Module 1

Inertia in the Power System

What does inertia do for us and where does it come from?
Module 1a

Intuition on Inertia’s Importance

What does inertia do for us and where does it come from?
The Power System Connects Centralized Generation to Distant Loads

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The Power System Connects Centralized Generation to Distant Loads

Fuel burned to power turbine generator (synchronous machine)
All **synchronized** to 60Hz
The Power System Connects Centralized Generation to Distant Loads

Hundreds or thousands of km separation
The Power System Connects Centralized Generation to Distant Loads

Distant load consumes power
The System with Widely Distributed Renewable Generation

Renewable resources (wind and solar) will “plug in” widely along this network.
The Power System Must be Resilient

Failure modes:
The Power System Must be Resilient

Failure modes:
• Transmission line outage
The Power System Must be Resilient

Failure modes:
• Transmission line outage
• Generator outage
The Power System Must be Resilient

Failure modes:

- Transmission line outage
- Generator outage
- Bus outage
Maintaining System Frequency is Crucial

Failure modes:
- Transmission line outage
- Generator outage
- Bus outage

Frequency limit:
59.5Hz min, 60.5Hz max
~0.8% variation
Turbine Generator Mechanical Model

Fuel → Boiler → Valve → Turbine → Speed control → Synchronous Generator → Electrical Power

\[ \omega_m \] → \[ \omega_e \]
Grid Frequency Set by Generator Rotational Speed

Fuel → Boiler → Valve → Turbine → \( \omega_m \) → Synchronous Generator → \( \omega_e \) → Electrical Power

\[ V_o \sin \omega_e t \]

\[ \omega_e = k \omega_m \]

e.g. rotate at 3600 RPM to produce 60Hz
Rotating Masses “Store” Kinetic Energy

Kinetic energy: \( \frac{1}{2} J \omega_m^2 \)

- \( J \): moment of inertia
- \( \omega_m \): angular speed
Nominal Operation, \( P_{in} = P_{out} \)

Power System

\[ \omega_e = k \omega_m \]

Kinetic Energy

\[ \frac{1}{2} J \omega_m^2 \]
Generator Outage, $P_{in} < P_{out}$

Power System

$\omega_e = k \omega_m$

Kinetic Energy

$\frac{1}{2} J \omega_m^2$

Supplies the difference
Generator Outage, $P_{in} < P_{out}$

Power System:

$\omega_e = k\omega_m$

Kinetic Energy:

$\frac{1}{2} I\omega_m^2$

Supplies the difference
Intuition from Energy Balance Perspective

• Power system **stores inertial energy** in generators

• When an outage occurs, this energy serves as a “buffer”
  • Decreases for $P_{in} < P_{out}$

• Generator speed is directly affected by outages

::: The system frequency is directly affected by power imbalances in the grid
Dynamic Synchronous Generator Model

A first step towards studying power system dynamics
Generator Power Balance

\[ P_m = P_e + \frac{d}{dt} W_{kinetic} + P_{friction} \]

(Ignore machine losses, except for mechanical friction)
Generator Power Balance

\[ P_m = P_e + \frac{d}{dt} W_{\text{kinetic}} + P_{\text{friction}} \]

(Ignore machine losses, except for mechanical friction)
Generator Modeling Assumptions

1. Balanced three-phase positive-sequence operation
Generator Modeling Assumptions

1. Balanced three-phase positive-sequence operation
2. Constant machine excitation

\[ N_S = i_{dc} \]
Generator Modeling Assumptions

1. Balanced three-phase positive-sequence operation
2. Constant machine excitation
3. Ignore saturation and saliency (so, round rotor, constant airgap)
Generator Modeling Assumptions

1. Balanced three-phase positive-sequence operation
2. Constant machine excitation
3. Ignore saturation and **saliency** (so, round rotor, constant airgap)
4. The turbine-generator has a very large moment of inertia
\[ P_e = 3 \frac{|E||V_t|}{X_d} \sin \delta \]

Per phase equivalent (Assumption 1)

Constant internal voltage (Assumption 2)

Assumption 3
Simplified Generator Electrical Model

\[ P_e = 3 \frac{|E||V_t|}{X_d} \sin\delta \]

Phase angle of phase “a” internal voltage
Rotor Position During Transients

\[ \theta(t) = \omega_m t + \theta_0 \]

Smooth rotation
Rotor Position During Transients

\[ \theta(t) = \omega_m t + \theta_0 + \Delta\theta(t) \]

Transient rotor angle
Rotor Position Relation to Electrical Angle

\[
\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)
\]

\[
\delta(t) = \theta_0 + \Delta\theta(t) - \frac{\pi}{2}
\]

When rotor angle is 90 deg, maximum coupling to phase "a"

Assuming equal number of rotor and stator poles
Generator Power Balance

\[ P_m = P_e + \frac{d}{dt}W_{\text{kinetic}} + P_{\text{friction}} \]

(Ignore machine losses, except for mechanical friction)
Rotor Speed and Acceleration

\[ W_{\text{kinetic}} = \frac{1}{2} J \dot{\theta}^2 \]

\[ \theta(t) = \omega_m t + \theta_0 + \Delta \theta(t) \]

\[ \frac{d}{dt} \theta(t) = \omega_m + \frac{d}{dt} \Delta \theta(t) \]
Rotor Speed and Acceleration

\[ \theta(t) = \omega_m t + \theta_0 + \Delta \theta(t) \]

\[ \frac{d}{dt} \theta(t) = \omega_m + \frac{d}{dt} \Delta \theta(t) \approx \omega_m \]

Speed of transient rotor angle small relative to shaft speed due to large inertia (Assumption 4)
Rotor Speed and Acceleration

\[ \theta(t) = \omega_m t + \theta_0 + \Delta\theta(t) \]

\[ \frac{d}{dt} \theta(t) = \omega_m + \frac{d}{dt} \Delta\theta(t) \approx \omega_m \]

\[ \frac{d^2}{dt^2} \theta(t) = \frac{d^2}{dt^2} \Delta\theta(t) \]
Kinetic Energy Variation

\[ W_{kinetic} = \frac{1}{2} J \dot{\theta}^2 \]

\[ \frac{d}{dt} W_{kinetic} = J \dot{\theta} \ddot{\theta} = J \omega_m \ddot{\theta} \]

= \[ J \omega_m \delta \]

Angular momentum of rotor  Rotor acceleration
The Per Unit Inertia Constant, \( H \)

Steady-state rotor energy

\[
H = \frac{W_{\text{kinetic}}^0}{S_{\text{base}}} = \frac{J\omega_m^2}{2S_{\text{base}}} \quad [s]
\]

Typ. 1-10 seconds

MVA rating of generator

\[
\frac{d}{dt} \frac{W_{\text{kinetic}}}{S_{\text{base}}} = \frac{H}{\pi f_m} \ddot{\delta} = M \ddot{\delta}
\]

Rotor speed in Hz
Friction Losses

\[ P_{\text{friction}} = k\dot{\theta}^2 = k\omega_m^2 + 2k\omega_m\delta \]

Static term, not critical – can be subtracted from input mech power

\[ P_{\text{friction}} \approx 2k\omega_m\delta \]

Define \( D = \frac{2k\omega_m}{S_{\text{base}}} \)
The “Swing Equation”

\[ P_m = 3 \frac{|E||V_t|}{X_d} \sin \delta + 2k \omega_m \dot{\delta} + J \omega_m \ddot{\delta} \]

\[ \frac{P_m}{S_{base}^3 \phi} = \frac{e v_t}{X_d} \sin \delta + D \dot{\delta} + M \ddot{\delta} \]

Non-linear differential equation describing “swings” in power angle during transients
Example: Increase in Mechanical Power

(Not practical, prime mover dynamics on order of seconds, but insightful)
Example: Increase in Mechanical Power

(Not practical, prime mover dynamics on order of seconds, but insightful)

Assume negligible friction
\( f_m = 60 \text{Hz} \)

Per unit quantities:
\( H = 5 \)
\( x_a = 0.2 \)
\( e = 1.2 \)
\( v = 1 \)
\( p_{m1} = 0.6, p_{m2} = 1.8 \)

How does \( \delta \) evolve?
Determine Initial Conditions

\[ \delta(0^-) = \delta(0^+) = \delta_0 \]
\[ \dot{\delta} = 0 \]

\[ p_{m1} = \frac{ev}{x_d} \sin \delta_0 \Rightarrow \delta_0 = 0.2527 \text{ rad.} = 14.5^\circ \]

Assume negligible friction
\[ f_m = 60 \text{Hz} \]

Per unit quantities:
\[ H = 5, x_d = 0.5, e = 1.2 \]
\[ v = 1, p_{m1} = 0.6, p_{m2} = 1.8 \]
Check Final Condition

\[ \delta(0^-) = \delta(0^+) = \delta_0 \]
\[ \dot{\delta} = 0 \]

\[ p_{m1} = \frac{ev}{x_d} \sin \delta_0 \Rightarrow \delta_0 = 0.2527 \text{ rad.} \]
\[ = 14.5^\circ \]

We know the final condition too:

\[ \delta_\infty = 0.848 \text{ rad.} = 48.6^\circ \]

Assume negligible friction

\[ f_m = 60\text{Hz} \]

Per unit quantities:

\[ H = 5, x_d = 0.5, e = 1.2 \]
\[ v = 1, p_{m1} = 0.6, p_{m2} = 1.8 \]
Define Governing Equation

Governing equation:

\[
p_m = p_{e,max} \sin \delta + D \dot{\delta} + M \ddot{\delta}
\]

Split into two first order equations:

\[
x_1 = \delta, \ x_2 = \dot{\delta}
\]

\[
\dot{x}_1 = \dot{\delta} = x_2
\]

\[
\dot{x}_2 = \frac{p_m - D x_2 - p_{e,max} \sin x_1}{M}
\]

Must solve numerically

Assume negligible friction

\[f_m = 60 \text{Hz}\]

Per unit quantities:

\[H = 5, \ x_d = 0.5, \ e = 1.2\]

\[v = 1, \ p_{m1} = 0.6, p_{m2} = 1.8\]
Oscillates Around $\delta_\infty$, Variation with D

$p_e$

$p_{m2}$

$p_{m1}$

$\pi/2$ $\pi$

94.5°

14.4°
Oscillates Around $\delta_\infty$, Variation with $H$
The “Equal-Area Criterion”

- In our example, these areas are equal
- Physical meaning
The “Equal-Area Criterion”

- In our example, these areas are equal
- Physical meaning

- Generator output > input
  - Rotor decelerates

- Rotor accelerates

- A useful way to check stability of a single machine
  (extendable to two-machine system)
Example: Generator Fault

Three-phase to ground bolted short on generator terminals

Assume negligible friction

\[ f_m = 60 \text{Hz} \]

Per unit quantities:

\[ H = 5 \]
\[ x_a = 0.5 \]
\[ e = 1.2 \]
\[ v = 1.0 \]
\[ p_m = 1.0 \]

How does \( \delta \) evolve?
**Condition for Instability**

In this example, $\delta_{max} = 91.7^\circ$.

Accelerating energy cannot be removed, shaft speed increases, lose synchronism.

If reconnected here, guaranteed instability.
Solution is Monotonic!

Worst-case example with no damping, but:

we’re on the clock

If fault cleared too late, generator loses synchronism

Higher inertia, more time to respond

\[ \delta = \frac{p_m}{2M} t^2 + \delta_1 \]
Conclusions

• Synchronous generator dynamic model derived from power balance

• Nonlinear swing equation defines rotor angle evolution
• System is stable when

• Higher inertia systems evolve more slowly
Multimachine Frequency Dynamics

A model for studying disturbances in the power system
Classical Model Used for “First Swing Analysis”

- Simplest model used in stability studies
- Limited to relatively short time-scales (order of seconds)

Traditional Primary Control

- Primary frequency control: first 30 seconds
Secondary and Tertiary Control

• Secondary frequency control, 30s to 10s of minutes

• Tertiary: After ~15 mins, adapt generator and load set points
Rate of Change of Frequency

- **ROCOF**
  - Inversely proportional to system inertia
- Provides time for primary frequency control to adjust prime mover output

Power System Classical Model
Power System Classical Model

\[ E_1 \angle \delta_1 + \beta_1 \]
\[ jX_{d1} \]
\[ I_1 \]
\[ + \]
\[ V_{a1} \angle \beta_1 \]
\[ - \]
\[ V_{an} \angle \beta_n \]

\[ E_n \angle \delta_n + \beta_n \]
\[ jX_{dn} \]
\[ I_n \]

Transmission Network

- Constant \( p_{mi} \)
- \( D=0 \)
Power System Classical Model

$$E_1 \angle \delta_1 + \beta_1$$

$$jX_{d1}$$

$$I_1$$

$$V_{a1} \angle \beta_1$$

$$jX_{dn}$$

$$I_n$$

$$V_{an} \angle \beta_n$$

Transmission Network

$$Z_{L1}$$

$$I_{L1}$$

$$Z_{L2}$$

$$I_{Lm}$$

- Constant impedance loads
Power System Classical Model

Internal machine node

Reference node

Transmission Network

Generator bus

Load bus

\[ E_1 \angle \delta_1 + \beta_1 \]

\[ E_n \angle \delta_n + \beta_n \]

\[ I_1 \]

\[ I_n \]

\[ jX_{d1} \]

\[ jX_{dn} \]

\[ V_{a1} \angle \beta_1 \]

\[ V_{an} \angle \beta_n \]

\[ Z_{L1} \]

\[ Z_{L2} \]
Obtain a System of Swing Equations

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators

\[ p_{ei} = \text{Re}\{e_i i_i^*\} \]

- Must solve network equation:

\[
\begin{bmatrix}
Y_{nn} & Y_{ns} \\
Y_{ns}^T & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
e_n \\
v_s
\end{bmatrix} =
\begin{bmatrix}
i_n \\
0
\end{bmatrix}
\]
Mathematical Network Description

\[ E_1 \angle \delta_1 + \beta_1 \]

\[ jX_{d1} \]

\[ I_1 \]

\[ jX_{dn} \]

\[ V_{a1} \angle \beta_1 \]

\[ V_{an} \angle \beta_n \]

\[ I_n \]

\[ I_{L1} \rightarrow Z_{L1} \]

\[ I_{Lm} \rightarrow Z_{L2} \]
Mathematical Network Description

\[ E_1 \angle \delta_1 + \beta_1 \]

\[ I_1 \]

\[ jX_{d1} \]

\[ V_{a1} \angle \beta_1 \]

\[ E_n \angle \delta_n + \beta_n \]

\[ jX_{dn} \]

\[ V_{an} \angle \beta_n \]

\[ I_n \]

\[ I = Y_{bus} V \]

Describes network

Vector of all currents

Vector of bus voltages

\[ I_L \]

\[ Z_{L1} \]

\[ Z_{L2} \]
Mathematical Network Description

\[ I = \hat{Y}E \]

Describes network, load, and generator impedances

Vector of generator currents

Vector of internal generator voltages

\[ E_1 \angle \delta_1 + \beta_1 \]

\[ E_n \angle \delta_n + \beta_n \]
Admittance Matrix Definition

\[
\hat{Y} = \begin{bmatrix}
\hat{G}_{11} + j\hat{B}_{11} & \cdots & \hat{G}_{1n} + j\hat{B}_{1n} \\
\vdots & \ddots & \vdots \\
\hat{G}_{n1} + j\hat{B}_{n1} & \cdots & \hat{G}_{nn} + j\hat{B}_{nn}
\end{bmatrix}
\]

For \( n \) generators

\[
I_i = \sum_{k=1}^{n} Y_{ik} E_k
\]
Generator Power a Bit More Involved

\[ \hat{Y} = \begin{bmatrix} \hat{G}_{11} + j\hat{B}_{11} & \cdots & \hat{G}_{1n} + j\hat{B}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{G}_{n1} + j\hat{B}_{n1} & \cdots & \hat{G}_{nn} + j\hat{B}_{nn} \end{bmatrix} \]

For \( n \) generators

\[ I_i = \sum_{k=1}^{n} Y_{ik} E_k \]

\[ S_i = V_i I_i^* = \sum_{k=1}^{n} Y_{ik}^* E_k^* \]

\[ P_i = \sum_{k=1}^{n} |E_i||E_k| [\hat{G}_{ik} \cos(\delta_i - \delta_j) + \hat{B}_{ik} \sin(\delta_i - \delta_j)] \]
Multimachine Swing Equation

\[ M_i \ddot{\delta}_i = p_{mi} - \sum_{k=1}^{n} |e_i||e_k| [\hat{g}_{ik} \cos(\delta_i - \delta_k) + \hat{b}_{ik} \sin(\delta_i - \delta_k)] \]

\[ p_{ei} \]

Now, multivariable definition:

\[ x_i = \delta_i \]
\[ x_{i+n} = \dot{\delta}_i \]

\[ \dot{x}_i = \dot{\delta}_i = x_{i+n} \]
\[ \dot{x}_{i+n} = \frac{p_{mi} - p_{ei}}{M} \]

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix} \]

\text{e.g. } n = 2
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers.
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Apply fault by modifying admittance matrix Y

Solve multimachine swing equation until fault cleared

Solve multimachine swing equation for new steady state
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Apply fault by modifying admittance matrix $Y$

Solve multimachine swing equation for fault duration
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Solve multimachine swing equation for fault duration

Apply post-fault conditions by modifying $Y$

Apply fault by modifying admittance matrix $Y$
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Apply fault by modifying admittance matrix $Y$

Solve multimachine swing equation for fault duration

Apply post-fault conditions by modifying $Y$

Solve multimachine swing equation for new steady state
Example Solution for 7 Bus, 3 Generator system
Conclusion

• Multimachine frequency dynamics are a straightforward conceptual extension of single-machine dynamics

• Classical model enables “first swing analysis” to determine inertial response of electromechanical system. Inertia buys us time.

• Looking ahead… we wouldn’t need so much inertia if we could respond more quickly!
Conclusion

• Multimachine frequency dynamics are a straightforward conceptual extension of single-machine dynamics.

• Classical model enables “first swing analysis” to determine inertial response of electromechanical system. **Inertia buys us time.**

• Looking ahead… we wouldn’t need so much inertia if we could respond more quickly.

• Much higher detail can be added (damper circuits, rotor and stator circuits, detailed flux linkages, higher level control) by extensions of the classical model principle.
Example Solution for 7 Bus, 3 Generator system
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

\[
E_1 \angle \delta_1 + \beta_1
\]

\[
I_1
\]

\[
1.04 \angle 0^\circ
\]

\[
j0.08
\]

\[
1.02 \angle -3.55^\circ
\]

\[
j0.18
\]

\[
I_2
\]

\[
E_2 \angle \delta_2 + \beta_2
\]

\[
0.65 + j0.2
\]

\[
0.99 \angle -7.5^\circ
\]
Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Determine pre-fault admittance matrix

Apply fault by modifying admittance matrix $Y$

Apply post-fault by modifying $Y$

Solve multimachine swing equation until fault cleared

Solve multimachine swing equation
Example of Applying Fault

\[
p_{e1} = |e_1|^2 \hat{g}_{11} + |e_1||e_2|[\hat{g}_{12} \cos(\delta_1 - \delta_2) + \hat{b}_{12} \sin(\delta_1 - \delta_2)]
\]

\[
p_{e2} = |e_2|^2 \hat{g}_{22} + |e_1||e_2|[\hat{g}_{21} \cos(\delta_2 - \delta_1) + \hat{b}_{21} \sin(\delta_2 - \delta_1)]
\]

Now, multivariable definition:

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix}
\]

\[
\dot{x}_i = \dot{\delta}_i = x_{i+n}
\]

\[
\dot{x}_{i+n} = \frac{p_{mi} - p_{ei}}{M}
\]
Network Component Descriptions

\[
\begin{bmatrix}
Y_{nn} & Y_{ns} \\
Y_{ns}^T & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
e_n \\
v_s
\end{bmatrix}
=
\begin{bmatrix}
i_n \\
0
\end{bmatrix}
\]

\[
i =
\begin{bmatrix}
i_1 \\
\vdots \\
i_n
\end{bmatrix}
\quad
\begin{bmatrix}
e_1 \\
\vdots \\
e_n
\end{bmatrix}
\quad
\begin{bmatrix}
v_1 \\
\vdots \\
v_m
\end{bmatrix}
\]

generator currents

Generator internal voltages

Bus voltages
(non-generator)
Obtain a System of Swing Equations

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators

\[ p_{ei} = \text{Re}\{e_i i_i^*\} \]

- Must solve network equation:

\[
\begin{bmatrix}
Y_{nn} & Y_{ns} \\
Y_{ns}^T & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
e_n \\
n_s
\end{bmatrix}
=
\begin{bmatrix}
i_n \\
0
\end{bmatrix}
\]
Proceeds as Before

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

\( V_t \angle 0^\circ \)

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators

\[
\begin{bmatrix}
Y_{nn} & Y_{ns} \\
Y_{ns}^T & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
e_n \\
v_s
\end{bmatrix}
= \begin{bmatrix} i_n \\
0
\end{bmatrix}
\]
Obtain a System of Swing Equations

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators

\[ p_{ei} = \text{Re}\{e_i i_i^*\} \]

- Must solve network equation:

\[
\begin{bmatrix}
Y_{nn} & Y_{ns} \\
Y_{ns}^T & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
e_n \\
n_s
\end{bmatrix}
=
\begin{bmatrix}
i_n \\
0
\end{bmatrix}
\]
Obtain a System of Swing Equations

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators
- \( p_{ei} = \text{Re}\{e_i i_i^*\} \)

- Must solve network equation:

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
\nu_a \\
e
\end{bmatrix}
= \begin{bmatrix}
0 \\
i
\end{bmatrix}
\]
Network Component Descriptions

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
v \\
e
\end{bmatrix}
=
\begin{bmatrix}
0 \\
i
\end{bmatrix}
\]

Assume \(N\) system busses and \(M\) internal machine buses

\[
i = \begin{bmatrix}
  i_1 \\
  \vdots \\
  i_m
\end{bmatrix}
\quad e = \begin{bmatrix}
  e_1 \\
  \vdots \\
  e_m
\end{bmatrix}
\quad v = \begin{bmatrix}
  v_1 \\
  \vdots \\
  v_n
\end{bmatrix}
\]

\(M\) vector of generator currents and internal voltages

\(N\) vector of bus voltages
Admittance Matrix, $Y_{22}$

N system busses and M internal machine buses

\[
\begin{bmatrix}
  Y_{11} & Y_{12} \\
  Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{v} \\
  \mathbf{e}
\end{bmatrix}
= 
\begin{bmatrix}
  0 \\
  \mathbf{i}
\end{bmatrix}
\]

$Y_{11}\mathbf{v} + Y_{12}\mathbf{e} = 0$

$Y_{12}^T\mathbf{v} + Y_{22}\mathbf{e} = \mathbf{i}$

\[
Y_{22} =
\begin{bmatrix}
  \frac{1}{jX_{d1}} & & & 0 \\
  & \frac{1}{jX_{d2}} & & \\
  & & \ddots & \\
  0 & & & \frac{1}{jX_{dM}}
\end{bmatrix}
\]

MxM diagonal matrix of reciprocal generator reactances
Network Component Descriptions

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
v_a \\
e
\end{bmatrix} =
\begin{bmatrix}
0 \\
i
\end{bmatrix}
\]

Assume \( N \) system busses and \( M \) internal machine buses

\[
i = \begin{bmatrix}
i_1 \\
\vdots \\
i_m
\end{bmatrix} \quad e = \begin{bmatrix}
e_1 \\
\vdots \\
e_m
\end{bmatrix} \quad v_a = \begin{bmatrix}
v_1 \\
\vdots \\
v_n
\end{bmatrix}
\]

\( M \) vector of generator currents and internal voltages

\( N \) vector of bus voltages
Mathematical Network Description

\[ E_1 \angle \delta_1 + \beta_1 \]

\[ E_n \angle \delta_n + \beta_n \]

\[ I_1 \]

\[ I_n \]

\[ V_{a1} \angle \beta_1 \]

\[ V_{an} \angle \beta_n \]

\[ I = Y_{bus} V \]

Describes network and loads

Vector of generator currents

Vector of bus voltages
Admittance Matrix, $Y_{22}$

N system busses and M internal machine buses

$$
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
v \\
e
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
i
\end{bmatrix}
$$

$$Y_{11}v + Y_{12}e = 0$$

$$Y_{12}^Tv + Y_{22}e = i$$

Diagonal matrix of reciprocal generator reactances

$$
Y_{22} =
\begin{bmatrix}
\frac{1}{jX_{d1}} & 0 \\
0 & \frac{1}{jX_{d2}} \\
& \ddots \\
0 & & \frac{1}{jX_{dM}}
\end{bmatrix}
$$
Network Component Descriptions

NxN
Relates

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y^{T}_{12} & Y_{22}
\end{bmatrix}
\]
Network Component Descriptions

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12}^T & Y_{22}
\end{bmatrix}
\begin{bmatrix}
v_a \\
e
\end{bmatrix}
= \begin{bmatrix}
0 \\
i
\end{bmatrix}
\]

\[
i = \begin{bmatrix}
i_1 \\
\vdots \\
i_n
\end{bmatrix}
\]

generator currents

\[
e = \begin{bmatrix}
e_1 \\
\vdots \\
e_n
\end{bmatrix}
\]

Generator internal voltages

\[
v_a = \begin{bmatrix}
v_{a1} \\
\vdots \\
v_{an}
\end{bmatrix}
\]

Bus voltages
Obtain a System of Swing Equations

\[ M_i \ddot{\delta}_i = p_{mi} - p_{ei} \]

- \( p_{ei} \) for each generator depends on network, loads, and actions of all other generators
- Network is defined by admittance matrix

\[ I_i = \sum_{k=1}^{n} Y_{ik} V_k \]

\[ i_i^* = v_{ai} \quad p_{ei} = \text{Re}\{e_i i_i^*\} \]
Power System Classical Model

- Start with generator model, assume $D=0$

\[ E_i \angle \delta_i + \beta_i \]

\[ I_i \]

\[ jX_{d,i} \]

\[ V_{a,i} \angle \beta_i \]
Exercises
Grid Frequency Set by Generator Rotational Speed

\[ \omega_e = k \omega_m \]

\[ V_o \sin \omega_e t \]

e.g. rotate at 3600 RPM to produce 60Hz
Determine Initial Conditions

\[ \delta(0^-) = \delta(0^+) = \delta_0 \]
\[ \dot{\delta} = 0 \]

\[ p_{m1} = \frac{ev}{x_d} \sin \delta_0 \Rightarrow \delta_0 = 0.4298 \text{rad.} \]
\[ = 24.6^\circ \]

Assume negligible friction
\[ f_m = 60 \text{Hz} \]

Per unit quantities:
\[ H = 5, x_d = 0.5, e = 1.2 \]
\[ v = 1, p_m = 1.0 \]
Define Governing Equation

Governing equation:

\[ p_m = p_{e,\text{max}} \sin \delta + D \dot{\delta} + M \ddot{\delta} \]

Split into two first order equations:

\[ x_1 = \delta, \quad x_2 = \dot{\delta} \]

\[ \dot{x}_1 = \dot{\delta} = x_2 \]

\[ \dot{x}_2 = \frac{p_m - D x_2 - p_{e,\text{max}} \sin x_1}{M} \]

Must solve numerically

Assume negligible friction

\[ f_m = 60 \text{Hz} \]

Per unit quantities:

\[ H = 5, \quad x_d = 0.5, \quad e = 1.2 \]

\[ v = 1, \quad p_{m1} = 0.6, \quad p_{m2} = 1.8 \]
Example: Increase in Mechanical Power

Governing equation:

\[ 2.4 \sin \delta + 0.0265 \ddot{\delta} = 1.8 \]

Split into two first order equations:

\[ \dot{\delta} = \omega(t) \]
\[ \dot{\omega}(t) = 67.92 - 90.57 \sin \delta \]

Use MATLAB to solve

Assume negligible friction
\[ f_m = 60 \text{Hz} \]

Per unit quantities:
\[ H = 5, x_d = 0.5, e = 1.2 \]
\[ v = 1, p_{m1} = 0.6, p_{m2} = 1.8 \]
Example: Increase in Mechanical Power

Assume negligible friction
Per unit quantities:
\[ H = 5 \]
\[ x_d = 0.2 \]
\[ e = 1.2 \]
\[ v = 1 \]

\[ M = \frac{H}{\pi} \]
Example: Increase in Mechanical Power

Assume negligible friction
Per unit quantities:

\[ H = 5 \]
\[ x_d = 0.2 \]
\[ e = 1.2 \]
\[ v = 1 \]
Example: Generator Terminal Fault

\[ \frac{P_m}{S_{base}} = D\dot{\delta} + M\ddot{\delta} \]

Assume negligible

\[ \delta(0) = \delta_1 \]
\[ \dot{\delta}(0) = 0 \]
Example: Generator Terminal Fault

Assume negligible

\[
\frac{P_m}{S_{base}} = D\dot{\delta} + M\ddot{\delta}
\]

\[
\delta(0) = \delta_1
\]

\[
\dot{\delta}(0) = 0
\]

\[
\delta = \frac{P_m}{2MS_{base}} t^2 + \delta_1
\]

Know what the angle will be the moment the fault is cleared. But is this angle acceptable?
Example: Generator Terminal Fault

\[ \frac{P_m}{S_{base}} = D\dot{\delta} + M\ddot{\delta} \]

\[ \delta(0) = \delta_1 \]
\[ \dot{\delta}(0) = 0 \]

\[ \delta = \frac{c_1 M}{D} e^{-\frac{D}{M}t} + \frac{P_m}{DS_{base}} t + c_2 \]

\[ \delta = \delta_1 + \frac{MP_m}{S_{base}D^2} e^{-\frac{D}{M}t} - \frac{MP_m}{S_{base}D^2} + \frac{MP_m}{S_{base}D} t \]
Goal: Understand Transient Stability in the Power System

- Transient stability: maintaining system frequency ("synchronism") after a disturbance
- The swing equation – describes "swing" in power angle during transients
- Linearizing the swing eqn? Not important
- Solving nonlinear swing eqn
- Equal-area stability criterion
Explore This Behaviour in a Single Generator

Mechanical power $\omega_m$ → Synchronous Generator $\omega_e$ → Electrical Power

$$P_{mech} - P_{elec} = I \frac{d\omega_m}{dt} \omega_m$$

$\omega_m = 3600 \text{ RPM} = 377 \text{ rad/s}$
Electrical Model of a Synchronous Machine

• State and provide intuition on the electrical model
• Emphasize the simplifications we are making
• Discuss $P = f(\delta)$, so there’s an inherent connection between the mechanical state and the output power
• We’ll ride this curve as faults happen
Dynamics of a Synchronous Machine

• Derive the dynamic equation of an SM
• In doing so, emphasize the assumptions we are making
• Highlight what the behavior is during a fault and how inertia provides damping
High Inertia Generators are “Stiff”

- Emphasizing the modeling benefits we can derive from the generators having high inertia
Equal Area Criterion?

• Worth covering?
Wind and Solar do Not Provide Inertia

- No inertia in solar
- There’s a spinning turbine in wind… is that useful inertia? No. We’ll explain later
- In general, we’ll discuss more in the next module
The Instability of a 100% Renewable Grid

- First, must have storage to ensure dispatchability
- No system inertia, what happens?
Conclusions

• Energy stored in rotating inertias is fundamental to how the power system handles transients
• Wind, solar, and battery sources do not provide system inertia
• This greatly hampers system stability, unless we do something about it.
• Looking ahead: power electronics can respond to disturbances quickly…