

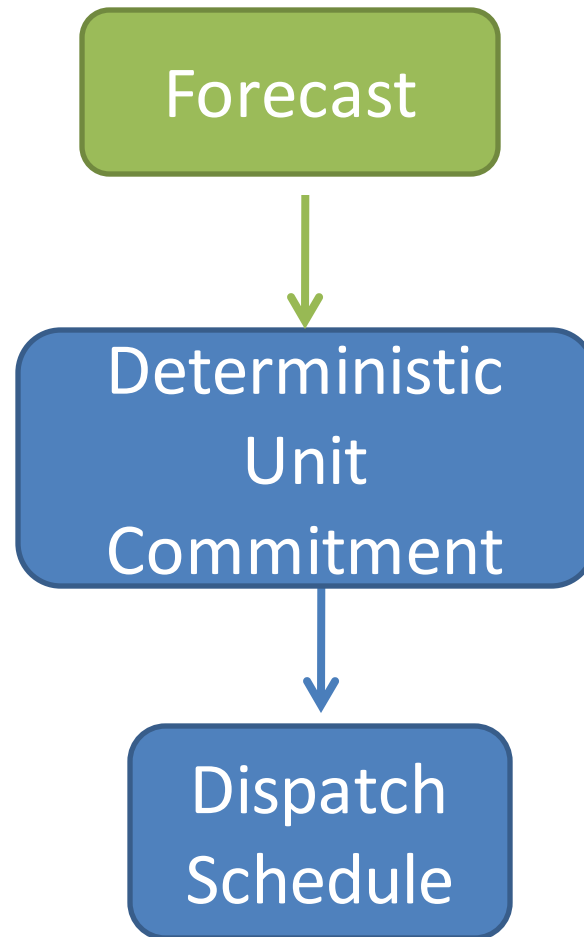
# Two Stage Programs



# Agenda

- What is a deterministic optimization problem?  
Stochastic optimization? Stochastic programming?
- Two-stage scenario-based (stochastic) programs

# Deterministic Security-Constrained Unit Commitment



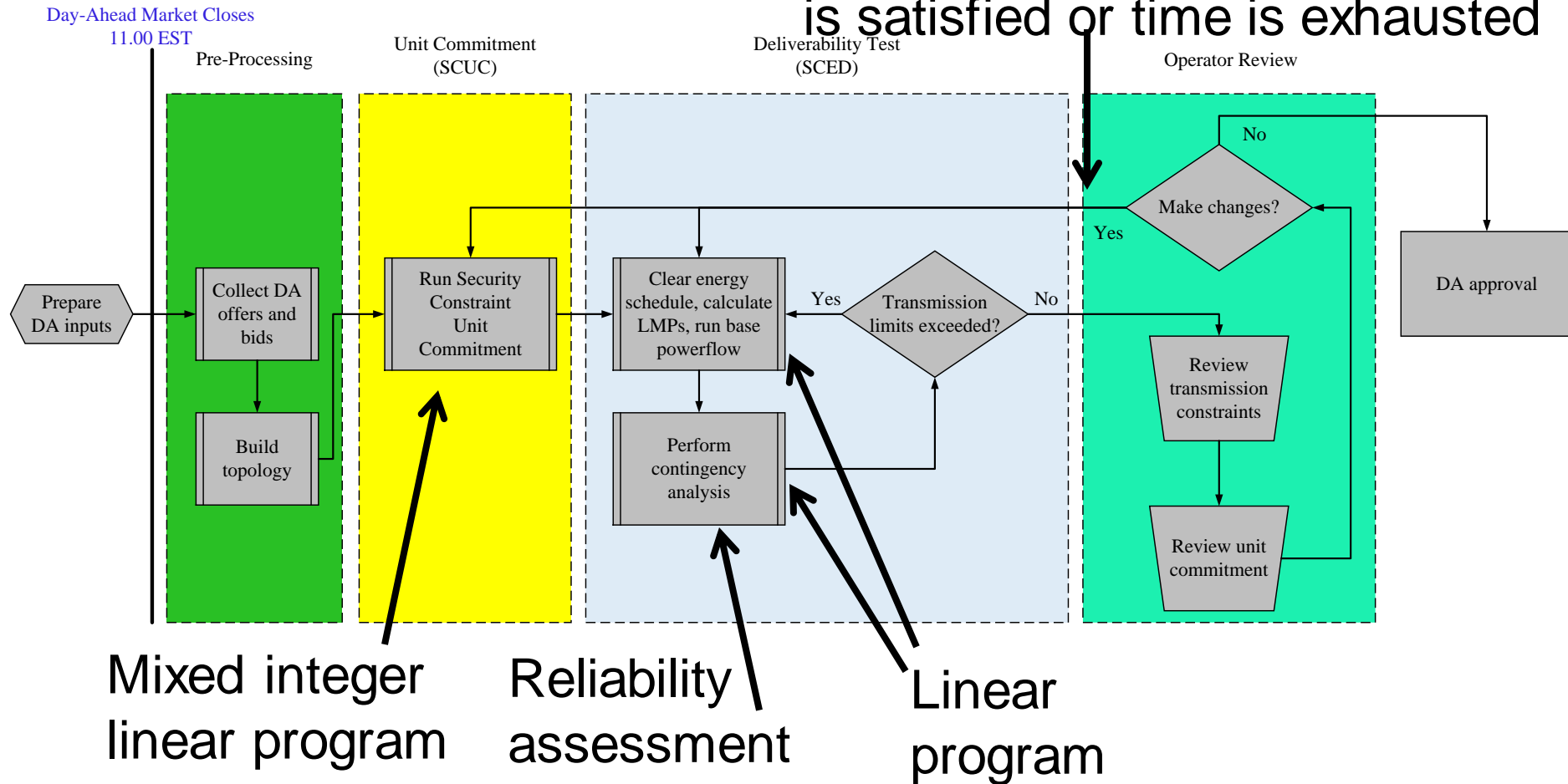
# Deterministic Security-Constrained Unit Commitment/OPF

- The industry today solves **deterministic** security constrained unit commitment (SCUC) and security-constrained ED (SCED) problems
- What is a deterministic optimization problem?
- Assumes perfect forecast / no uncertainty (explicitly modeled)
- While we do not capture the true uncertainty of the problem, we model reserve requirements within SCUC and SCED, which approximate a two-stage program

Question: Is there a way to explicitly model a contingency to alleviate this problem?

# Day-Ahead Scheduling in Midcontinent Independent System Operator (MISO)

Iterative process until operator is satisfied or time is exhausted



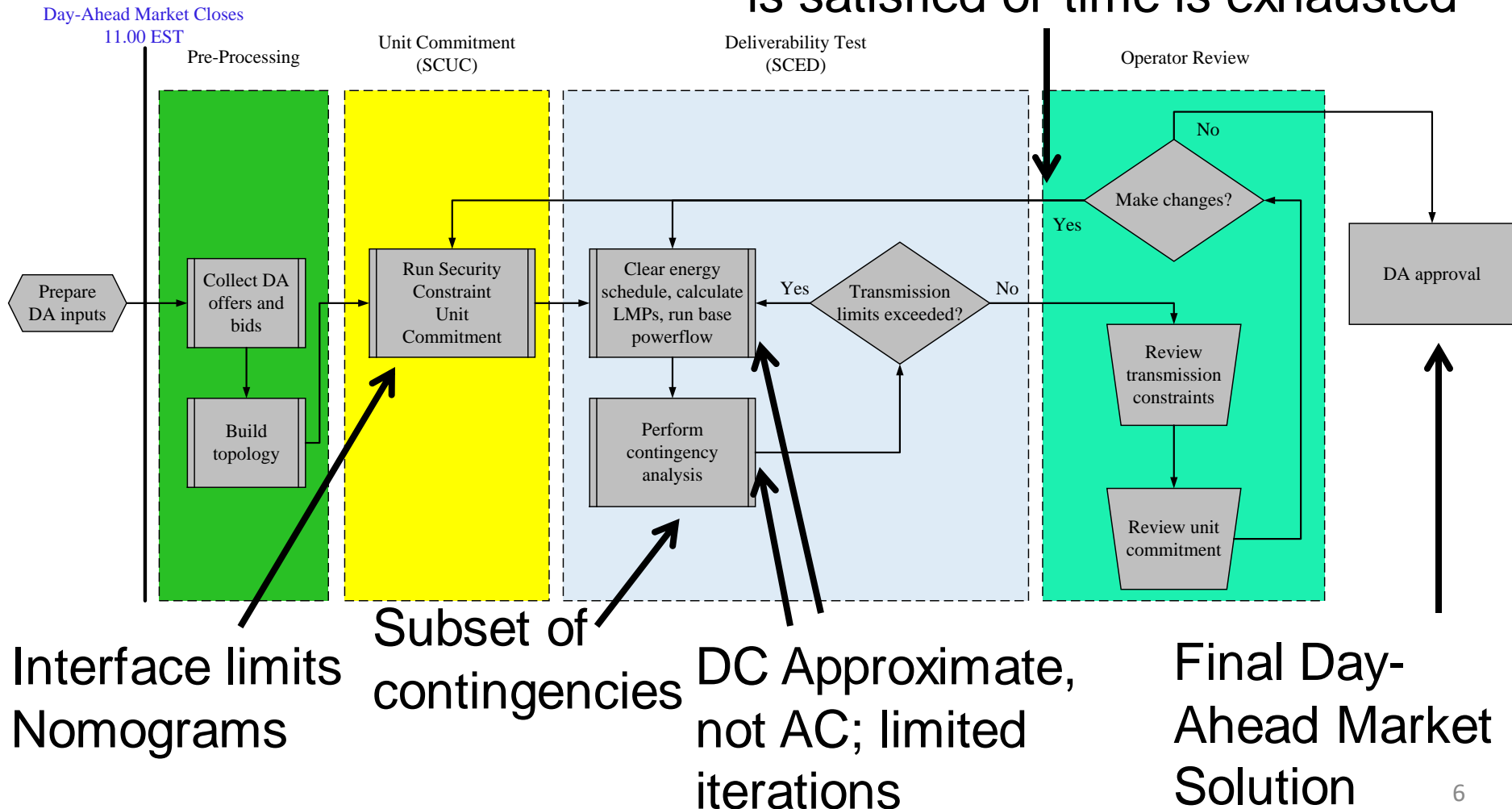
MISO Day-Ahead Scheduling Procedure

[1] Aaron Casto, "Overview of MISO day-ahead markets," *Midwest ISO*, [Online].

Available: [http://www.atcllc.com/oasis/Customer\\_Notices/NCM\\_MISO\\_DayAhead111507.pdf](http://www.atcllc.com/oasis/Customer_Notices/NCM_MISO_DayAhead111507.pdf).

# Day-Ahead Scheduling in Midcontinent Independent System Operator (MISO)

Iterative process until operator is satisfied or time is exhausted



Break

# Two Stage Programs

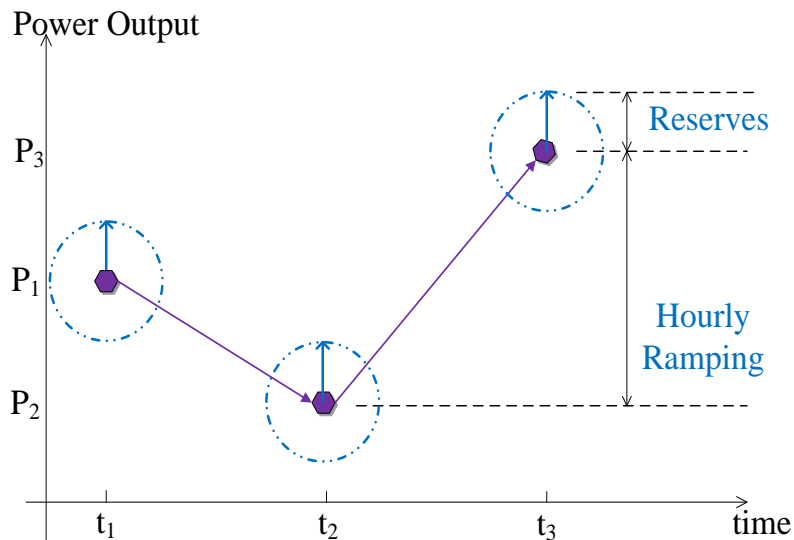




This picture shows how we generally model N-1 for stochastic unit commitment

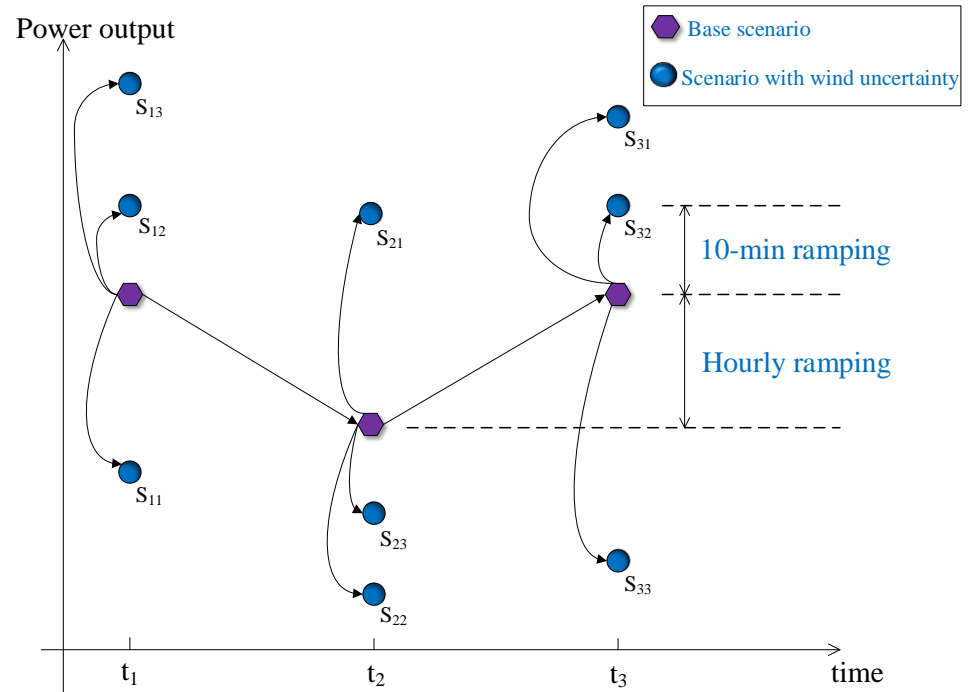
There is a single base-case pre-contingency output  
The two-stage stochastic program must ramp from the base-case to a new output for the contingency

### Deterministic



Question: If you assume an infinite bus model (no OPF), how does that change the deterministic model?

### Stochastic

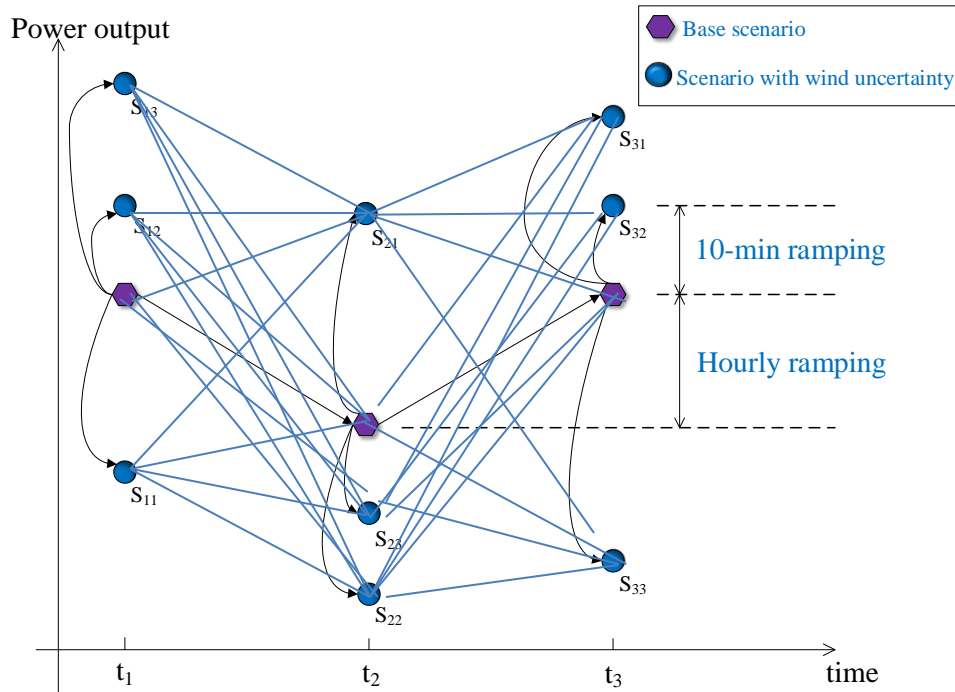


Two-stage stochastic program

# Stochastic Unit Commitment

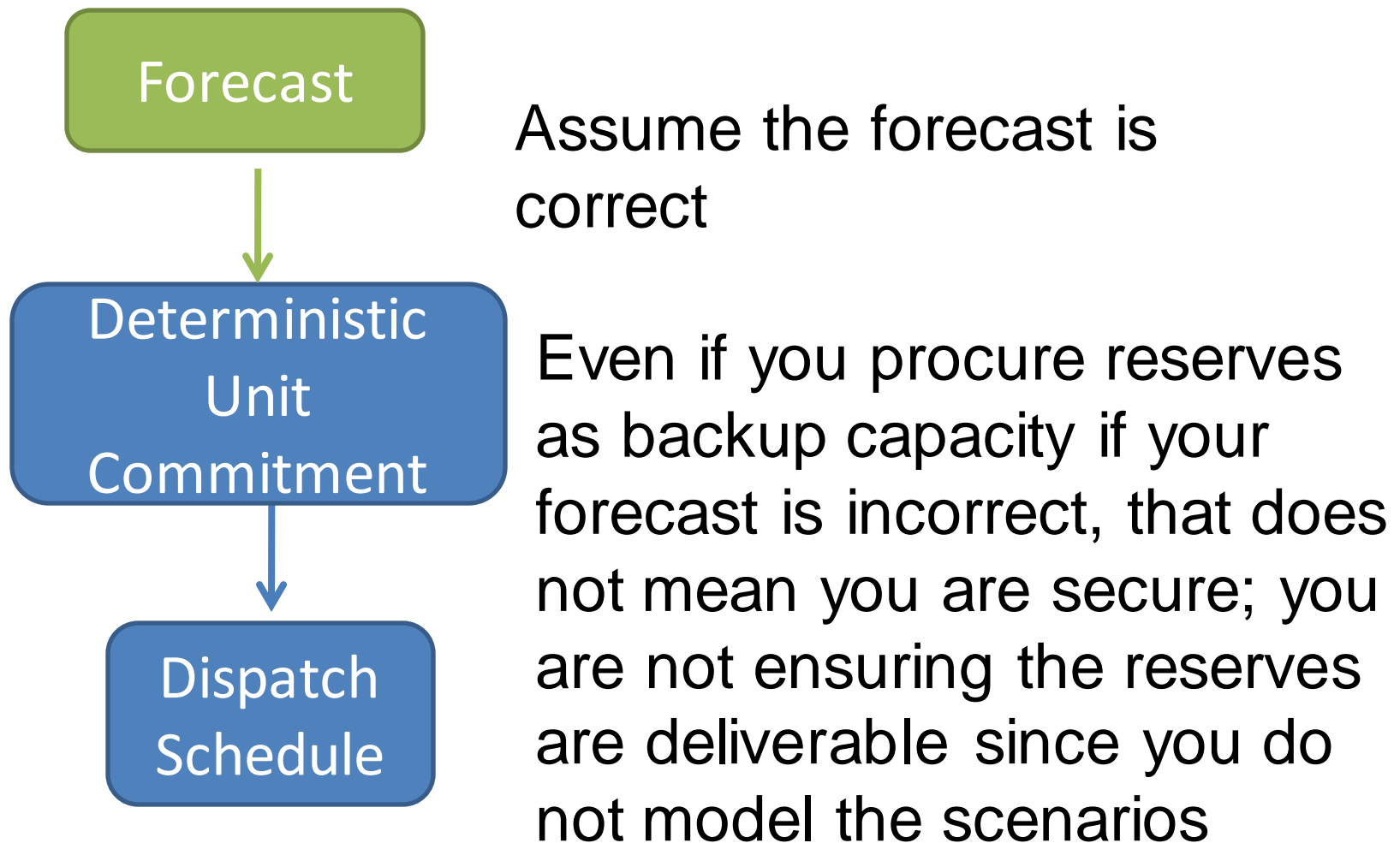
Stochastic unit commitment (another modeling approach)

**Stochastic**

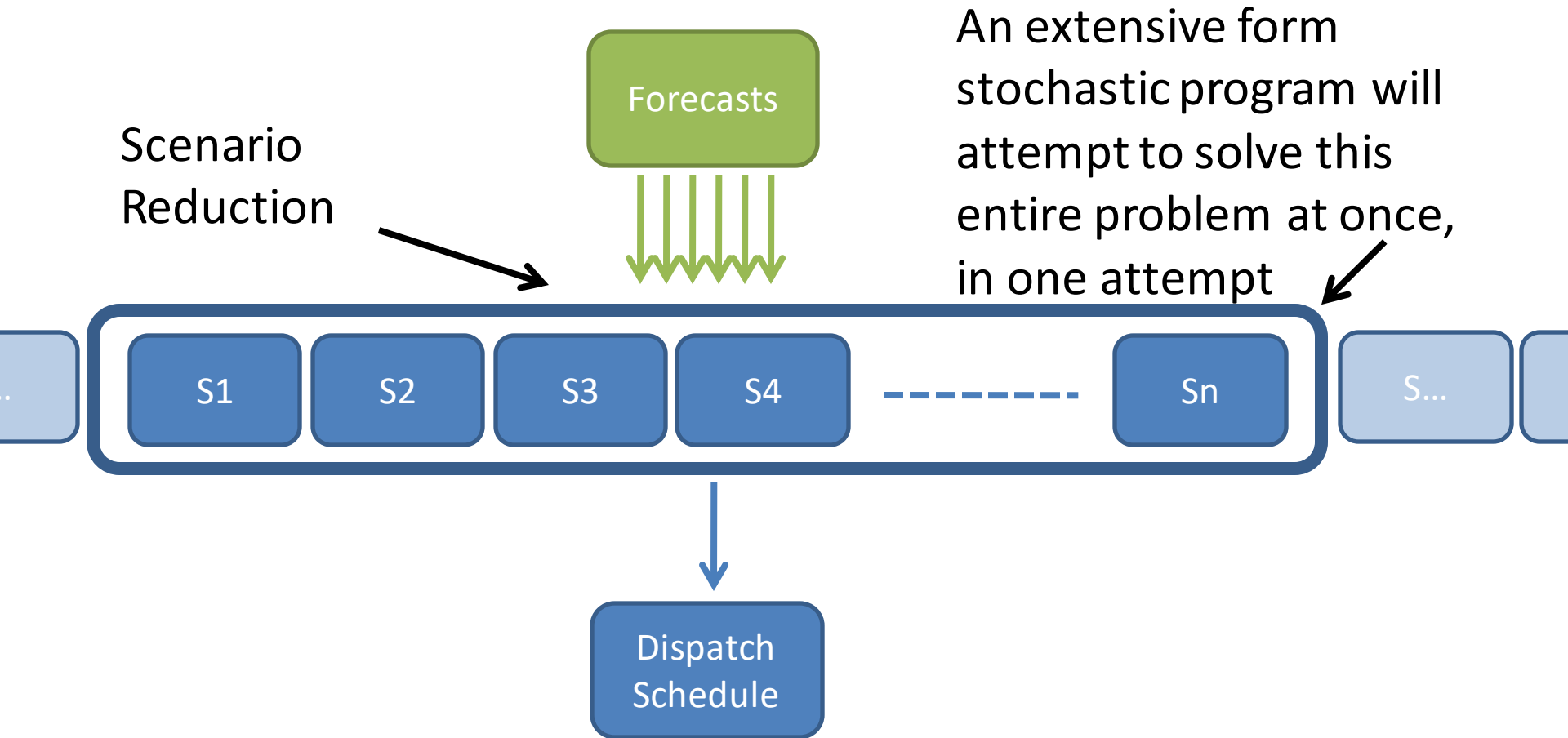


Two-stage stochastic program

# Deterministic Security-Constrained Unit Commitment



# Extensive Form Stochastic Unit Commitment



# Stochastic Unit Commitment

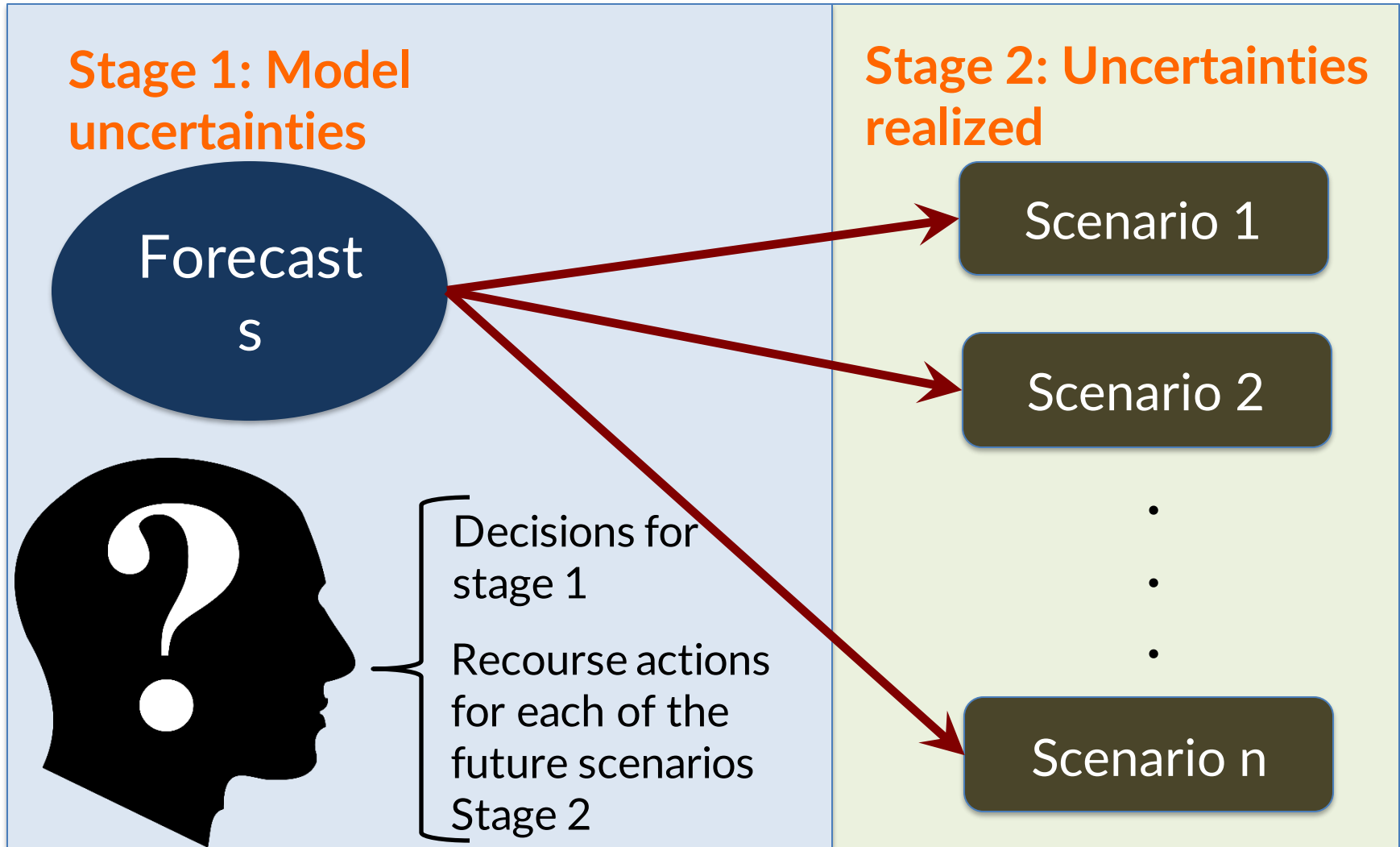
One form of stochastic unit commitment is:

- Stochastic programming
  - Captures uncertainty / multiple potential states
- Two-stage stochastic unit commitment
  - The **first stage** models the **pre-contingency** state
  - The **second stage** models the **post-contingency** state
  - The two states are linked (by constraints) that govern how the system will respond (transition from the pre-contingency to the post-contingency state) to the contingency
  - The **“optimal dispatch”** to this problem must not only account for the pre-contingency state but also the post-contingency state

# Stochastic Unit Commitment

- Why don't we just implement stochastic programming?
- **Computational complexities**
  - How many contingencies does a large system have? How does the optimization problem change as a result?
- Instead, we create proxy requirements for how much reserve (and where) we need in the grid
- Note that there is a lot of research on improving the computational performance of stochastic unit commitment
  - For both N-1 reliability and the integration of renewable resources

# Two-Stage Scenario-Based Stochastic Programs



Break



# Two Stage Programs



# Deterministic Program

- Deterministic Problem

- Minimize total cost

$$\min C^T x$$

$$\text{st } Ax = b$$

- How to consider uncertainty?

- Assumption

: Set of realization of uncertain variables with corresponding probabilities is available

# Stochastic Programming Problem

- Stochastic SCED: **Extensive Form**

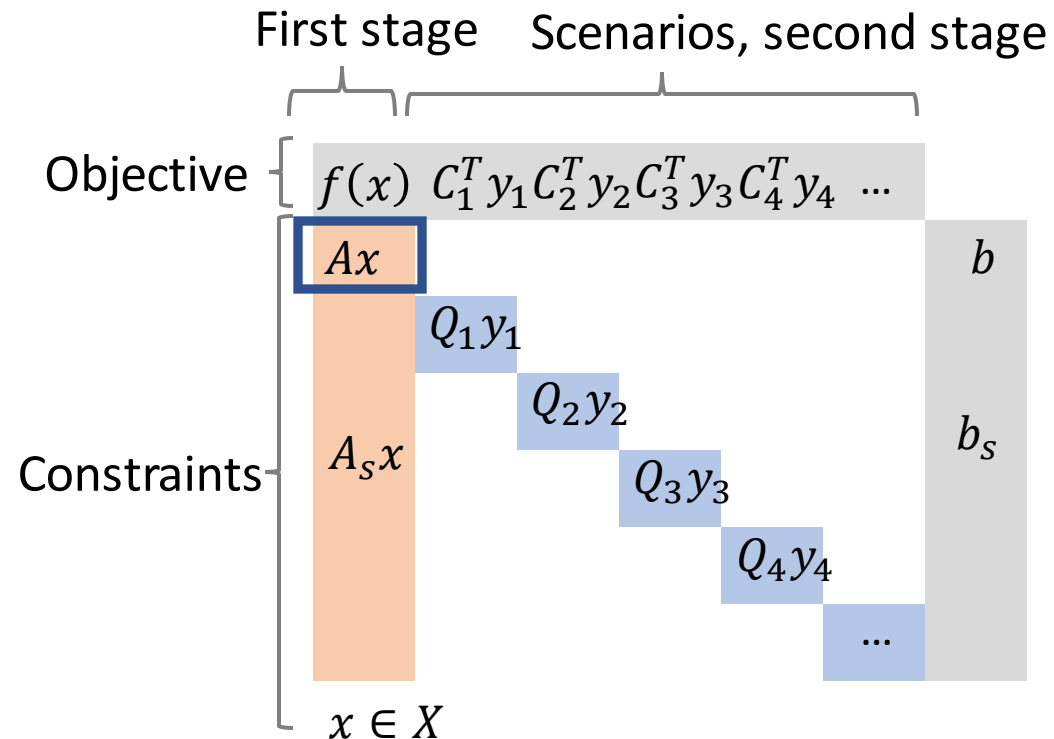
$$\min \sum_{s \in S} \pi_s [C^T x + C_s y_s]$$

Subject to (s.t.):

$$Ax = b$$

$$A_s x + Q_s y_s = b_s$$

- How to solve???



## First Stage Decision Variables

- First stage decision variables:
  - Decisions made prior to the uncertainty realization
  - Preventive actions
  - Base-case, pre-contingency
- First stage decisions:
  - Base-case real power generator dispatch values
  - Ancillary services procured
  - (Resulting base-case pre-contingency line flows)

## Second-Stage Decision Variables

- Second stage decision variables:
  - Corrective control capability implemented only after the event/scenario/contingency occurs
- Second stage decisions:
  - Activation of reserve – how much (gen redispatch)
  - (Resulting base-case pre-contingency line flows)

## Agreement Across Scenarios

- First stage decisions must agree across scenarios
- Second stage decisions are free to vary but are linked together through the first stage decisions

Break

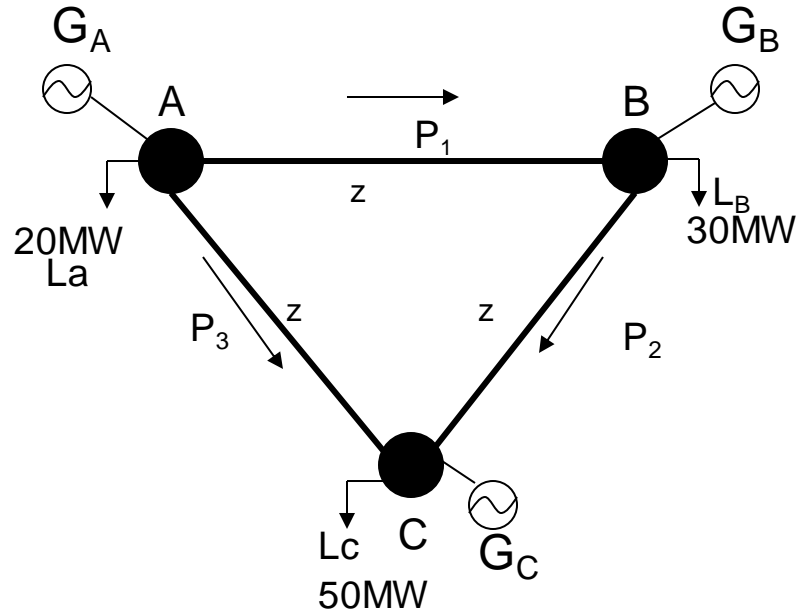
# Two Stage Programs





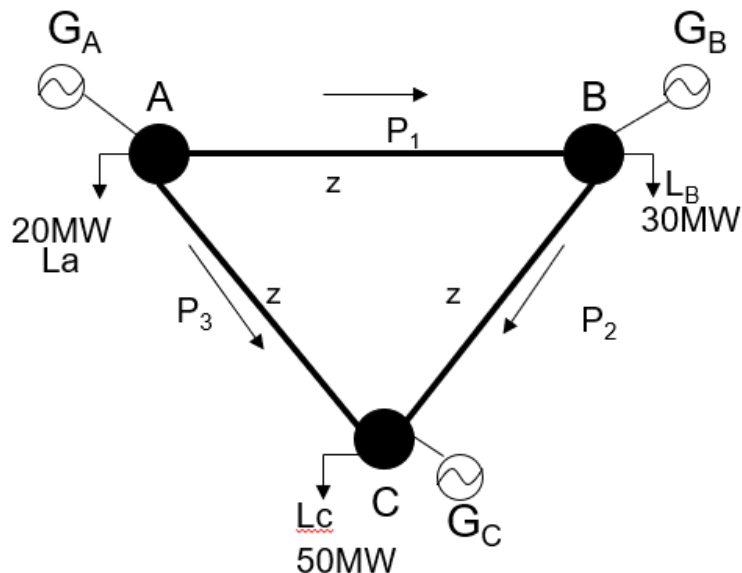
Example: SCED

# SCED



The presented formulation is a SCED as it incorporates security criteria in the form of explicit representation of transmission contingencies as well as generator contingency modeling.

Example: SCED  
Base-Case  
Pre-contingency  
First-Stage Modeling



## Gen Modeling (example):

$$0 \leq G_A, 0 \leq G_B, 0 \leq G_C$$

$$G_A + r_A \leq 80$$

$$G_B + r_B \leq 70$$

$$G_C + r_C \leq 60$$

$$0 \leq r_A \leq 40; 0 \leq r_B \leq 30; 0 \leq r_C \leq 20$$

$$r_A + r_B + r_C \geq G_A + r_A$$

$$r_A + r_B + r_C \geq G_B + r_B$$

$$r_A + r_B + r_C \geq G_C + r_C$$

## Gen Modeling (generic):

$$P_g^{min} \leq p_{gt} \quad \forall g \in G, t \in T$$

$$p_{gt} + r_{gt}^{spin} \leq P_g^{max} \quad \forall g, t$$

$$0 \leq r_{gt}^{spin} \leq R_g^{10\ spin} \quad \forall g, t$$

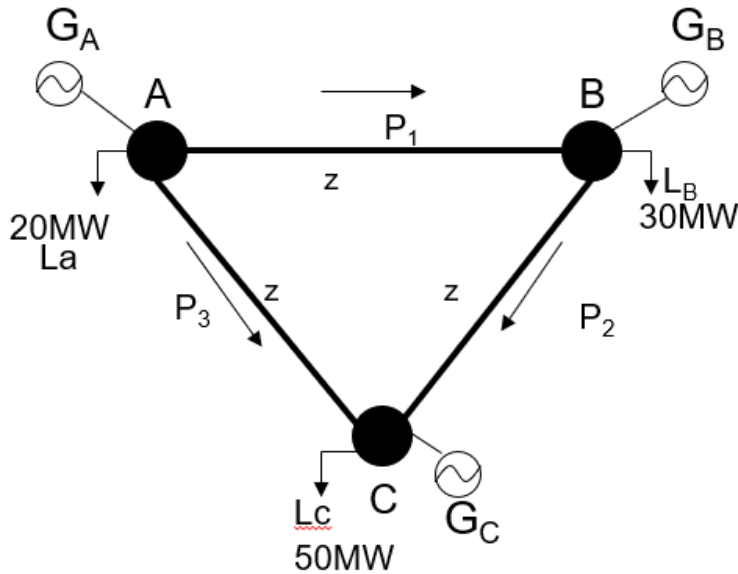
$$\sum_{\forall s \in G} r_{st}^{spin} \geq p_{gt} + r_{gt}^{spin} \quad \forall g, t \quad (x)$$

$p_{gt}$ : **First stage pre-contingency** dispatch setpoint, gen  $g$ , period  $t$

$r_{gt}^{spin}$ : **First stage pre-contingency** 10-minute spin reserve from gen  $g$ ;

note that  $s$  references the generator set just as  $g$  does; note that you could write (x) as (xx) instead

$$\sum_{\substack{\forall s \in G \\ s \neq g}} r_{st}^{spin} \geq p_{gt} \quad \forall g, t \quad (xx)$$



### Base Case Power Flow Modeling (example):

$$-40 \leq \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30) \leq 40$$

$$-40 \leq \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30) \leq 40$$

$$-40 \leq \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30) \leq 40$$

$$G_A + G_B + G_C = 100$$

Bus c is the reference bus

### Base Case Power Flow Modeling (generic):

$$P_k^{min} \leq \sum_{\forall n} PTDF_{k,n}^R (\sum_{\forall g \in G^n} (p_{gt}) - D_{nt}) \leq P_k^{max} \quad \forall k, t$$

$$p_{k,t}^0 = \sum_{\forall n} PTDF_{k,n}^R (\sum_{\forall g \in G^n} (p_{gt}) - D_{nt}) \quad \forall k, t$$

$$\sum_{\forall g} (p_{gt}) = \sum_{\forall n} (D_{nt}) \quad \forall t$$

$p_{gt}$ : **First stage pre-contingency** dispatch setpoint, gen g, period t

$p_{kt}^0$ : **First stage pre-contingency** power flow on branch k, gen g, period t;

this var is not needed truly but I am defining it to make a follow-on equation easier to write.

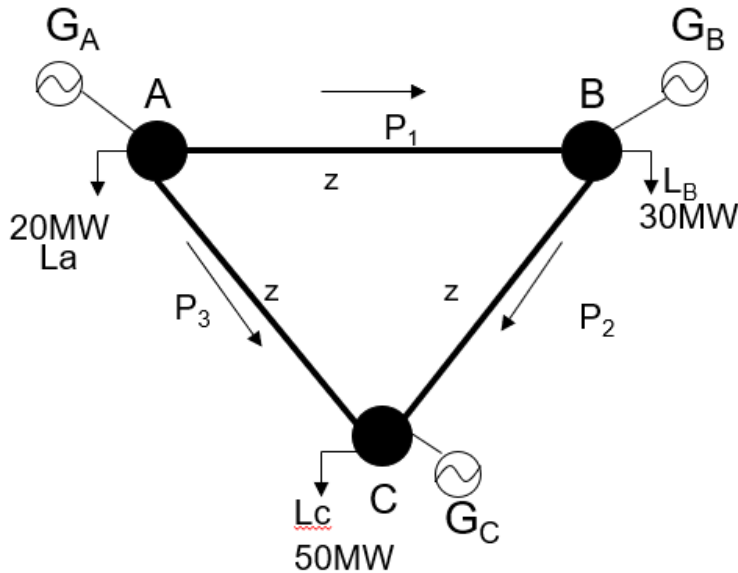
Break

# Two Stage Programs



Example: SCED  
Post-contingency  
Transmission Contingency  
Second-Stage Modeling





### Post Contingency (Trans) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \leq P_k^{min,rate c} \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$$p_{k\ell t} = p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$p_{kt}^0$ : **First stage pre-contingency** power flow on branch k, period t

$p_{k\ell t}$ : **Second stage post-contingency** power flow on branch k, period t, **for the loss of line  $\ell$**

$$P_k^{min,rate c} \leq \sum_{\forall n} PTDF_{k,n}^{R \text{ loss of } \ell} (\sum_{\forall g \in G^n} (p_{g\ell t}) - D_{nt}) \leq P_k^{max,rate c} \quad \forall k, \ell, t$$

$$p_{g\ell t}^{loss \text{ of } \ell} = p_{gt} : \text{non-anticipativity constraint}$$

Another way to formulate (above)

Note that there are limitations when LODFs can be used; refer to Power Generation, Operation, and Control textbook

Very important: most security criteria modeling for transmission contingencies assume no movement away from the pre-contingency dispatch setpoints. Power system engineers often impose that the pre-contingency setpoints satisfy post-contingency limits for a single transmission contingency. If you go back to the prior slide,  $p_{kt}^0$  (and  $p_{\ell t}^0$  as well) are defined based only on first-stage pre-contingency dispatch setpoint injections. There is no need to declare a variable  $p_{k\ell t}$ , which can be seen to have a new index. Many people see the first equation and since it is based on the pre-contingency flows only (determined then by the pre-contingency dispatch setpoints again), they don't declare this to be a two-stage program. But it is. Look to the right. The sets defining how this equation is declared specify that you have  $K \times K$  equations being included – you are directly representing an unknown event, a loss of a line, and then determining the impact on all lines.

### Post Contingency (Trans) Power Flow Modeling (generic):

$$P_k^{min,rate\ c} \leq p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \leq P_k^{min,rate\ c} \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$$p_{k\ell t} = p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$p_{kt}^0$ : **First stage pre-contingency** power flow on branch k, period t

$p_{k\ell t}$ : **Second stage post-contingency** power flow on branch k, period t, **for the loss of line  $\ell$**

$$P_k^{min,rate\ c} \leq \sum_{\forall n} PTDF_{k,n}^{R\ loss\ of\ \ell} (\sum_{\forall g \in G^n} (p_{g\ell t}) - D_{nt}) \leq P_k^{max,rate\ c} \quad \forall k, \ell, t$$

$$p_{g\ell t}^{loss\ of\ \ell} = p_{gt} : \text{non-anticipativity constraint}$$

Another way to formulate (above)

Note that there are limitations when LODFs can be used; refer to the 577 textbook (available online via ASU Library)

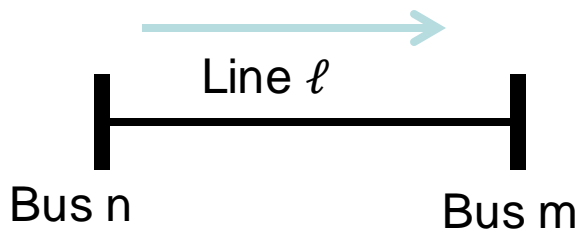
# Line Outage Distribution Factors

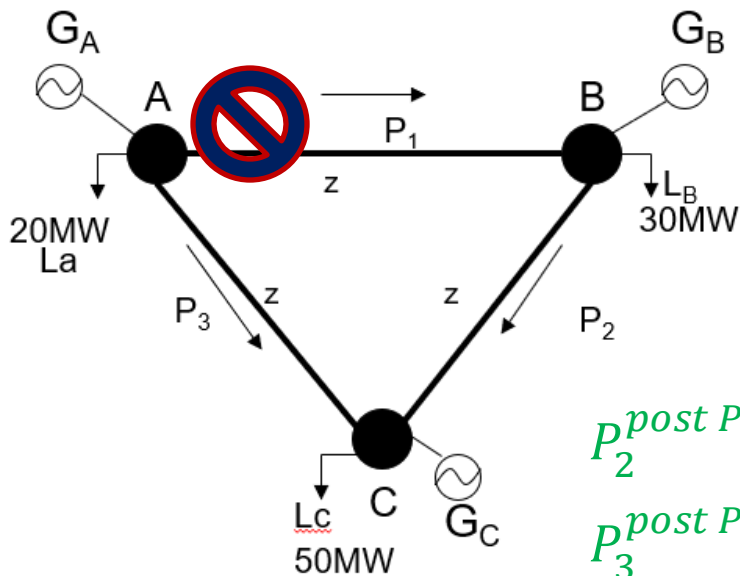
- An LODF allows you to determine the resulting flow on a particular line due to the loss of a different line
- When monitoring line  $k$ , what is the new flow on line  $k$  after you lose line  $\ell$

0 represents state 0, the base case, the pre-contingency state

$$P_k^{new} = P_k^0 + LODF_{k,\ell} P_\ell^0$$

$$LODF_{k,\ell} = PTDF_{n,k}^m \left( \frac{1}{1 - PTDF_{n,\ell}^m} \right)$$





## Post Cont (Trans) Power Flow Modeling: (example loss of line 1):

$$P_1^{pre} = \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30)$$

$$P_2^{pre} = \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30)$$

$$P_3^{pre} = \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30)$$

$$P_2^{post P1} = P_2^{pre} + \left(-\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_1^{pre}$$

$$P_3^{post P1} = P_3^{pre} + \left(\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_1^{pre}$$

$$-60 \leq P_2^{post P1}$$

$$P_2^{post P1} \leq 60$$

$$-60 \leq P_3^{post P1}$$

$$P_3^{post P1} \leq 60$$

LODFs are in purple

## Post Contingency (Trans) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \leq P_k^{min,rate c} \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

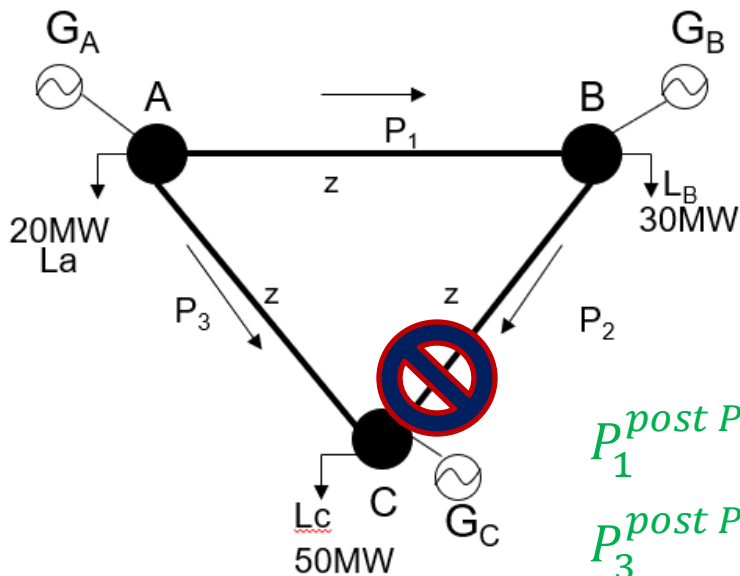
$$p_{k\ell t} = p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$p_{kt}^0$ : First stage pre-contingency power flow on branch k, period t

$p_{k\ell t}$ : Second stage post-contingency power flow on branch k, period t, for the loss of line  $\ell$

$$P_2^{post P1} = \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30) + (-1) \left( \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30) \right) = (G_B - 30)$$

$$P_3^{post P1} = \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30) + (1) \left( \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30) \right) = (G_A - 20)$$



## Post Cont (Trans) Power Flow Modeling: (example loss of line 2):

$$P_1^{pre} = \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30)$$

$$P_2^{pre} = \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30)$$

$$P_3^{pre} = \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30)$$

$$P_1^{post P2} = P_1^{pre} + \left(-\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_2^{pre}$$

$$P_3^{post P2} = P_3^{pre} + \left(\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_2^{pre}$$

$$-60 \leq P_1^{post P2}$$

$$P_1^{post P2} \leq 60$$

$$-60 \leq P_3^{post P2}$$

$$P_3^{post P2} \leq 60$$

LODFs are in purple

## Post Contingency (Trans) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \leq P_k^{min,rate c} \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$$p_{k\ell t} = p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

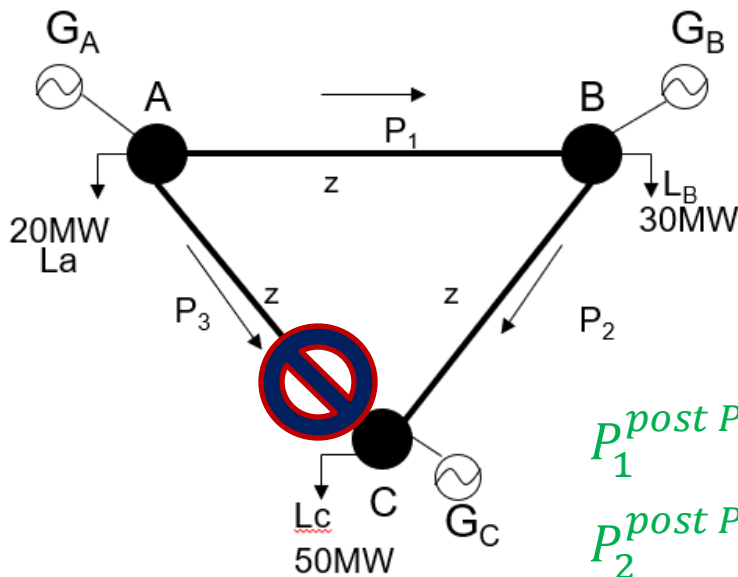
$p_{kt}^0$ : **First stage pre-contingency** power flow on branch k, period t

$p_{k\ell t}$ : **Second stage post-contingency** power flow on branch k, period t, **for the loss of line  $\ell$**

$$P_1^{post P2} = \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30) + (-1) \left( \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30) \right) = -(G_B - 30)$$

$$P_3^{post P2} = \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30) + (1) \left( \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30) \right)$$

$$= (G_A - 20) + (G_B - 30)$$



## Post Cont (Trans) Power Flow Modeling: (example loss of line 3):

$$P_1^{pre} = \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30)$$

$$P_2^{pre} = \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30)$$

$$P_3^{pre} = \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30)$$

$$P_1^{post P3} = P_1^{pre} + \left(\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_3^{pre}$$

$$P_2^{post P3} = P_2^{pre} + \left(\frac{1}{3}\right) \left(\frac{1}{1-2/3}\right) P_3^{pre}$$

$$-60 \leq P_1^{post P3}$$

$$P_1^{post P3} \leq 60$$

$$-60 \leq P_2^{post P3}$$

$$P_2^{post P3} \leq 60$$

LODFs are in purple

## Post Contingency (Trans) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \leq P_k^{min,rate c} \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$$p_{k\ell t} = p_{kt}^0 + LODF_{k,\ell} p_{\ell t}^0 \quad \forall k \in K, k \neq \ell, \ell \in K, t \in T$$

$p_{kt}^0$ : **First stage pre-contingency** power flow on branch k, period t

$p_{k\ell t}$ : **Second stage post-contingency** power flow on branch k, period t, **for the loss of line  $\ell$**

$$P_1^{post P3} = \frac{1}{3} (G_A - 20) - \frac{1}{3} (G_B - 30) + (1) \left( \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30) \right) = (G_A - 20)$$

$$P_2^{post P3} = \frac{1}{3} (G_A - 20) + \frac{2}{3} (G_B - 30) + (1) \left( \frac{2}{3} (G_A - 20) + \frac{1}{3} (G_B - 30) \right)$$

$$= (G_A - 20) + (G_B - 30)$$

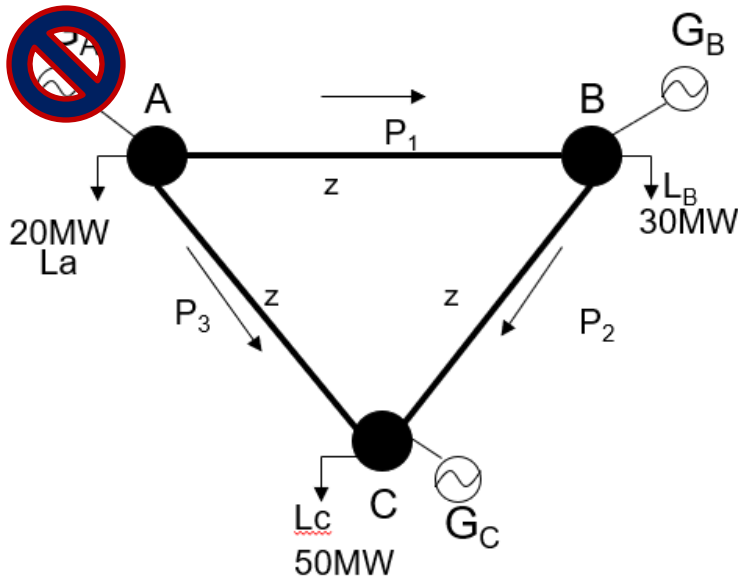
Break

# Two Stage Programs





Example: SCED  
Post-contingency  
Generator Contingency  
Second-Stage Modeling



## Post Cont (gen) Modeling (example loss of gen A):

$$r_B^{loss A} + r_C^{loss A} = G_A$$

$$-30 \leq r_B^{loss A} \leq r_B; -20 \leq r_C^{loss A} \leq r_C$$

$$0 \leq r_B^{loss A} + G_B; 0 \leq r_C^{loss A} + G_C$$

$$-60 \leq \frac{1}{3}(-20) - \frac{1}{3}(G_B + r_B^{loss A} - 30) \leq 60$$

$$-60 \leq \frac{1}{3}(-20) + \frac{2}{3}(G_B + r_B^{loss A} - 30) \leq 60$$

$$-60 \leq \frac{2}{3}(-20) + \frac{1}{3}(G_B + r_B^{loss A} - 30) \leq 60$$

## Post Contingency (Gen) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kht} \leq P_k^{max,rate c} \quad \forall k \in K, h \in G, t \in T$$

$$p_{kht} = \sum_{\forall n} [PTDF_{k,n}^R (\sum_{\forall g \in G^n} (p_{gt} + r_{ght}^{spin}) - D_{nt})] - PTDF_{k,h@n}^R p_{g=h,t}$$

$$\sum_{\forall g} r_{ght}^{spin} = p_{g=h,t} \quad \forall g \in G, h \in G, t \in T \quad P_g^{min} \leq p_{gt} + r_{ght}^{spin} \quad \forall g \in G, h \in G, t \in T$$

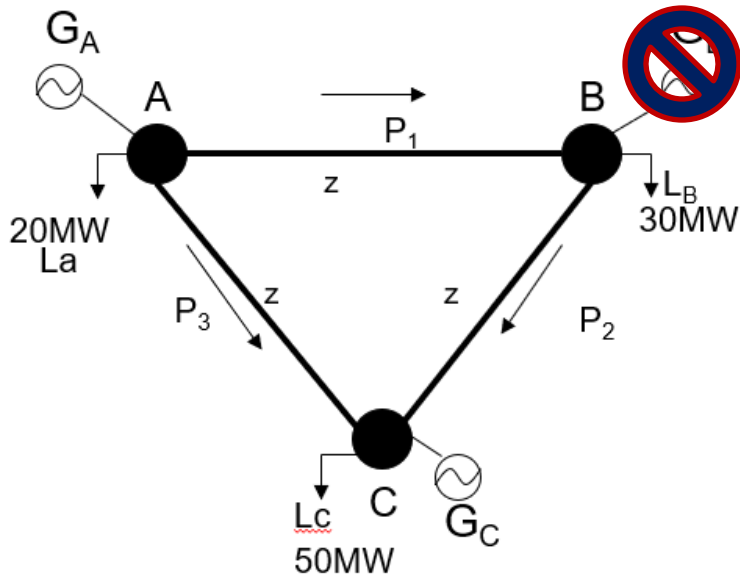
$$r_{g=h,h,t}^{spin} = 0 \quad \forall h \in G, t \in T \quad -R_g^{10 spin} \leq r_{ght}^{spin} \leq r_{gt}^{spin} \quad \forall g \in G, h \in G, t \in T$$

$p_{gt}$ : **First stage pre-contingency** gen dispatch setpoint for gen g, period t

$r_{gt}^{spin}$ : **First stage pre-contingency** gen 10-min spin reserve for gen g, period t

$r_{ght}^{spin}$ : **Second stage post-contingency** spin reserve activated from gen g in response to loss of gen h, in period t

$p_{kht}$ : **Second stage post-contingency** power flow on line k, outage of gen h, period t



## Post Cont (gen) Modeling (example loss of gen B):

$$r_A^{loss B} + r_C^{loss B} = G_B$$

$$-40 \leq r_A^{loss B} \leq r_A; -20 \leq r_C^{loss B} \leq r_C$$

$$0 \leq r_A^{loss B} + G_A; 0 \leq r_C^{loss B} + G_C$$

$$-60 \leq \frac{1}{3}(G_A + r_A^{loss B} - 20) - \frac{1}{3}(-30) \leq 60$$

$$-60 \leq \frac{1}{3}(G_A + r_A^{loss B} - 20) + \frac{2}{3}(-30) \leq 60$$

$$-60 \leq \frac{2}{3}(G_A + r_A^{loss B} - 20) + \frac{1}{3}(-30) \leq 60$$

## Post Contingency (Gen) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kht} \leq P_k^{min,rate c} \quad \forall k \in K, h \in G, t \in T$$

$$p_{kht} = \sum_{\forall n} [PTDF_{k,n}^R (\sum_{\forall g \in G^n} (p_{gt} + r_{ght}^{spin}) - D_{nt})] - PTDF_{k,h@n}^R p_{g=h,t}$$

$$\sum_{\forall g} r_{ght}^{spin} = p_{g=h,t} \quad \forall g \in G, h \in G, t \in T \quad P_g^{min} \leq p_{gt} + r_{ght}^{spin} \quad \forall g \in G, h \in G, t \in T$$

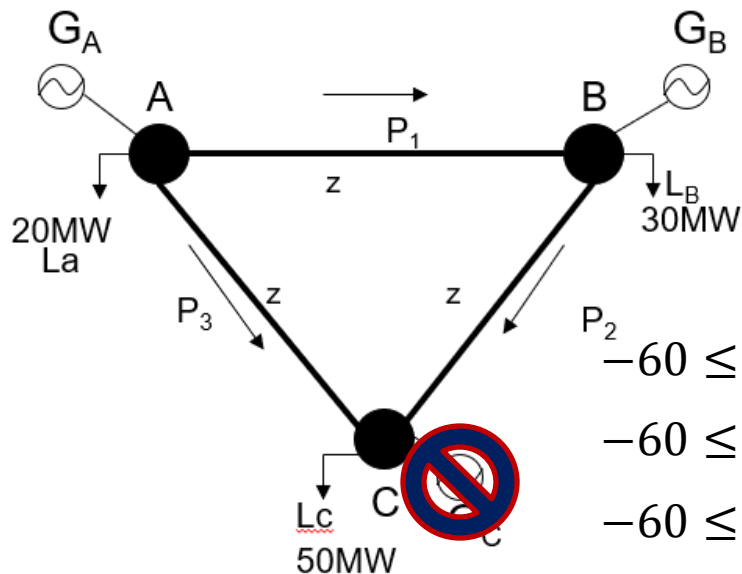
$$r_{g=h,h,t}^{spin} = 0 \quad \forall h \in G, t \in T \quad -R_g^{10 spin} \leq r_{ght}^{spin} \leq r_{gt}^{spin} \quad \forall g \in G, h \in G, t \in T$$

$p_{gt}$ : **First stage pre-contingency** gen dispatch setpoint for gen g, period t

$r_{gt}^{spin}$ : **First stage pre-contingency** gen 10-min spin reserve for gen g, period t

$r_{ght}^{spin}$ : **Second stage post-contingency** spin reserve activated from gen g in response to loss of gen h, in period t

$p_{kht}$ : **Second stage post-contingency** power flow on line k, outage of gen h, period t



## Post Cont (gen) Modeling (example loss of gen C):

$$r_A^{loss C} + r_B^{loss C} = G_C$$

$$-40 \leq r_A^{loss C} \leq r_A; -30 \leq r_B^{loss C} \leq r_B$$

$$0 \leq r_A^{loss C} + G_A; 0 \leq r_B^{loss C} + G_B$$

$$-60 \leq \frac{1}{3}(G_A + r_A^{loss C} - 20) - \frac{1}{3}(G_B + r_B^{loss C} - 30) \leq 60$$

$$-60 \leq \frac{1}{3}(G_A + r_A^{loss C} - 20) + \frac{2}{3}(G_B + r_B^{loss C} - 30) \leq 60$$

$$-60 \leq \frac{2}{3}(G_A + r_A^{loss C} - 20) + \frac{1}{3}(G_B + r_B^{loss C} - 30) \leq 60$$

## Post Contingency (Gen) Power Flow Modeling (generic):

$$P_k^{min,rate c} \leq p_{kht} \leq P_k^{min,rate c} \quad \forall k \in K, h \in G, t \in T$$

$$p_{kht} = \sum_{\forall n} [PTDF_{k,n}^R (\sum_{\forall g \in G^n} (p_{gt} + r_{ght}^{spin}) - D_{nt})] - PTDF_{k,h@n}^R p_{g=h,t}$$

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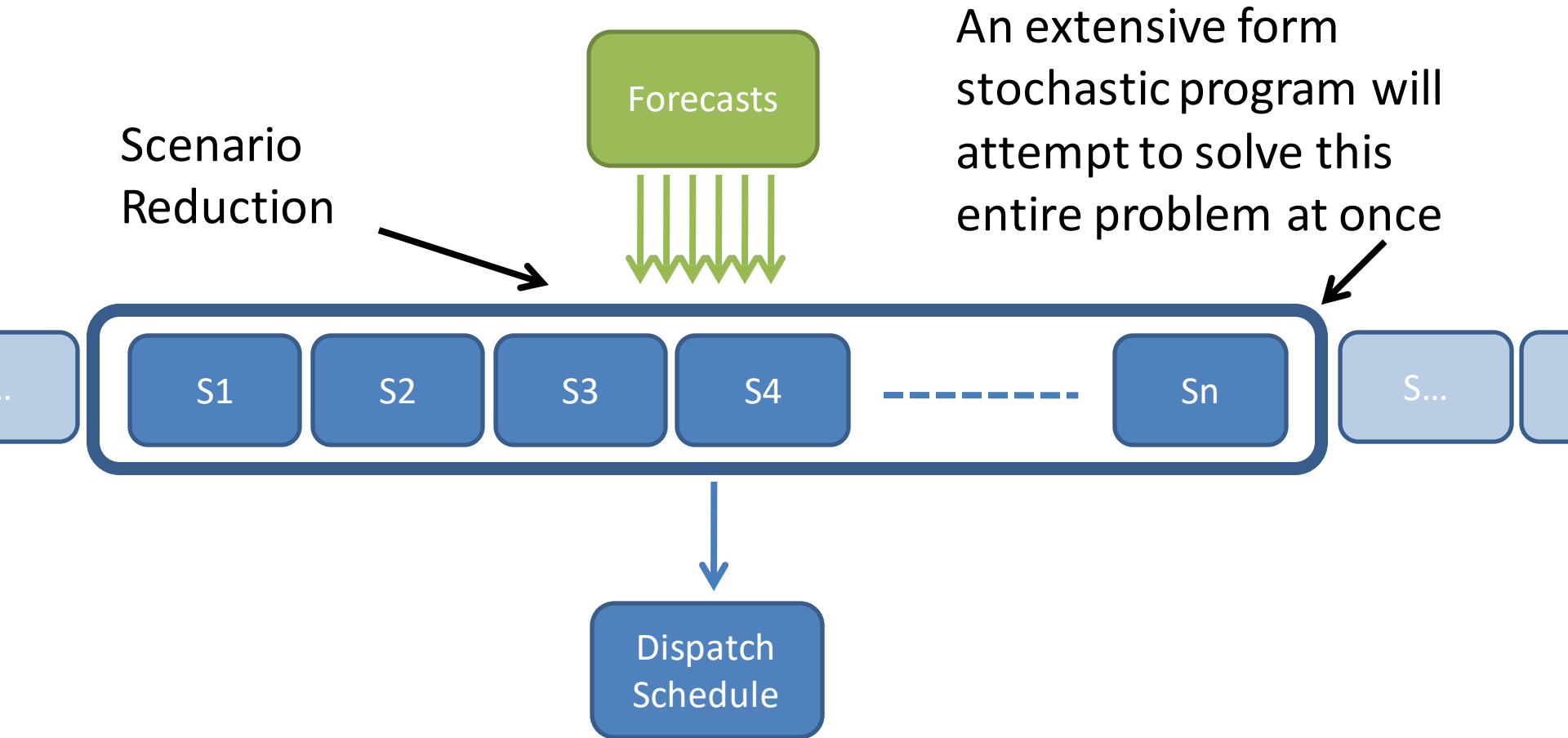
$p_{kht}$ : **Second stage post-contingency** power flow on line k, outage of gen h, period t

Break

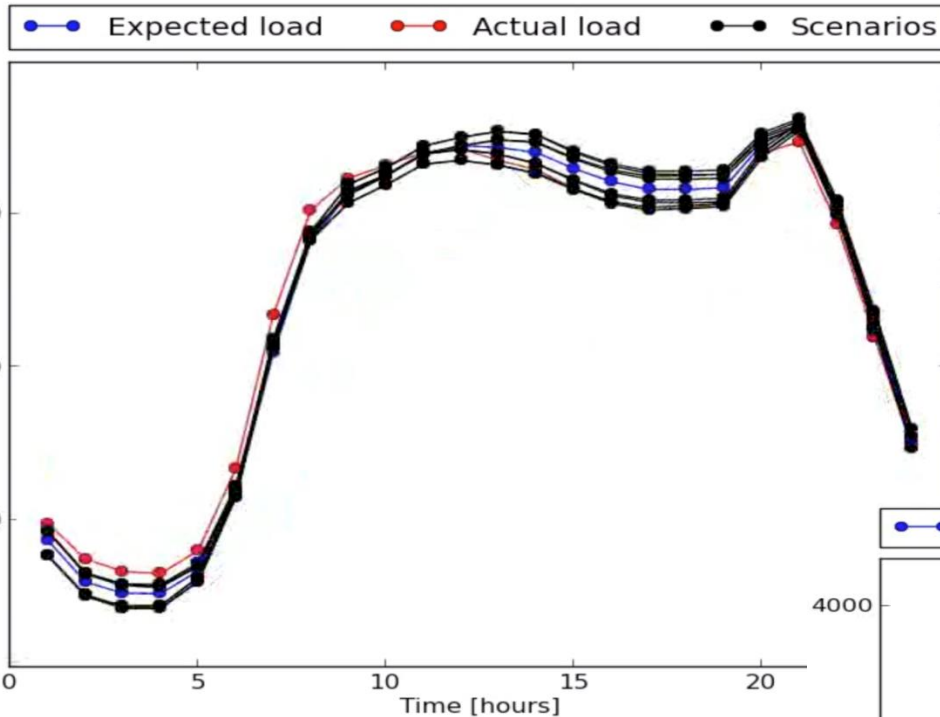
# Two Stage Programs



# Extensive Form Stochastic Unit Commitment

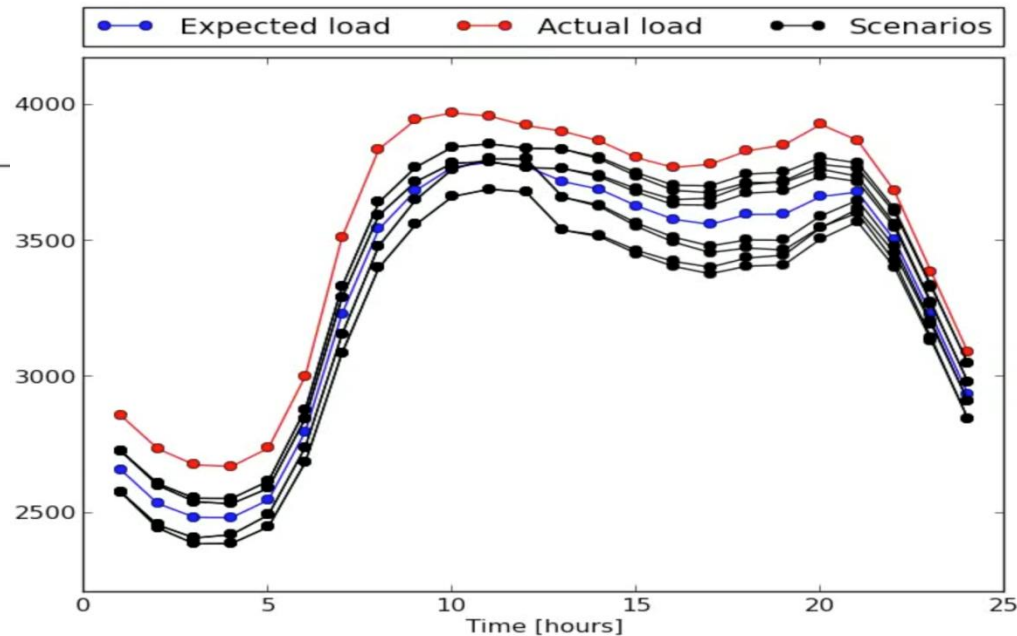


# Illustrative Load Scenarios: ISO-NE



If the historical data indicates no variability, then the scenarios will reflect that consistency

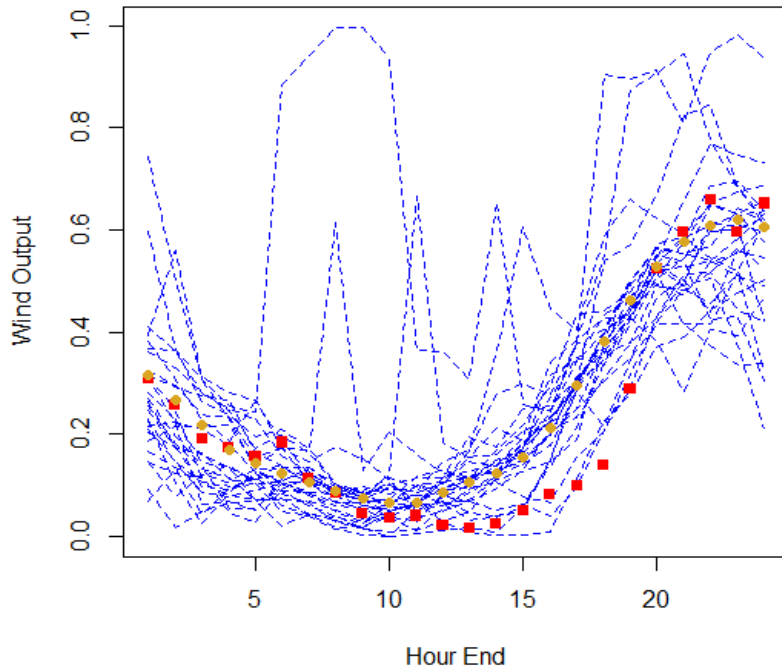
Captures variability in load when present – but predictions are not perfect!



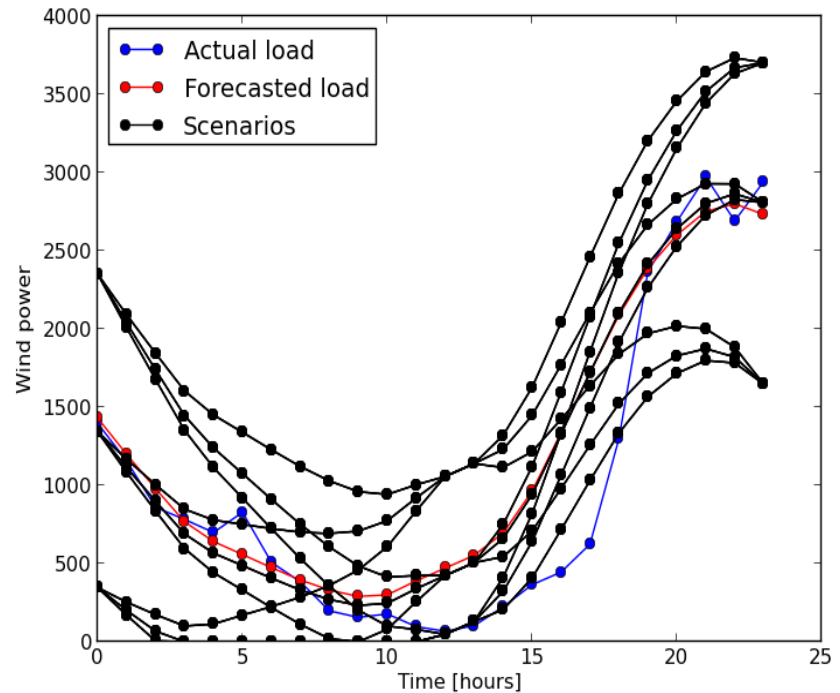


# Wind Scenario Generation Examples

Scenarios generated using Pinson et al. method



Scenarios generated using epi-spline approach

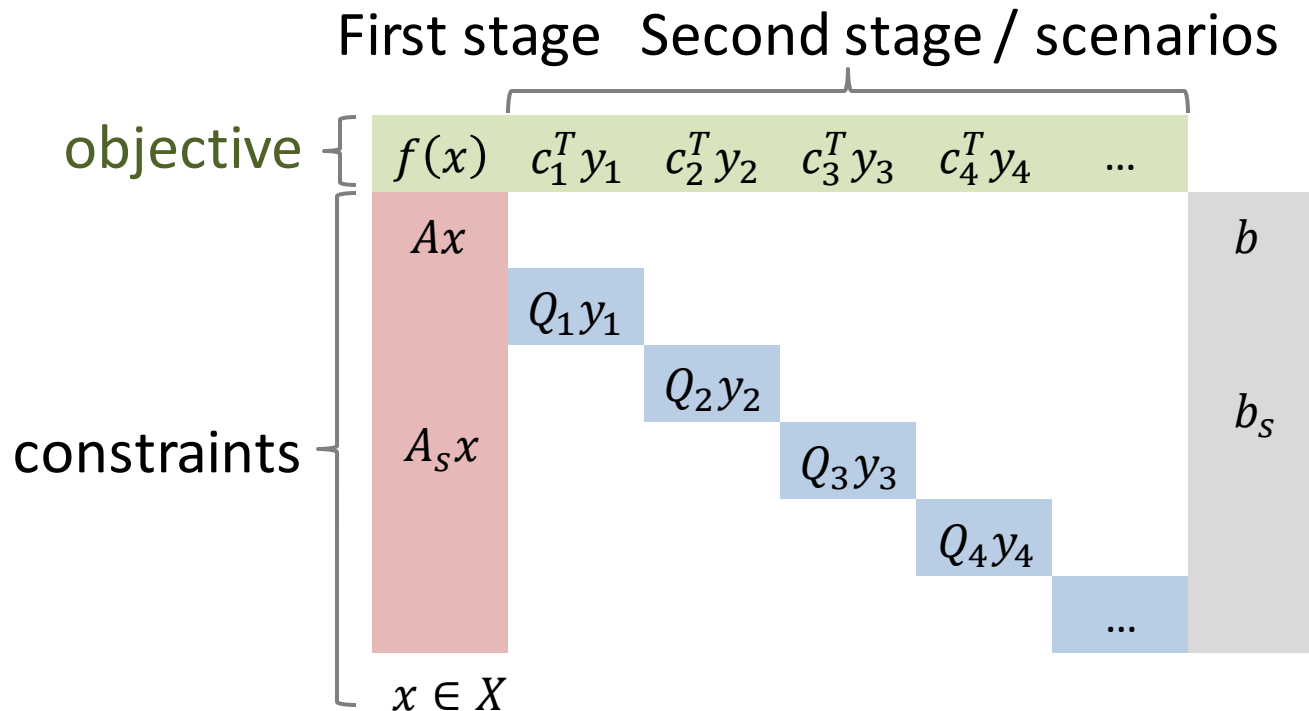


Note: Real wind profiles show significant ramps, but not as extreme as those obtained using (e.g.,) the Pinson et al. method

# Block Diagonal Structure

Two stage-stochastic programs

- **Stage one** ( $x$ ): base-case decisions made **here and now**
- **Stage two** ( $y$ ): recourse decisions that can be **deferred**



# Block Diagonal Structure

Two stage-stochastic programs

- **Stage one** ( $x$ ): base-case decisions made **here and now**
- **Stage two** ( $y$ ): recourse decisions that can be **deferred**

First stage    Second stage / scenarios

objective {  $f(x)$   $c_1^T y_1$   $c_2^T y_2$   $c_3^T y_3$   $c_4^T y_4$  ... }

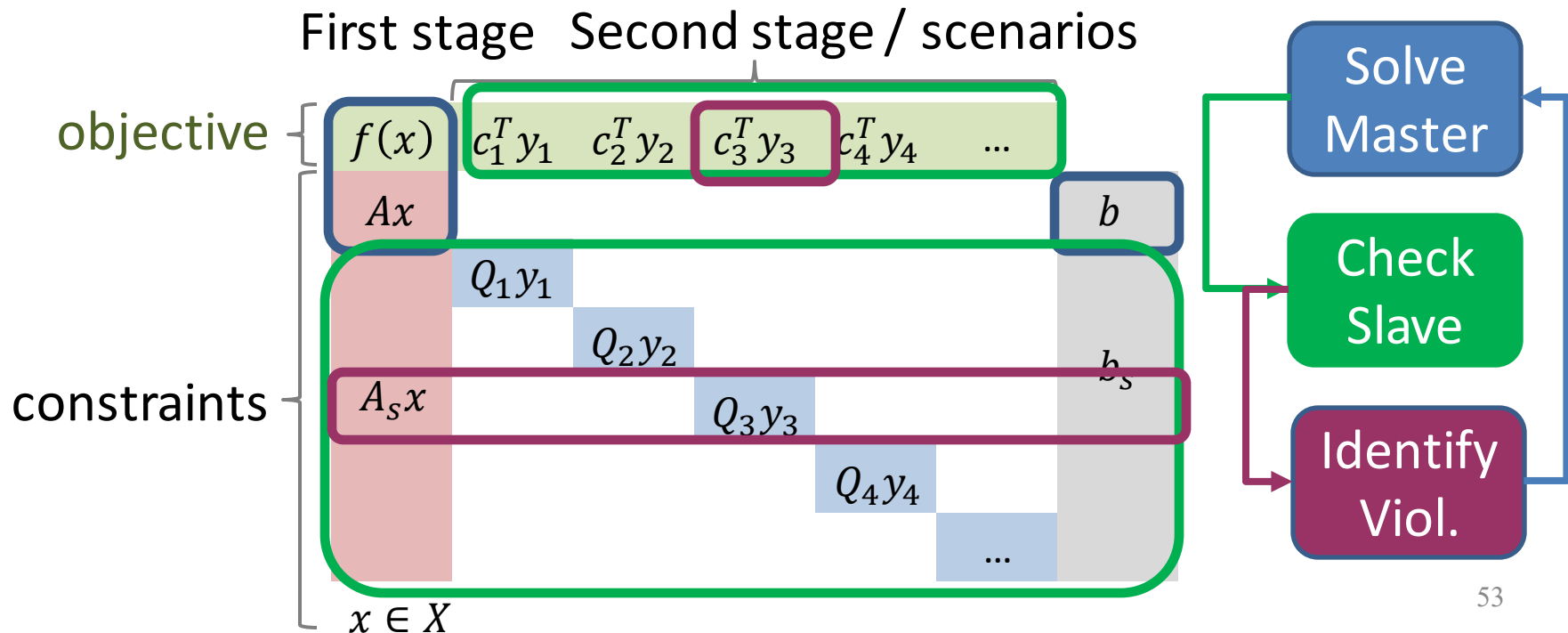
## Non-anticipativity constraints:

- Link first stage decisions with second stage decisions
- For unit commitment: commitment status of non fast-start generators (generator  $g$ , period  $t$ , scenario  $s$ ):

$$u_{gt} = u_{gts}$$

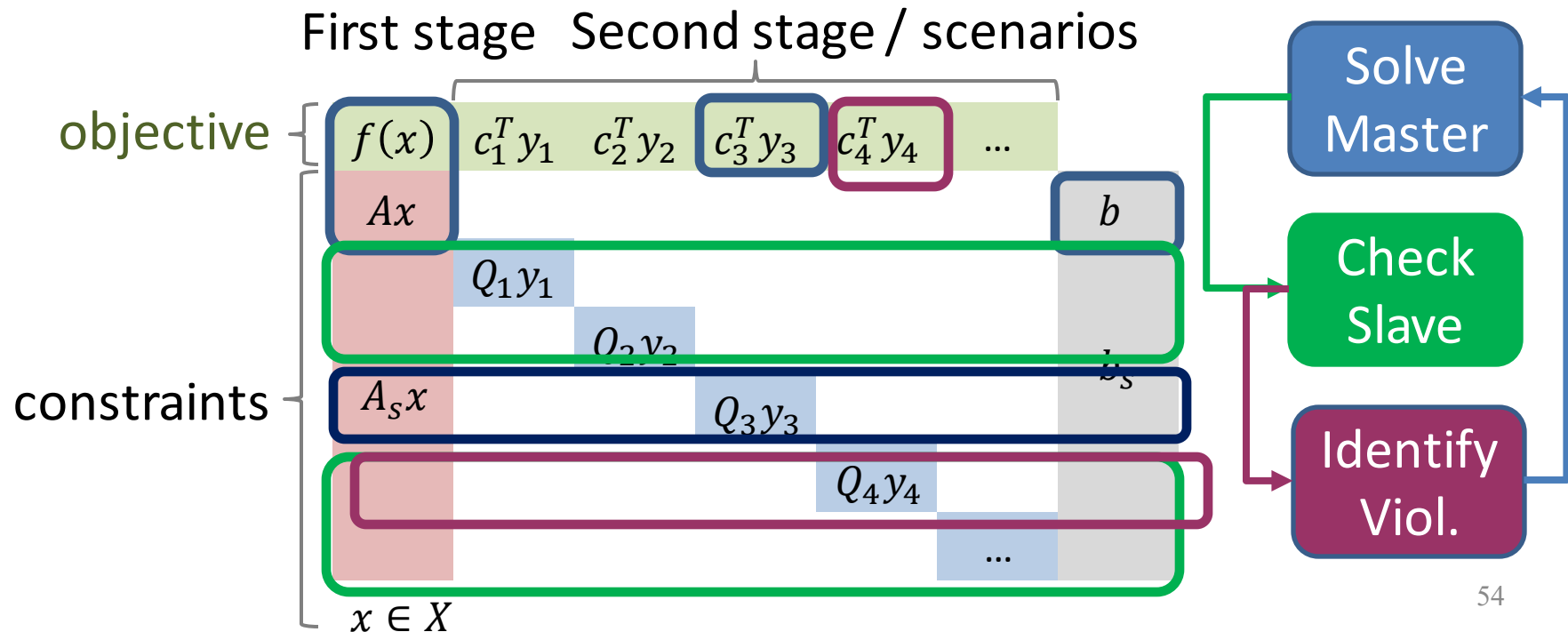
## Block Diagonal Decomposition

- Two-stage stochastic programs
  - **Stage one** ( $x$ ): base-case decisions made **here and now**
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- **Obstacle I - Computational Complexity**
  - Size of the problem: **OPF**  $\times$  **Scenarios**



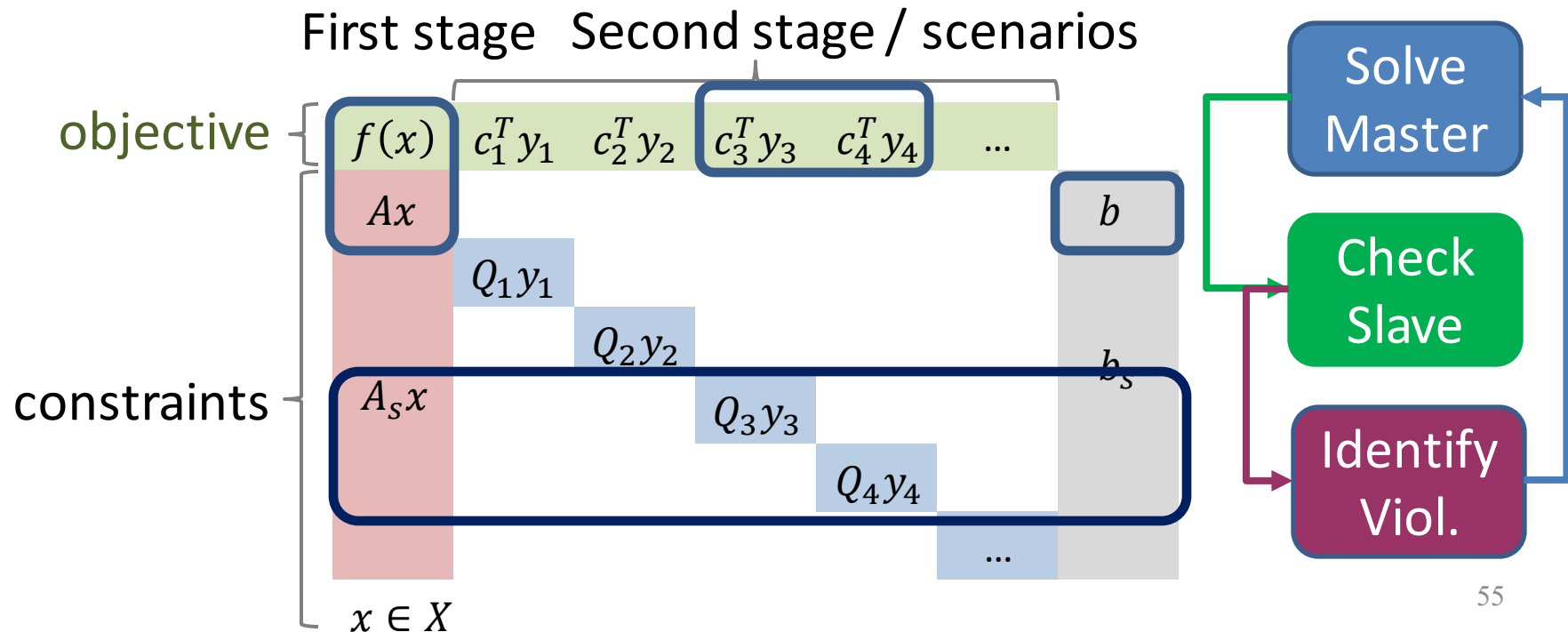
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## Block Diagonal Decomposition

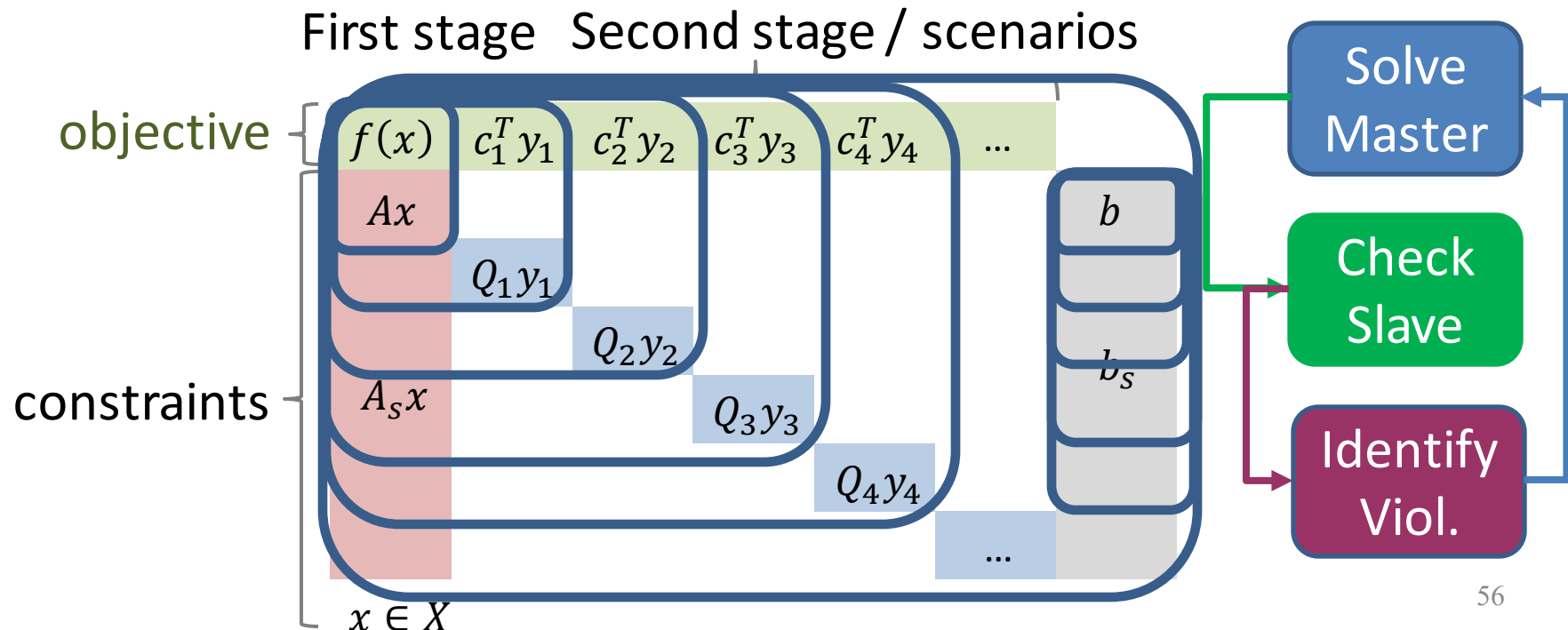
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  - Size of the problem: **OPF**  $\times$  **Scenarios**



# Block Diagonal Decomposition

*Of course the hope is that you terminate before full expansion*

- Two-stage stochastic programs
  - **Stage one** ( $x$ ): base-case decisions made **here and now**
  - **Stage two** ( $y$ ): recourse decisions that can be **deferred**
- **Obstacle I - Computational Complexity**
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Break



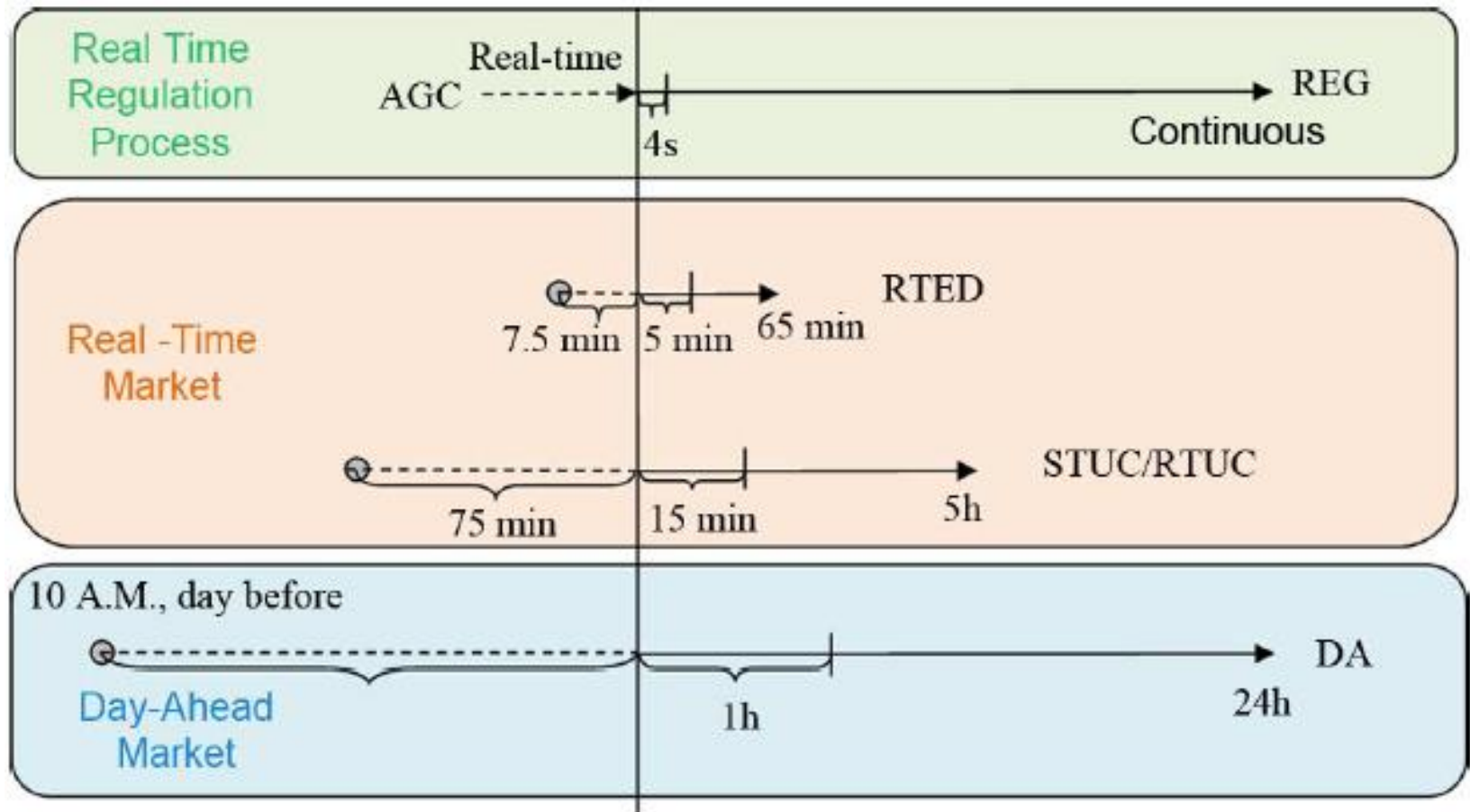
# Two Stage Programs



# Challenges for Day-Ahead Scheduling & Markets

# Operational Timeline

Source: Y. Makarov, et al., "Incorporating uncertainty of wind power generation forecast into power system operation, dispatch, and unit commitment procedures," *IEEE Trans. Sust. Energy*, 2011.



# Preferred Day-Ahead Scheduling Problem for Energy Markets

## Potential system size:

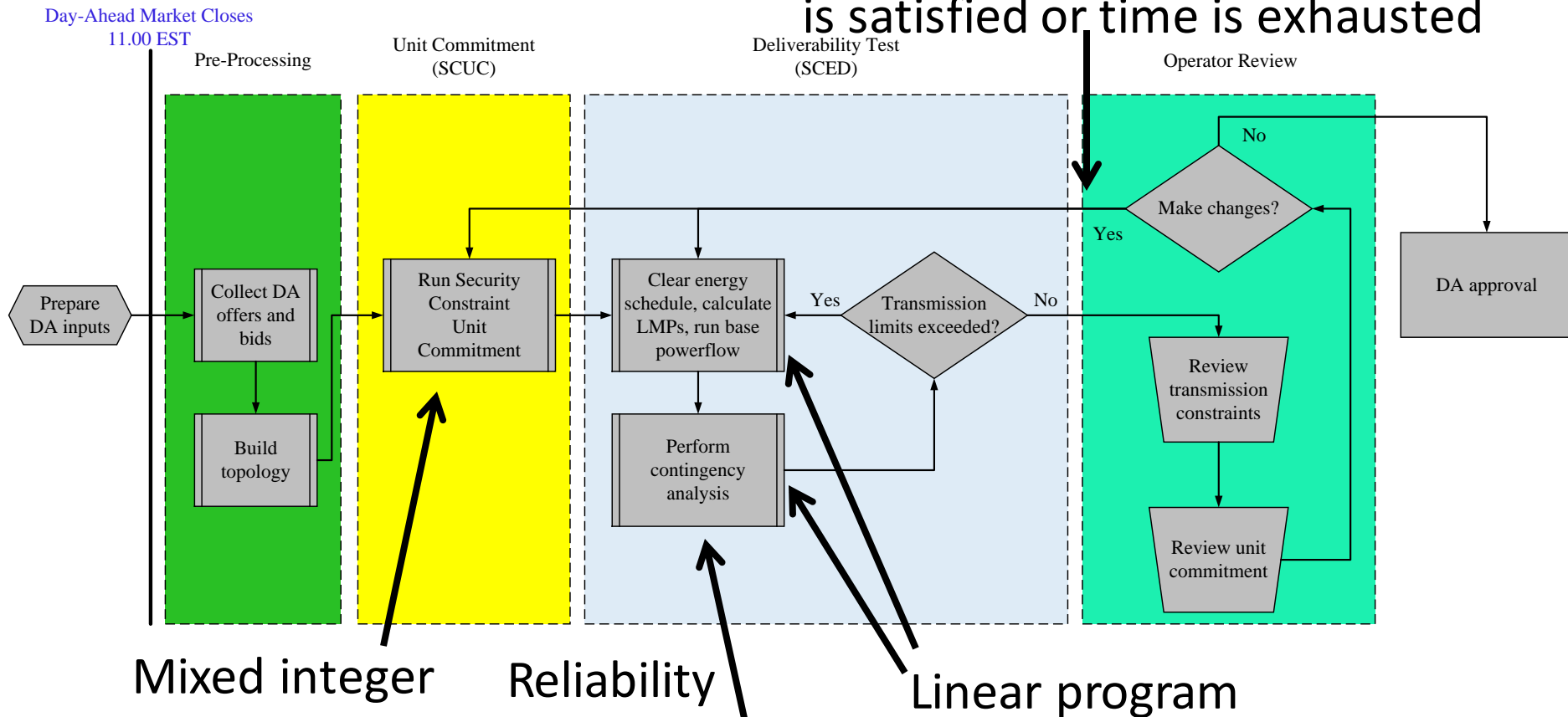
- 10,000 – 20,000 buses (nodes)
- >20,000 transmission lines (branches), transformer tap settings, flexible ac transmission systems devices, etc.
- Generators: 500-2000; flexible loads; virtual bidders (banks, financial institutions): thousands of bids
- Multiple day time horizon

**Optimization problem:** Stochastic mixed integer non-linear program (non-convex constraints)

**Uncertainties:** contingencies: >20,000; renewable (wind/solar) scenarios: >500; uncertain demand response: >?

# Day-Ahead Scheduling in Midcontinent Independent System Operator (MISO)

Iterative process until operator is satisfied or time is exhausted



MISO Day-Ahead Scheduling Procedure

[1] Aaron Casto, "Overview of MISO day-ahead markets," *Midwest ISO*, [Online].

Available: [http://www.atcllc.com/oasis/Customer\\_Notices/NCM\\_MISO\\_DayAhead111507.pdf](http://www.atcllc.com/oasis/Customer_Notices/NCM_MISO_DayAhead111507.pdf).

# Final Day-Ahead Market Solution

What is guaranteed within the market SCUC solution?

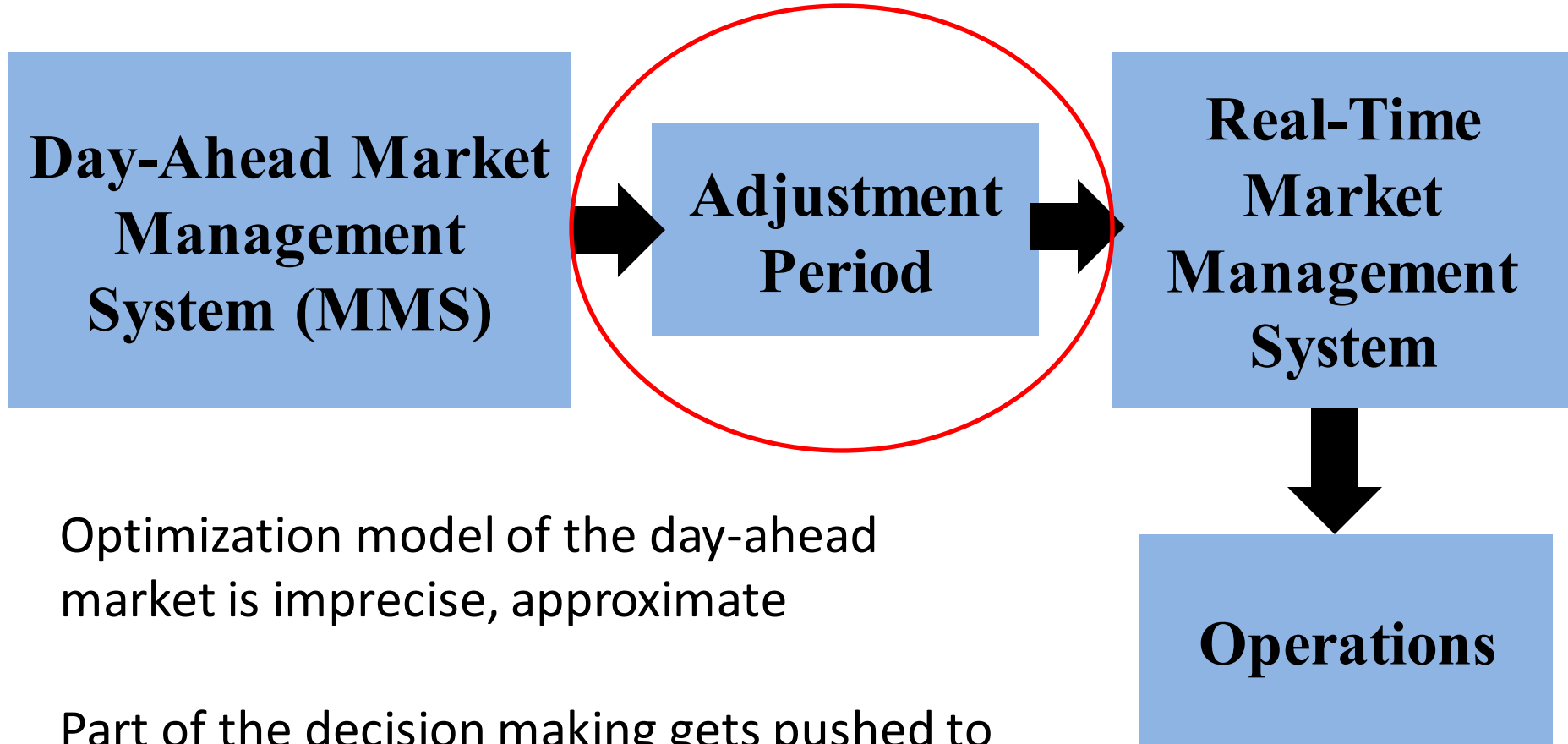
- AC feasible?
- N-1 Reliable?
- Renewable uncertainty?
- Stability?

Approximations within SCUC models result in either infeasible solutions that must be corrected outside of the market engine or overly conservative solutions

# Existing Model Simplifications

- **Modeled** system size: out of 20k lines and as many as 30k contingencies, the number of monitored base case and post-contingency flow limitations are **fewer than 10k**
- MMS and Energy Management Systems (EMS) rely on many assumptions regarding system operating conditions
  - Which network flow constraints to impose; which to ignore
- **Operators are observing never before seen flow patterns within the grid due to renewables (e.g., within BPA)**
- Unprecedented levels of variability and uncertainty caused by renewables **undermine such assumptions**
  - More challenging to **predict** the operational limitations
  - Existing practices and software must be changed

# Market Adjustment Process



Optimization model of the day-ahead market is imprecise, approximate

Part of the decision making gets pushed to the adjustment period – engineering / operator adjustment (not optimized) takes over to obtain feasibility



# Adjustment Period

Market operators must adjust market solutions to create realistic, feasible solutions

- Many different terms: Uneconomic adjustments; supplemental dispatch; out-of-merit capacity; out-of-merit energy; exceptional dispatches; reserve disqualification; reserve downflags
- We call these adjustments: **out-of-market corrections**
- Accounting for such corrections is key when evaluating new algorithms (e.g., stochastic programs)

Break

# Two Stage Programs



Separation between academic models  
and reality and what this means for the  
transition from deterministic to  
stochastic

# Representation of the Network Flow Model

- The actual network flow model is approximated
- Key benefit of stochastic programming is the ability to **locate reserves on a nodal basis**
- With approximate network flow models, you get approximate solutions as to where to locate reserves
  
- Stochastic programming must not only overcome the complexity of modeling uncertainty but also the complexity to go from such coarse approximate network flow models to more precise network flow models
  - **Huge challenge**

# Operating Reserve Quantity

- Ad-hoc rules
- Typical 10-minute operating reserve quantity requirements in SCUC/SCED:
  - Single largest contingency
  - Proportional to demand/renewables
- CAISO:
  - Largest contingency
  - 5% of load met by hydro + 7% load met by non-hydro

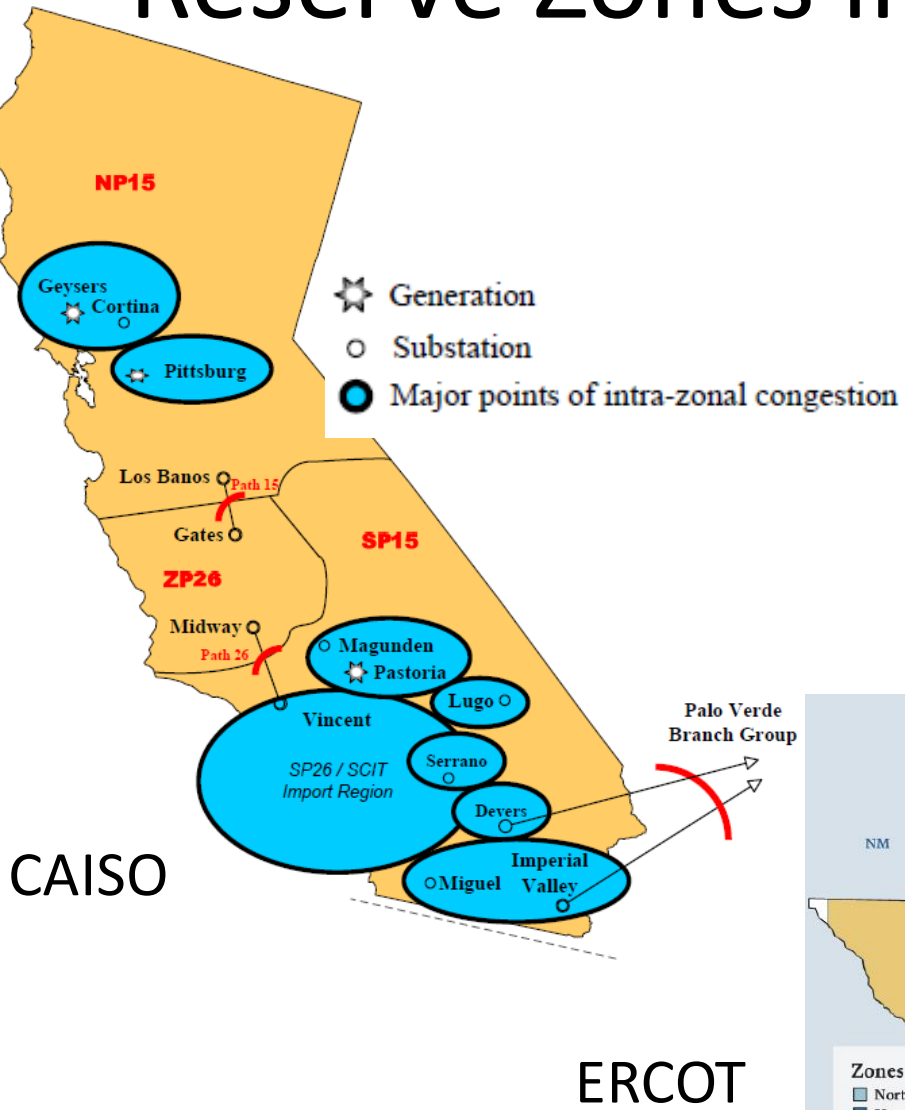
**CAISO is the central auctioneer running the CA market and operating the CA grid**

# Reserve Location

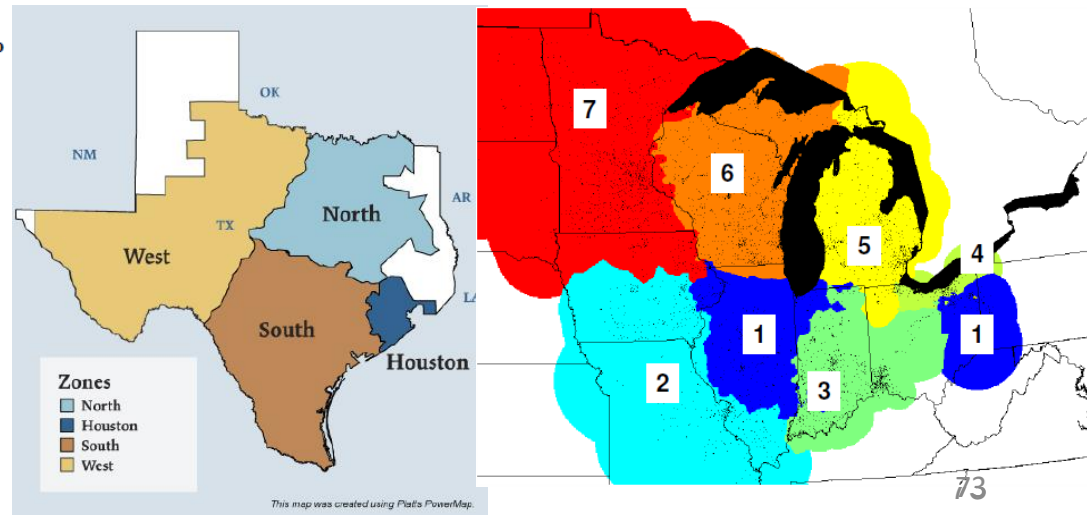
- Stochastic programming
  - Implicitly locate reserve (and determine quantity)
  - Curse of dimensionality
  - Stakeholder issues; Pricing issues
- Reserve zones
  - Traditionally based on ad-hoc rules such as utilities ownership, or geographical boundaries, and treated as static
  - Blindly choose reserve inside reserve zones
  - It is assumed that reserve can be delivered without congestion inside the zone

# Reserve Zones in Existing Markets

- CAISO has 3 reserve zones
- Their reserve rules do not account for intra-zonal congestion
- Intra-zonal congestion is account for by other rules



Area 1 is a part of PJM MISO





# Advanced Algorithms

- Lagrange Relaxation
  - Used to be the main approach for deterministic SCUC before being replaced by MIP (e.g., B&B) based methods
  - Now being considered to assist with stochastic SCUC
- Benders' Decomposition
- Progressive Hedging (an augmented Lagrangian approach)
- ADMM (another augmented Lagrangian approach)
  - Alternating Direction Method of Multipliers

# Summary

- Model complexity will grow with computational capability
- Smart, well-designed reserve policies will benefit near-term operations and future advances
- Expert systems balanced with advanced optimization algorithms
  - Can improve efficiency while also improving scalability
- Commercial grade changes to planning, operational planning, and real-time operational optimization software will include such expert system based approaches; there will not be a direct jump to stochastic programming