Module 1

Inertia in the Power System

What does inertia do for us and where does it come from?

Arizona State University

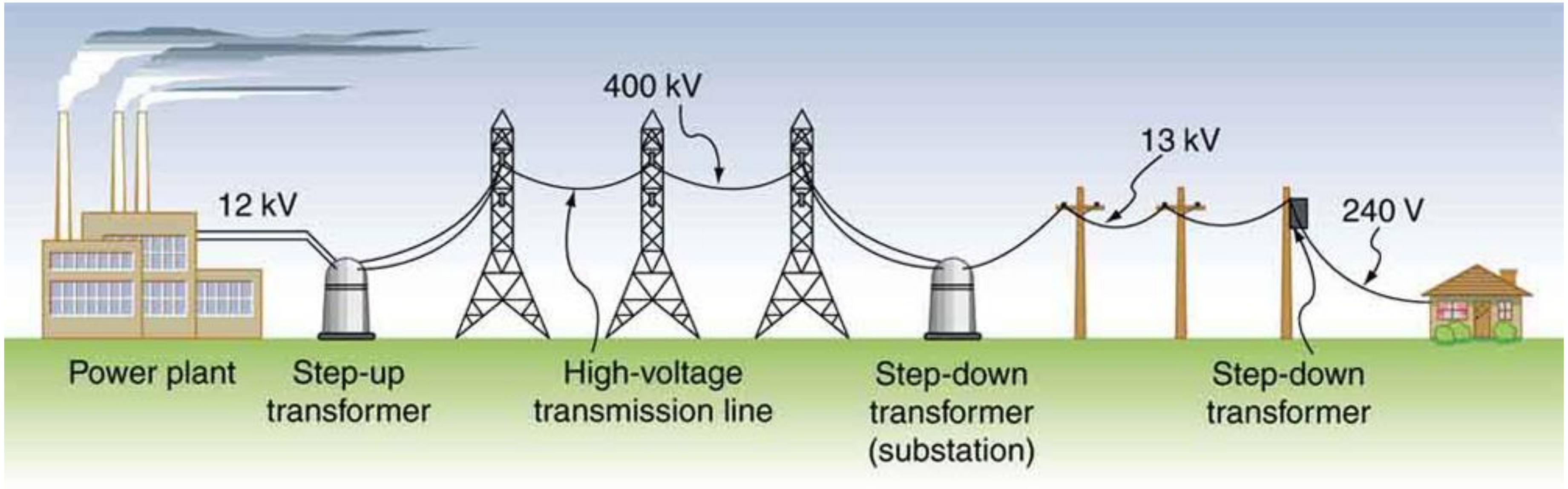
Module 1a

Intuition on Inertia’s Importance

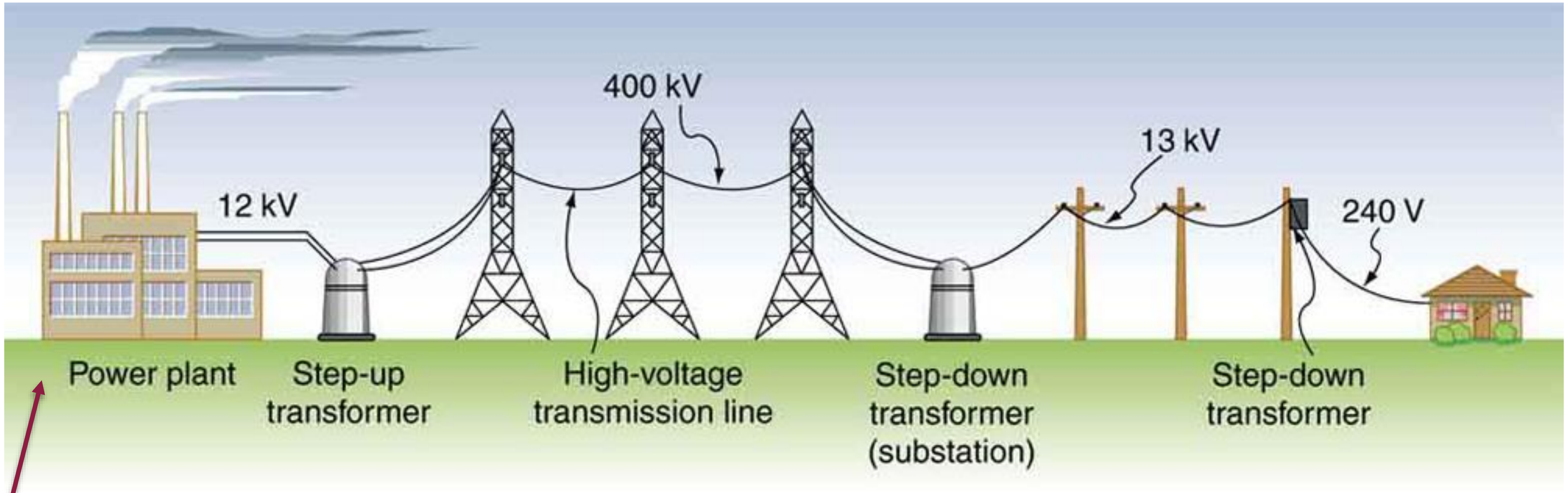
What does inertia do for us and where does it come from?

Arizona State University

The Power System Connects Centralized Generation to Distant Loads

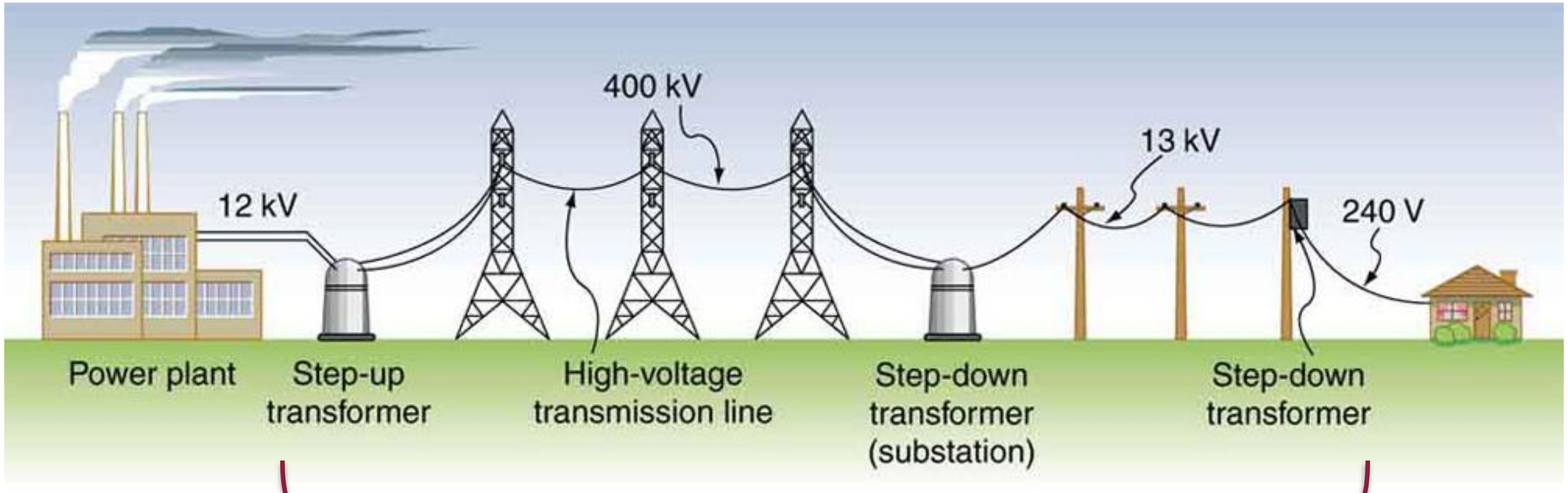


The Power System Connects Centralized Generation to Distant Loads



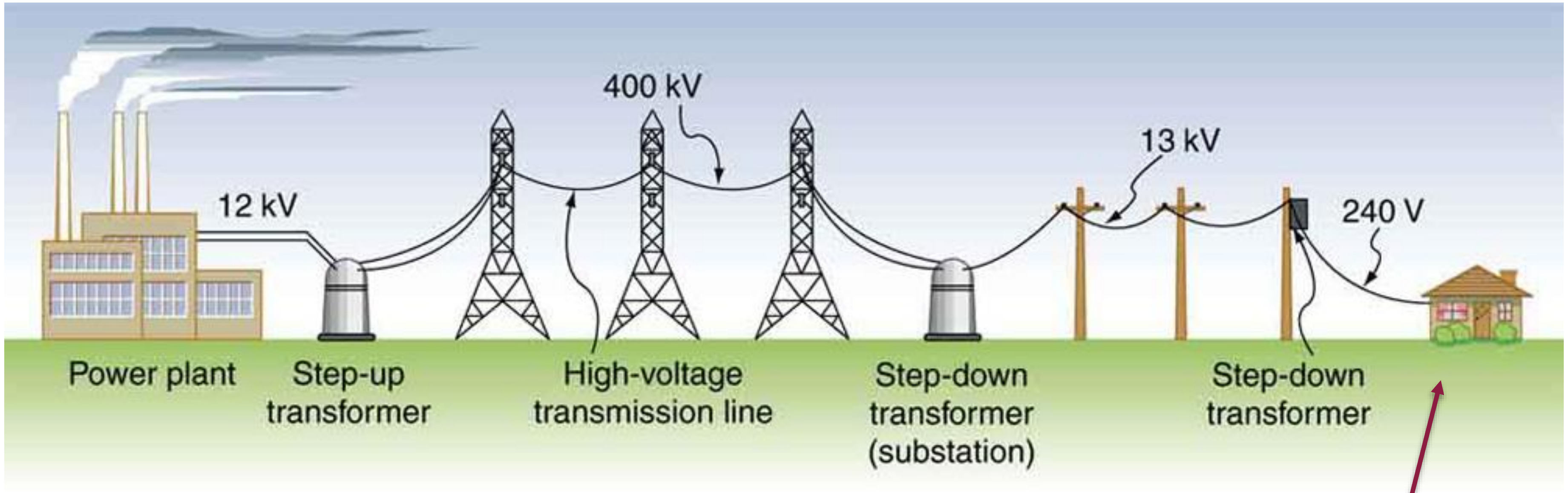
Fuel burned to power turbine generator (synchronous machine)
All **synchronized** to 60Hz

The Power System Connects Centralized Generation to Distant Loads



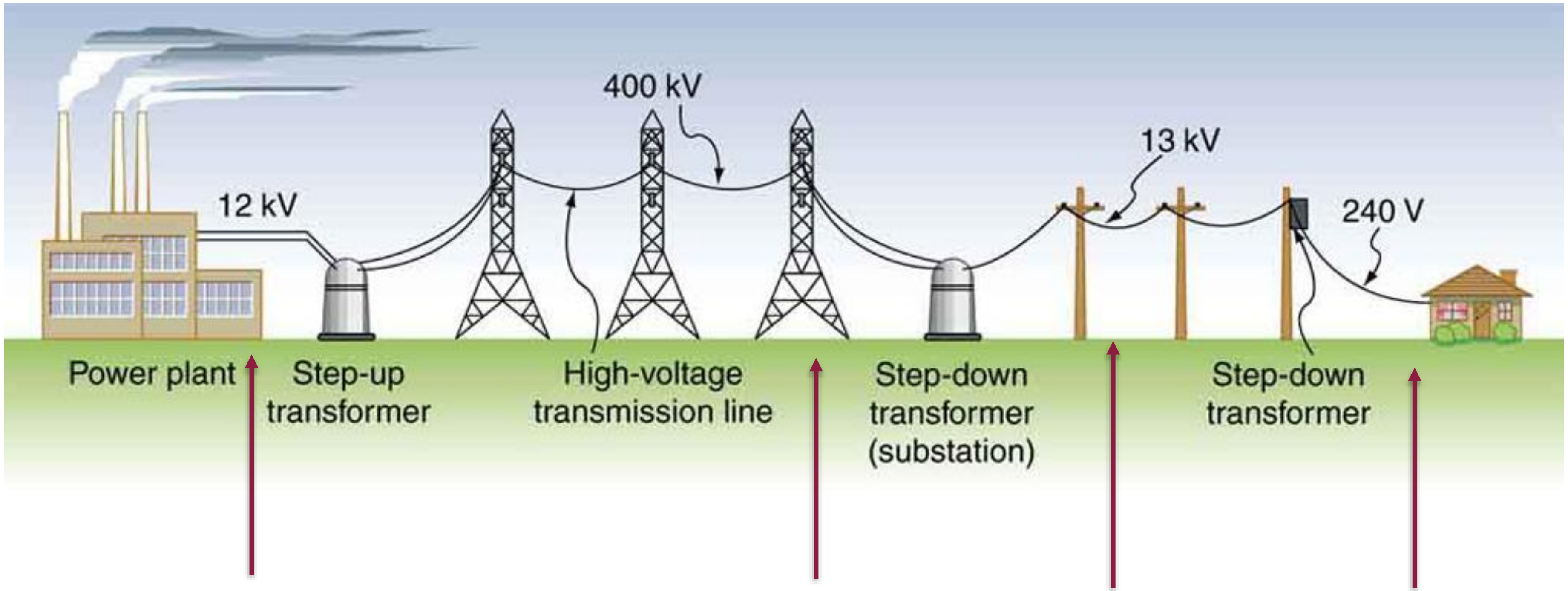
Hundreds or thousands of km separation

The Power System Connects Centralized Generation to Distant Loads



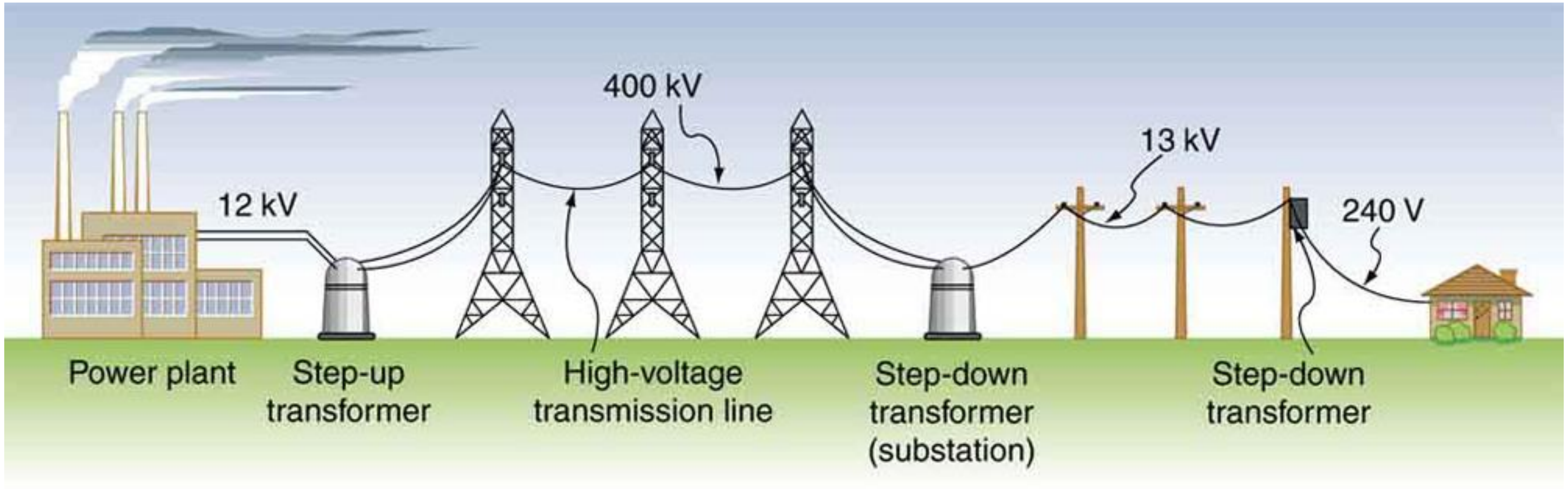
Distant load consumes power

The System with Widely Distributed Renewable Generation



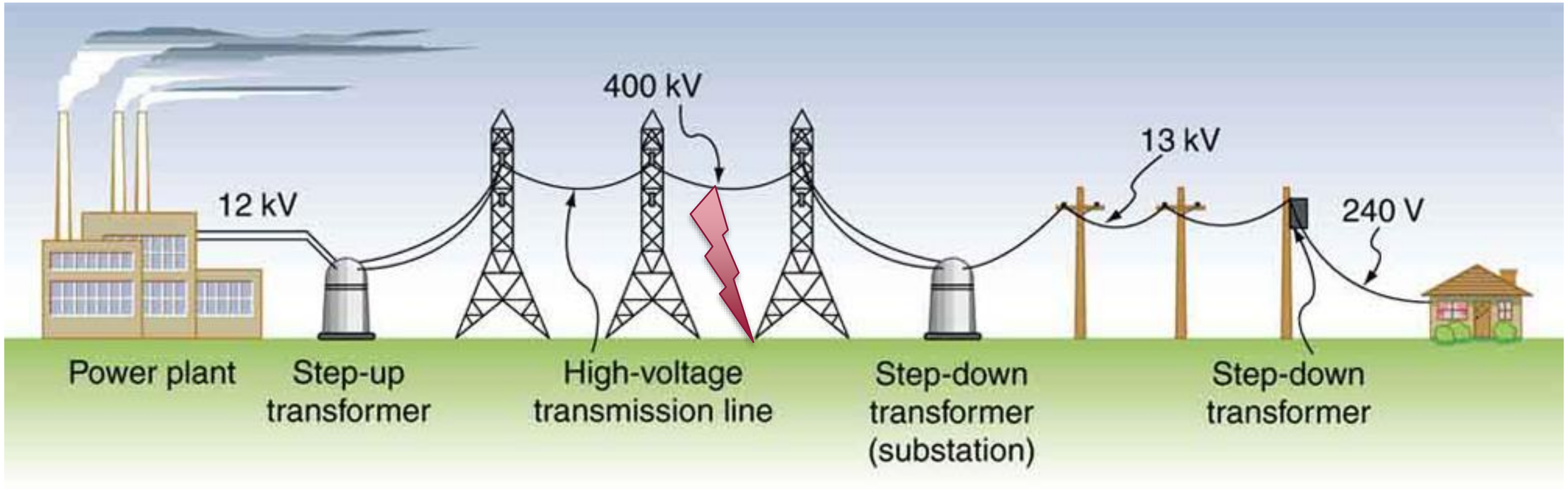
Renewable resources (wind and solar) will “plug in” widely along this network

The Power System Must be <u>Resilient</u>



Failure modes:

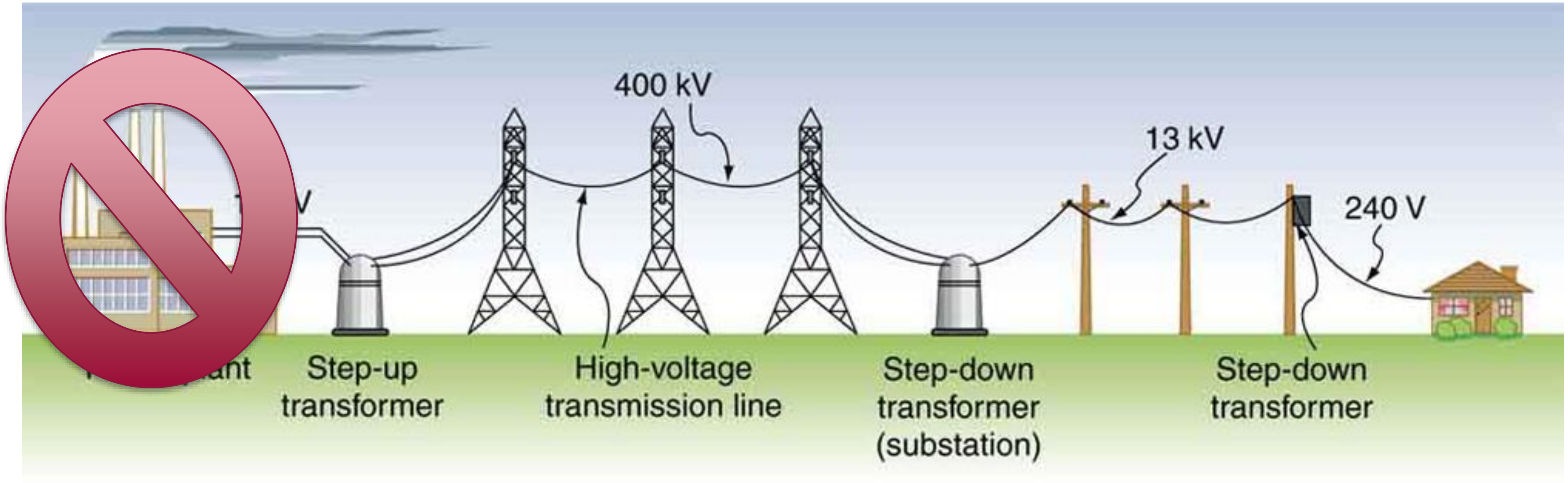
The Power System Must be Resilient



Failure modes:

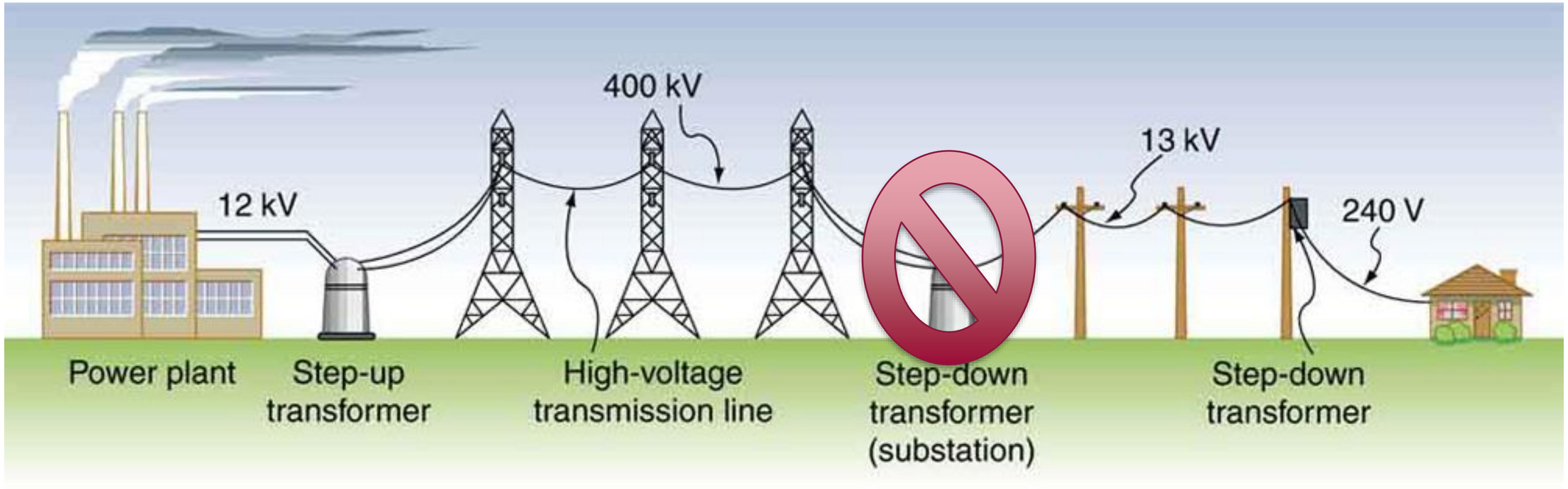
- Transmission line outage

The Power System Must be Resilient



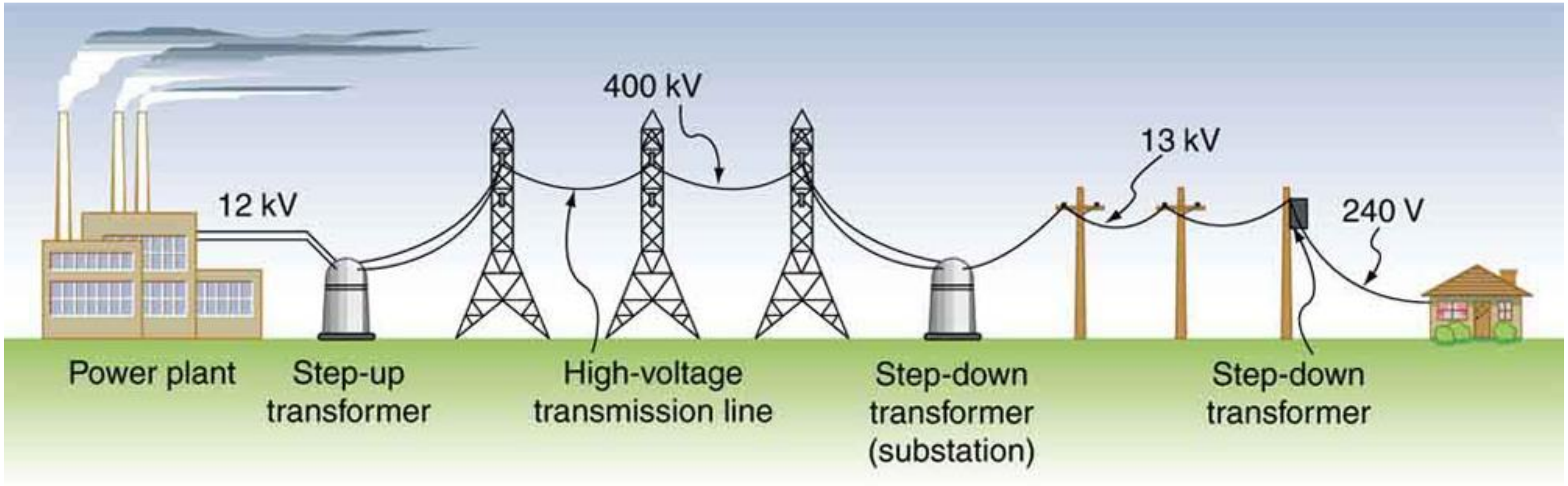
Failure modes:
- Transmission line outage
- Generator outage

The Power System Must be Resilient



Failure modes:
- Transmission line outage
- Generator outage
- Bus outage

Maintaining System Frequency is Crucial

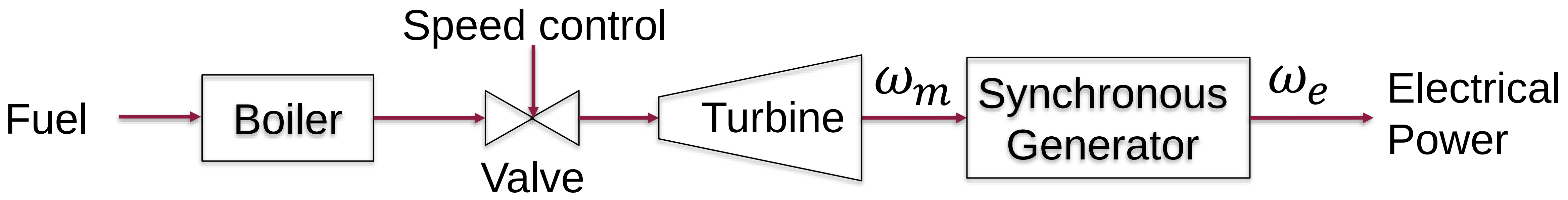


Failure modes:

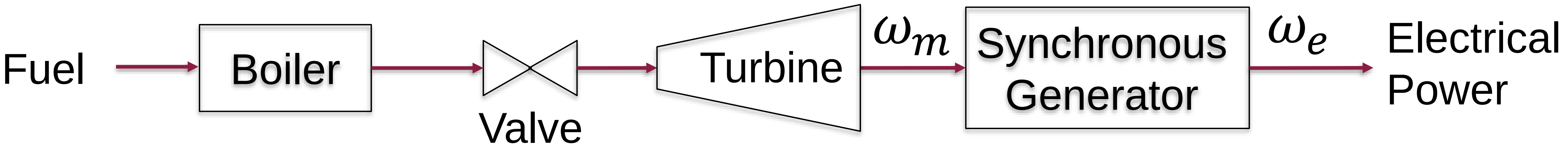
- Transmission line outage
- Generator outage
- Bus outage

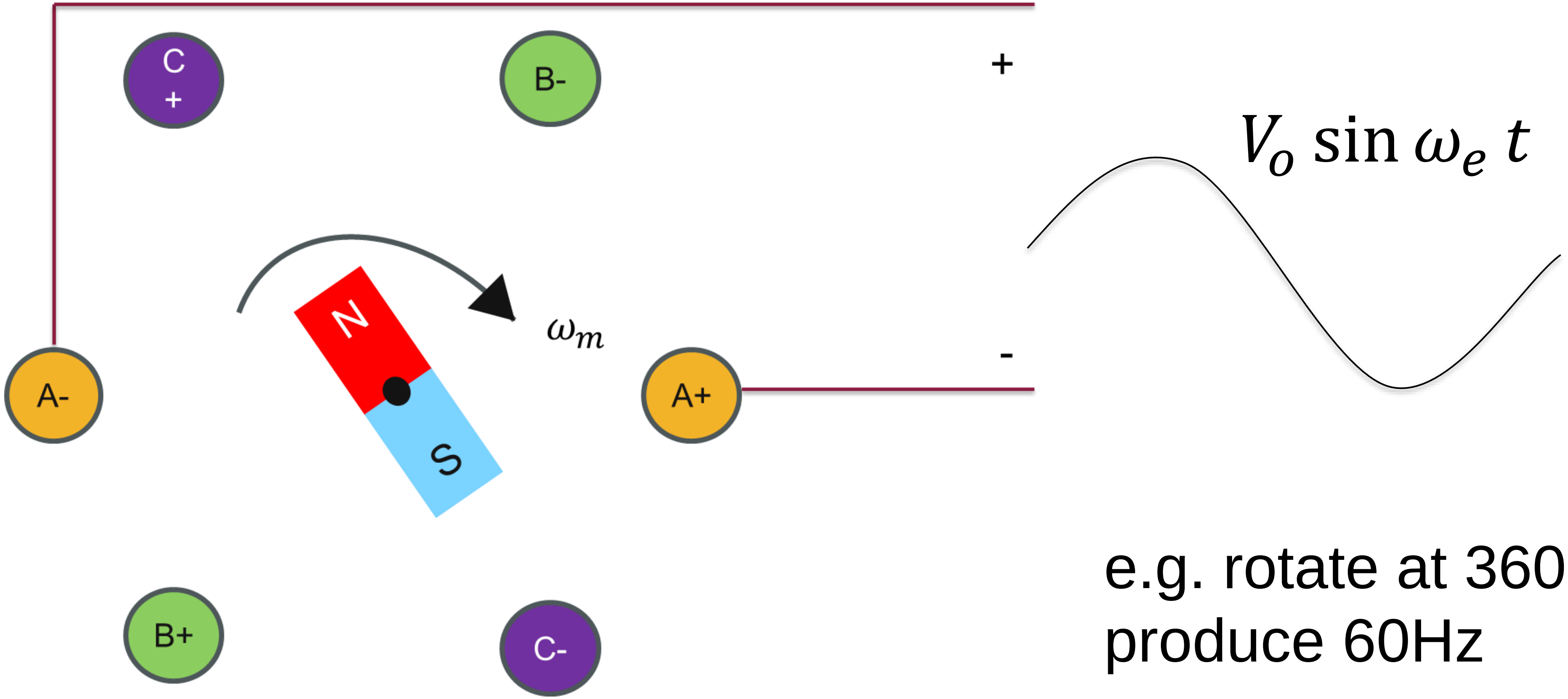
Frequency limit:
59.5Hz min, 60.5Hz max
~0.8% variation

Turbine Generator Mechanical Model



Grid Frequency Set by Generator Rotational Speed

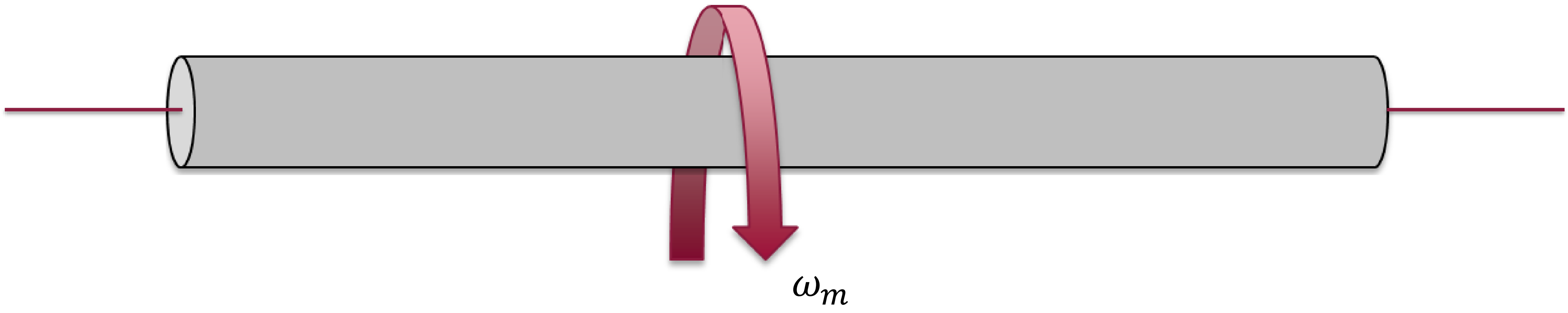




$$\omega_e = k\omega_m$$

e.g. rotate at 3600 RPM to produce 60Hz

Rotating Masses "Store" Kinetic Energy

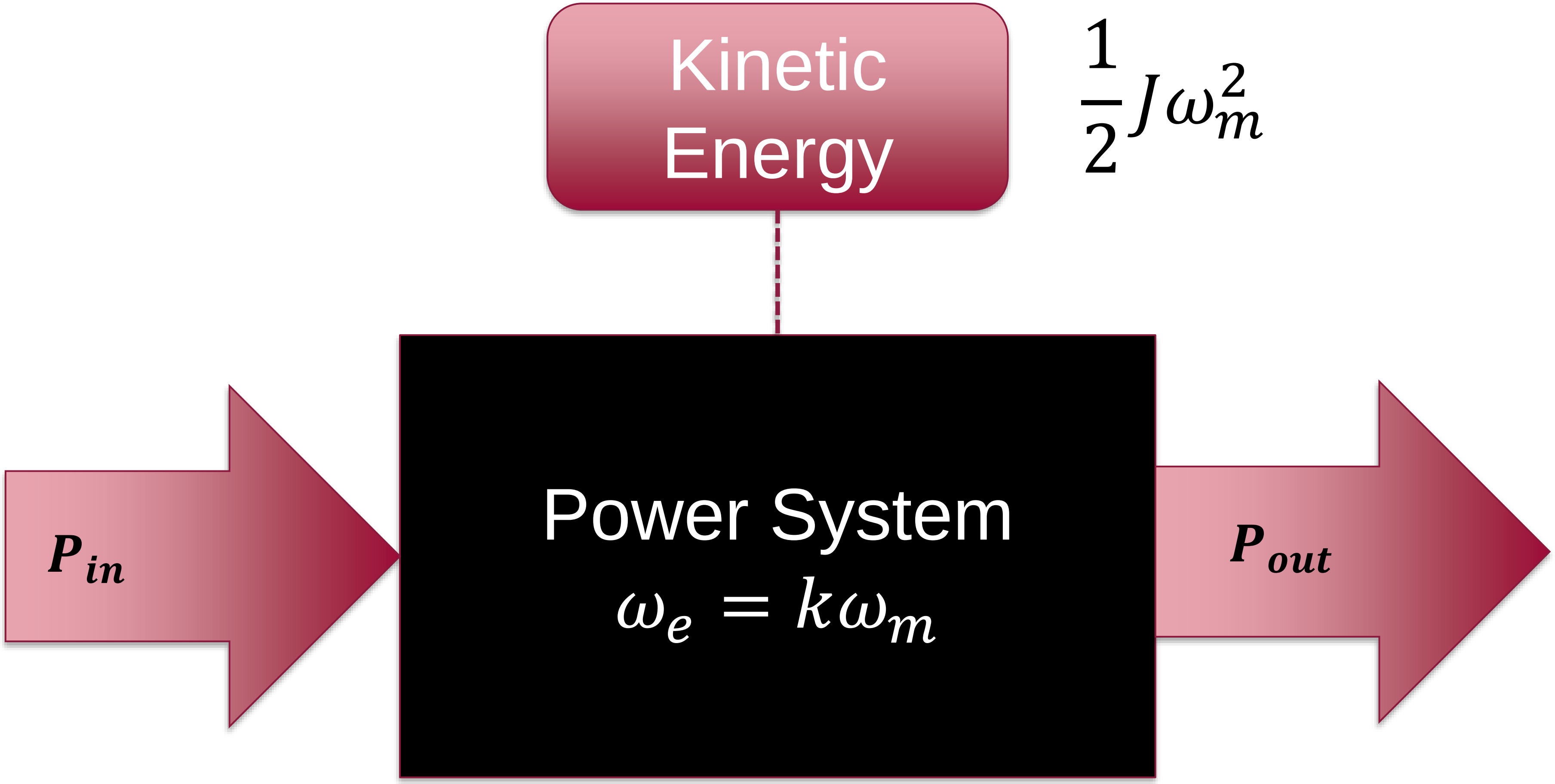


J: moment of inertia

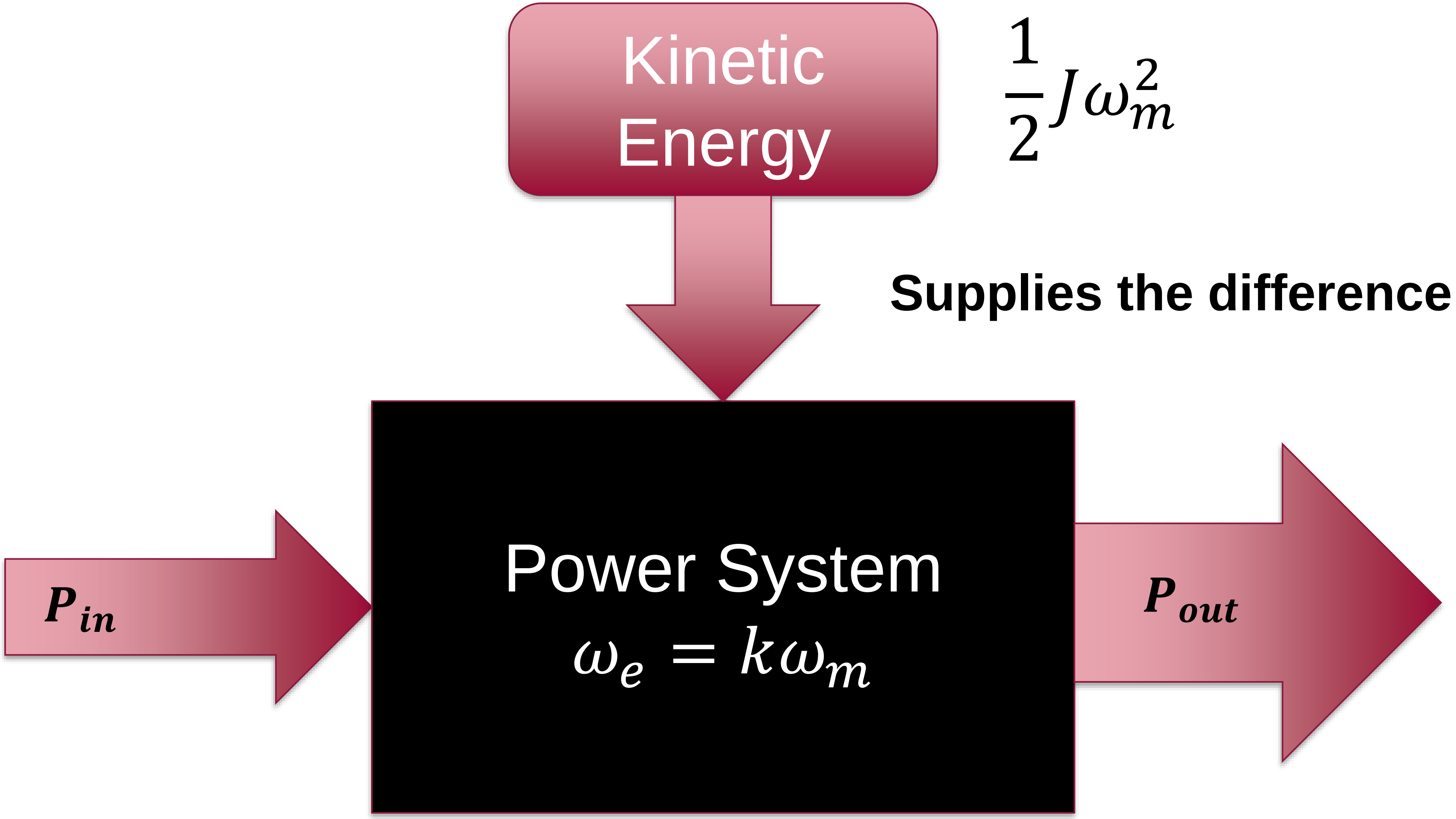
ω_m: angular speed

Kinetic energy: $\frac{1}{2} J \omega_m^2$

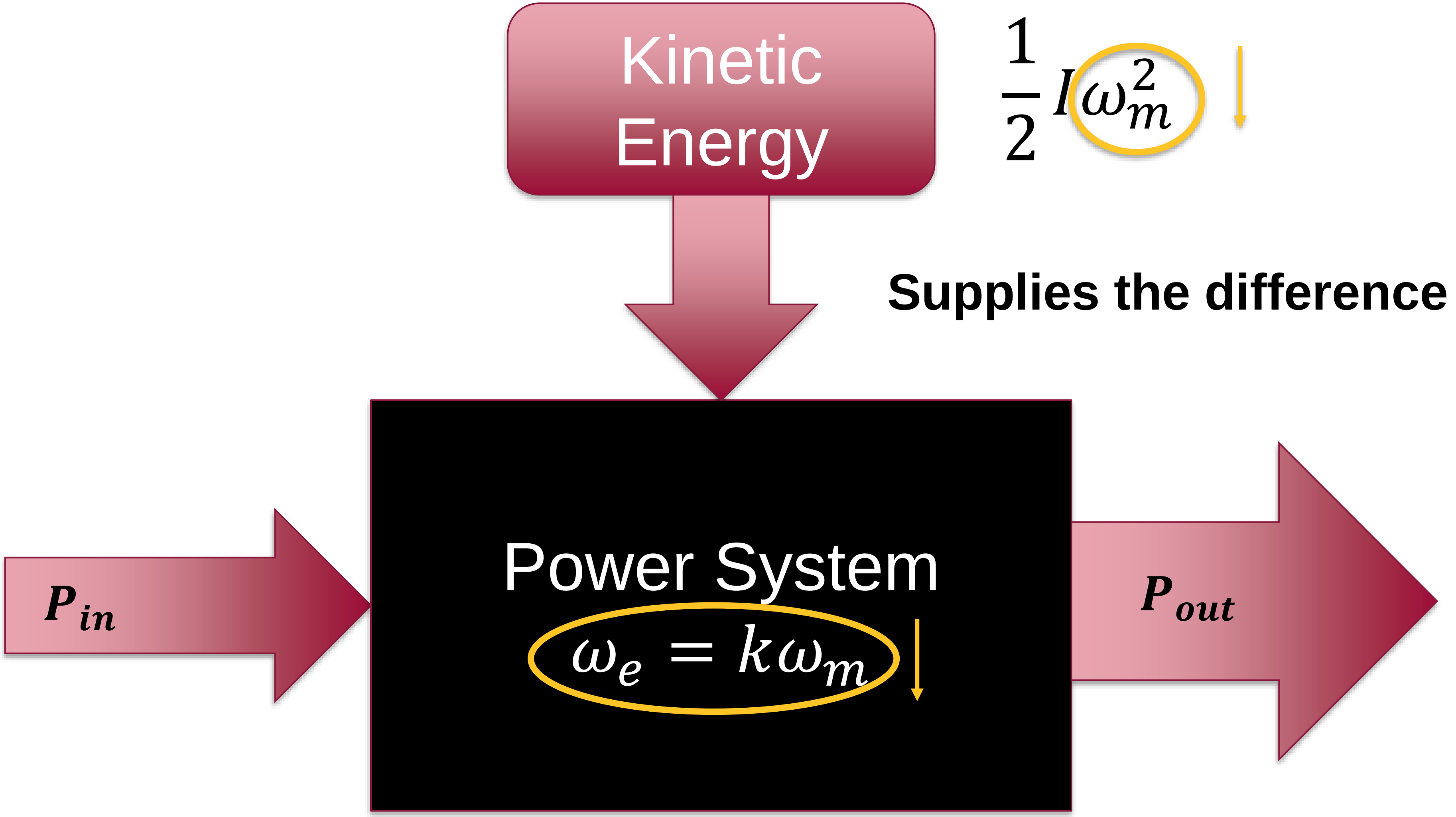
Nominal Operation, $P_{in} = P_{out}$



Generator Outage, $P_{in} < P_{out}$



Generator Outage, $P_{in} < P_{out}$



Intuition from Energy Balance Perspective

- Power system **stores inertial energy** in generators

- When an outage occurs, this energy serves as a "buffer"
  - Decreases for $P_{in} < P_{out}$

- Generator speed is directly affected by outages

$\therefore$The system frequency is directly affected by power imbalances in the grid

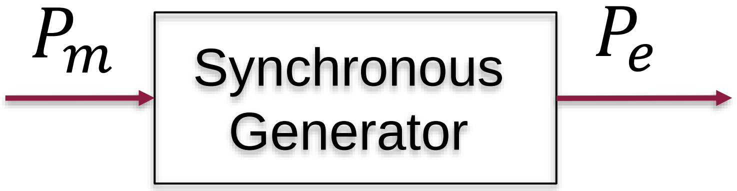
Module 1b

Dynamic Synchronous Generator Model

A first step towards studying power system dynamics

Arizona State University

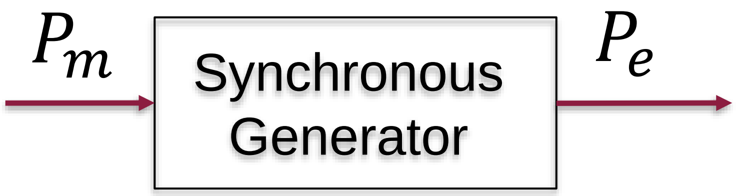
Generator Power Balance



$$P_m = P_e + \frac{d}{dt} W_{kinetic} + P_{friction}$$

(Ignore machine losses, except for mechanical friction)

Generator Power Balance

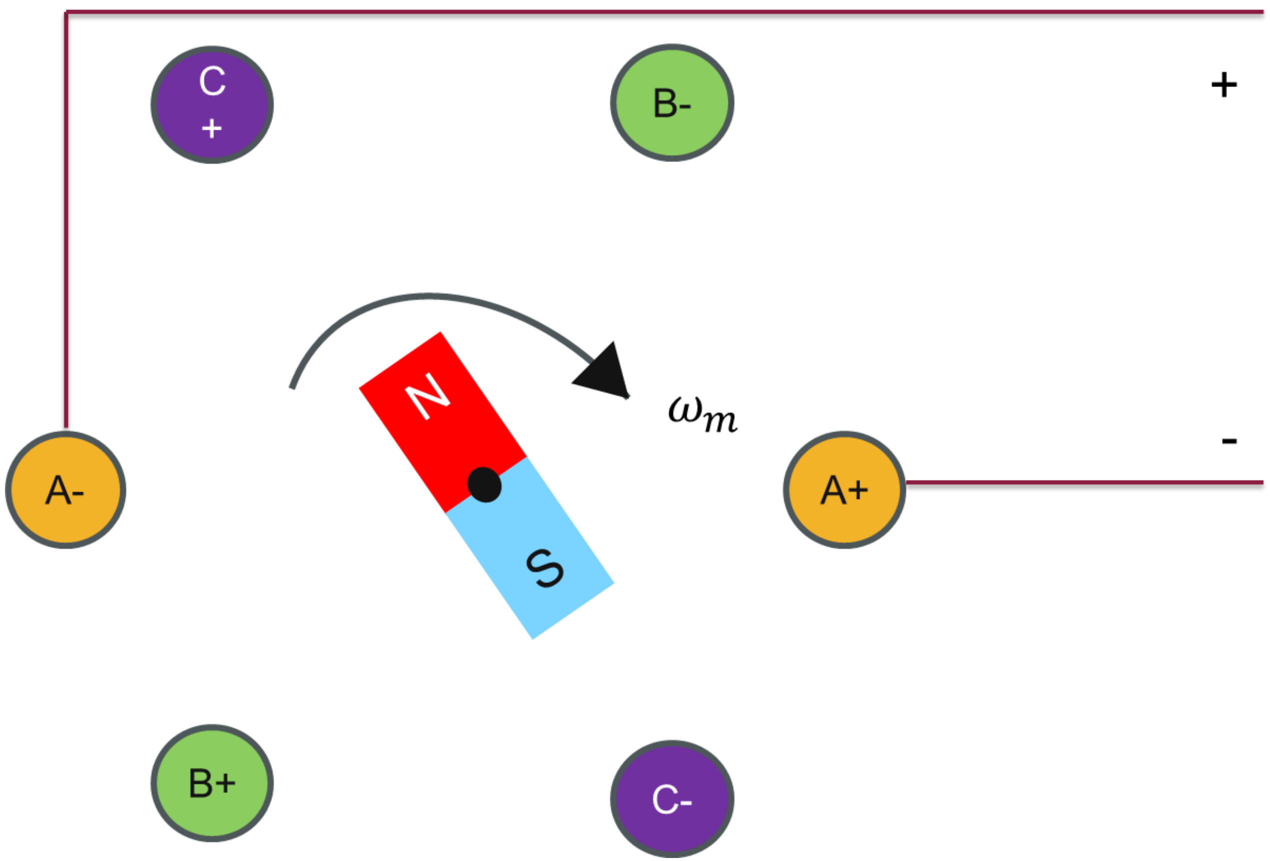


$$P_m = \boxed{P_e} + \frac{d}{dt} W_{kinetic} + P_{friction}$$

(Ignore machine losses, except for mechanical friction)

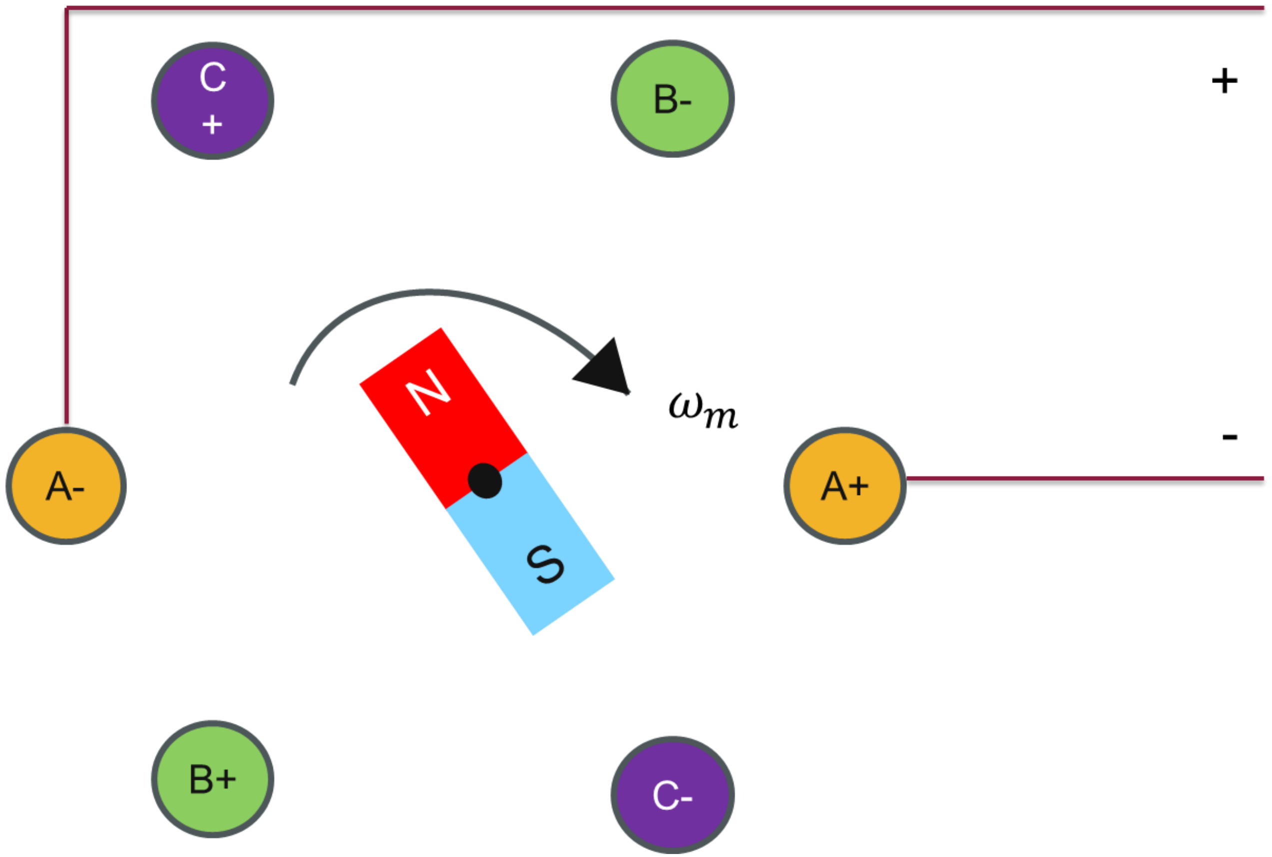
Generator Modeling Assumptions

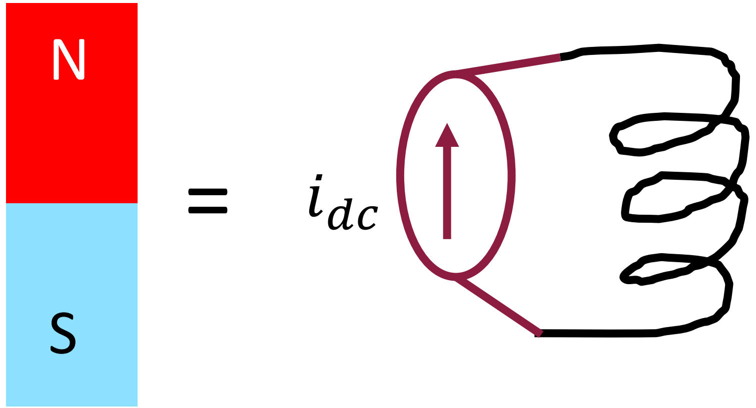
1. Balanced three-phase positive-sequence operation



Generator Modeling Assumptions

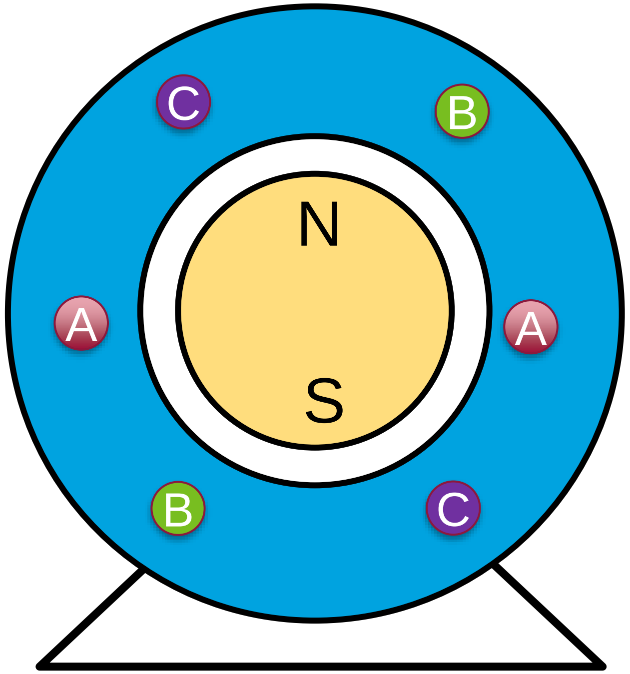
1. Balanced three-phase positive-sequence operation
2. Constant machine excitation





Generator Modeling Assumptions

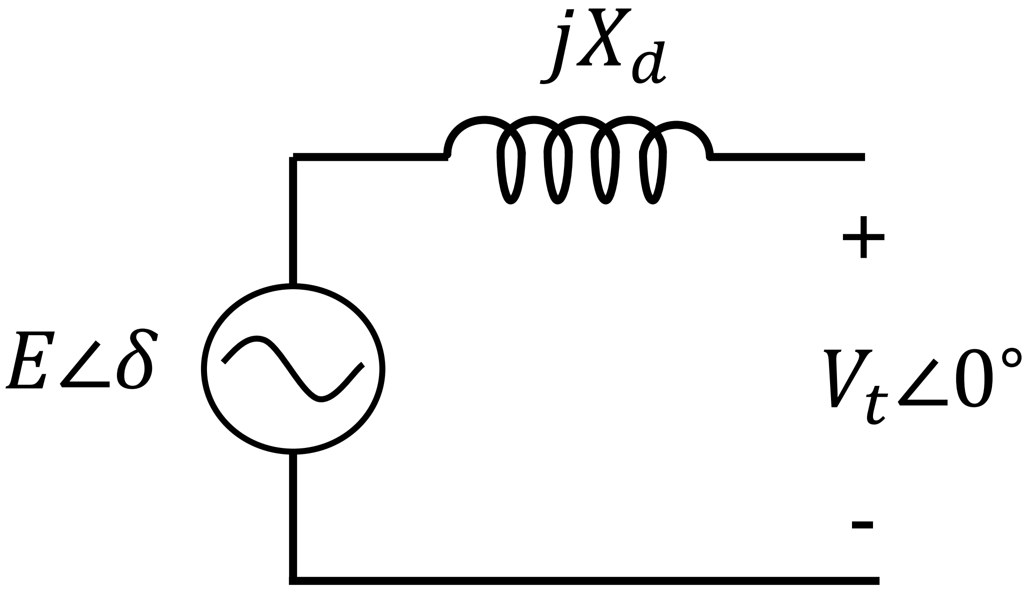
1. Balanced three-phase positive-sequence operation
2. Constant machine excitation
3. Ignore saturation and **saliency** (so, round rotor, constant airgap)



Generator Modeling Assumptions

1. Balanced three-phase positive-sequence operation
2. Constant machine excitation
3. Ignore saturation and **saliency** (so, round rotor, constant airgap)
4. The turbine-generator has a very large moment of inertia

Simplified Generator Electrical Model

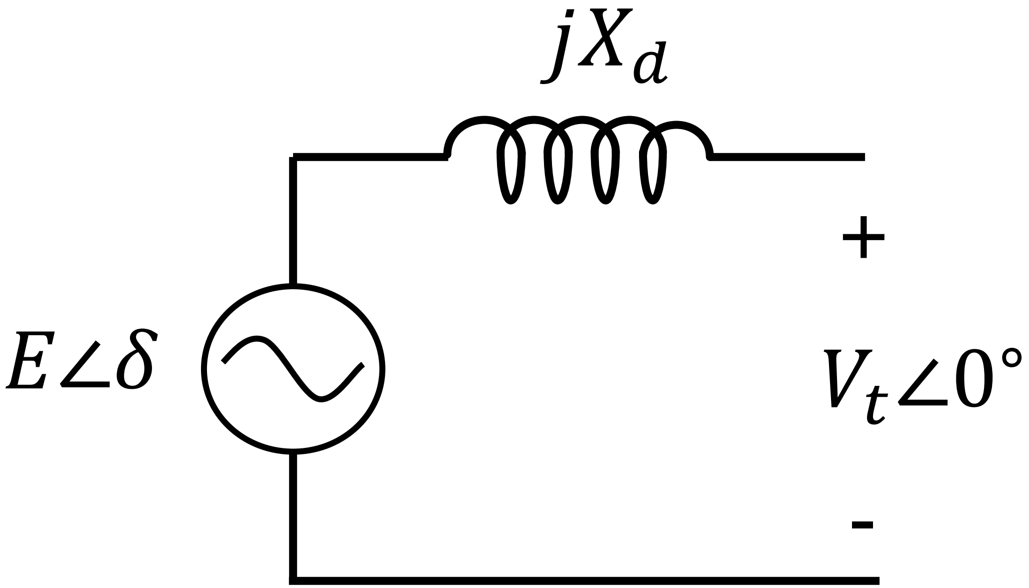


per phase equivalent
(Assumption 1)

Constant internal voltage
(Assumption 2)

$$P_e = 3\underbrace{\frac{|E||V_t|}{X_d}\sin\delta}_{\text{Assumption 3}}$$

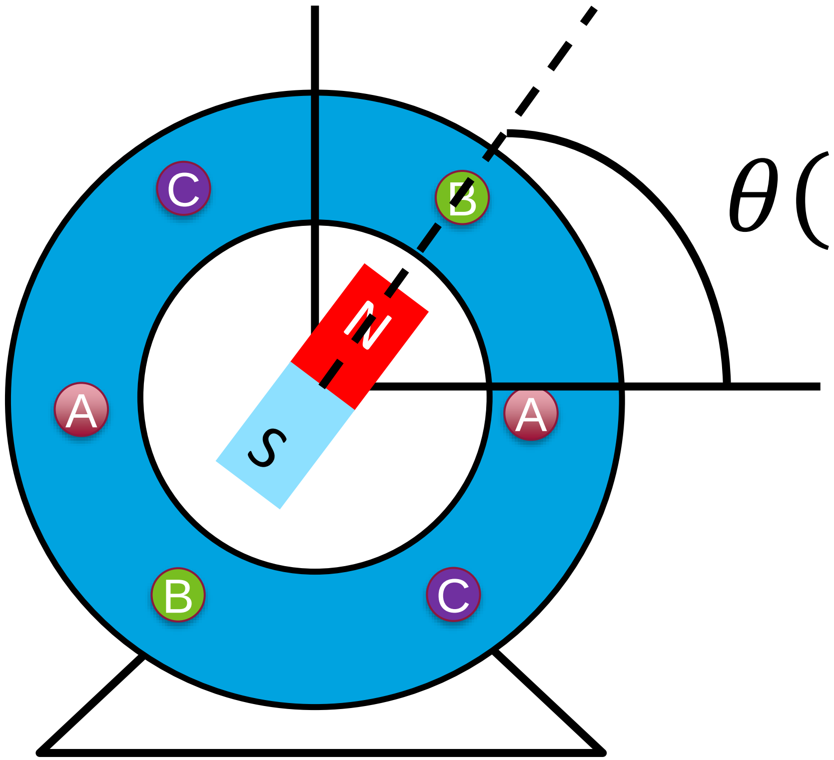
Simplified Generator Electrical Model



$$P_e = 3\,\frac{|E||V_t|}{X_d}\,\sin\delta$$

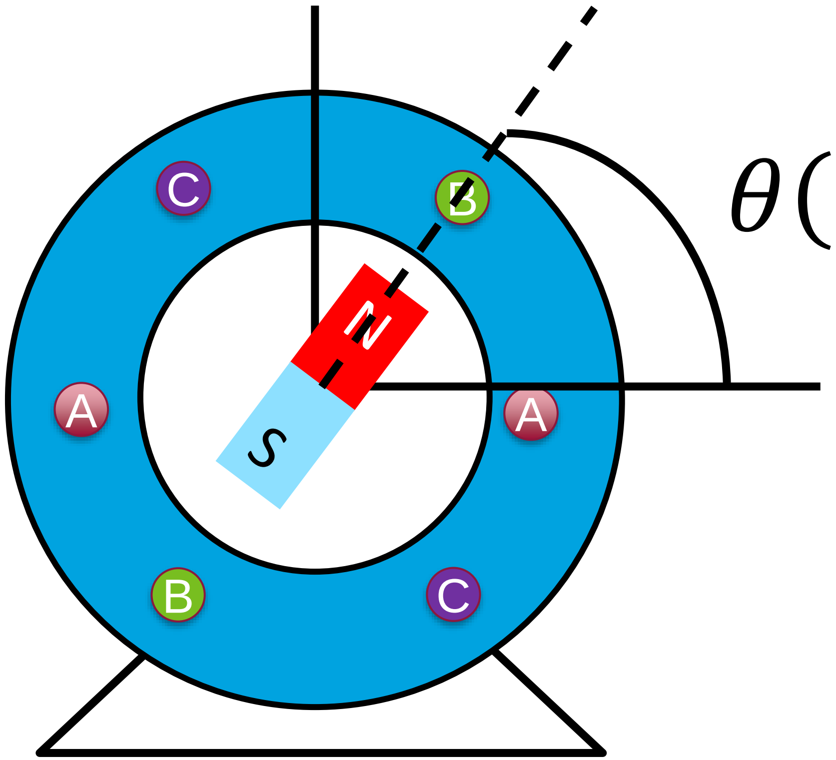
Phase angle of phase “a” internal voltage

Rotor Position During Transients



$$\theta(t) = \underbrace{\omega_m t + \theta_0}_{\text{Smooth rotation}}$$

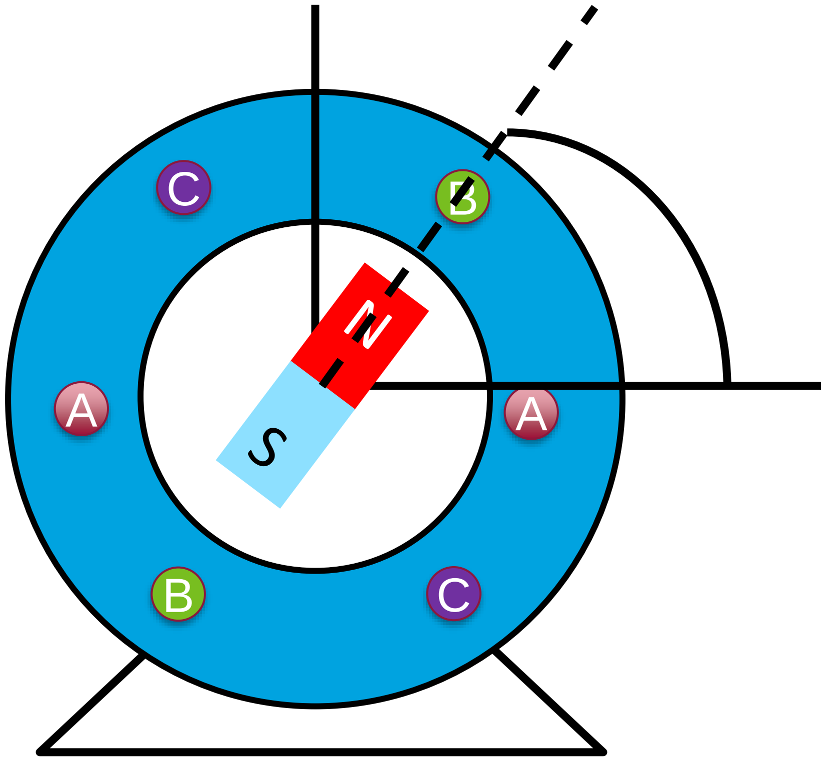
Rotor Position During Transients



$$\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)$$

Transient rotor angle

Rotor Position Relation to Electrical Angle



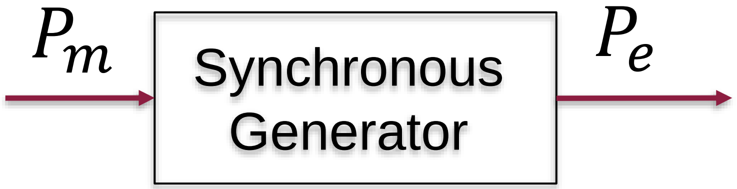
$$\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)$$

$$\delta(t) = \theta_0 + \Delta\theta(t) - \frac{\pi}{2}$$

When rotor angle is 90 deg,
maximum coupling to phase “a”

Assuming equal number of rotor and stator poles

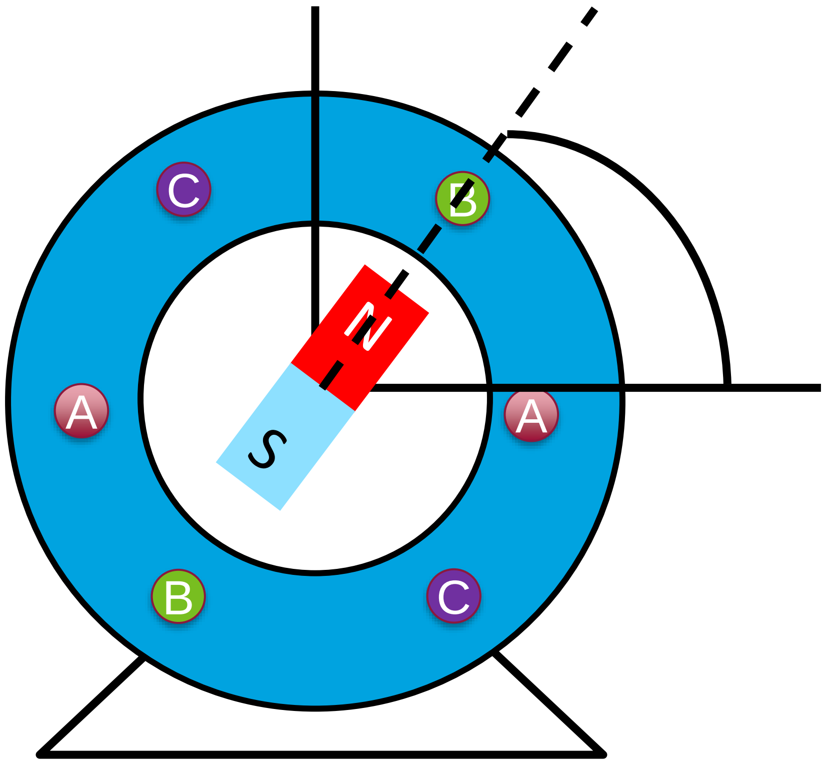
Generator Power Balance



$$P_m = P_e + \boxed{\frac{d}{dt} W_{kinetic}} + P_{friction}$$

(Ignore machine losses, except for mechanical friction)

Rotor Speed and Acceleration

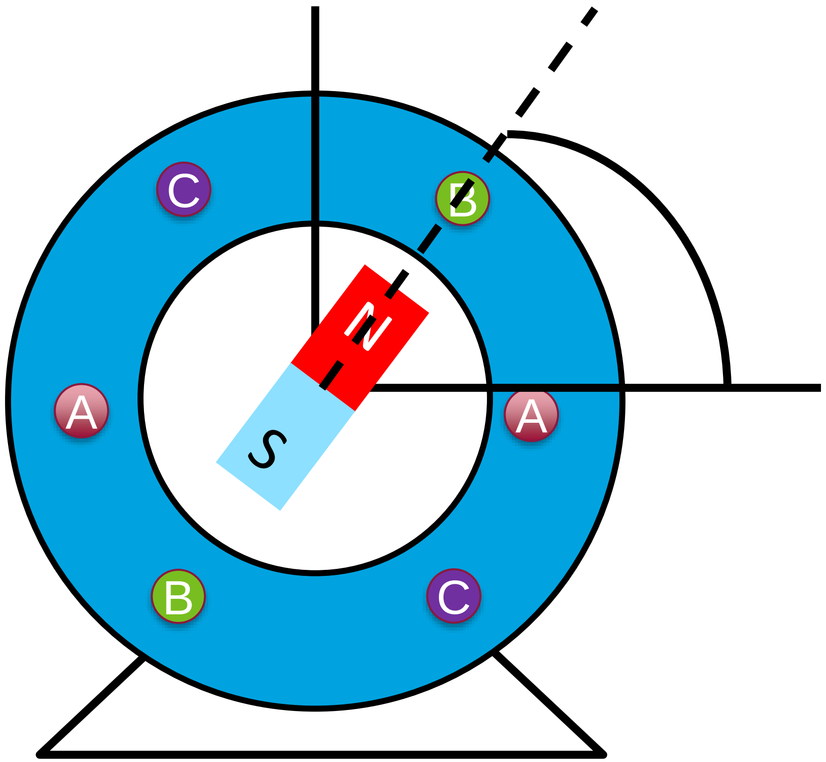


$$W_{kinetic} = \frac{1}{2} J \dot{\theta}^2$$

$$\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)$$

$$\frac{d}{dt}\theta(t) = \omega_m + \frac{\mathrm{d}}{\mathrm{dt}}\Delta\theta(t)$$

Rotor Speed and Acceleration

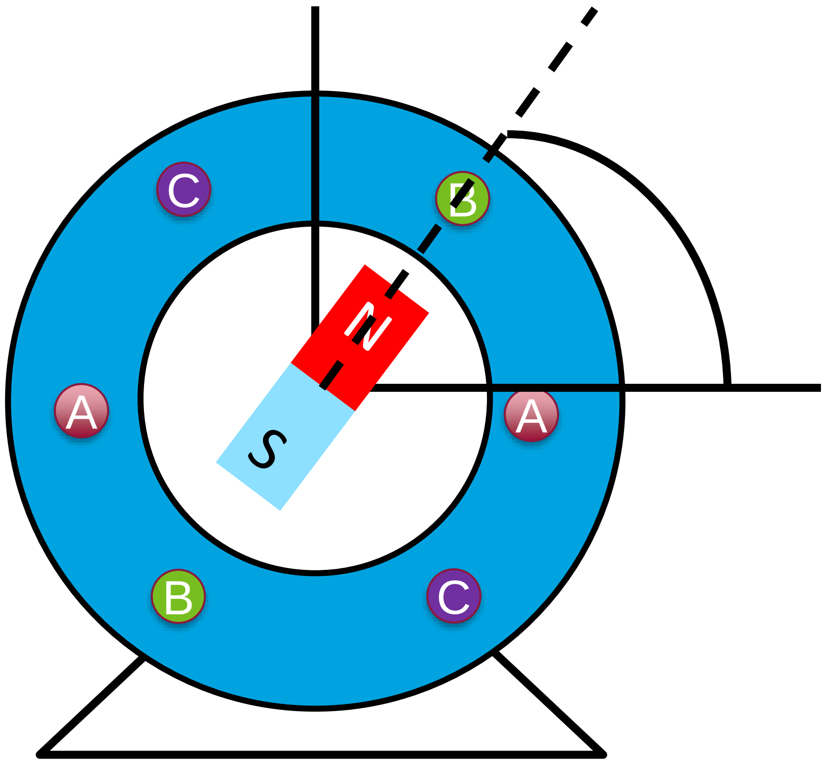


$$\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)$$

$$\frac{d}{dt}\theta(t) = \omega_m + \cancelto{0}{\frac{\mathrm{d}}{\mathrm{d}t}\Delta\theta(t)} \approx \omega_m$$

Speed of transient rotor angle small relative to shaft speed due to large inertia (Assumption 4)

Rotor Speed and Acceleration



$$\theta(t) = \omega_m t + \theta_0 + \Delta\theta(t)$$

$$\frac{d}{dt}\theta(t) = \omega_m + \frac{\mathrm{d}}{\mathrm{dt}}\Delta\theta(t) \approx \omega_m$$

$$\frac{d^2}{dt^2}\theta(t) = \frac{\mathrm{d}^2}{\mathrm{dt}^2}\Delta\theta(t)$$

Kinetic Energy Variation

$$W_{kinetic} = \frac{1}{2} J \dot{\theta}^2$$

$$\frac{d}{dt} W_{kinetic} = J\dot{\theta}\ddot{\theta} = J\omega_m \ddot{\theta}$$

$$= \underbrace{J\omega_m}_{\text{Angular momentum of rotor}} \ddot{\delta}$$

Rotor acceleration

The Per Unit Inertia Constant, H

Steady-state rotor energy

$$H = \frac{W^0_{kinetic}}{S_{base}} = \frac{J\omega_m^2}{2S_{base}} \quad [s]$$

Typ. 1-10 seconds

MVA rating of generator

$$\frac{\frac{d}{dt}W_{kinetic}}{S_{base}} = \frac{H}{\pi f_m}\ddot{\delta} = M\ddot{\delta}$$

Rotor speed in Hz

Friction Losses

$$P_{friction} = k\dot{\theta}^2 = k\omega_m^2 + 2k\omega_m\dot{\delta}$$

Static term, not critical – can be subtracted from input mech power

$$\boxed{P_{friction} \approx 2k\omega_m\dot{\delta}}$$

Define $D = \frac{2k\omega_m}{S_{base}}$

The “Swing Equation”

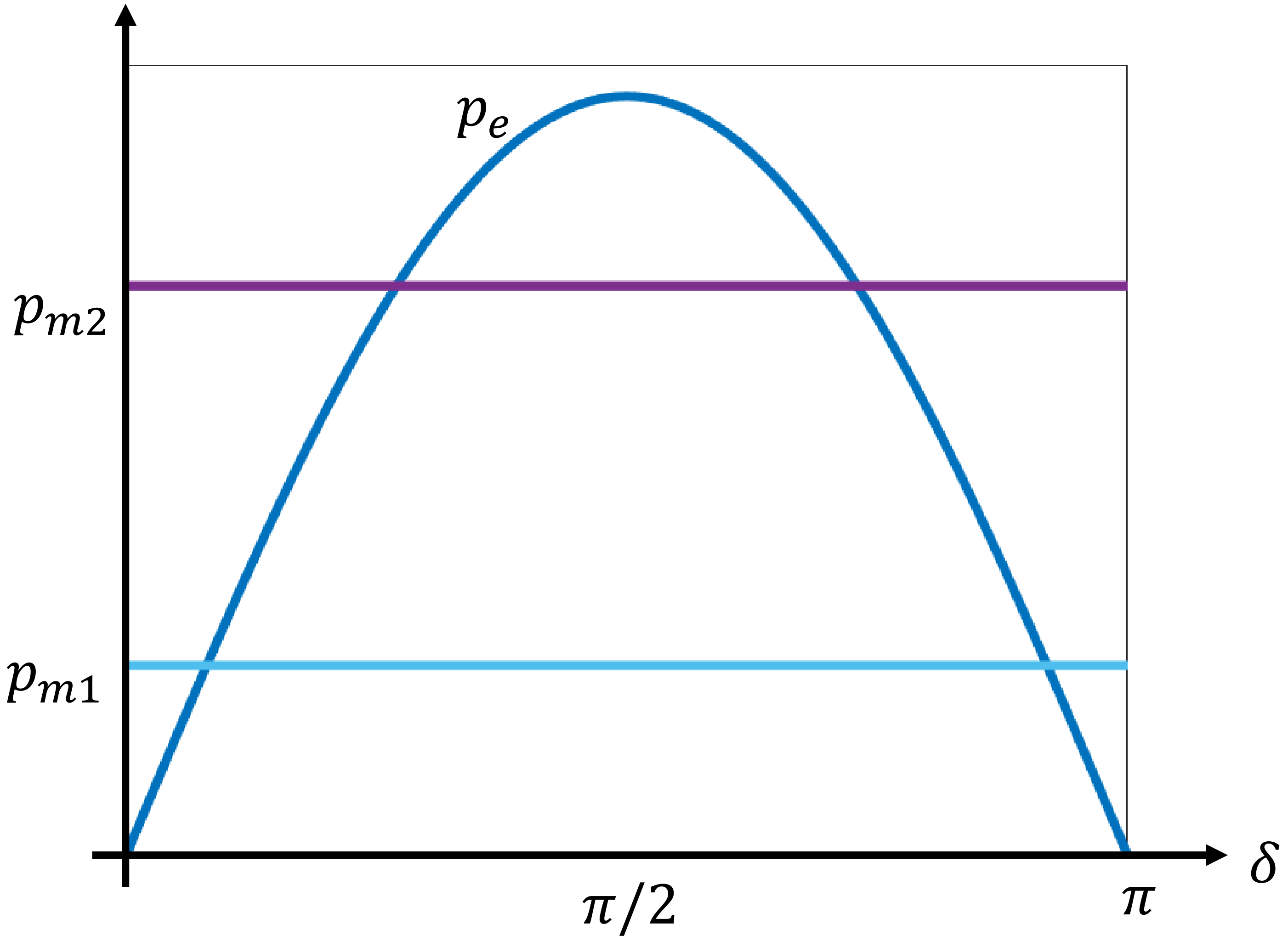
$$P_m = 3\,\frac{|E||V_t|}{X_d}\sin\delta + 2k\omega_m\dot{\delta} + J\omega_m\ddot{\delta}$$

$$\boxed{\frac{P_m}{S_{base}^{3\phi}} = \frac{ev_t}{x_d}\sin\delta + D\dot{\delta} + M\ddot{\delta}}$$

Non-linear differential equation describing “swings” in power angle during transients

Example: Increase in Mechanical Power

(Not practical, prime mover dynamics on order of seconds, but insightful)



Example: Increase in Mechanical Power

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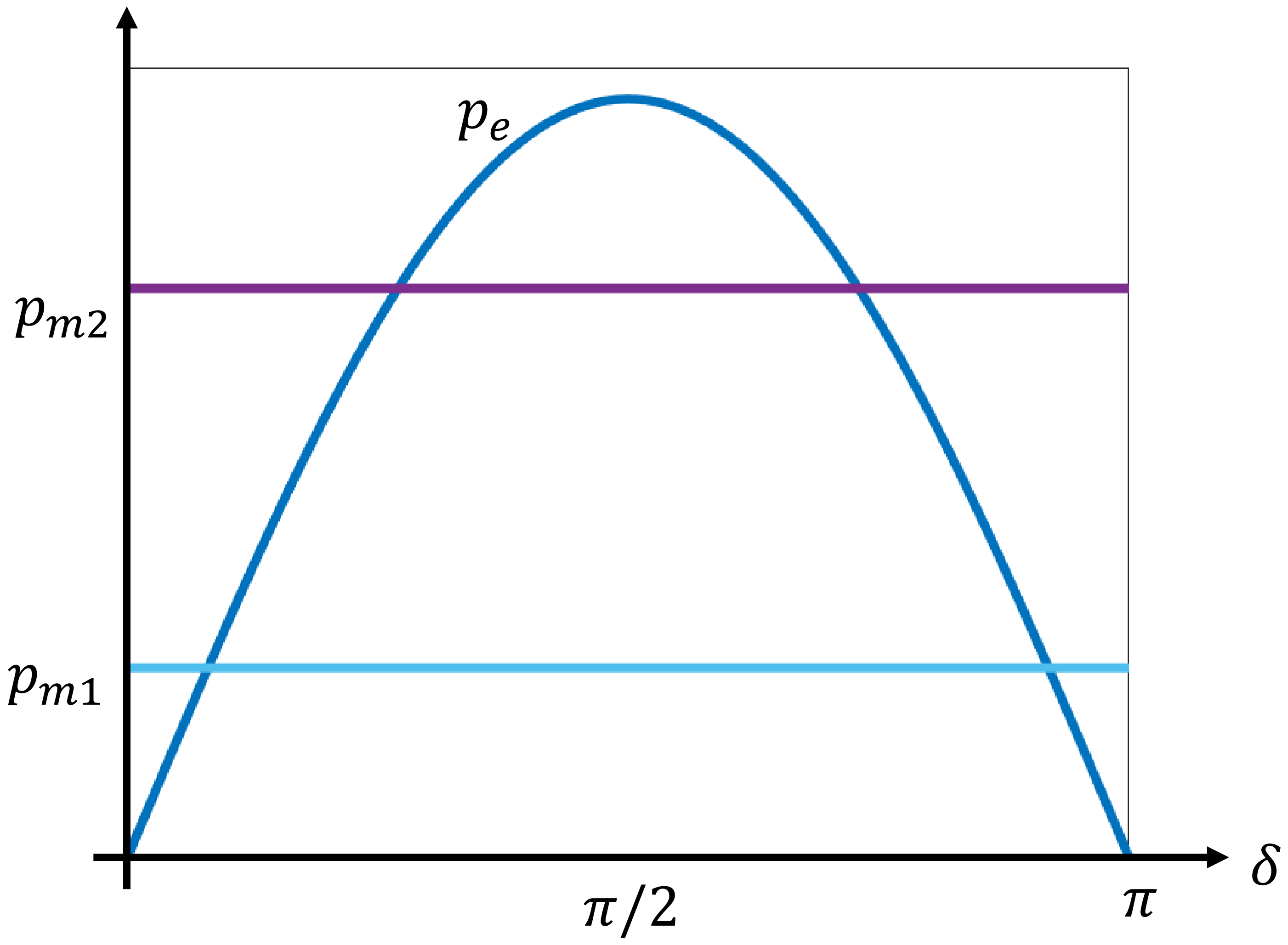


Assume negligible friction
$f_m = 60\text{Hz}$

Per unit quantities:

$$H = 5$$
$$x_d = 0.2$$
$$e = 1.2$$
$$\mathrm{v} = 1$$
$$p_{m1} = 0.6, p_{m2} = 1.8$$

How does δ evolve?

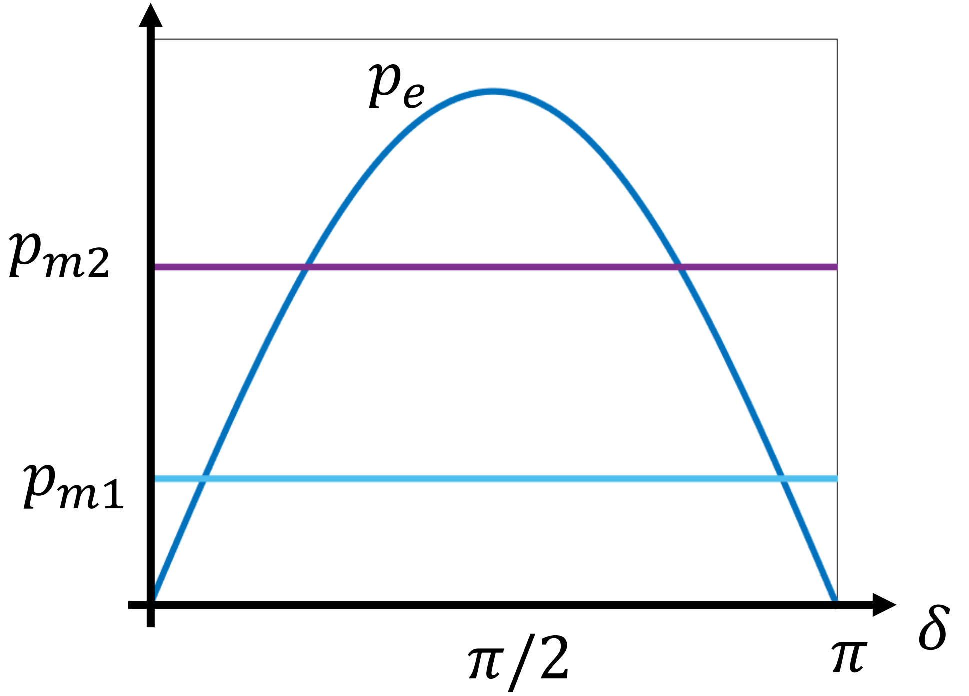
Determine Initial Conditions

$$\delta(0^-) = \delta(0^+) = \delta_0$$
$$\dot{\delta} = 0$$

$$p_{m1} = \frac{ev}{x_d} \sin\delta_o \Rightarrow \delta_0 = 0.2527 \text{ rad.}$$
$$= 14.5°$$

Assume negligible friction
$f_m = 60\text{Hz}$

Per unit quantities:
$H = 5, x_d = 0.5, e = 1.2$
$\text{v} = 1, p_{m1} = 0.6, p_{m2} = 1.8$



Check Final Condition

$$\delta(0^-) = \delta(0^+) = \delta_0$$
$$\dot{\delta} = 0$$

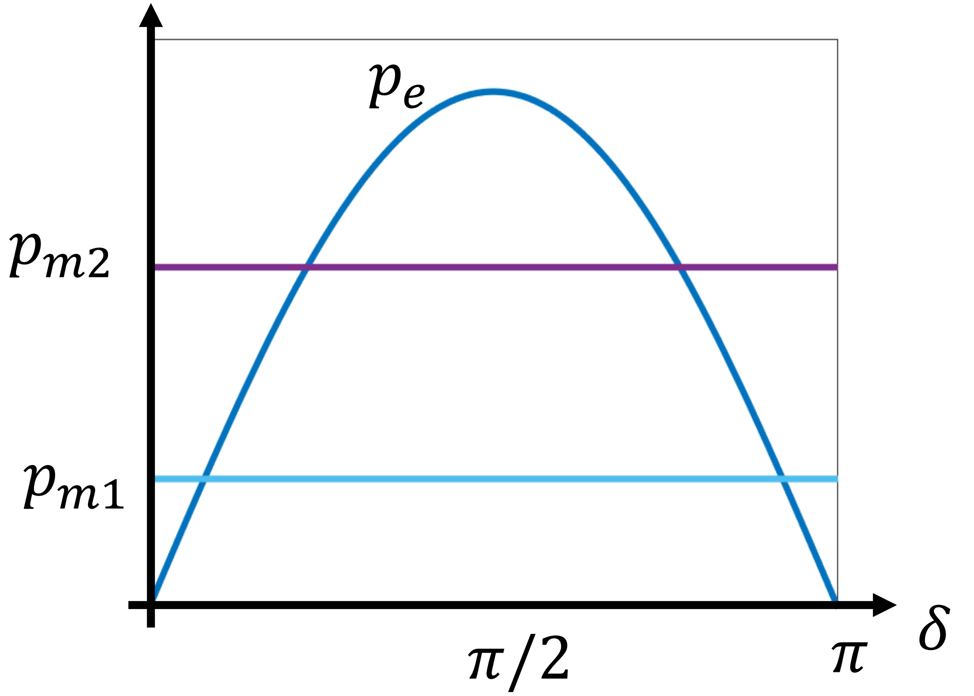
$$p_{m1} = \frac{ev}{x_d} \sin \delta_o \Rightarrow \delta_0 = 0.2527 \text{ rad.}$$
$$= 14.5°$$

We know the final condition too:

$\delta_\infty = 0.848$ rad. $= 48.6°$

Assume negligible friction
$f_m = 60\text{Hz}$

Per unit quantities:
$H = 5, x_d = 0.5, e = 1.2$
$\text{v} = 1, p_{m1} = 0.6, p_{m2} = 1.8$



Define Governing Equation

Governing equation:

$$\boxed{p_m = p_{e,max}\sin\delta + D\dot{\delta} + M\ddot{\delta}}$$

Split into two first order equations:

$$x_1 = \delta,\ x_2 = \dot{\delta}$$

$$\dot{x_1} = \dot{\delta} = x_2$$

$$\dot{x_2} = \frac{p_m - Dx_2 - p_{e,max}\sin x_1}{M}$$

Must solve numerically

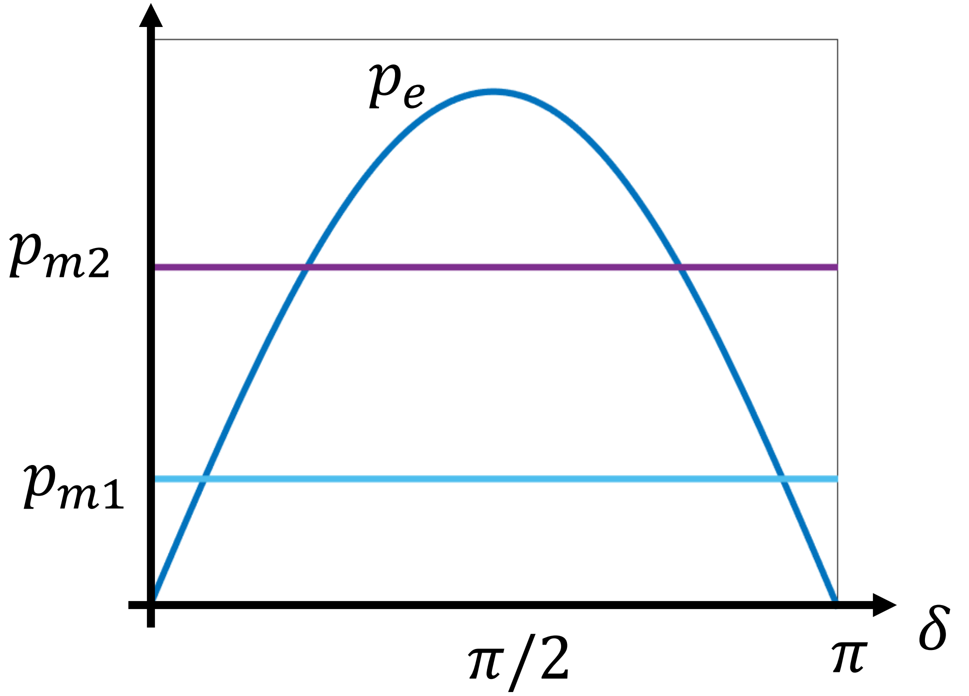
Assume negligible friction

$f_m = 60\text{Hz}$

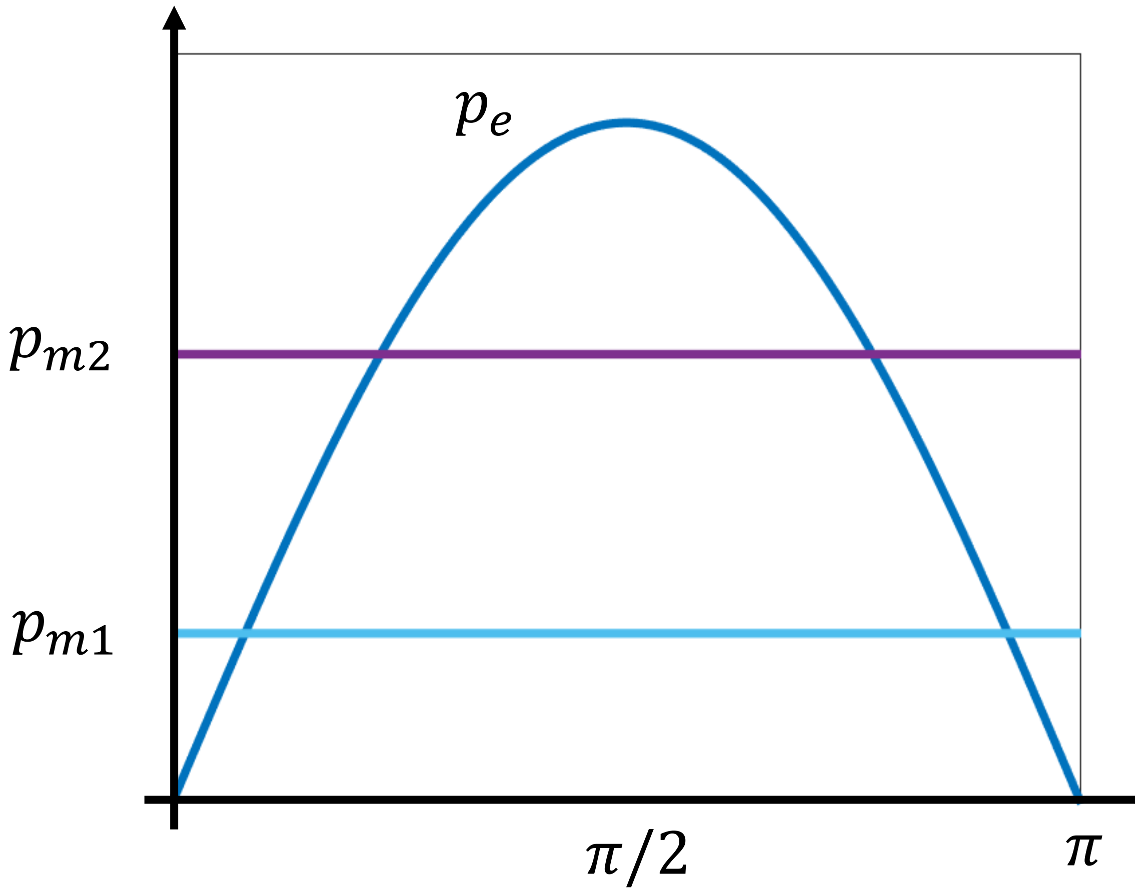
Per unit quantities:

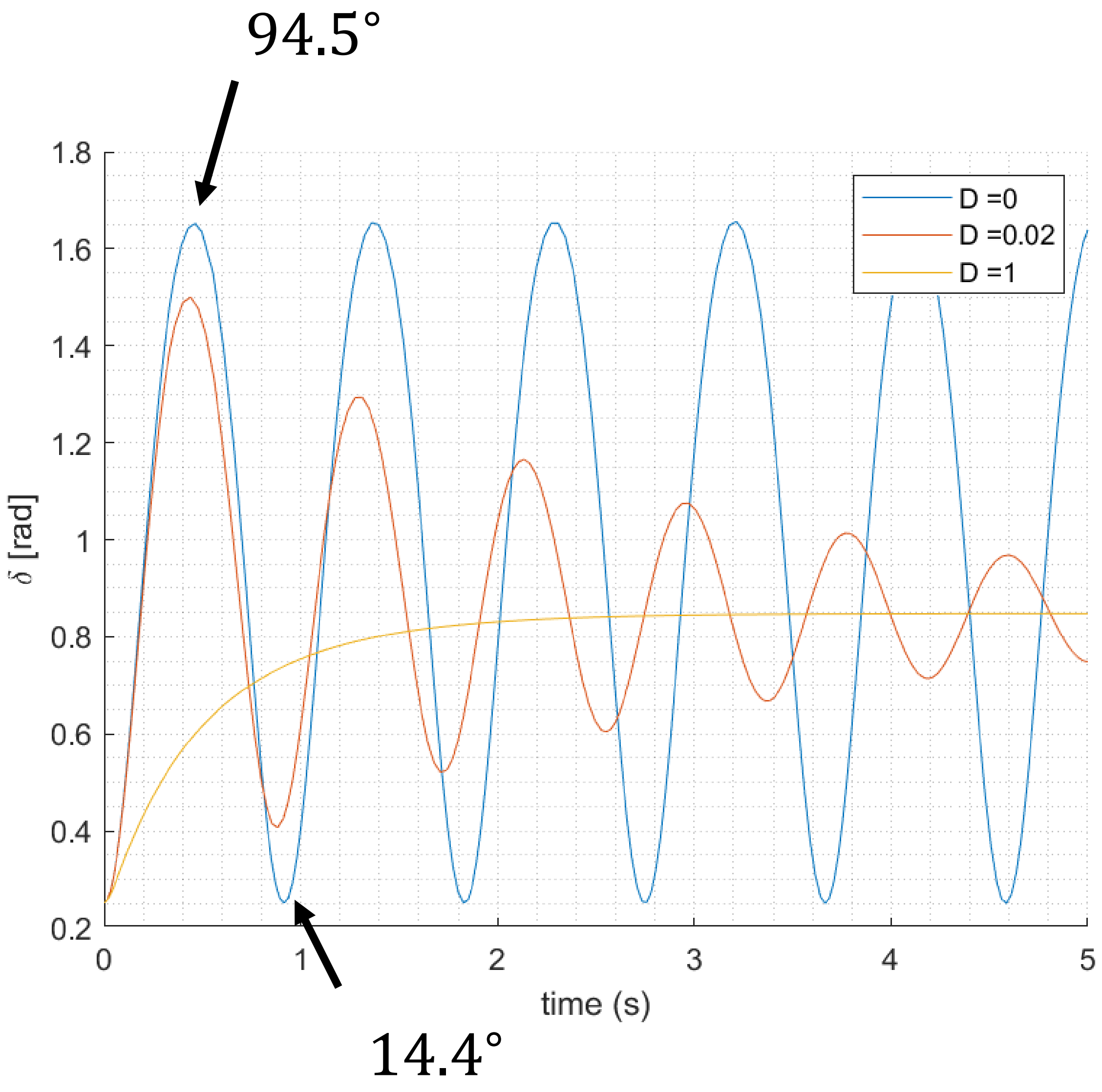
$H = 5, x_d = 0.5, e = 1.2$

$\text{v} = 1, p_{m1} = 0.6, p_{m2} = 1.8$

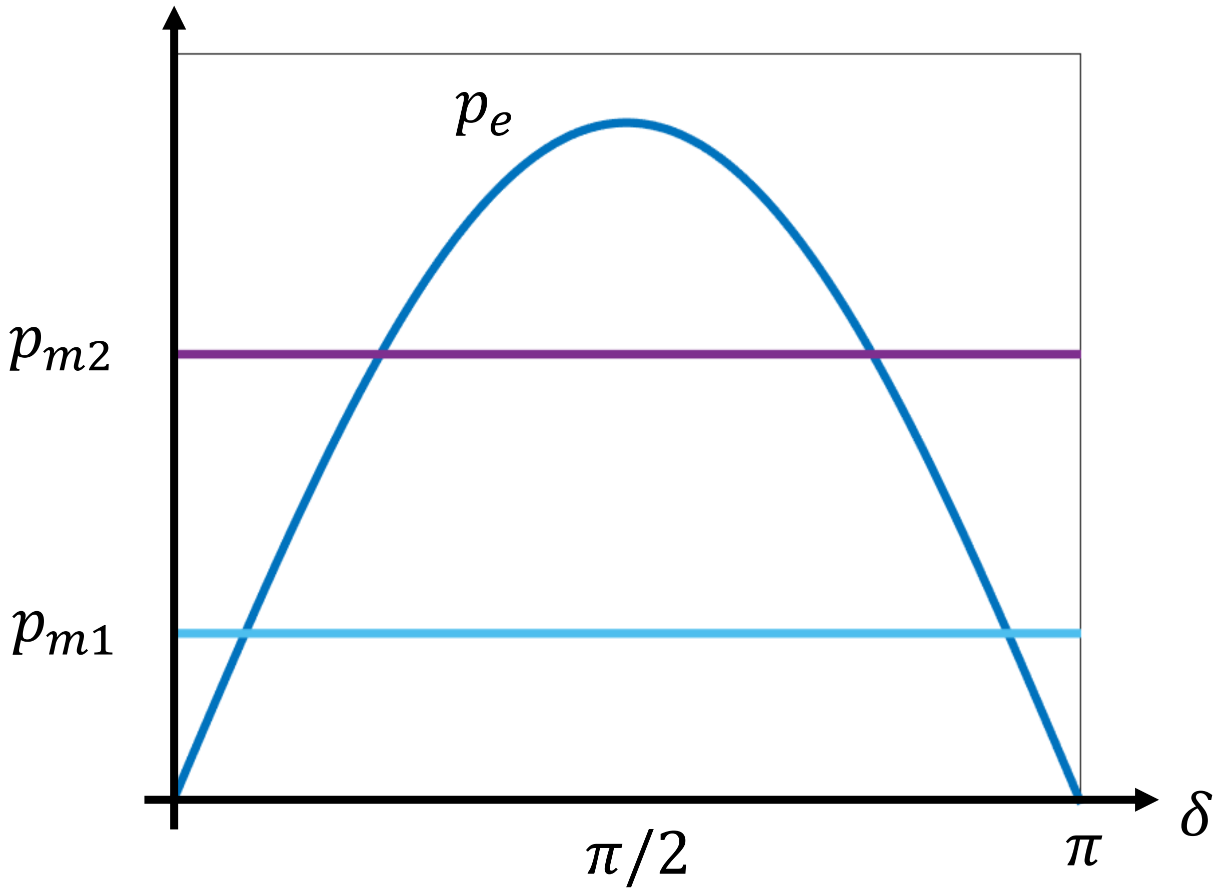


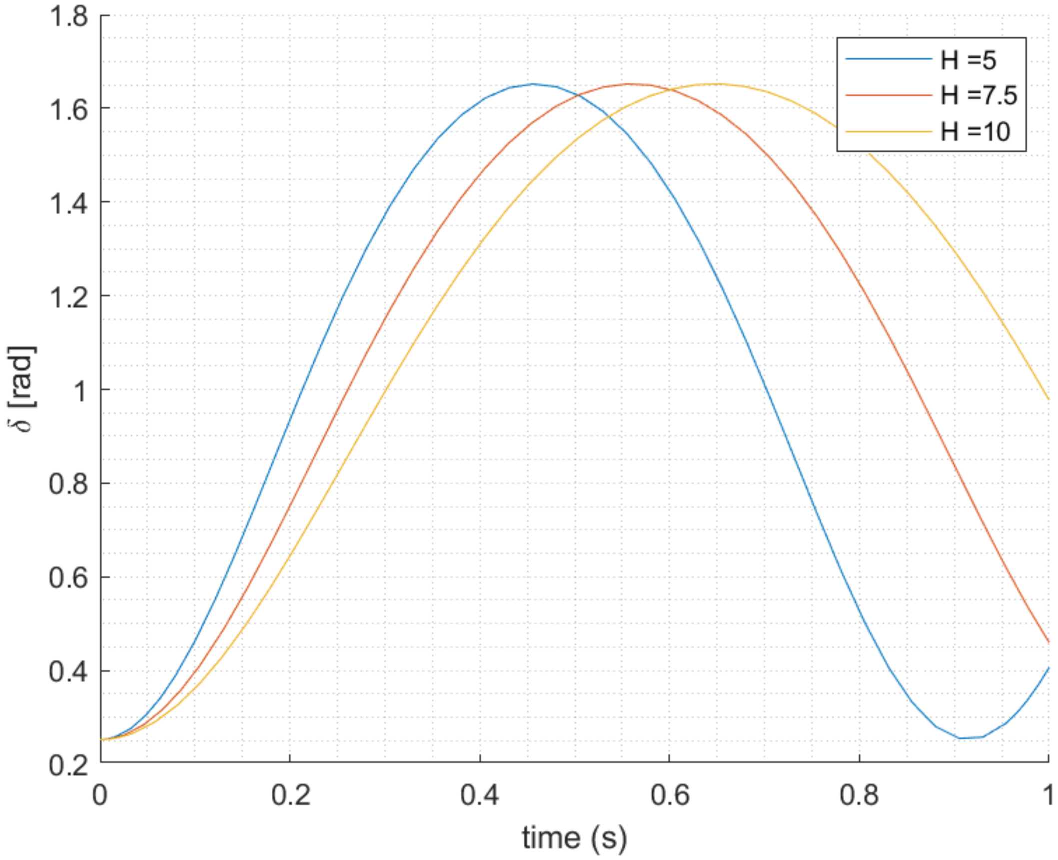
Oscillates Around δ_∞, Variation with D



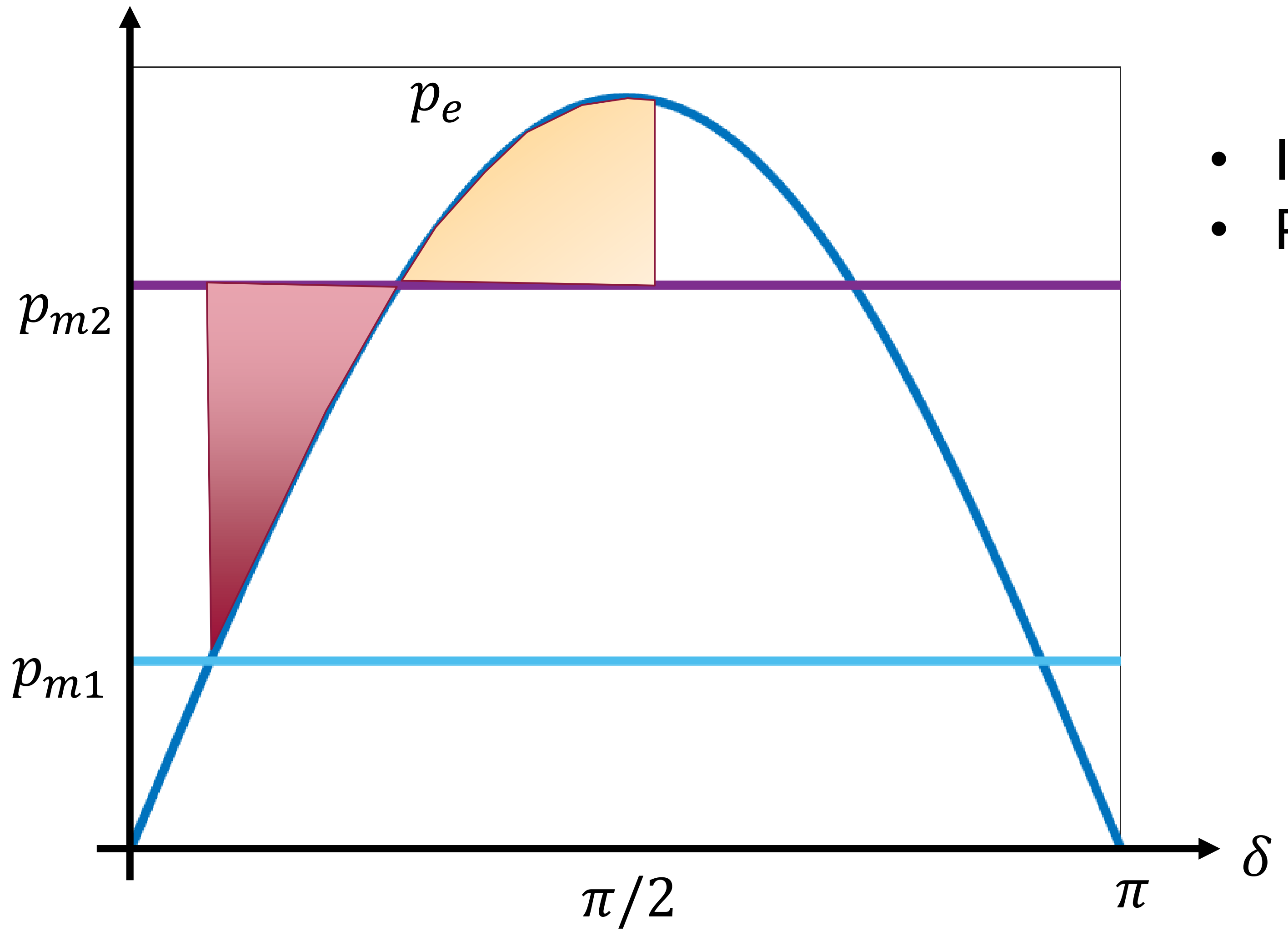


Oscillates Around δ_∞, Variation with H



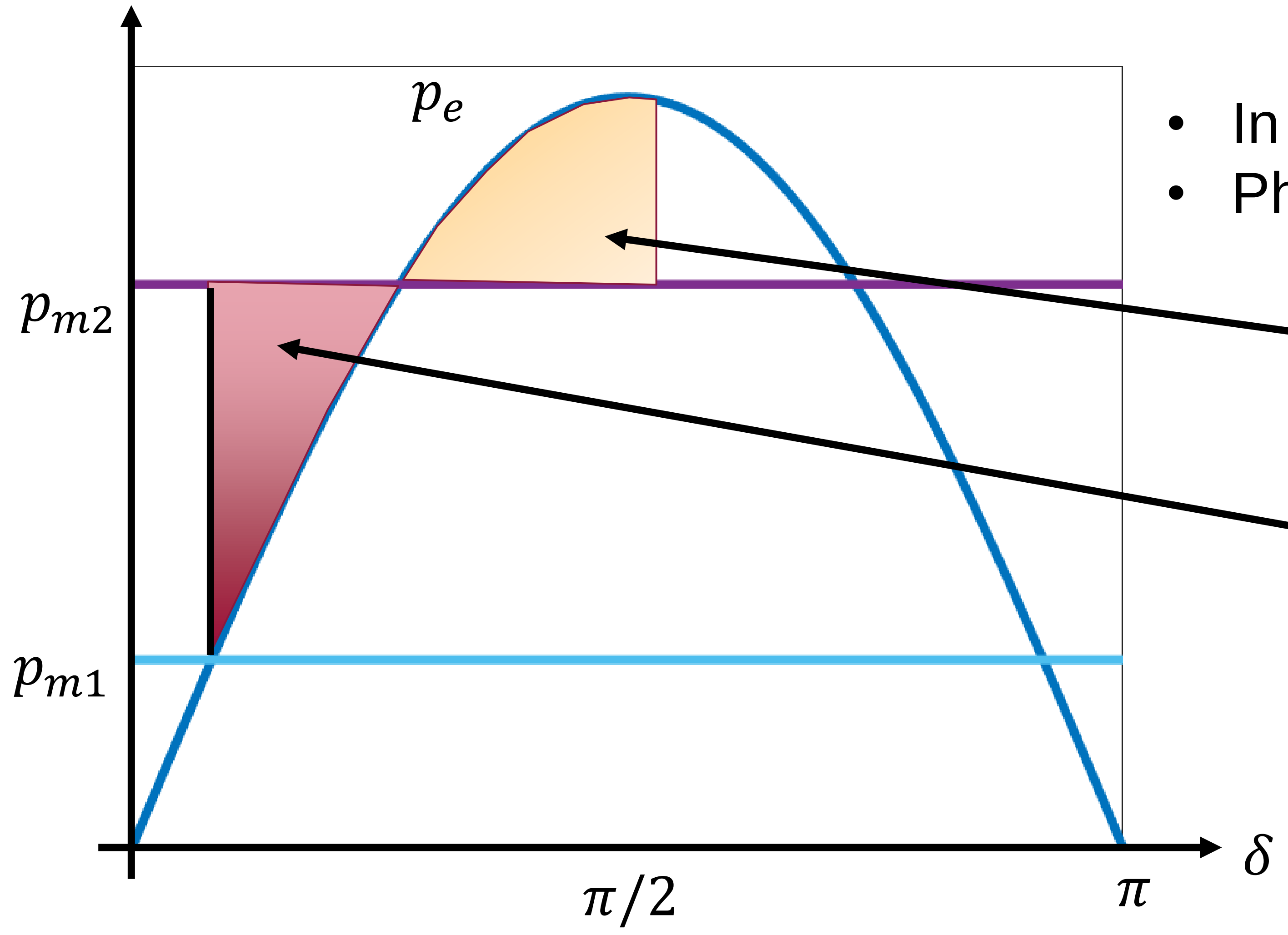


The “Equal-Area Criterion”



- In our example, these areas are equal
- Physical meaning

The "Equal-Area Criterion"



- In our example, these areas are equal
- Physical meaning

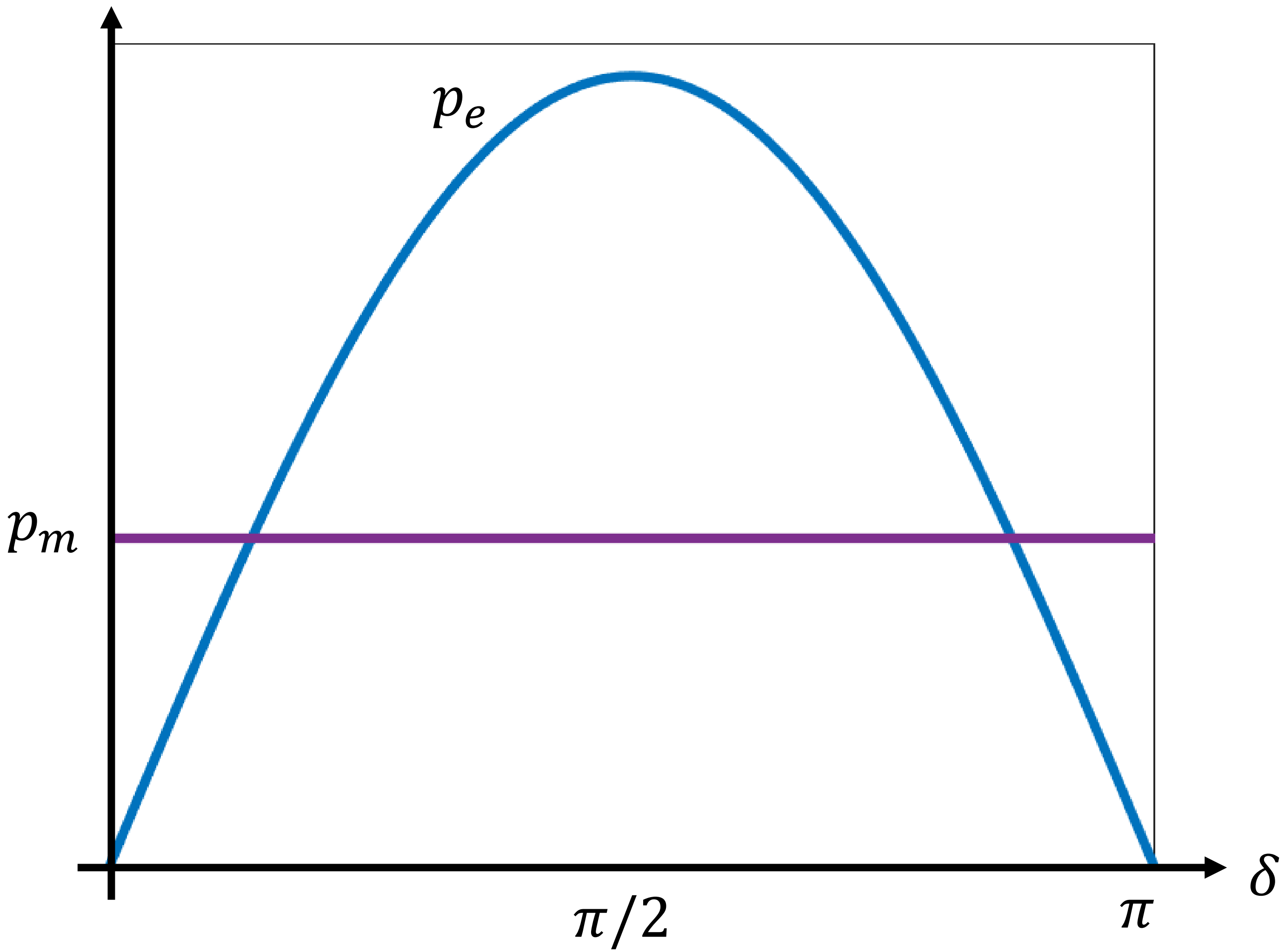
Generator output > input
Rotor decelerates

Rotor accelerates

A useful way to check stability of a single machine
(extendable to two-machine system)

Example: Generator Fault

Three-phase to ground bolted short on generator terminals



Assume negligible friction

$f_m = 60\text{Hz}$

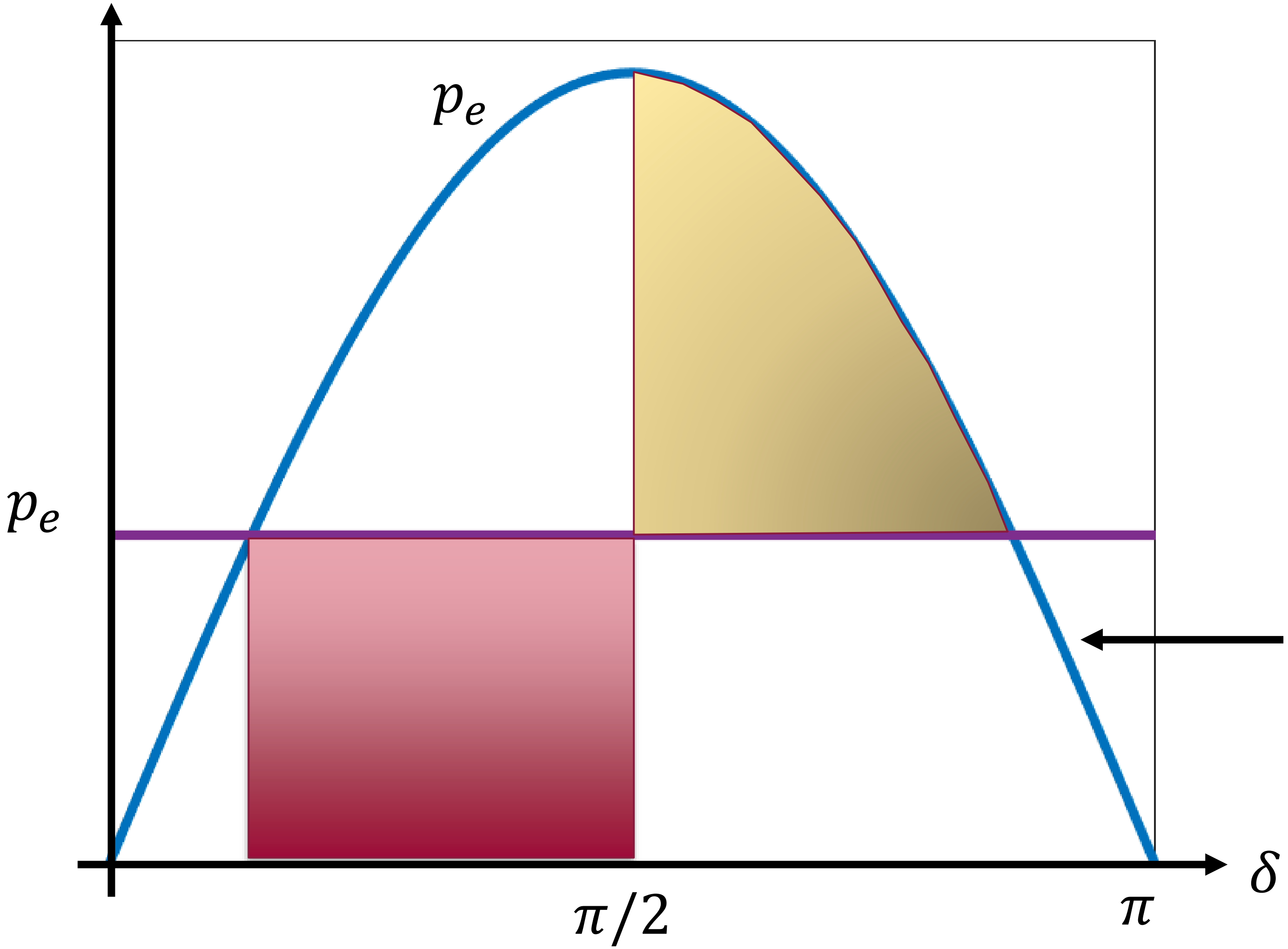
Per unit quantities:

$$H = 5$$
$$x_d = 0.5$$
$$e = 1.2$$
$$\mathrm{v} = 1.0$$
$$p_m = 1.0$$

How does δ evolve?

Condition for Instability

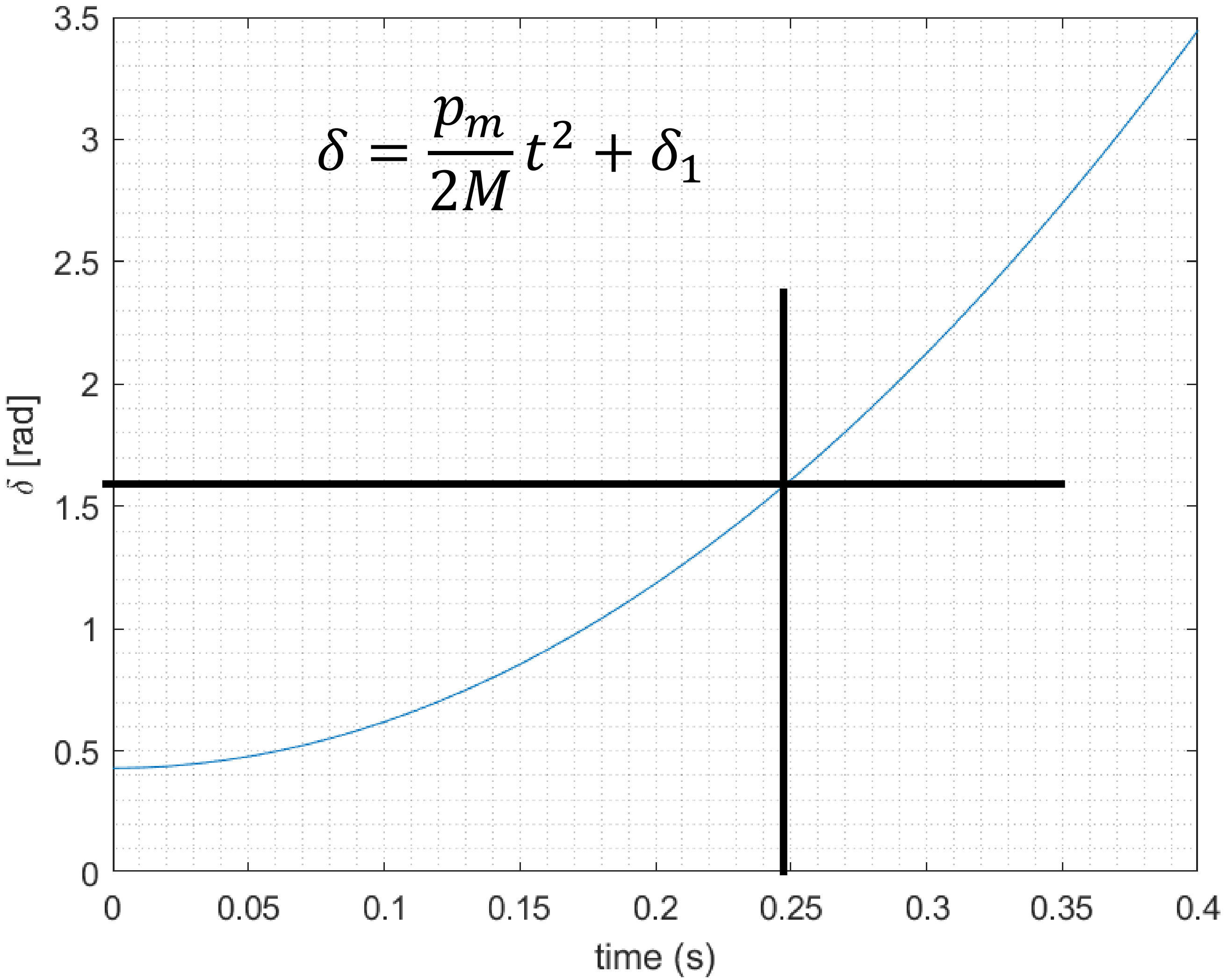
Accelerating energy cannot be removed, shaft speed increases, lose synchronism



In this example, $\delta_{max} = 91.7°$

If reconnected here, guaranteed instability

Solution is Monotonic!



Worst-case example with no damping, but:

we're on the clock

If fault cleared too late, generator loses synchronism

Higher inertia, more time to respond

Conclusions

- Synchronous generator dynamic model derived from power balance

- Nonlinear swing equation defines rotor angle evolution
- System is stable when

- Higher inertia systems evolve more slowly

Module 1c

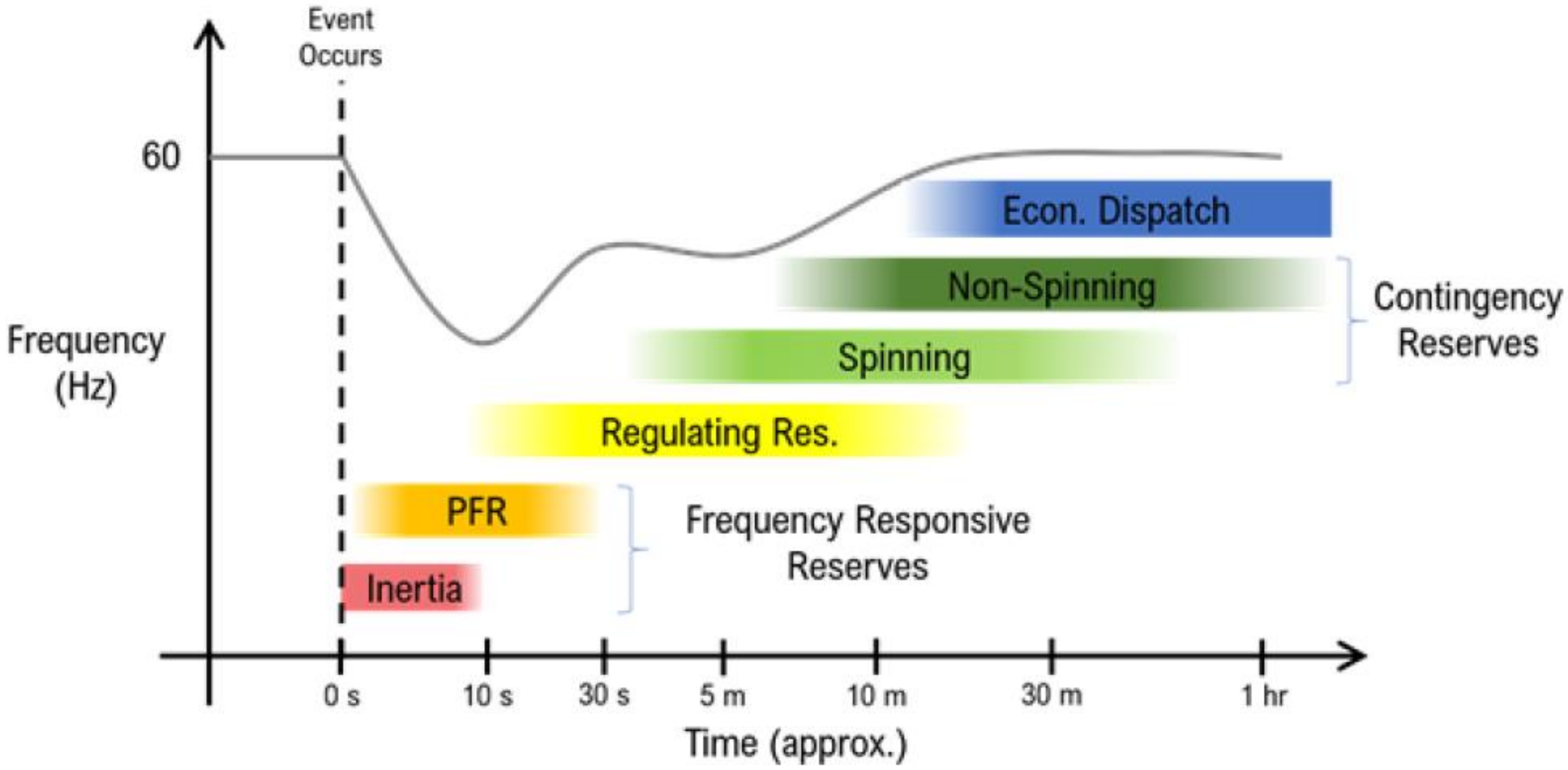
Multimachine Frequency Dynamics

A model for studying disturbances in the power system

Arizona State University

Classical Model Used for “First Swing Analysis”

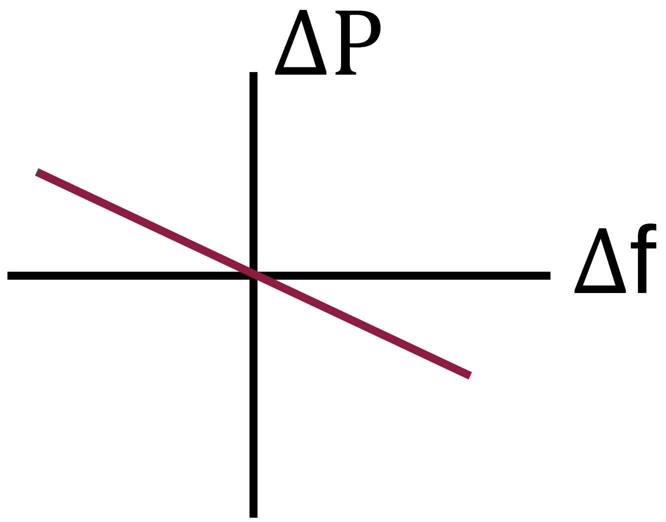
- Simplest model used in stability studies
- Limited to relatively short time-scales (order of seconds)

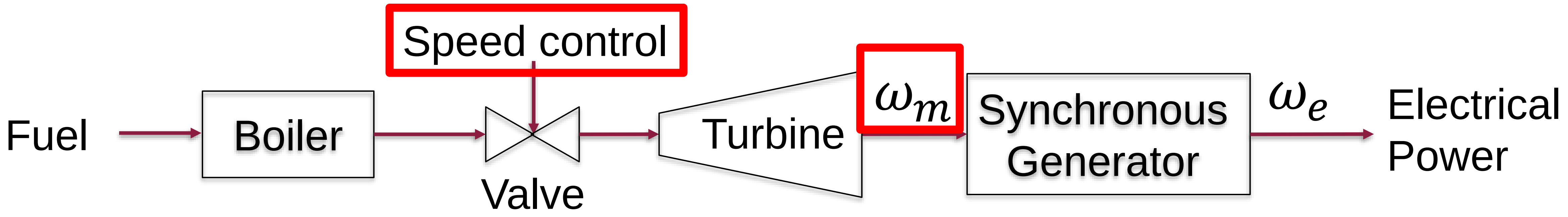


J. L. Jorgenson and P. L. Denholm, “Modeling Primary Frequency Response for Grid Studies,” NREL/TP-6A20-72355, 1489895, Jan. 2019. doi: 10.2172/1489895.

Traditional Primary Control

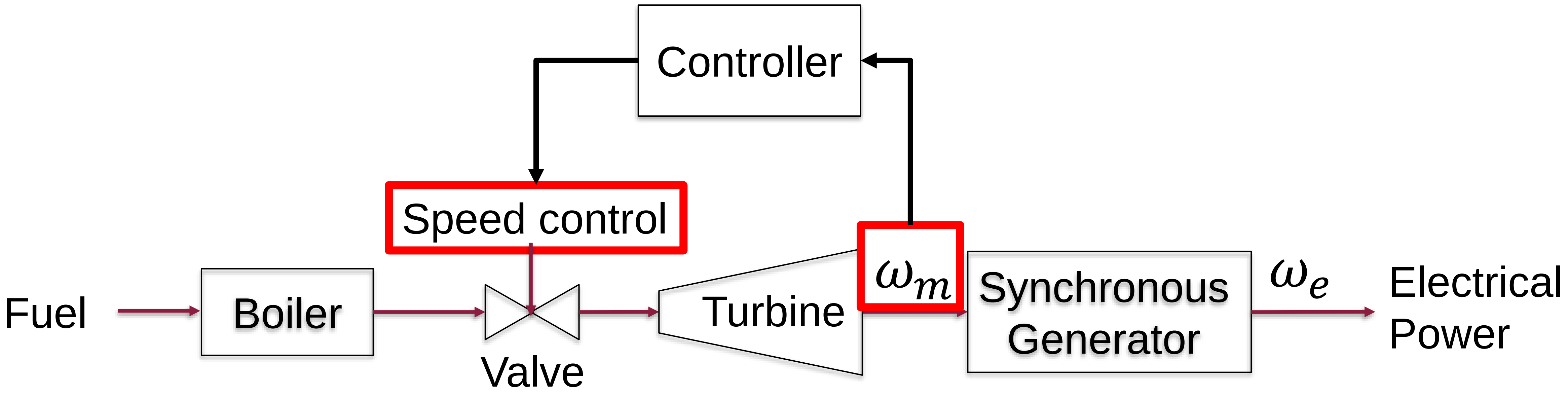
- Primary frequency control: first 30 seconds





Secondary and Tertiary Control

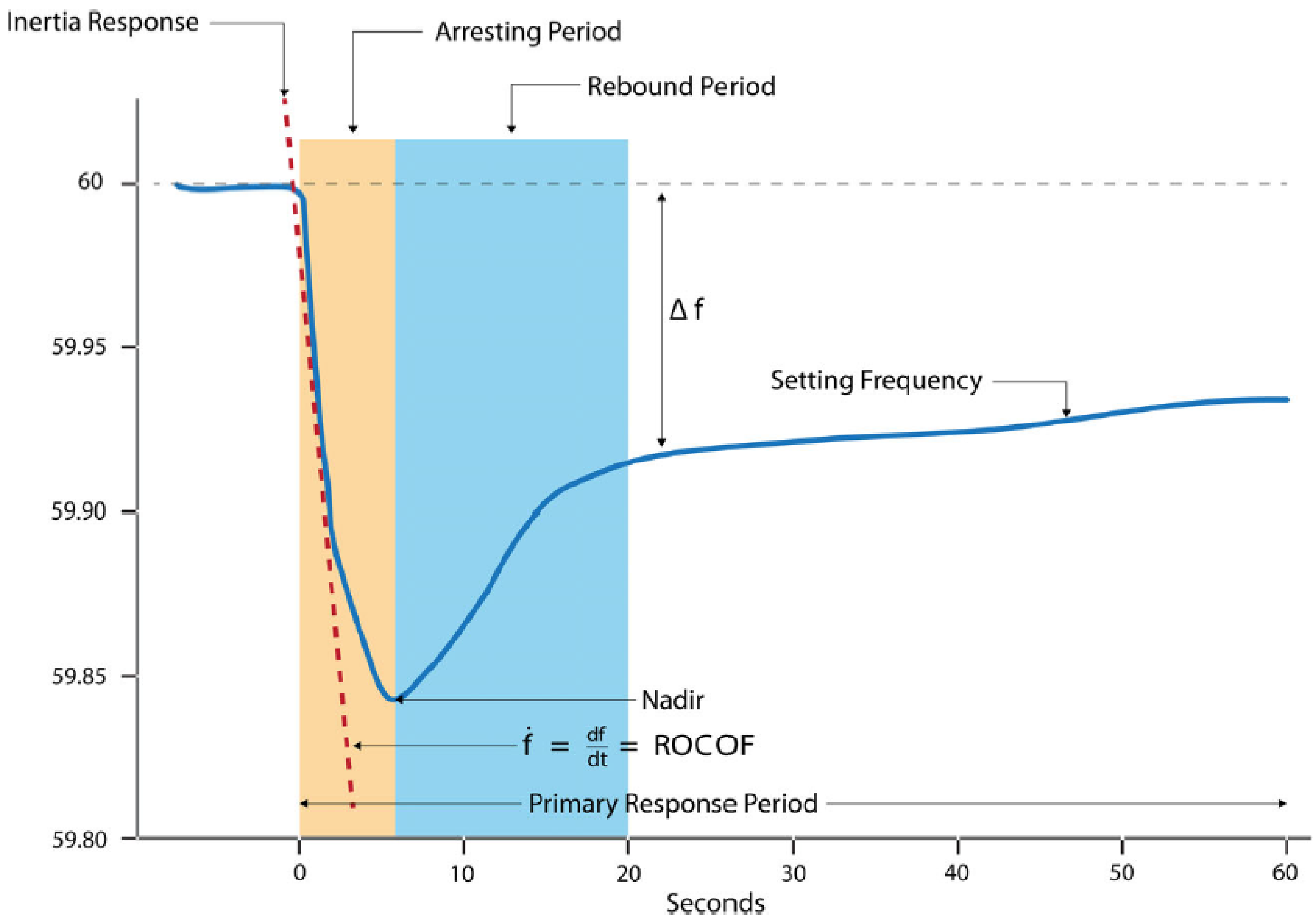
- Secondary frequency control, 30s to 10s of minutes



- Tertiary: After ~15 mins, adapt generator and load set points

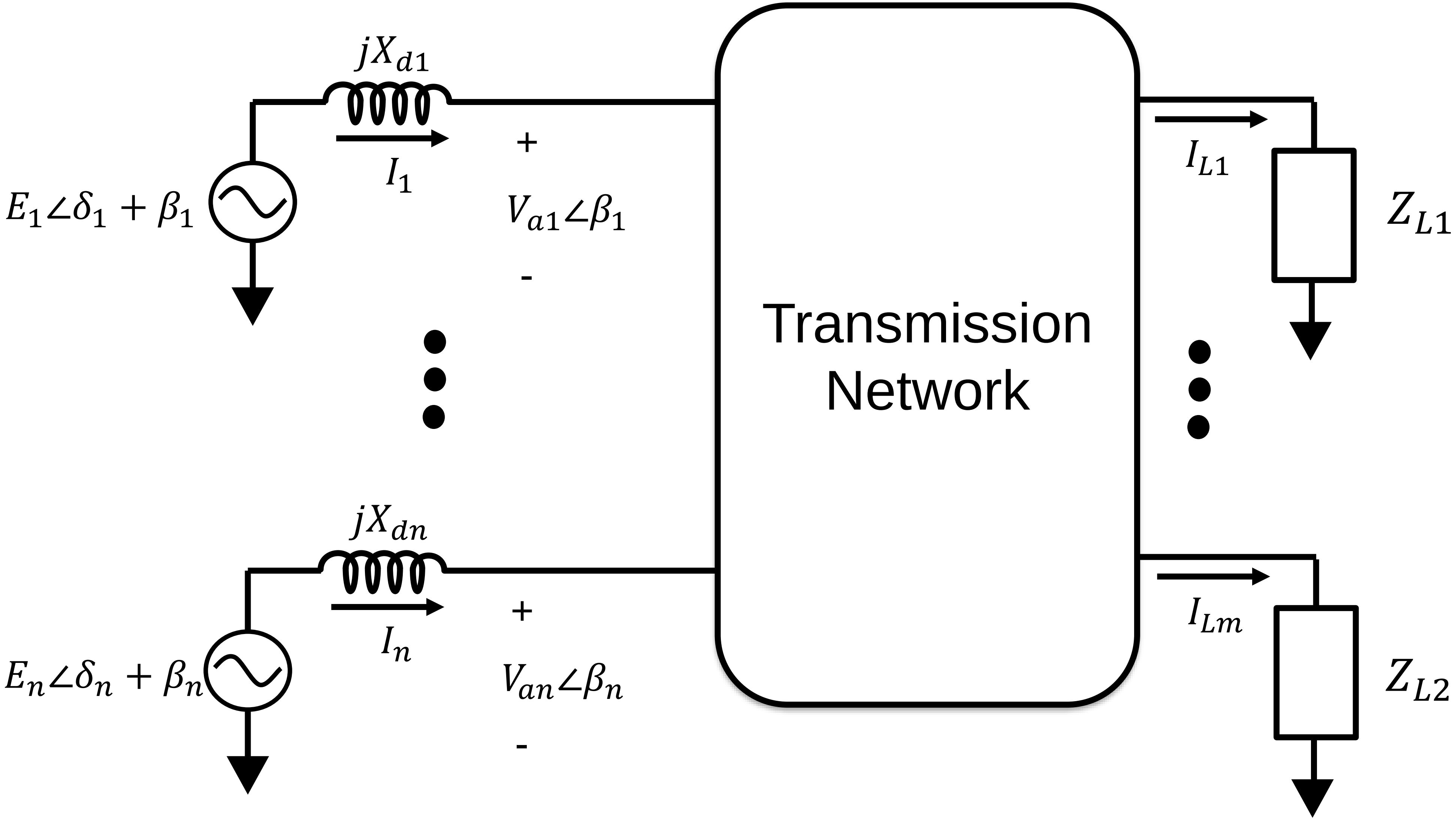
Rate of Change of Frequency

- ROCOF
  - Inversely proportional to system inertia
- Provides time for primary frequency control to adjust prime mover output

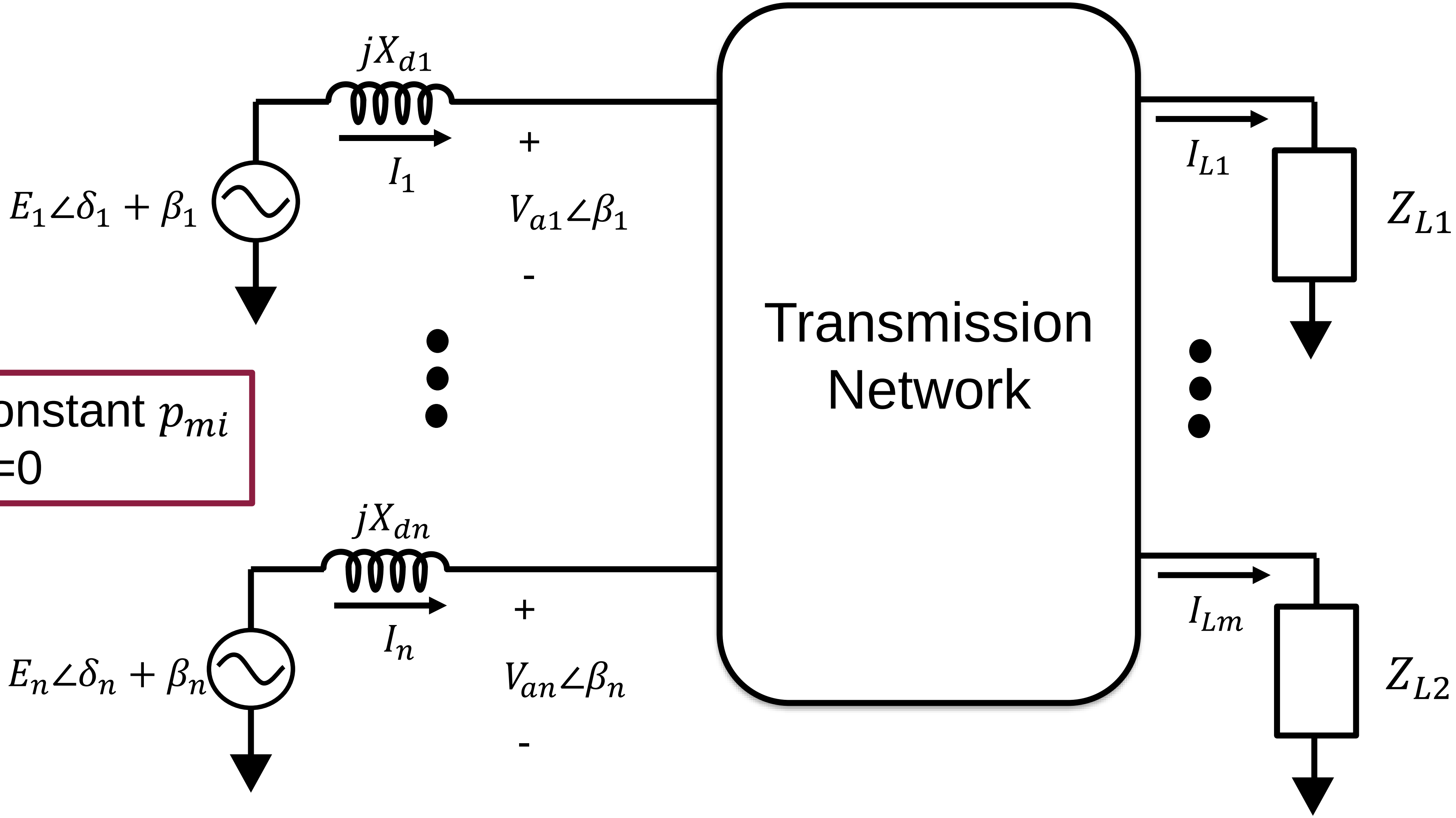


B.-M. S. Hodge *et al.*, "Addressing technical challenges in 100% variable inverter-based renewable energy power systems," *WIREs Energy and Environment*, vol. 9, no. 5, p. e376, 2020, doi: 10.1002/wene.376.

Power System Classical Model

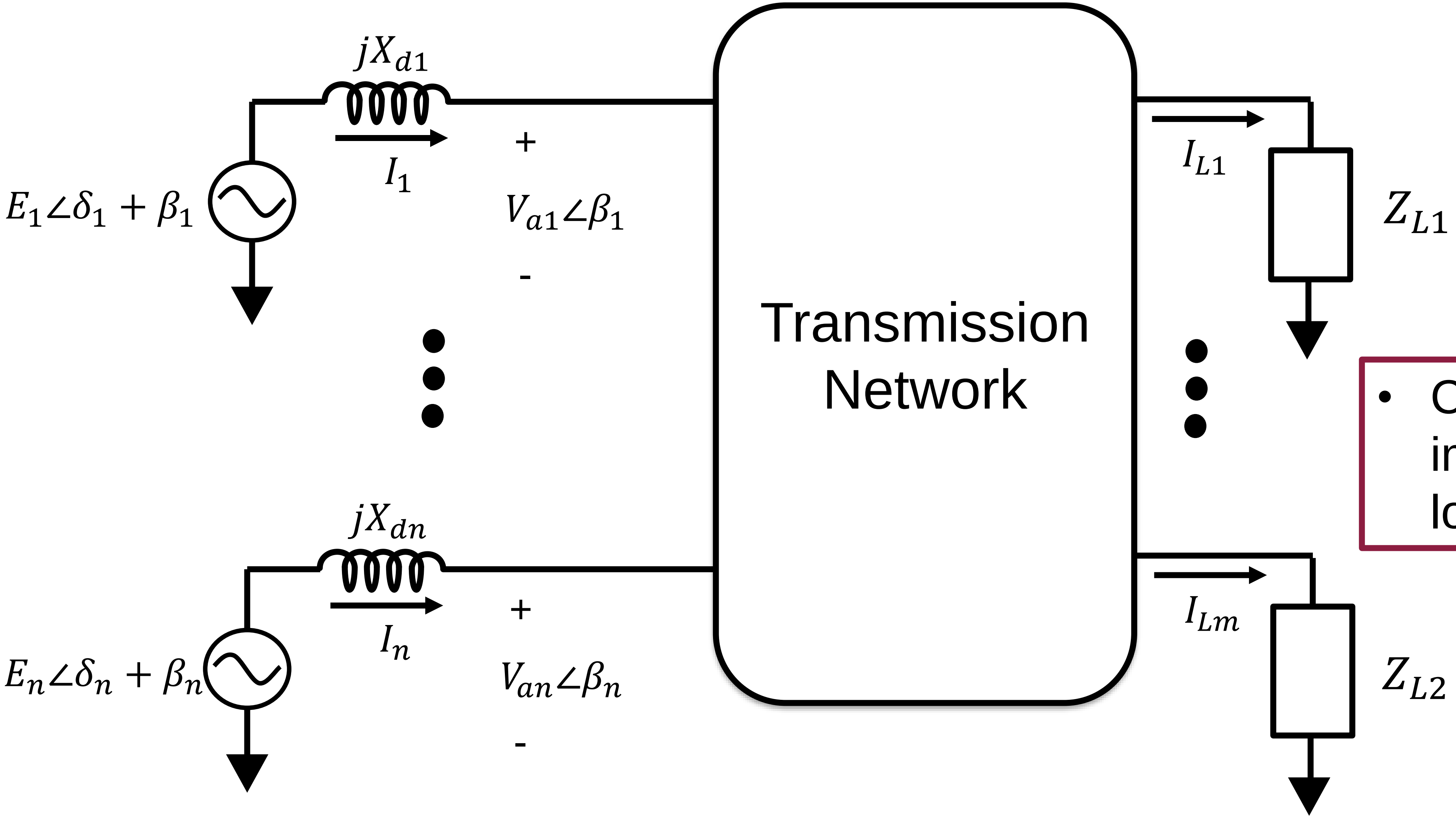


Power System Classical Model



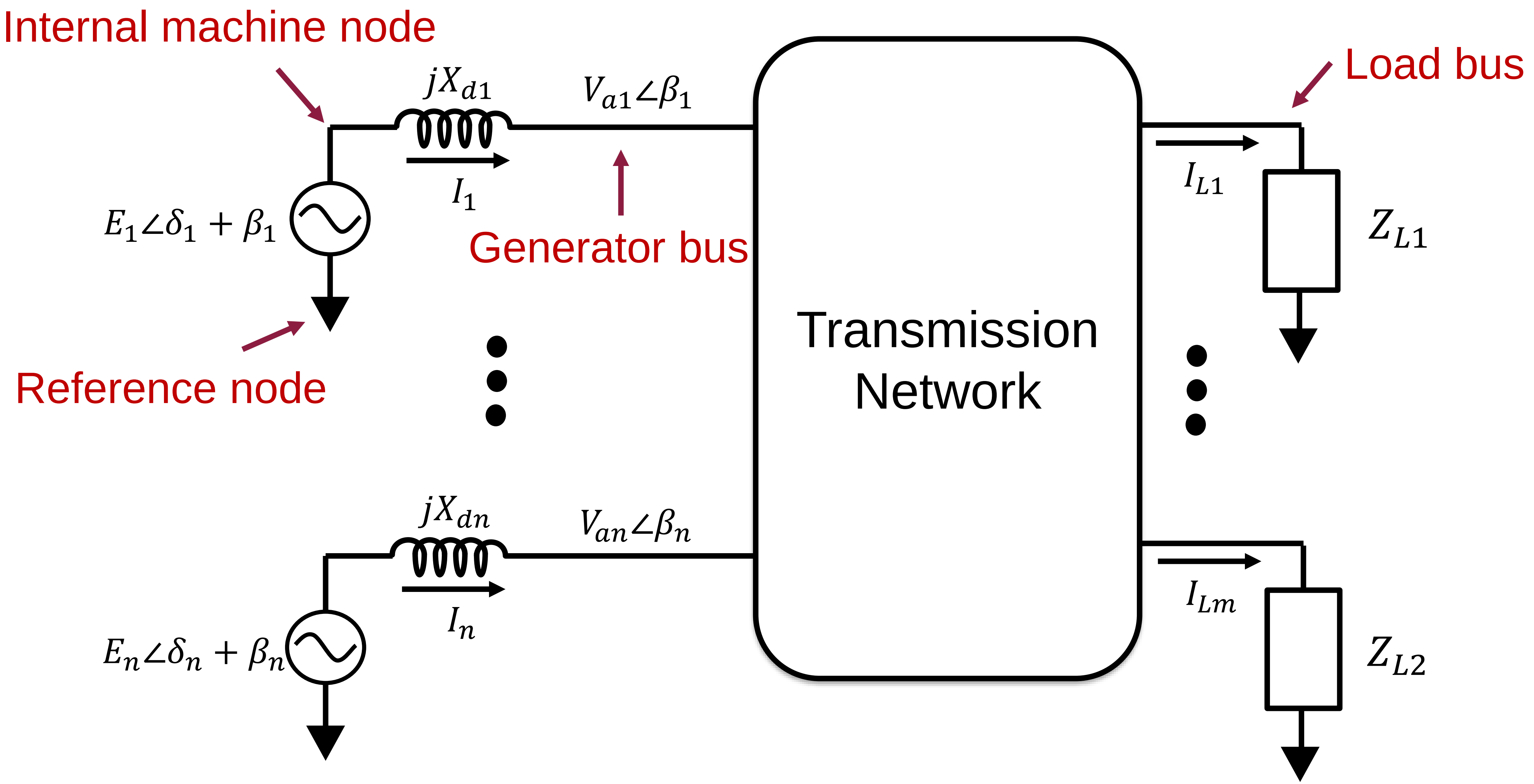
- Constant p_{mi}
- D=0

Power System Classical Model



- Constant impedance loads

Power System Classical Model



Obtain a System of Swing Equations

$$M_i \ddot{\delta}_i = p_{mi} - p_{ei}$$

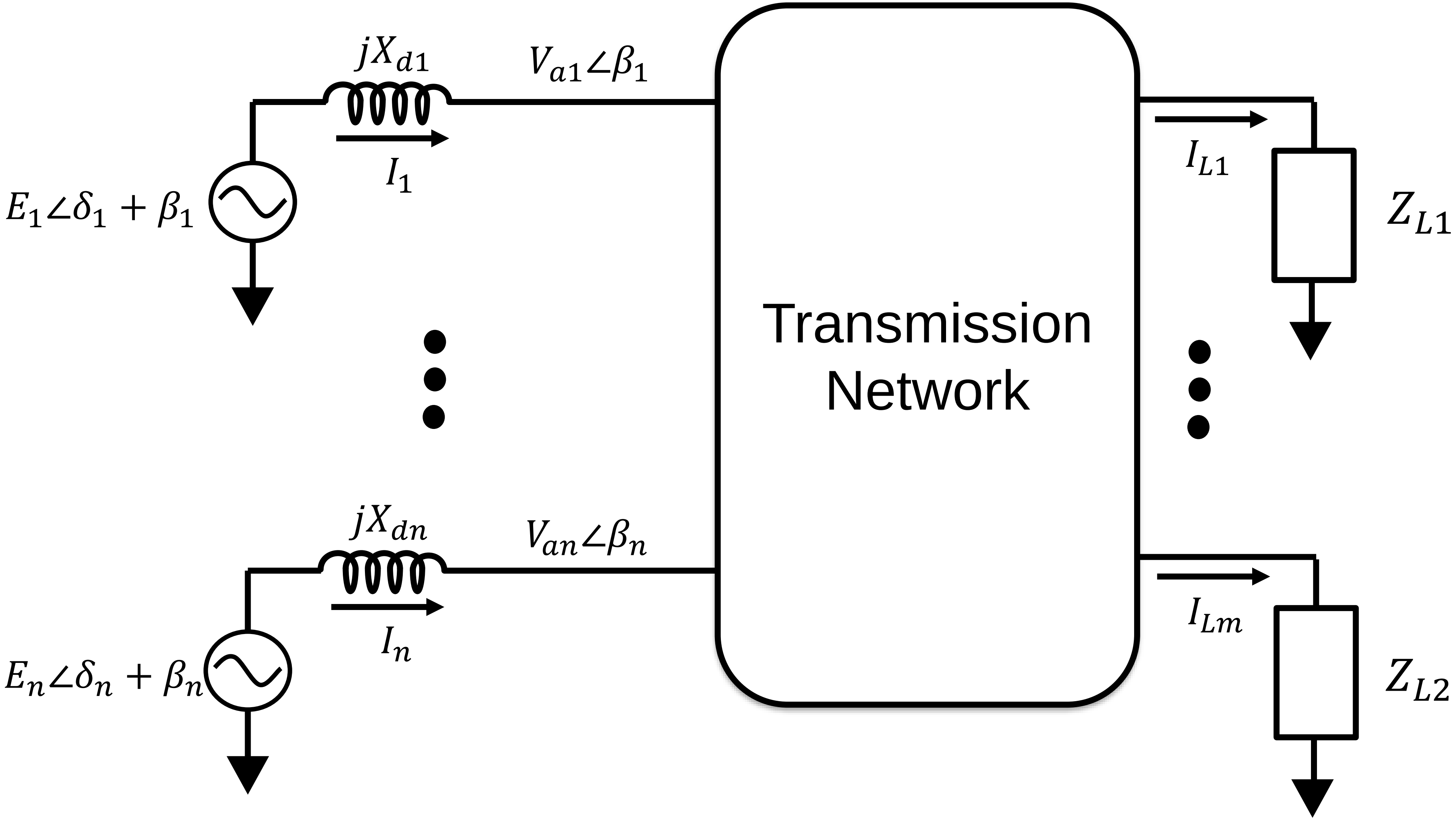
- p_{ei} for each generator depends on network, loads, and actions of all other generators

$$p_{ei} = \mathrm{Re}\{e_i i_i^*\}$$

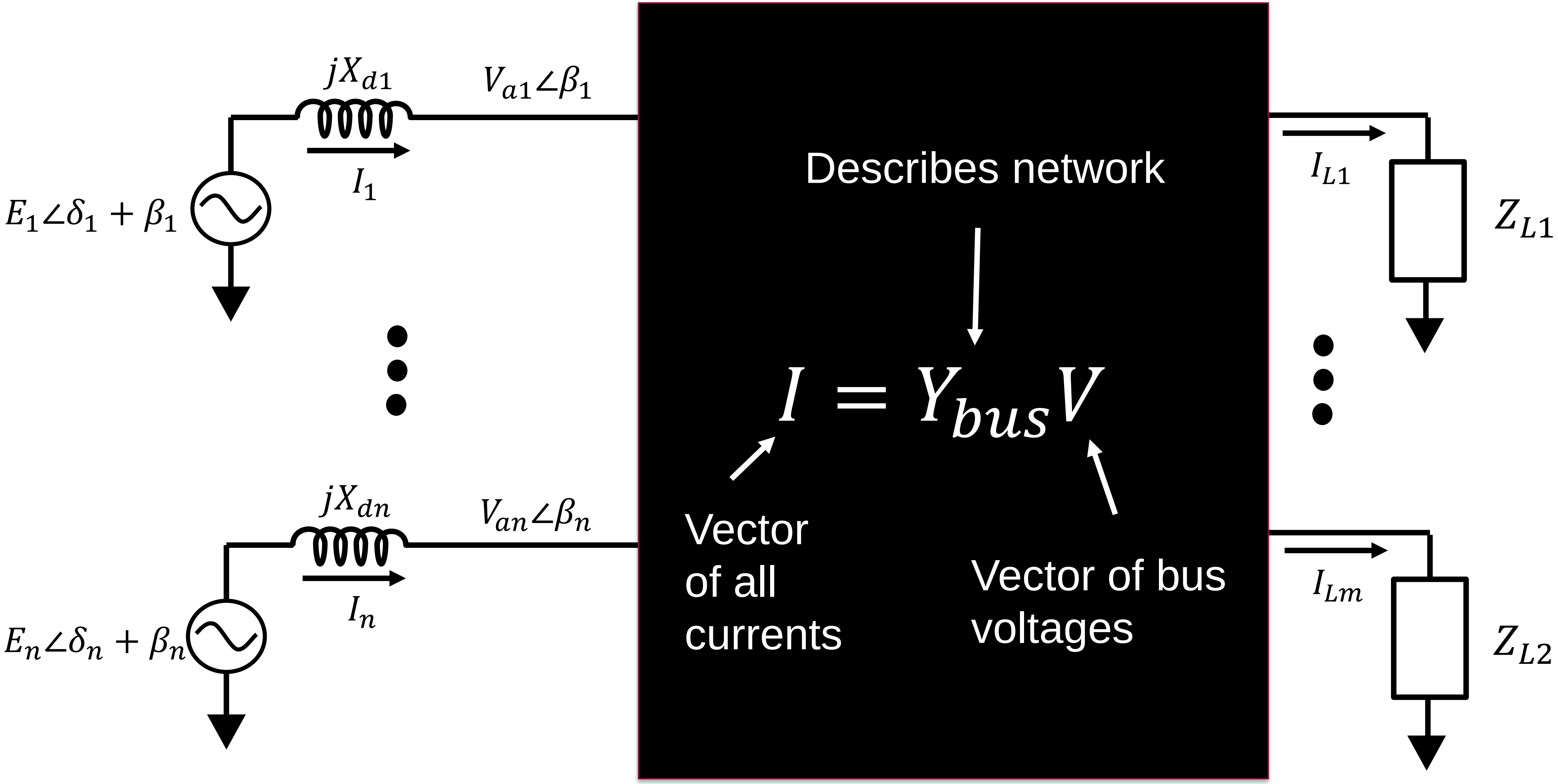
- Must solve network equation:

$$\begin{bmatrix} Y_{nn} & Y_{ns} \\ Y_{ns}^T & Y_{ss} \end{bmatrix} \begin{bmatrix} \boldsymbol{e_n} \\ \boldsymbol{v_s} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i_n} \\ 0 \end{bmatrix}$$

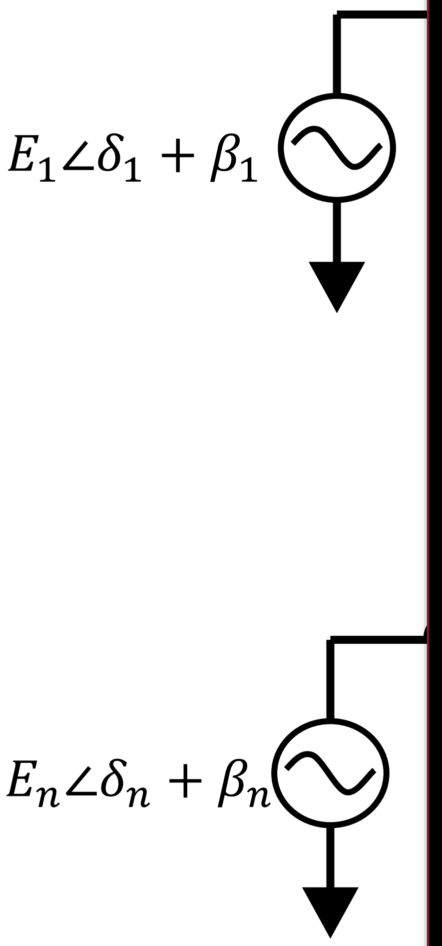
Mathematical Network Description



Mathematical Network Description



Mathematical Network Description



Describes network, load, and generator impedances

$$I = \hat{Y}E$$

Vector of generator currents

Vector of internal generator voltages

Admittance Matrix Definition

For *n* generators

$$\widehat{\boldsymbol{Y}} = \begin{bmatrix} \widehat{G}_{\mathbf{11}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\mathbf{11}} & \cdots & \widehat{G}_{\mathbf{1}\boldsymbol{n}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\mathbf{1}\boldsymbol{n}} \\ \vdots & \ddots & \vdots \\ \widehat{G}_{\boldsymbol{n}\mathbf{1}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\boldsymbol{n}\mathbf{1}} & \cdots & \widehat{G}_{\boldsymbol{nn}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\boldsymbol{nn}} \end{bmatrix}$$

$$I_i = \sum_{k=1}^{n} Y_{ik} E_k$$

Generator Power a Bit More Involved

For *n* generators

$$\widehat{\boldsymbol{Y}} = \begin{bmatrix} \hat{G}_{\mathbf{11}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\mathbf{11}} & \cdots & \hat{G}_{\mathbf{1}\boldsymbol{n}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\mathbf{1}\boldsymbol{n}} \\ \vdots & \ddots & \vdots \\ \hat{G}_{\boldsymbol{n}\mathbf{1}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\boldsymbol{n}\mathbf{1}} & \cdots & \hat{G}_{\boldsymbol{nn}} + \boldsymbol{j}\widehat{\boldsymbol{B}}_{\boldsymbol{nn}} \end{bmatrix}$$

$$I_i = \sum_{k=1}^{n} Y_{ik} E_k$$

$$S_i = V_i I_i^* = \sum_{k=1}^{n} Y_{ik}^* E_k^*$$

$$P_i = \sum_{k=1}^{n} |E_i||E_k|\left[\hat{G}_{ik}\cos\left(\delta_i - \delta_j\right) + \hat{B}_{ik}\sin\left(\delta_i - \delta_j\right)\right]$$

Multimachine Swing Equation

$$M_i\ddot{\delta}_i = p_{mi} - \underbrace{\sum_{k=1}^{n} |e_i||e_k|\left[\hat{g}_{ik}\cos(\delta_i - \delta_k) + \hat{b}_{ik}\sin(\delta_i - \delta_k)\right]}_{p_{ei}}$$

Now, multivariable definition:

$$x_i = \delta_i$$
$$x_{i+n} = \dot{\delta}_i$$

$$\dot{x}_i = \dot{\delta}_i = x_{i+n}$$

$$\dot{x}_{i+n} = \frac{p_{mi} - p_{ei}}{M}$$

e.g. $n = 2$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix}$$

Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

→

Apply fault by modifying **admittance matrix Y**

Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

Apply fault by modifying **admittance matrix Y**

Solve multimachine swing equation for fault duration

Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

→

Apply fault by modifying **admittance matrix Y**

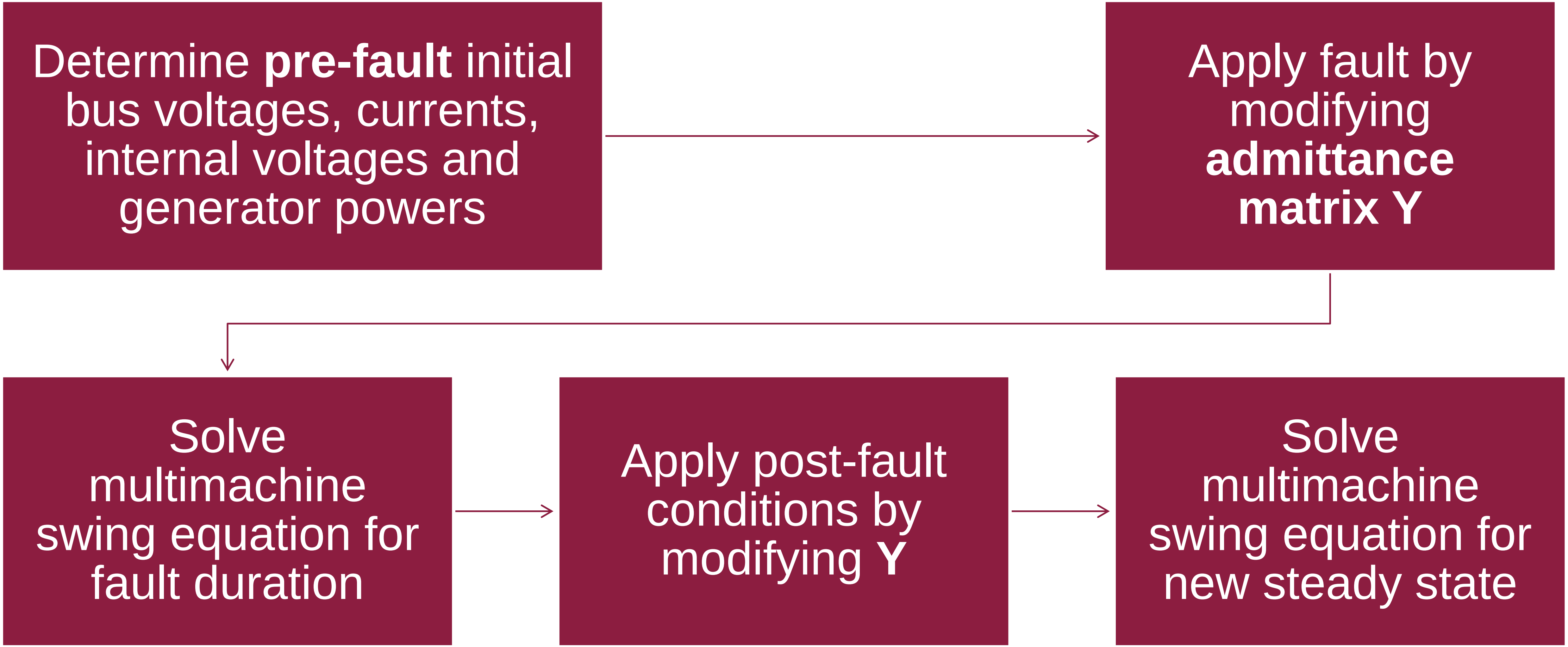
→

Solve multimachine swing equation for fault duration

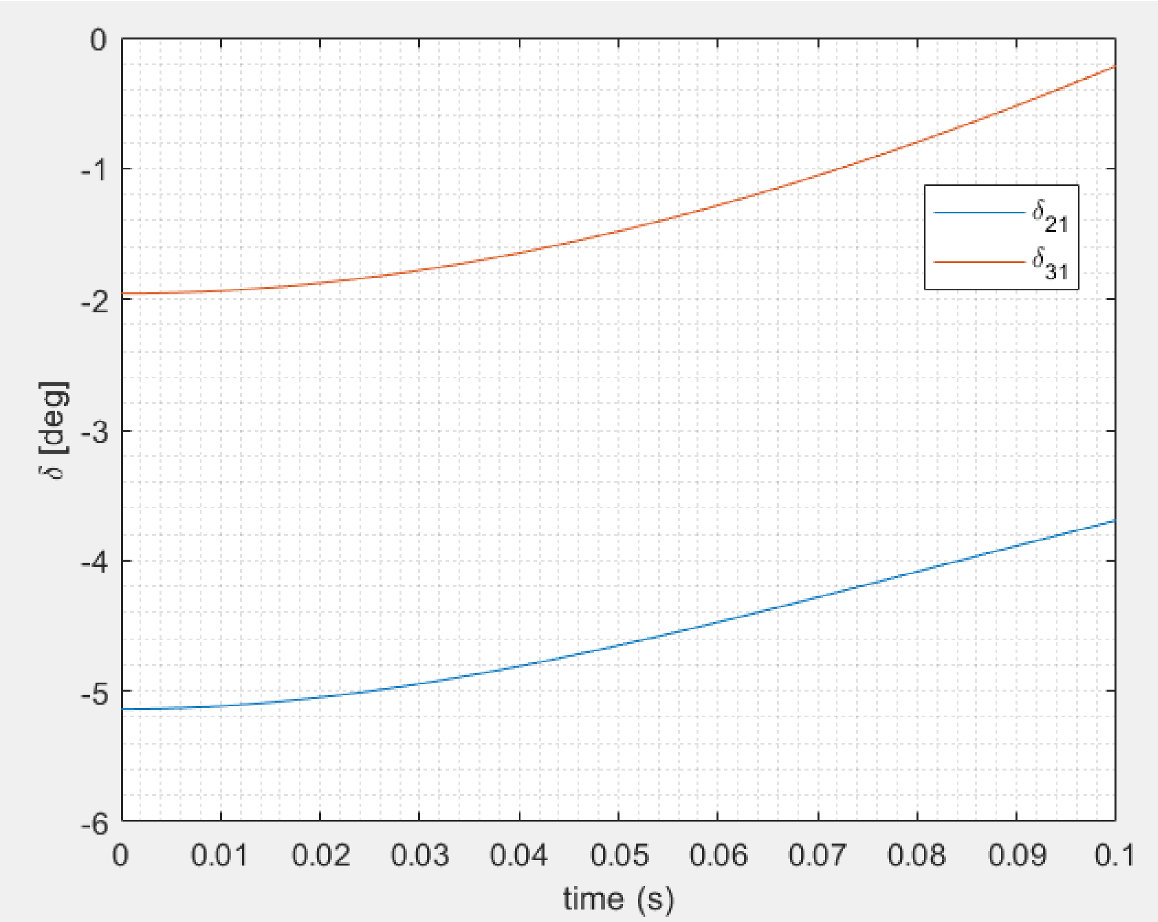
→

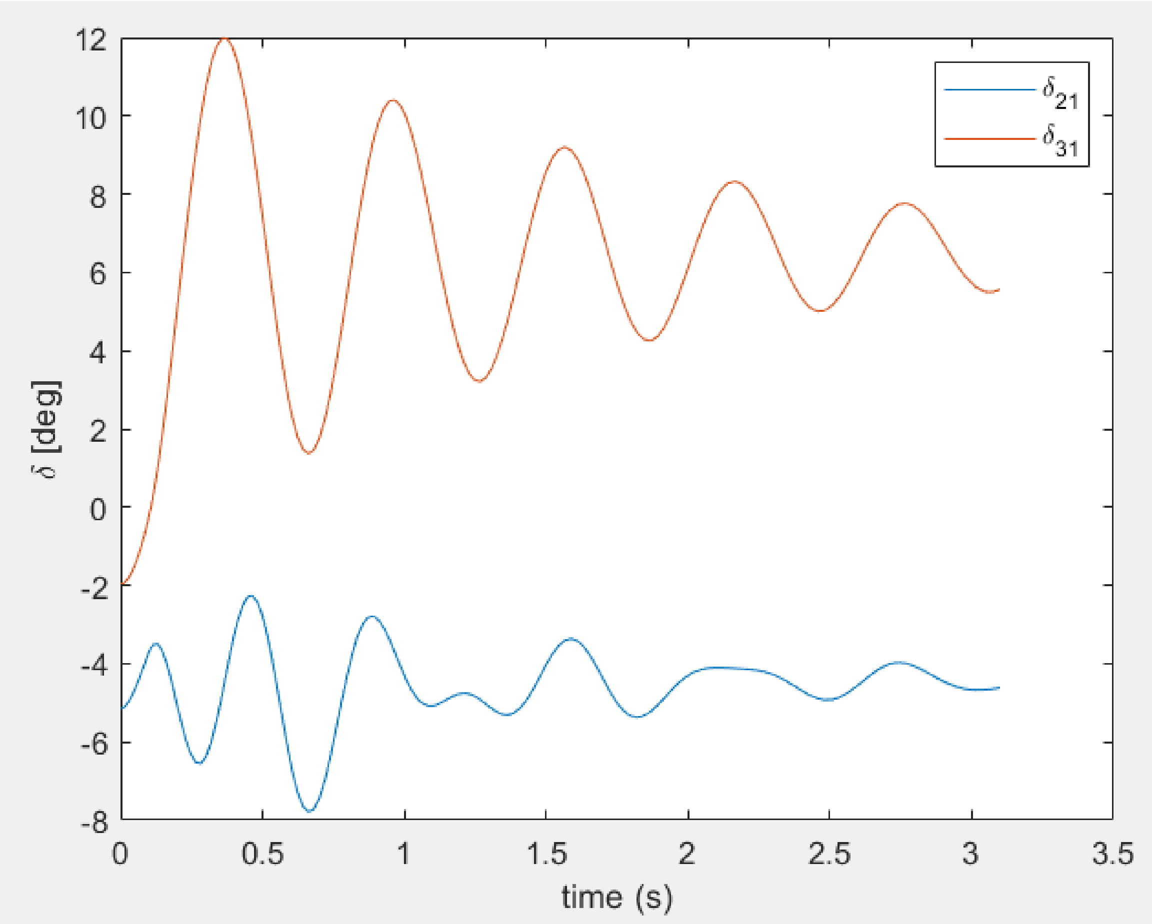
Apply post-fault conditions by modifying **Y**

Multimachine Fault Analysis

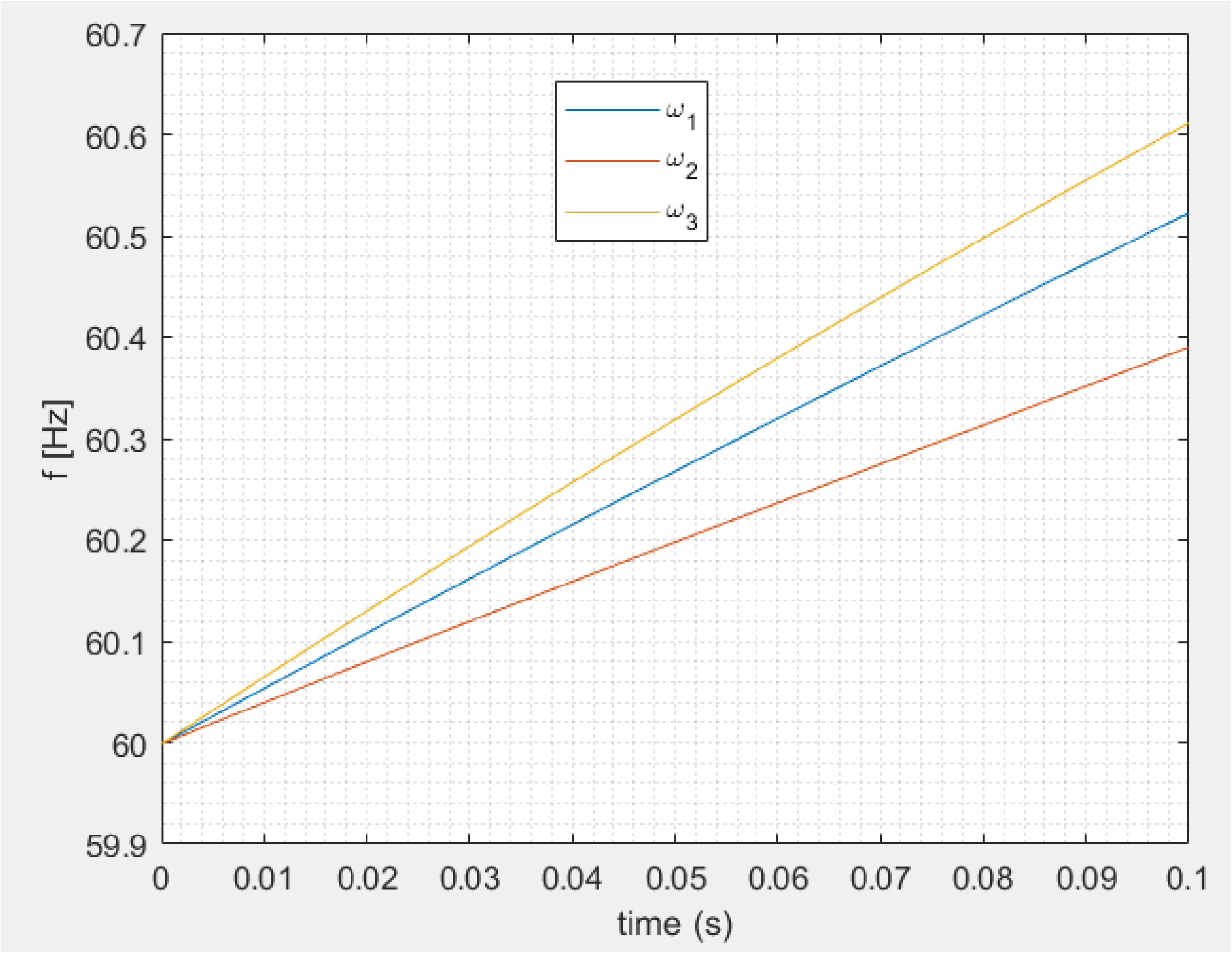


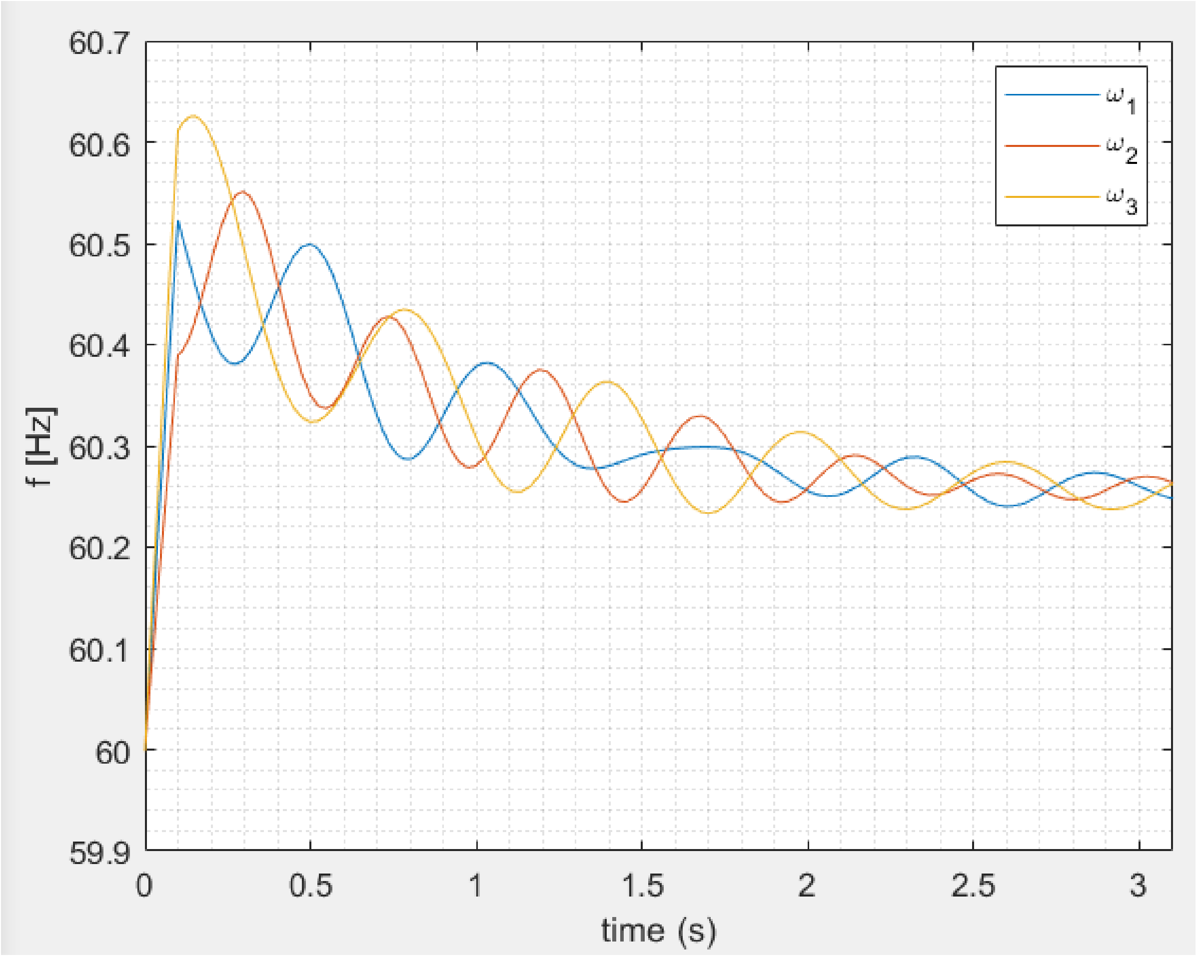
Example Solution for 7 Bus, 3 Generator system





Example Solution for 7 Bus, 3 Generator system





Conclusion

- Multimachine frequency dynamics are a straightforward conceptual extension of single-machine dynamics

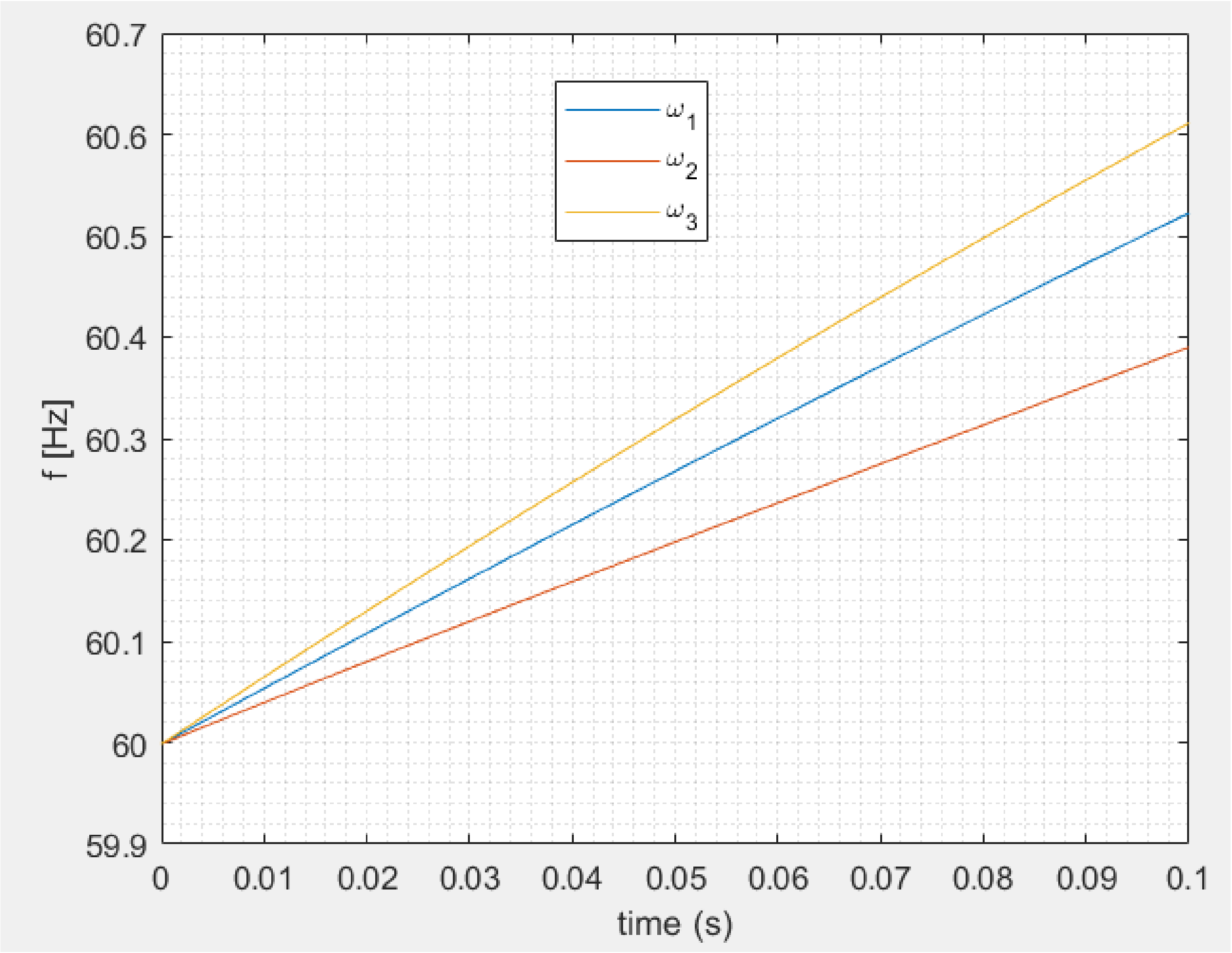
- Classical model enables “first swing analysis” to determine inertial response of electromechanical system. **Inertia buys us time.**

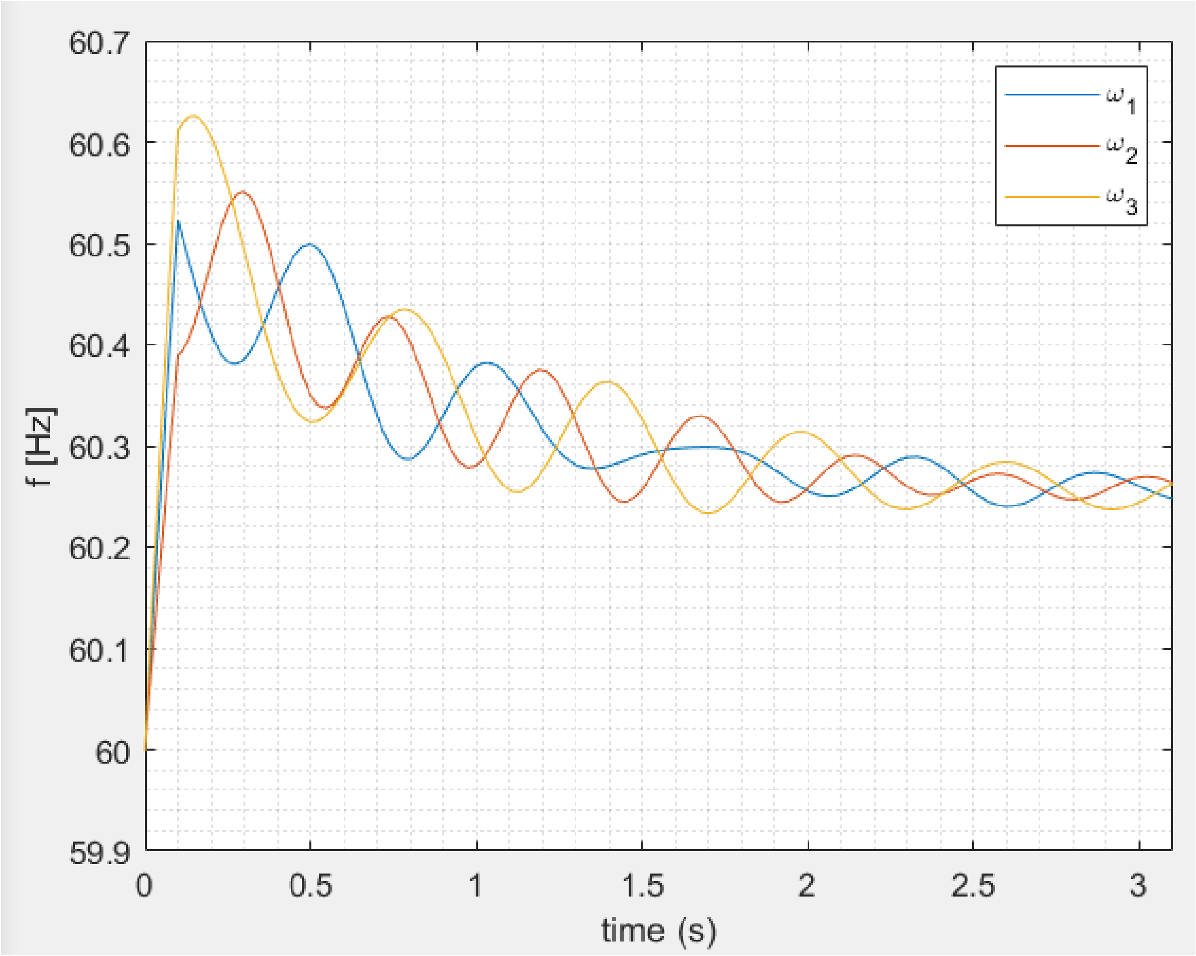
- Looking ahead… we wouldn’t need so much inertia if we could respond more quickly!

Conclusion

- Multimachine frequency dynamics are a straightforward conceptual extension of single-machine dynamics
- Classical model enables “first swing analysis” to determine inertial response of electromechanical system. **Inertia buys us time.**
- Looking ahead… we wouldn’t need so much inertia if we could respond more quickly
- Much higher detail can be added (damper circuits, rotor and stator circuits, detailed flux linkages, higher level control) by extensions of the classical model principle

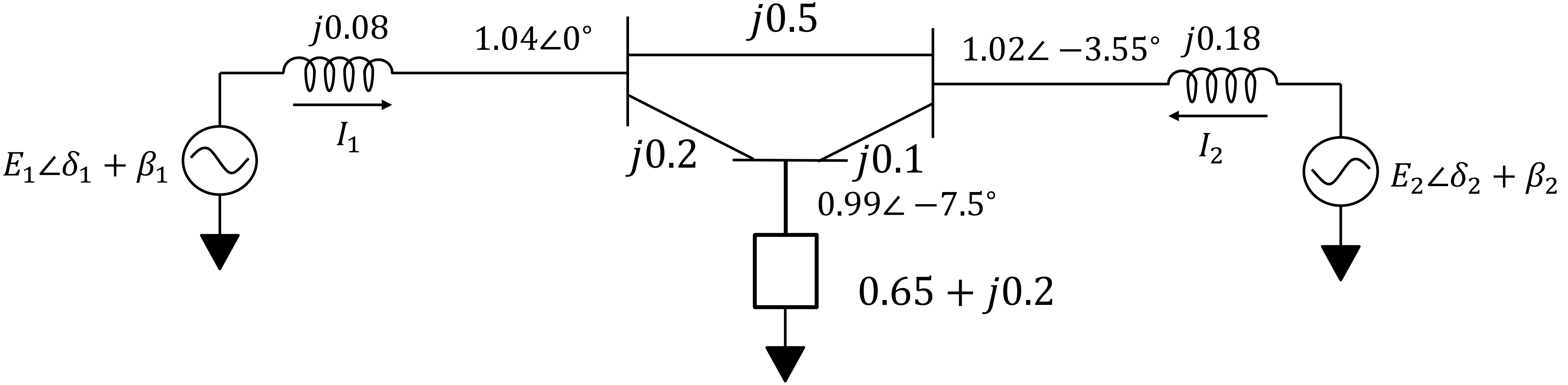
Example Solution for 7 Bus, 3 Generator system





Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers



Multimachine Fault Analysis

Determine **pre-fault** initial bus voltages, currents, internal voltages and generator powers

→ Determine pre-fault admittance matrix

→ Apply fault by modifying **admittance matrix Y**

→ Solve multimachine swing equation until fault cleared

→ Apply post-fault by modifying Y

→ Solve multimachine swing equation

Example of Applying Fault

$$p_{e1} = |e_1|^2 \hat{g}_{11} + |e_1||e_2|\left[\hat{g}_{12}\cos(\delta_1 - \delta_2) + \hat{b}_{12}\sin(\delta_1 - \delta_2)\right]$$

$$p_{e2} = |e_2|^2 \hat{g}_{22} + |e_1||e_2|\left[\hat{g}_{21}\cos(\delta_2 - \delta_1) + \hat{b}_{21}\sin(\delta_2 - \delta_1)\right]$$

Now, multivariable definition:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix}$$

$$\dot{x}_i = \dot{\delta}_i = x_{i+n}$$

$$\dot{x}_{i+n} = \frac{p_{mi} - p_{ei}}{M}$$

Network Component Descriptions

$$\begin{bmatrix} Y_{nn} & Y_{ns} \\ Y_{ns}^T & Y_{ss} \end{bmatrix} \begin{bmatrix} \boldsymbol{e_n} \\ \boldsymbol{v_s} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i_n} \\ 0 \end{bmatrix}$$

$$\boldsymbol{i} = \begin{bmatrix} i_1 \\ \vdots \\ . \\ i_n \end{bmatrix}$$

generator currents

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ \vdots \\ . \\ e_n \end{bmatrix}$$

Generator internal voltages

$$\boldsymbol{v_a} = \begin{bmatrix} v_1 \\ \vdots \\ . \\ v_m \end{bmatrix}$$

Bus voltages

(non-generator)

Obtain a System of Swing Equations

$$M_i \ddot{\delta}_i = p_{mi} - p_{ei}$$

- p_{ei} for each generator depends on network, loads, and actions of all other generators

$$p_{ei} = \mathrm{Re}\{e_i i_i^*\}$$

- Must solve network equation:

$$\begin{bmatrix} Y_{nn} & Y_{ns} \\ Y_{ns}^T & Y_{ss} \end{bmatrix} \begin{bmatrix} \boldsymbol{e_n} \\ \boldsymbol{v_s} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i_n} \\ 0 \end{bmatrix}$$

Proceeds as Before

+

$V_t\angle 0°$

-

ent

$$M_i\ddot{\delta}_i = p_{mi} - p_{ei}$$

- p_{ei} for each generator depends on network, loads, and actions of all other generators

$$\begin{bmatrix} Y_{nn} & Y_{ns} \\ Y_{ns}^T & Y_{ss} \end{bmatrix} \begin{bmatrix} \boldsymbol{e_n} \\ \boldsymbol{v_s} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i_n} \\ 0 \end{bmatrix}$$

Obtain a System of Swing Equations

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- p_{ei} for each generator depends on network, loads, and actions of all other generators

$$p_{ei} = \mathrm{Re}\{e_i i_i^*\}$$

- Must solve network equation:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v_a} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

Network Component Descriptions

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

Assume **N system busses** and **M internal machine buses**

$$\boldsymbol{i} = \begin{bmatrix} i_1 \\ \vdots \\ . \\ i_m \end{bmatrix} \qquad \boldsymbol{e} = \begin{bmatrix} e_1 \\ \vdots \\ . \\ e_m \end{bmatrix}$$

M vector of generator currents and internal voltages

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ \vdots \\ . \\ v_n \end{bmatrix}$$

N vector of bus voltages

Admittance Matrix, Y_{22}

N system busses and **M internal machine buses**

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

$$Y_{11}\boldsymbol{v} + \mathrm{Y}_{12}\mathbf{e} = 0$$
$$Y_{12}^T\boldsymbol{v} + Y_{22}\boldsymbol{e} = \boldsymbol{i}$$

MxM diagonal matrix of reciprocal generator reactances

$$\boldsymbol{Y}_{22} = \begin{bmatrix} \frac{1}{j\mathrm{X}_{d1}} & & & 0 \\ & \frac{1}{j\mathrm{X}_{d2}} & & \\ & & \ddots & \\ 0 & & & \frac{1}{j\mathrm{X}_{dM}} \end{bmatrix}$$

Network Component Descriptions

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v_a} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

Assume **N system busses** and **M internal machine buses**

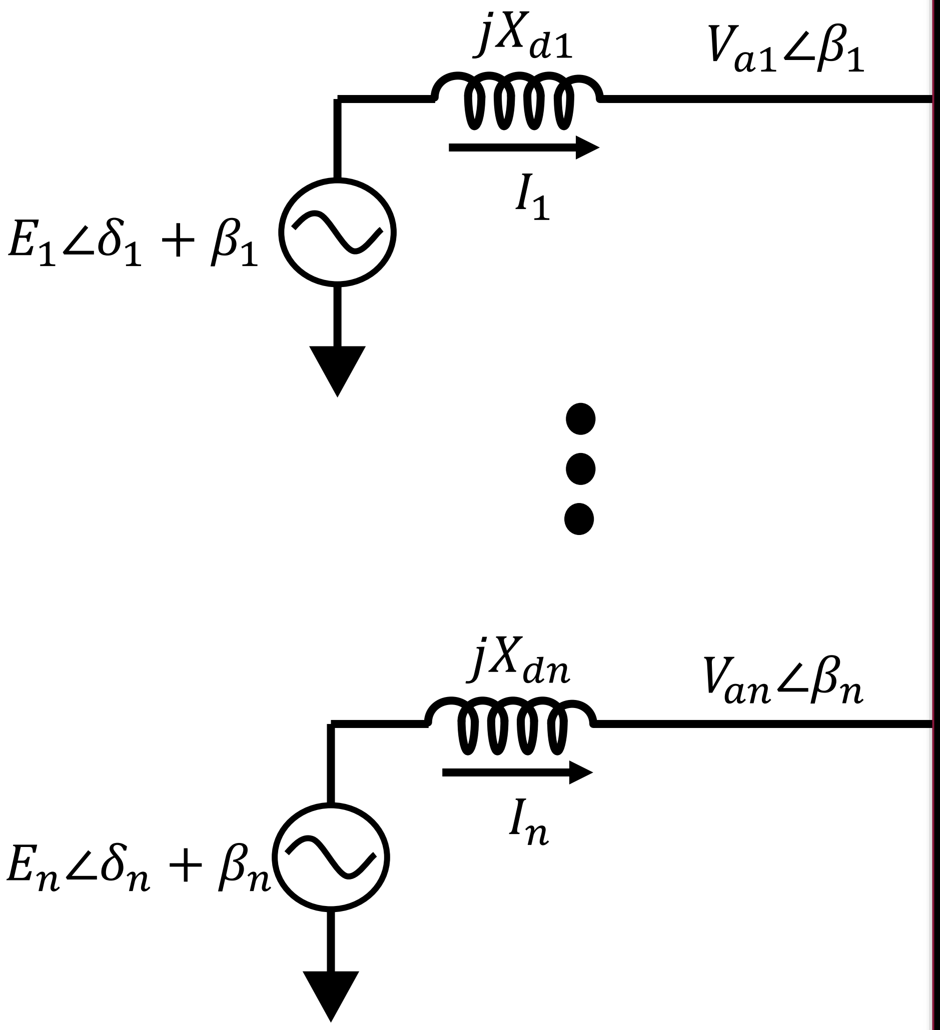
$$\boldsymbol{i} = \begin{bmatrix} i_1 \\ \vdots \\ . \\ i_m \end{bmatrix} \qquad \boldsymbol{e} = \begin{bmatrix} e_1 \\ \vdots \\ . \\ e_m \end{bmatrix}$$

M vector of generator currents and internal voltages

$$\boldsymbol{v_a} = \begin{bmatrix} v_1 \\ \vdots \\ . \\ v_n \end{bmatrix}$$

N vector of bus voltages

Mathematical Network Description



Describes network and loads

$$I = Y_{bus}V$$

Vector of generator currents

Vector of bus voltages

Admittance Matrix, Y_{22}

N system busses and **M internal machine buses**

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

$$Y_{11}\boldsymbol{v} + \mathrm{Y}_{12}\mathbf{e} = 0$$
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Diagonal matrix of reciprocal generator reactances

$$\boldsymbol{Y}_{22} = \begin{bmatrix} \frac{1}{j\mathrm{X}_{d1}} & & & 0 \\ & \frac{1}{j\mathrm{X}_{d2}} & & \\ & & \ddots & \\ 0 & & & \frac{1}{j\mathrm{X}_{dM}} \end{bmatrix}$$

Network Component Descriptions

NxN
Relates

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}$$

Network Component Descriptions

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{v_a} \\ \boldsymbol{e} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{i} \end{bmatrix}$$

$$\boldsymbol{i} = \begin{bmatrix} i_1 \\ \vdots \\ . \\ i_n \end{bmatrix}$$

generator currents

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ \vdots \\ . \\ e_n \end{bmatrix}$$

Generator internal voltages

$$\boldsymbol{v_a} = \begin{bmatrix} v_{a1} \\ \vdots \\ . \\ v_{an} \end{bmatrix}$$

Bus voltages

Obtain a System of Swing Equations

$$M_i \ddot{\delta}_i = p_{mi} - p_{ei}$$

- p_{ei} for each generator depends on network, loads, and actions of all other generators
- Network is defined by admittance matrix

Admittance between the i_{th} bus and the k_{th}

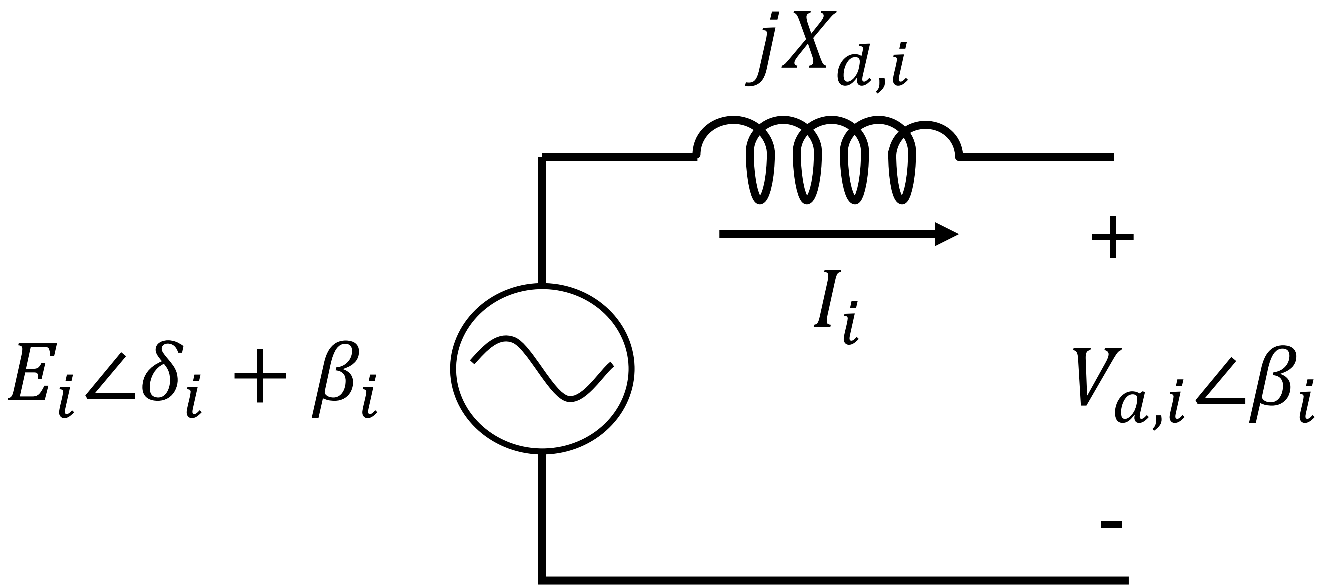
$$I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

$$i_i^* = v_{ai}$$

$$p_{ei} = \mathrm{Re}\{e_i i_i^*\}$$

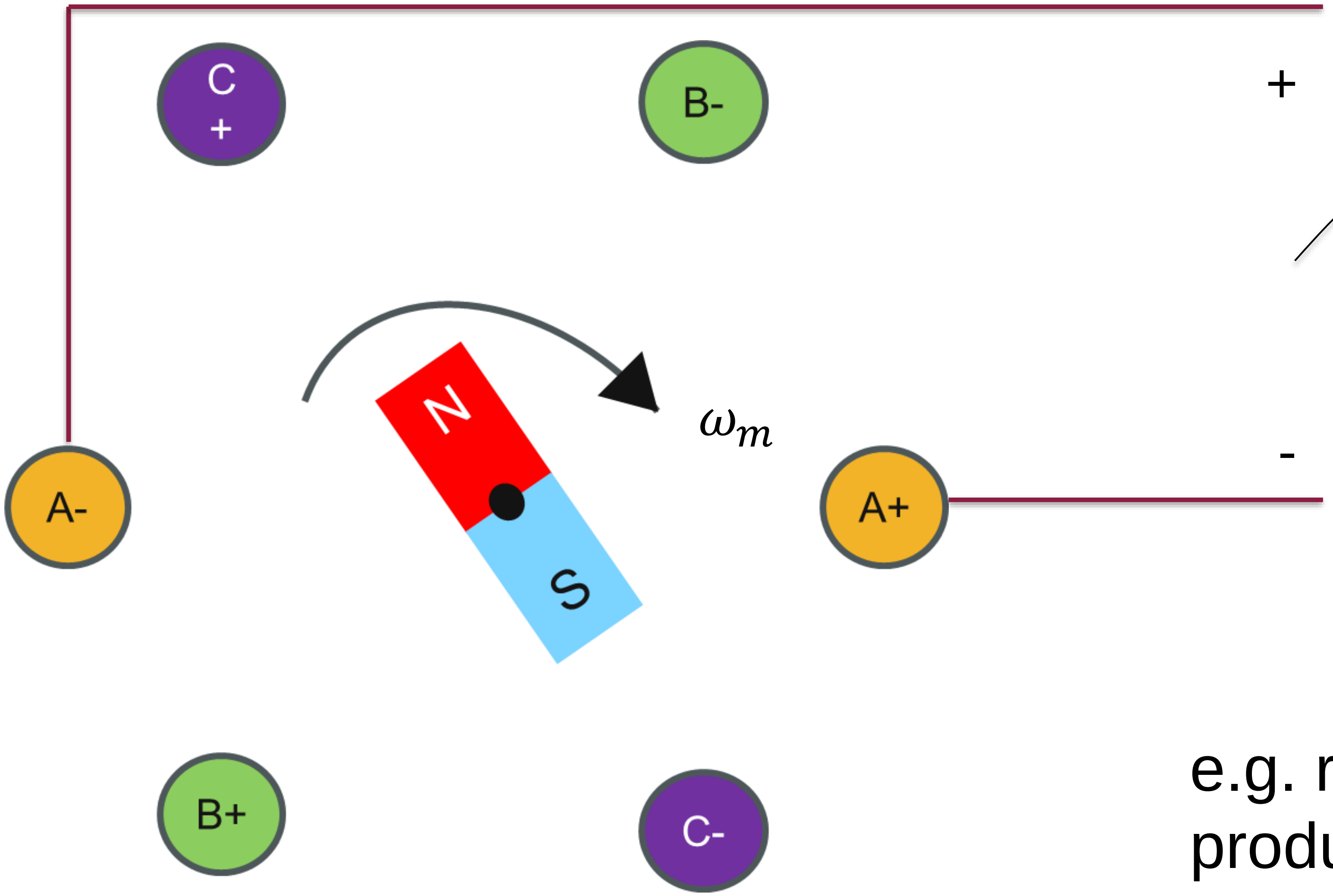
Power System Classical Model

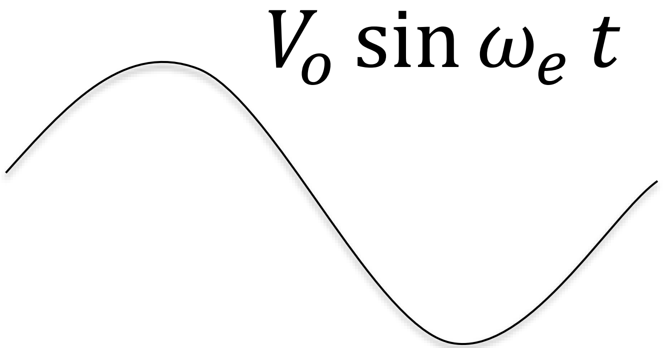
- Start with generator model, assume D=0



Exercises

Grid Frequency Set by Generator Rotational Speed





$$\omega_e = k\omega_m$$

e.g. rotate at 3600 RPM to produce 60Hz

Determine Initial Conditions

$$\delta(0^-) = \delta(0^+) = \delta_0$$
$$\dot{\delta} = 0$$

$$p_{m1} = \frac{ev}{x_d}\sin\delta_o \Rightarrow \delta_0 = 0.4298\text{rad.}$$
$$= 24.6°$$

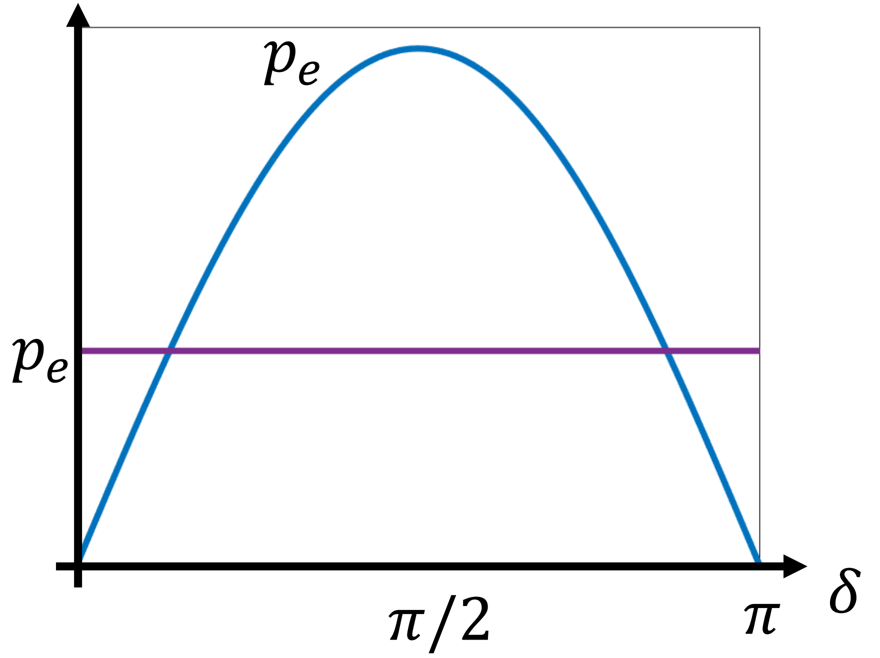
Assume negligible friction

$f_m = 60\text{Hz}$

Per unit quantities:

$H = 5, x_d = 0.5, e = 1.2$

$\text{v} = 1, p_m = 1.0$



Define Governing Equation

Governing equation:

$$\boxed{p_m = p_{e,max} \sin\delta + D\dot{\delta} + M\ddot{\delta}}$$

Split into two first order equations:

$$x_1 = \delta,\ x_2 = \dot{\delta}$$

$$\dot{x_1} = \dot{\delta} = x_2$$

$$\dot{x_2} = \frac{p_m - Dx_2 - p_{e,max}\sin \mathrm{x}_1}{M}$$

Must solve numerically

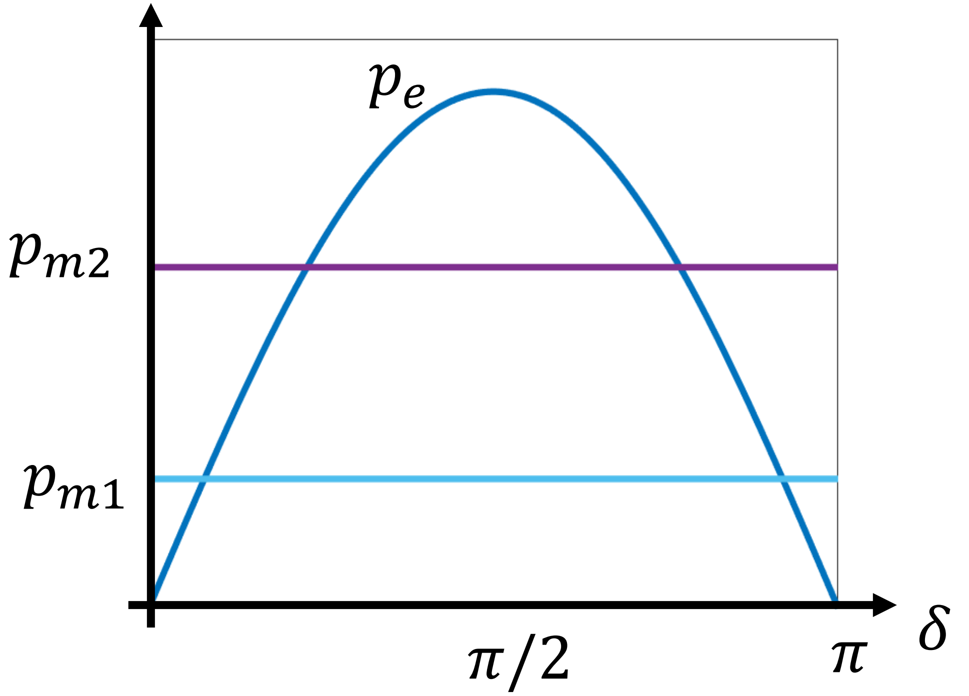
Assume negligible friction

$f_m = 60\text{Hz}$

Per unit quantities:

$H = 5, x_d = 0.5, e = 1.2$

$\mathrm{v} = 1, p_{m1} = 0.6, p_{m2} = 1.8$



Example: Increase in Mechanical Power

Governing equation:

$$2.4\sin\delta + 0.0265\ddot{\delta} = 1.8$$

Split into two first order equations:

$$\dot{\delta} = \omega(t)$$

$$\dot{\omega}(t) = 67.92 - 90.57\sin\delta$$

Use MATLAB to solve

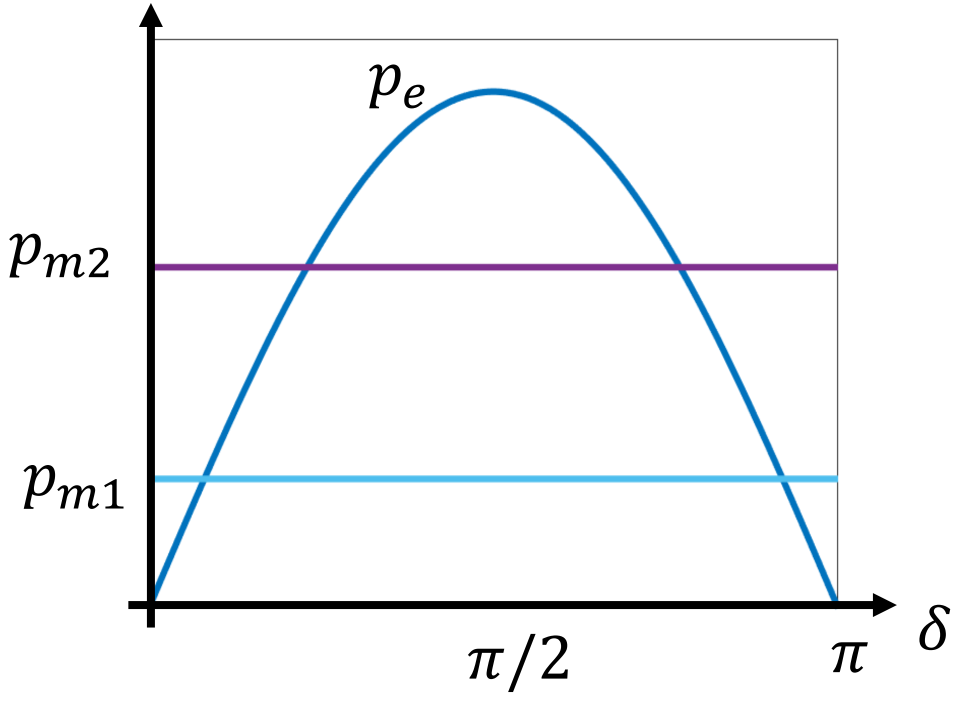
Assume negligible friction

$f_m = 60\text{Hz}$

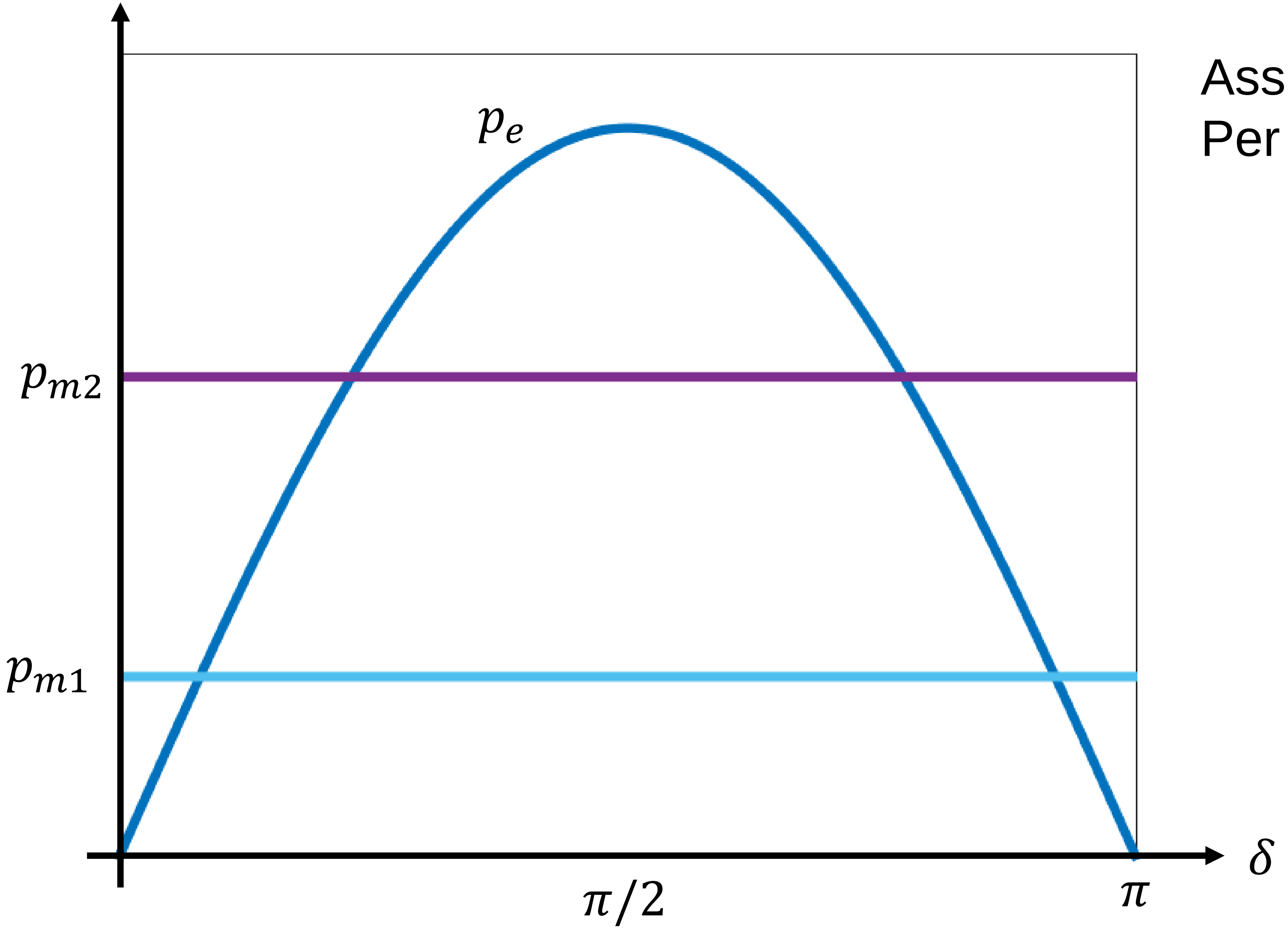
Per unit quantities:

$H = 5, x_d = 0.5, e = 1.2$

$\text{v} = 1, p_{m1} = 0.6, p_{m2} = 1.8$



Example: Increase in Mechanical Power

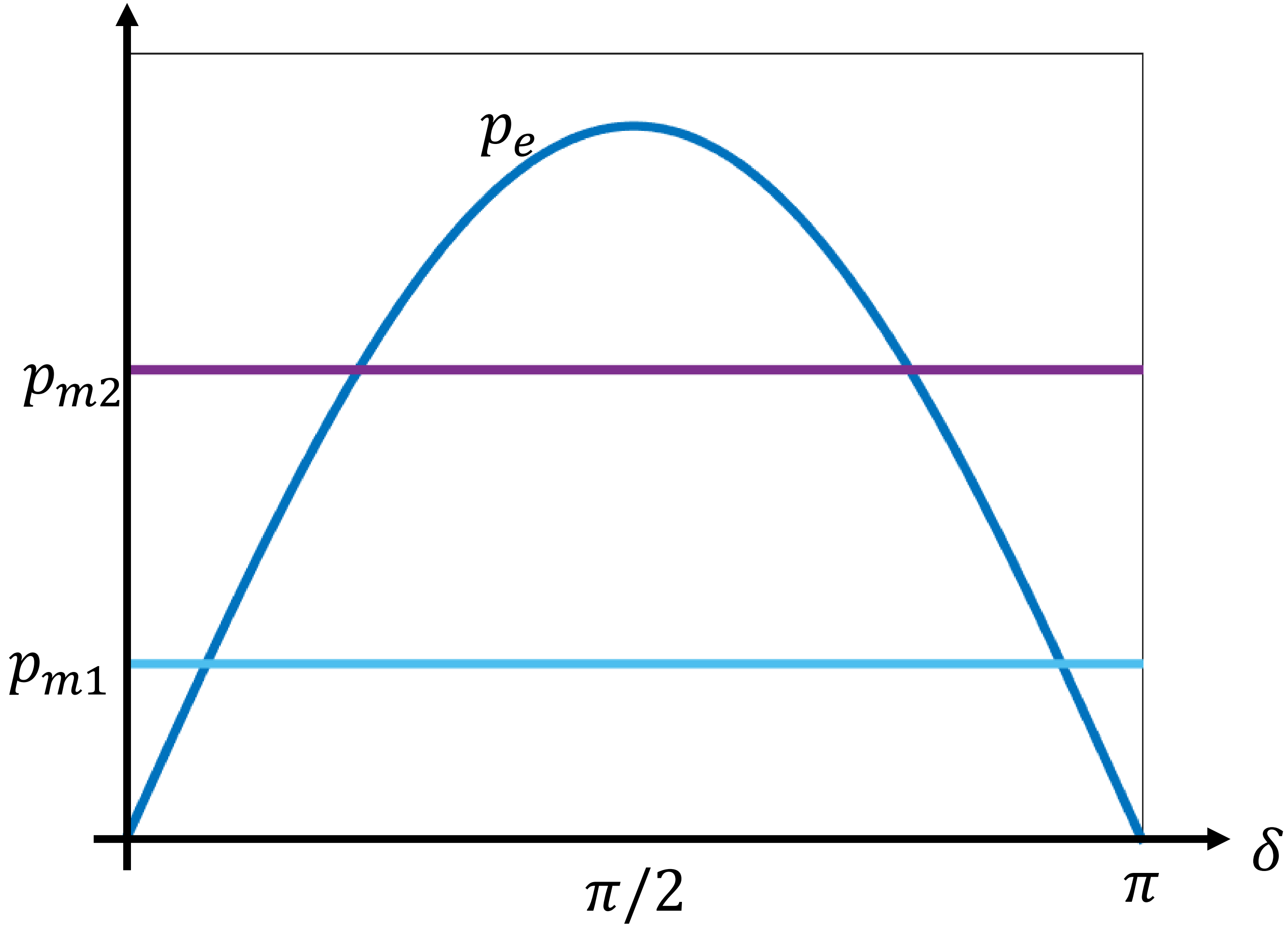


Assume negligible friction
Per unit quantities:

$$H = 5$$
$$x_d = 0.2$$
$$e = 1.2$$
$$\mathrm{v} = 1$$

$$M = \frac{H}{}$$

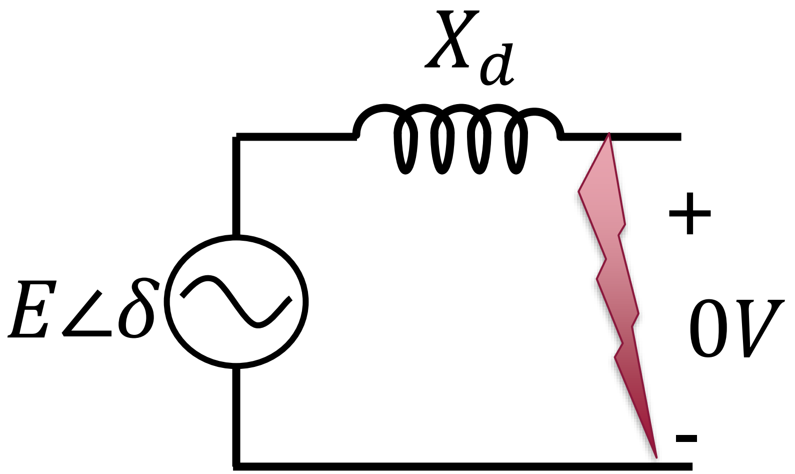
Example: Increase in Mechanical Power



Assume negligible friction
Per unit quantities:

$$H = 5$$
$$x_d = 0.2$$
$$e = 1.2$$
$$\mathrm{v} = 1$$

Example: Generator Terminal Fault

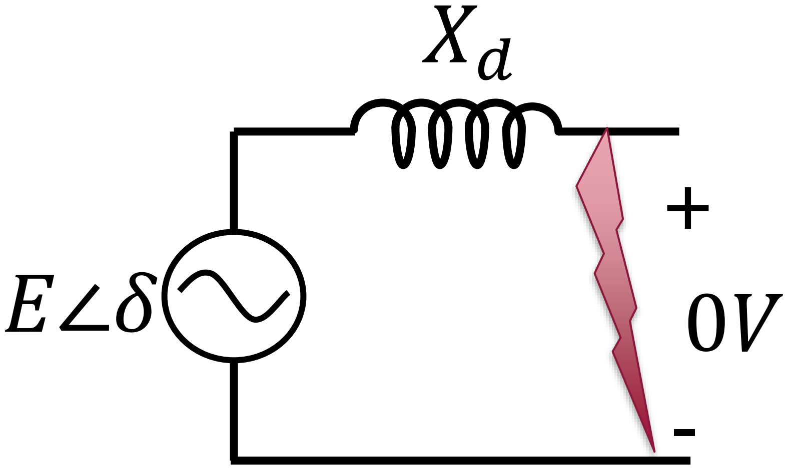


Assume negligible

$$\frac{P_m}{S_{base}} = \cancel{D}\dot{\delta} + M\ddot{\delta}$$

$$\delta(0) = \delta_1$$
$$\dot{\delta}(0) = 0$$

Example: Generator Terminal Fault



Assume negligible

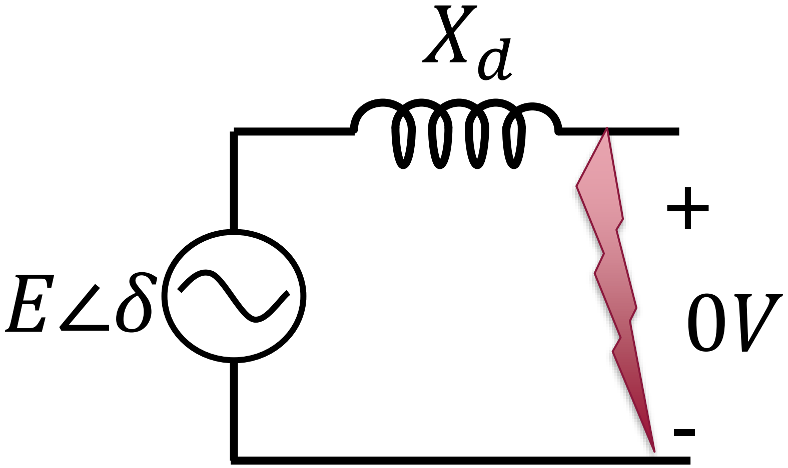
$$\frac{P_m}{S_{base}} = \cancel{D}\dot{\delta} + M\ddot{\delta}$$

$$\delta(0) = \delta_1$$
$$\dot{\delta}(0) = 0$$

$$\delta = \frac{P_m}{2MS_{base}} t^2 + \delta_1$$

Know what the angle will be the moment the fault is cleared. But is this angle acceptable?

Example: Generator Terminal Fault



$$\frac{P_m}{S_{base}} = \cancel{D}\dot{\delta} + M\ddot{\delta}$$

$$\delta(0) = \delta_1$$

$$\dot{\delta}(0) = 0$$

$$\delta = \frac{c_1 M}{D} e^{-\frac{D}{M}t} + \frac{P_m}{D S_{base}} t + c_2$$

$$\delta = \delta_1 + \frac{M P_m}{S_{base} D^2} e^{-\frac{D}{M}t} - \frac{M P_m}{S_{base} D^2} + \frac{M P_m}{S_{base} D} t$$

Goal: Understand Transient Stability in the Power System

- Transient stability: maintaining system frequency (“synchronism”) after a disturbance
- The swing equation – describes “swing” in power angle during transients
- Linearizing the swing eqn? Not important
- Solving nonlinear swing eqn
- Equal-area stability crtierion

Explore This Behaviour in a Single Generator

$$P_{mech} - P_{elec} = I\frac{d\omega_m}{dt}\omega_m$$

ω_m = 3600 RPM = 377 rad/s

Electrical Model of a Synchronous Machine

- State and provide intuition on the electrical model
- Emphasize the simplifications we are making
- Discuss P = f(delta), so there’s an inherent connection between the mechanical state and the output power
- We’ll ride this curve as faults happen

Dynamics of a Synchronous Machine

- Derive the dynamic equation of an SM
- In doing so, emphasize the assumptions we are making
- Highlight what the behavior is during a fault and how inertia provides <u>damping</u>

High Inertia Generators are "Stiff"

- Emphasizing the modeling benefits we can derive from the generators having high inertia

Equal Area Criterion?

- Worth covering?

Wind and Solar do Not Provide Inertia

- No inertia in solar
- There’s a spinning turbine in wind… is that useful inertia? No. We’ll explain later
- In general, we’ll discuss more in the next module

The Instability of a 100% Renewable Grid

- First, must have storage to ensure dispatchability
- No system inertia, what happens?

Conclusions

- Energy stored in rotating inertias is fundamental to how the power system handles transients
- Wind, solar, and battery sources do not provide system inertia
- This greatly hampers system stability, unless we do something about it.
- Looking ahead: *power electronics* can respond to disturbances quickly…