into account the effects of limits that change the equilibrium equations and the sequence in which the limits are encountered. In particular, the continuation program should use a predictor-corrector method so that equilibrium solutions are found at successive limit events.

## 4 Sensitivity formulas

Sensitivity computations can be used to estimate the effects of varying assumptions on the nominal transfer capability. This section states the sensitivity formulas and sketches some of the computations. The formulas are derived and further explained in [11]. This section also comments on the path dependence of the transfer capability.

For the computations of this paper, it is sufficient to model the power system with static equations

$$0 = F(z, \lambda, p) \quad (3)$$

where $z$ is the vector of $N$ equilibrium states, $\lambda$ a vector of interarea transfers, and $p$ a vector of parameters. For example, $p$ can be a vector of bus power injections, load parameters or generator participation factors [12]. $F$ should include area interchange, generator dispatch, and any other static controls. If a differential-algebraic or differential equation model of the power system is available then $F$ can be chosen as the right hand side of those equations.

The security requirements are usually modeled as inequalities on the equilibrium states:

$$z_i^{\mathrm{min}} \le z_i \le z_i^{\mathrm{max}} \qquad , i = 1 \cdots N \quad (4)$$

When a limit is encountered, one of the following equations holds for some $i$:

$$z_i = z_i^{\mathrm{min}} \quad (5)$$

$$z_i = z_i^{\mathrm{max}} \quad (6)$$

and we write the applicable equation in the general form

$$0 = E(z, \lambda, p) \tag{7}$$

If voltage collapse determines the security limit, then different sensitivity formulas apply. These formulas are not presented here and are presented in detail in [12].

In general, the transfer schedule should be assumed to be a piecewise linear curve; the successive endpoints of the linear portions are $\lambda_0, \lambda_1, \lambda_2, .., \lambda_n$. That is, $\lambda_0, \lambda_1, \lambda_2, .., \lambda_n$ are points on the transfer schedule and the transfer is assumed to be linear in between these points.

It is important to recognize that the transfer capability does not only depend on the endpoints $\lambda_0$ and $\lambda_n$, but also on the curve joining $\lambda_0$ and $\lambda_n$; that is, it depends on how the transfer changes from $\lambda_0$ to $\lambda_n$. One reason for this is that the normal or emergency limits encountered can vary depending on the curve [11]. In particular, the emergency limits encountered when a contingency is assumed can vary with the curve. Thus a sensible definition of transfer capability must specify not only the base case and a final transfer, but the way in which the transfer changes from the base transfer to the final transfer. This point is of considerable importance in specifying how several transfers are to be executed: the order of the transfers (when done sequentially) or the way in which the transfers are combined (when done at the same time) does make a difference to the transfer capability. For simplicity, this paper states the formulas for the case of a linear transfer schedule specified by the endpoints $\lambda_0, \lambda_1$. This corresponds to simultaneous transfers occurring in a fixed proportion.

Suppose that at transfer $\lambda_*$ a limit is encountered. The vector of state, transfers, and parameters at the limit is $(z_*, \lambda_*, p_*)$ and

$$F(z_*, \lambda_*, p_*) \quad = 0 \tag{8}$$

$$E(z_*, \lambda_*, p_*) \quad = 0 \tag{9}$$

The first step in the sensitivity computation is to obtain the direction $\hat{k}$ of transfer variation. For the simple case of a linear transfer schedule,

$$\hat{k} = \frac{\lambda_1 - \lambda_0}{|\lambda_1 - \lambda_0|} \tag{10}$$

Note that $\hat{k}$ is a unit vector in whatever norm is used to measure the transfer capability, usually the $\mathrm{L}^1$ norm.

The sensitivity of the transfer capability $T$ to the parameter $p$, denoted $T_p$, is computed using [11]:

$$T_p = -\frac{w \begin{pmatrix} F_p \\ E_p \end{pmatrix}\Big|_{(z_*, \lambda_*, p_*)}}{w \begin{pmatrix} F_\lambda \hat{k} \\ E_\lambda \hat{k} \end{pmatrix}\Big|_{(z_*, \lambda_*, p_*)}} \tag{11}$$

where,

- $F_p$ and $E_p$ are the derivatives of the equilibrium and limit equations with respect to parameter $p$.
- $F_\lambda \hat{k}$ and $E_\lambda \hat{k}$ are the vectors representing the derivatives of the equilibrium and limit equations with respect to the interarea transfers in the direction of scheduled interchange.
- $w$ is a nonzero row vector orthogonal to the range of the Jacobian matrix $J$ of the equilibrium and limit equations, where

$$J = \begin{pmatrix} F_z \\ E_z \end{pmatrix}\Big|_{(z_*, \lambda_*, p_*)} \tag{12}$$

The row vector $w$ is found by solving the linear system

$$wJ = 0 \tag{13}$$

The Jacobian matrix $J$ has $n$ columns and $n+1$ rows. Since every set of $n+1$ vectors in $\mathbf{R}^n$ is linearly dependent, there is always a nonzero vector $w$ that solves (13). $J$ generically has rank $n$, so that $w$ is unique up to a scalar multiple. This scalar multiple does not affect (11).

The first order estimate of the change in transfer capability corresponding to the change in $p$ of $\Delta p$ is

$$\Delta T = T_p \Delta p \tag{14}$$

The estimate of the transfers at the critical limit for a change in $p$ is

$$\lambda_*(p) = \lambda_n + \hat{k}\Delta T \tag{15}$$

Computation of the linear estimates is fast. For example, once a nominal transfer capability has been obtained, for large systems (1000 buses or more) it is expected that the sensitivity of the transfer capability to every generator output or simultaneous transfer could be computed in less time than is required to perform a single iteration of the loadflow.

## 5 Conclusions and Future Work

The paper addresses transfer capabilities caused by voltage magnitude or voltage collapse limits. It suggests how the sensitivity of the transfer capability can be computed and used to estimate the effect on the transfer capability of variations in parameters such as those describing other transfers, operating conditions or data. A continuation method is used to find the transfer capability due to voltage magnitude or voltage collapse limits. The first order sensitivity of this transfer capability to a wide range of parameters can then be quickly computed. The objective is to use these sensitivities to quickly extract the maximum possible engineering information from each continuation. The sensitivity methods could contribute to the quick update of transfer capabilities when operating conditions or other transfers change and to the computation of a transfer reserve margin.

There is clearly a need to test and assess the proposed sensitivity formulas on power system models. Preliminary test results will be presented at the workshop.

The paper has addressed the use of sensitivity methods after the transfer capability has been determined. There is also much challenge in determining the available transfer capability and in particular determining the combination of voltage limit and contingency which most constrains the transfer. There could be opportunities to reduce the number of continuations to be run by approximating the effects of contingencies by sensitivity methods. (Related work has shown that the effect of contingencies on loading margin to voltage collapse can be *ranked* by sensitivity methods [13].) Problems to be addressed include the best method of parameterizing the contingencies, the accuracy of the ranking, and the combined selection of binding limits and contingencies.

This paper has concentrated on sensitivity methods when the transfer capability is limited by voltage magnitude or voltage collapse limits. Generation injection or line outage distribution factors can be understood as computing the sensitivity of transfer capabilities limited by line flows to generator injections or line outages. It would be interesting to investigate computing the sensitivities of transfer capabilities limited by line flows to parameters other than generator injections or outages. Recent progress in computing the sensitivity of line flows to other line flows is reported in [20].

To summarize, areas for future work include:

- Test the proposed sensitivity methods on transfer capabilities limited by voltage magnitude or voltage collapse limits on large power systems.
- Explore sensitivity methods for transfer capabilities limited by line flow limits.
- Investigate whether approximating the effect of contingencies with sensitivity methods can contribute to the computation of available transfer capability by reducing the number of continuations.

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