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Power Flow Review

PSERC

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Start of Session: Power Flow

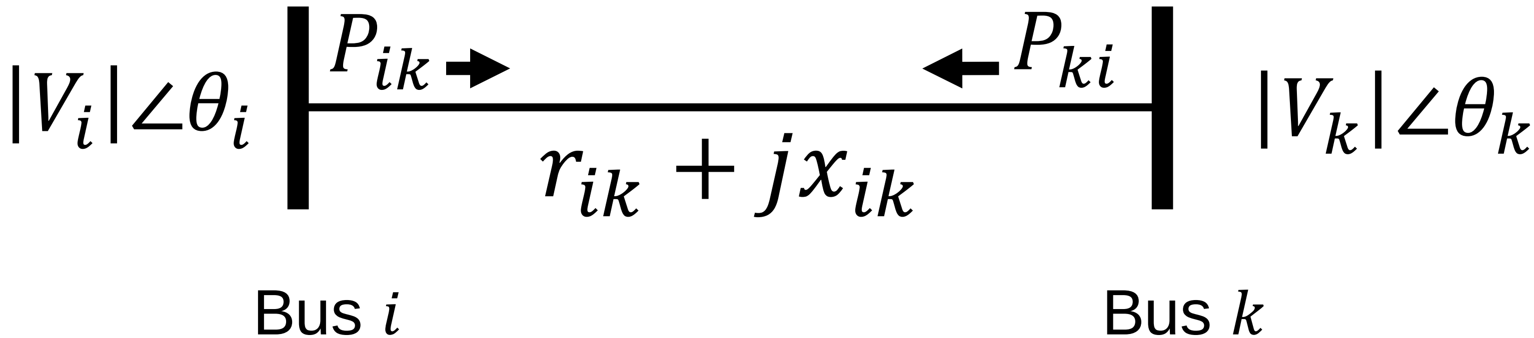
- Review of power flow basics
- Derivation of the linearized power flow (referred to as the DC power flow)
- Determination of shift factors

Examples to Learn Linearized Power Flow

AC Power Flow Equation

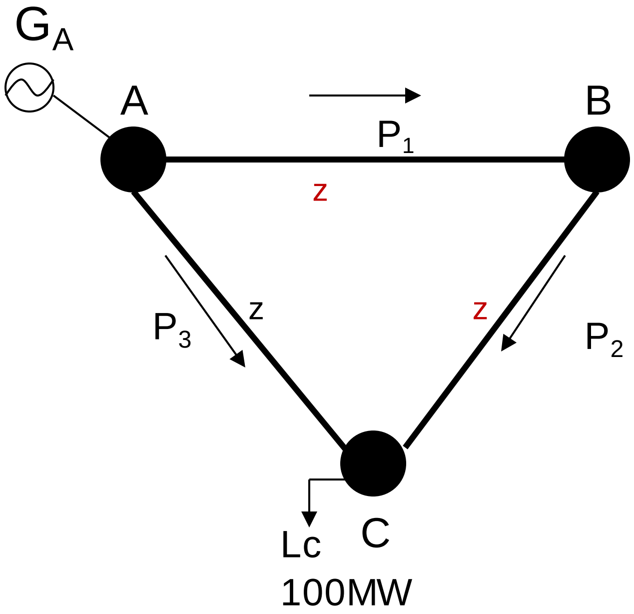
- AC real power line flow equation:

$$P_{ik} = |V_i|^2 g_{ik} - |V_i||V_k|(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$



$$g_{ik} + jb_{ik} = \frac{1}{r_{ik} + jx_{ik}} = \frac{r_{ik} - jx_{ik}}{r_{ik}^2 + x_{ik}^2} = \frac{r_{ik}}{r_{ik}^2 + x_{ik}^2} + j\frac{-x_{ik}}{r_{ik}^2 + x_{ik}^2}$$

Example 1



P3 = [(2z)/(2z+z)]*100

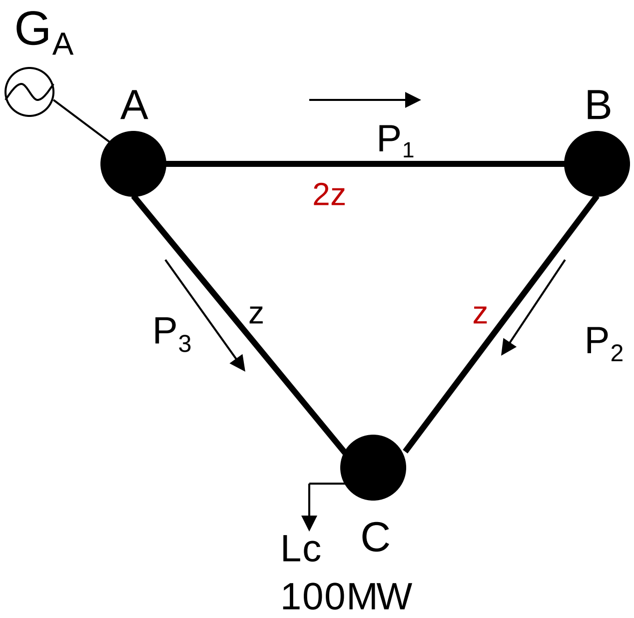
P1 = [z/(2z+z)]*100

P2 = [z/(2z+z)]*100

What are the flows on the lines?

G_A is the only supply

Example 2



P3 = [(3z)/(3z+z)]*100

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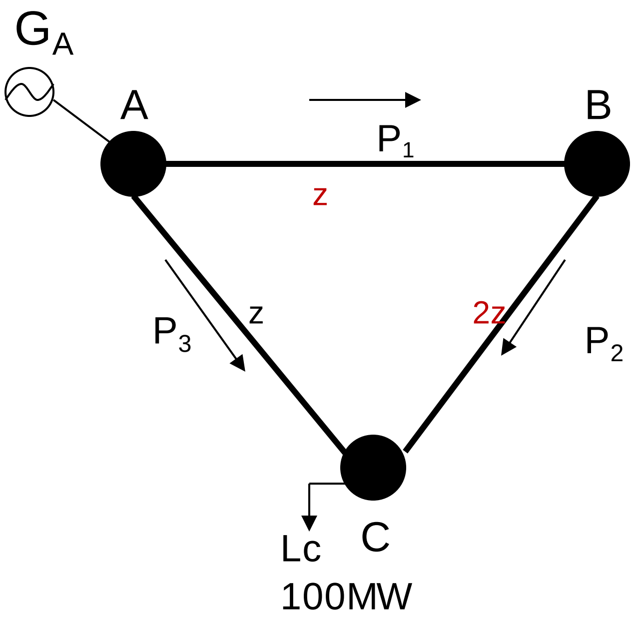
Example 3



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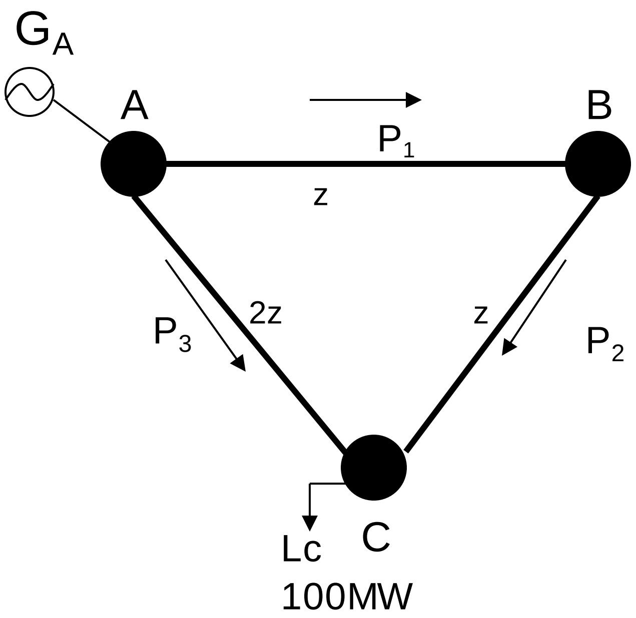
Example 4



What are the flows on the lines?

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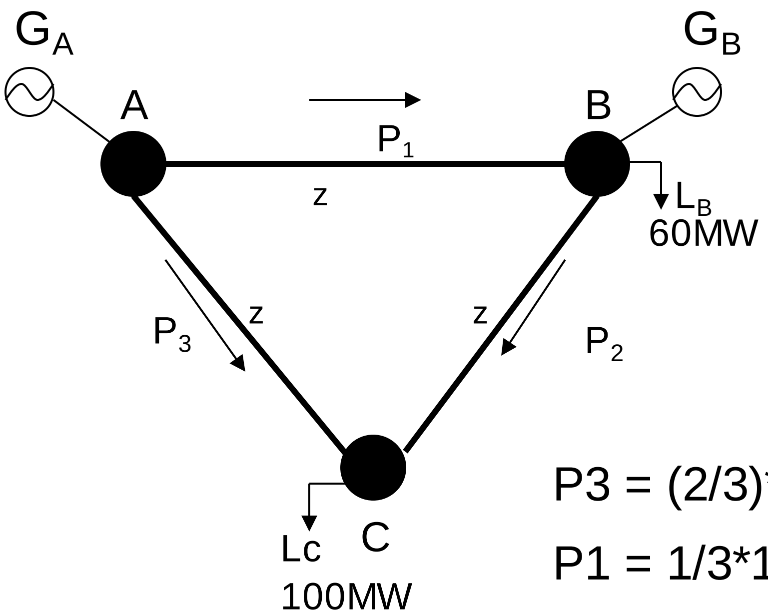
Example 5



P3 = (2/3)*120+(1/3)(-20)=73.33

P1 = 1/3*120+(-1)*(1/3)*(-20)=46.67

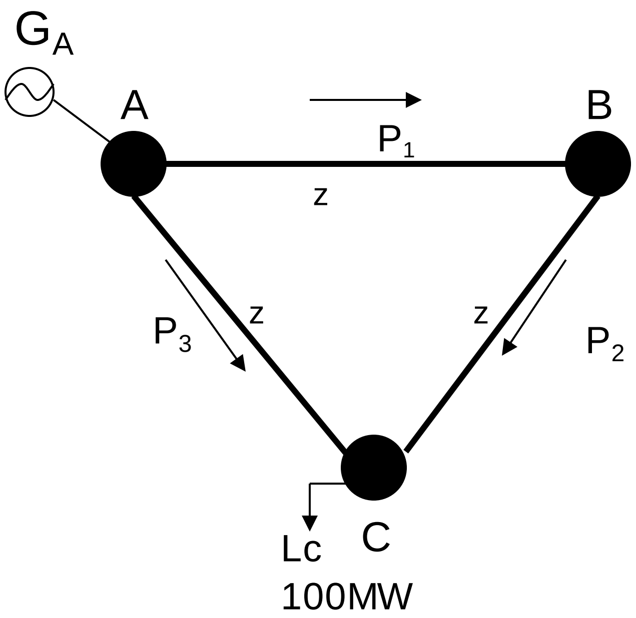
P2 = 1/3*120+(2/3)*(-20)=26.67

What are the flows on the lines?

$G_A = 120$

$G_B = 40$

Example 5



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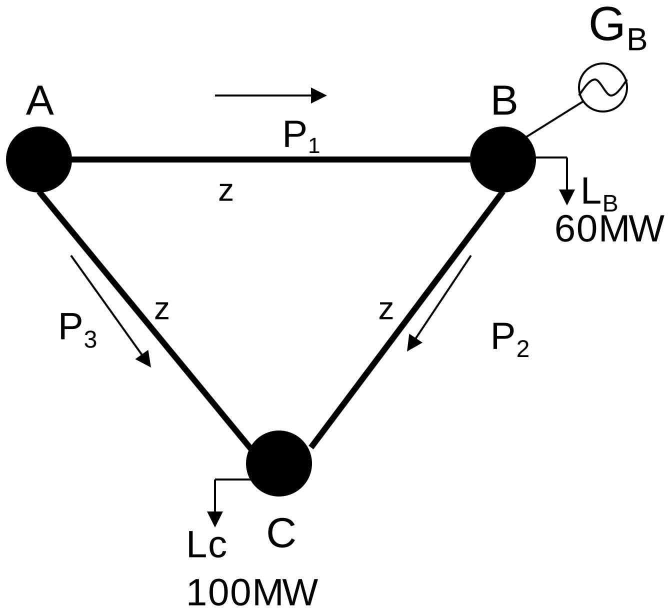
Example 5

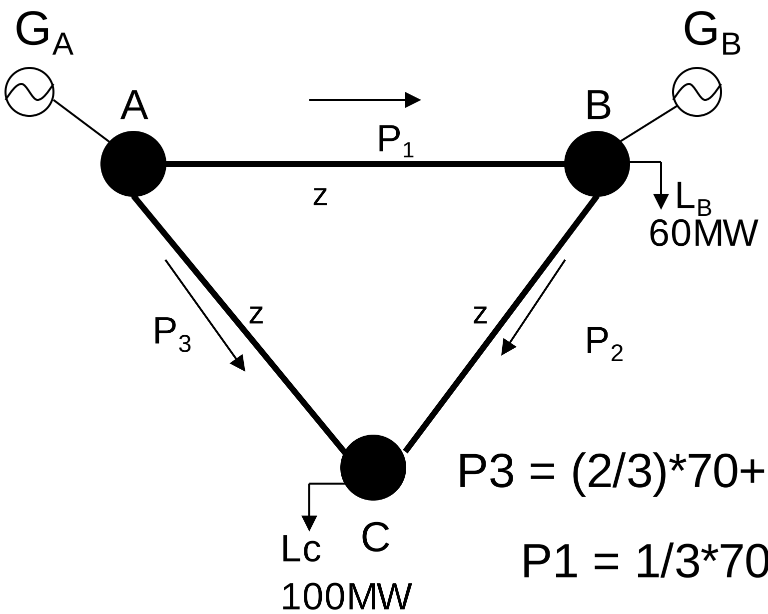


What are the flows on the lines?

$G_A = 120$

$G_B = 40$

Example 6



P3 = (2/3)*70+(1/3)(30)=56.67=23.33*2+10

P1 = 1/3*70+(-1)*(1/3)*(30)=13.33

P2 = 1/3*70+(2/3)*(30)=43.33

What are the flows on the lines?

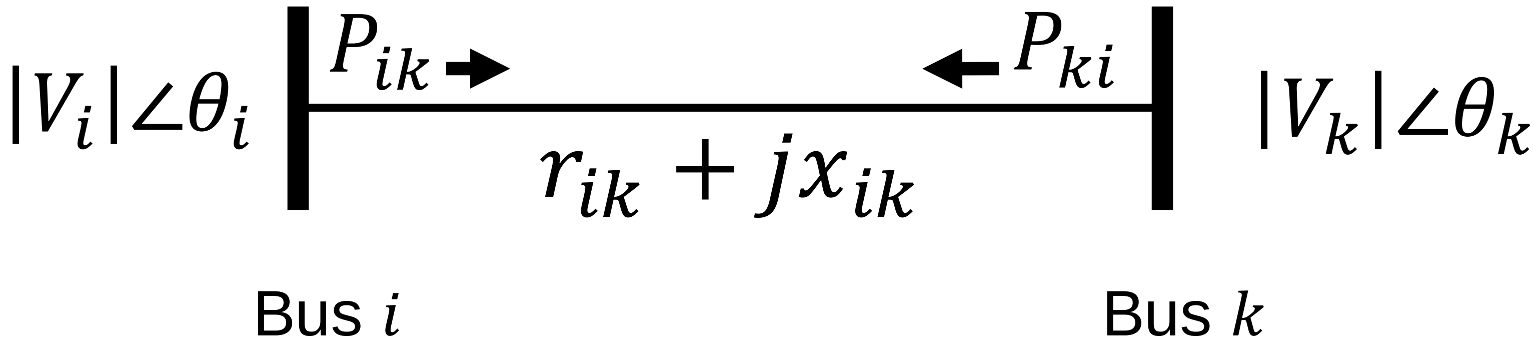
$G_A = 70$

$G_B = 90$

AC Power Flow Equation

- AC real power line flow equation:

$$P_{ik} = |V_i|^2 g_{ik} - |V_i||V_k|(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$



$$g_{ik} + jb_{ik} = \frac{1}{r_{ik} + jx_{ik}} = \frac{r_{ik} - jx_{ik}}{r_{ik}^2 + x_{ik}^2} = \frac{r_{ik}}{r_{ik}^2 + x_{ik}^2} + j\frac{-x_{ik}}{r_{ik}^2 + x_{ik}^2}$$

Break

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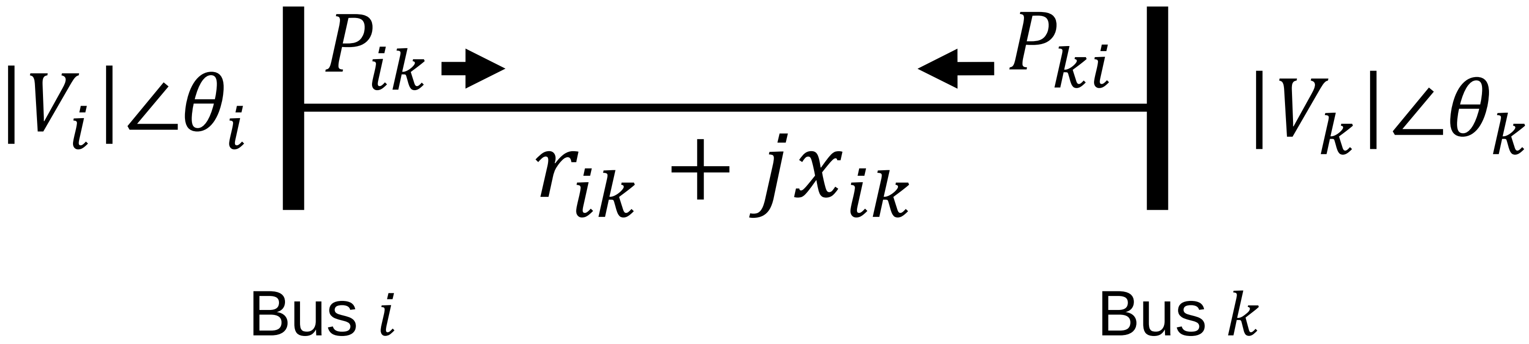
Power Flow Review

Derivation of Decoupled and DC Power Flow

AC Power Flow Equation

- AC real power line flow equation:

$$P_{ik} = |V_i|^2 g_{ik} - |V_i||V_k|(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$



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Decoupled Power Flow

- Proposed by Brian Stott and Ongun Alsac
- Also called the Stott decoupled power flow

- Both live in the Phoenix metro area and provide consulting for Nexant (they have a location in Chandler, AZ)

- Brian Stott was a professor at ASU (for a short period, many years ago)

Assumptions of the Decoupled Power Flow

Primary premises behind the decoupled power flow:

- **Weak** coupling between Real Power **(P)** and Voltage magnitude **(|V|)**
- **Weak** coupling between Reactive Power **(Q)** and Voltage Phase Angle θ
- **P** has a **strong** coupling to θ
- **Q** has a **strong** coupling to **|V|**

Additional Assumptions of the Decoupled Power Flow

- The difference in voltage angles between two buses that are connected by a transmission line is typically small:

$$\cos(\theta_i - \theta_k) \cong 1$$

$$\sin(\theta_i - \theta_k) \cong \theta_i - \theta_k$$

Additional Assumptions of the Decoupled Power Flow

- The conductance of the transmission line is generally smaller than the susceptance, then multiplied by the sine of a small angle:

$$G_{ik}\sin(\theta_i - \theta_k) \ll B_{ik}$$

- And the reactive power is small relative to:

$$Q_i \ll B_{ii}|V_i|^2$$

“DC” Power Flow Derivation

DC Power Flow

- The DC Power Flow is a further simplification beyond the Decoupled power flow
- Assumptions:
  - All voltages are 1 pu
  - Voltage angle differences across a line are small
  - The resistance of a line (r_j) << reactance (x_j)
  - Ignore reactive power (or you can de-rate transmission lines to account for anticipated reactive power flow across lines)

DC Power Flow Assumptions

- Original AC line flow equation:

$$P_{ik} = |V_i|^2 g_{ik} - |V_i||V_k|(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$

- Assume all voltages are 1 pu:

$$P_{ik} = g_{ik} - (g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$

- Assume $\theta_i - \theta_k$ is small

$$= g_{ik}(1) \qquad = b_{ik}(\theta_i - \theta_k)$$

$$P_{ik} = g_{ik} - (g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k))$$

- The real power line flow equation is now:

$$P_{ik} = -b_{ik}(\theta_i - \theta_k) = b_{ik}(\theta_k - \theta_i), \quad P_{ik} = \frac{1}{x_{ik}}(\theta_i - \theta_k)$$

- We also assume: $r_{ik} \ll x_{ik}$, $b_{ik} = \frac{-1}{x_{ik}}$, $Q_{ik} = 0$

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DC Optimal Power Flow (DCOPF)

A Linear OPF

A Basic DCOPF

- This formulation includes a linear objective, generator min and max production levels, line flow limits, angle difference constraints, and node-balance constraints

$$Min: \sum_{\forall g} c_g P_g$$

Objective: Minimize cost (in this example, it is linear)

Subject to:

$$P_g^{min} \le P_g \le P_g^{max}$$

Generator min and max capacity constraints

$$P_{ik}^{min} \le P_{ik} \le P_{ik}^{max}$$

Line flow capacity constraints; generally, the $P^{min} = -P^{max}$

$$P_{ik} - b_{ik}(\theta_k - \theta_i) = 0$$

Linear approximation of the line flow constraint

This formulation states that line *ik* is from *i* to *k*

$$\theta_{ik}^{min} \le \theta_i - \theta_k \le \theta_{ik}^{max}$$

Angle difference constraint

Do we really need this constraint?

$$\sum_{\forall Lines \rightarrow i} P_{ik} - \sum_{\forall Lines \leftarrow i} P_{ik} + \sum_{\forall g@i} P_g = d_i$$

Node balance constraint

Another DCOPF Formulation

- This formulation is more generic by allowing for multiple generators at a bus, multiple lines between the same two buses; however, it becomes a bit more complicated when representing the node balance constraint

$$Min: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_g^{min} \leq P_g \leq P_g^{max}$$

$$P_\ell^{min} \leq P_\ell \leq P_\ell^{max}$$

$$P_\ell - b_\ell(\theta_k - \theta_i) = 0$$

$$\sum_{\forall \ell \epsilon \delta(i)^+} P_\ell - \sum_{\forall \ell \epsilon \delta(i)^-} P_\ell + \sum_{\forall g \epsilon g(i)} P_g = d_i$$

P_g	Generator *g*'s real power production
P_ℓ	Flow on line ℓ, defined to be *from bus i to bus k*
θ_i	Bus voltage angle *at bus i*
$\delta(i)^+$	Set of lines defined to be *to bus i*
$\delta(i)^-$	Set of lines defined to be *from bus i*
$g(i)$	Set of generators *at bus i*

Complete Nomenclature

$$Min: \sum_{\forall g} c_g P_g$$

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$$P_g^{min} \leq P_g \leq P_g^{max}$$

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$\delta(i)^+$	Set of lines defined to be *to bus i*
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$g(i)$	Set of generators *at bus i*
c_g	Linear operating cost for generator *g*
b_ℓ	Susceptance of line ℓ (assumed to be $-1/x_\ell$)
P_g^{min}, P_g^{max}	Min and Max capacity of generator *g*
$P_\ell^{min}, P_\ell^{max}$	Min and Max line flow rating for line ℓ
d_i	Real power demand at bus *i*

Objective

$$Min: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_g^{min} \le P_g \le P_g^{max}$$

$$P_\ell - b_\ell(\theta_k - \theta_i) = 0$$

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d_i	Real power demand at bus *i*

Note that the objective can be easily changed to be a quadratic function or a piecewise linear function. Right now it is a simple linear function. If the objective is linear, the basic DCOPF is a linear program.

Representation of a Line Flow

$$Min: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_g^{min} \le P_g \le P_g^{max}$$

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The DCOPF is a Lossless model. Thus, for each line we only need one flow variable. We do not need two flow variables because the flow going from *i* to *k* is equal to the negative of the flow going from *k* to *i.* In the ACOPF formulation, we need two variables for each line but that is not the case for the DCOPF. So we use only one line flow variable *but you must state an assumed direction of the line's flow.* You then need to account for this direction when writing the node balance constraint to ensure you include the right variables *with the right sign* in the node balance constraint

For the DCOPF: $\boldsymbol{P_{ik} = -P_{ki}}$

Why is the DCOPF called “DC”

- The grid is AC

- Why does this linear OPF get this name?

- Let’s revisit the first examples

Break

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DCOPF Examples 1

Representation of a Line Flow

$$Min: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_g^{min} \le P_g \le P_g^{max}$$

$$P_\ell - b_\ell(\theta_k - \theta_i) = 0$$

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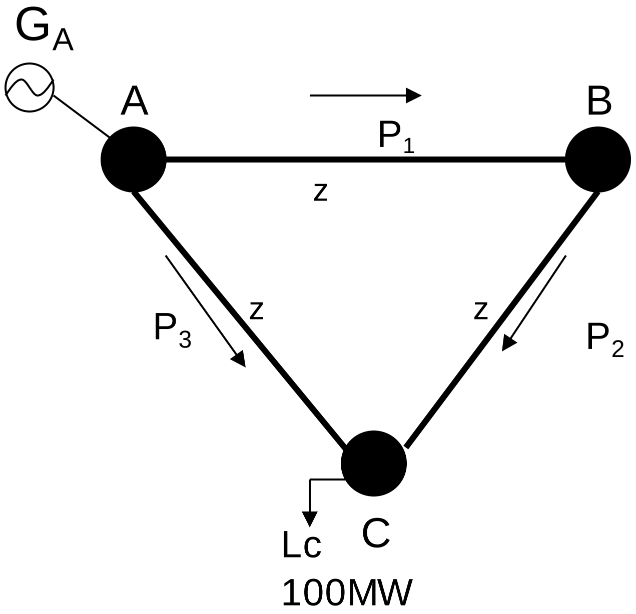
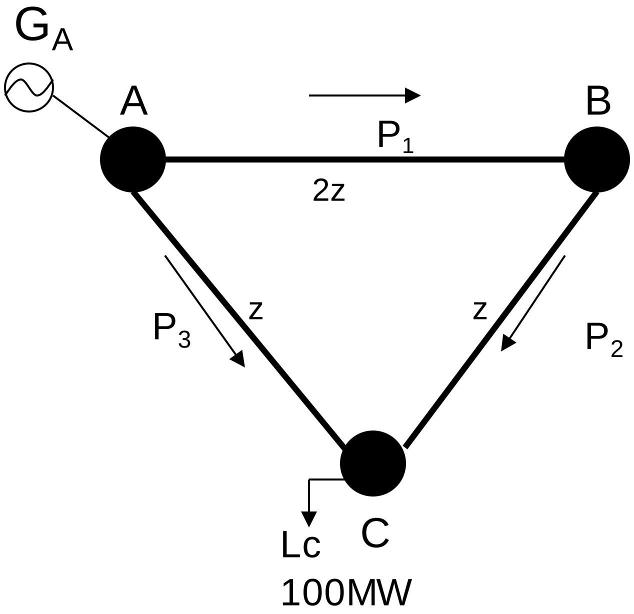


What are the flows on the lines?

G_A is the only supply

Example 2



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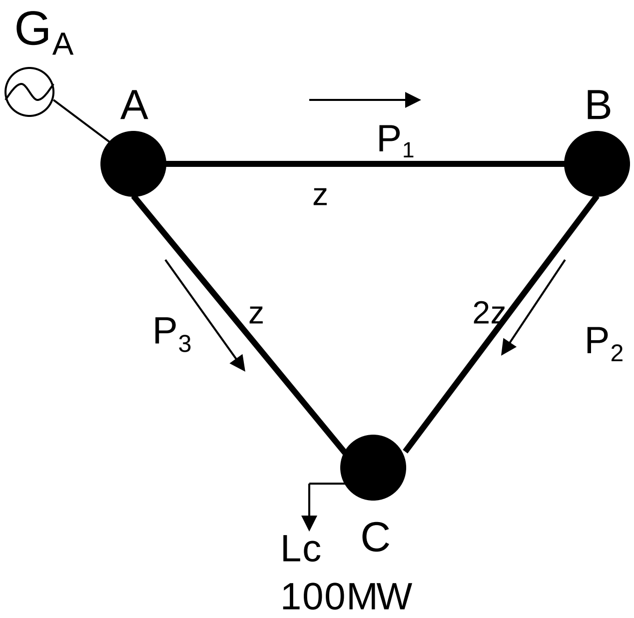

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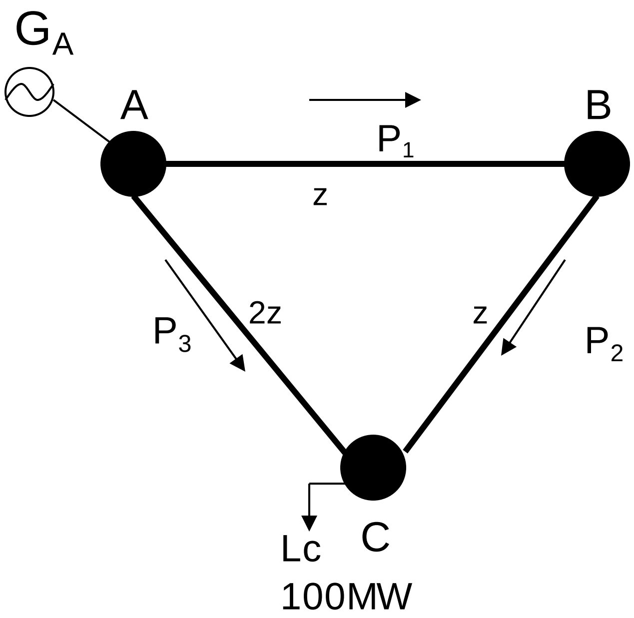
Example 4



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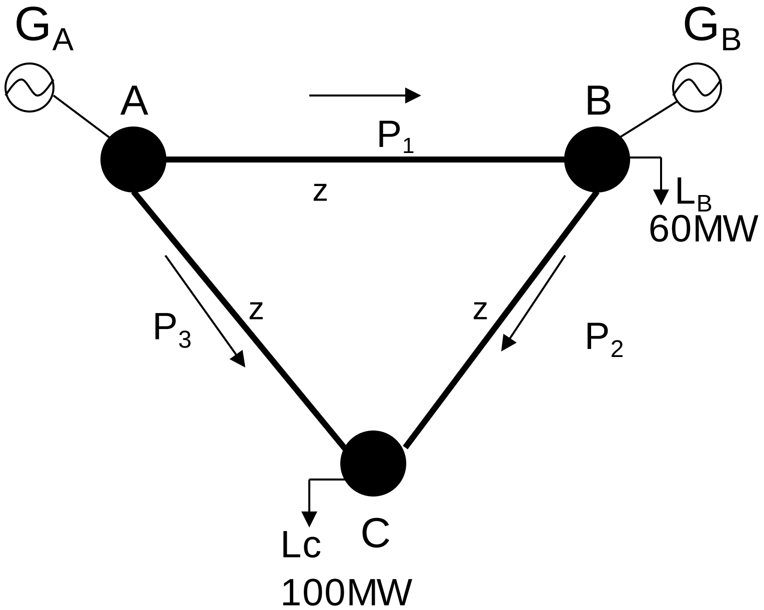
Example 5

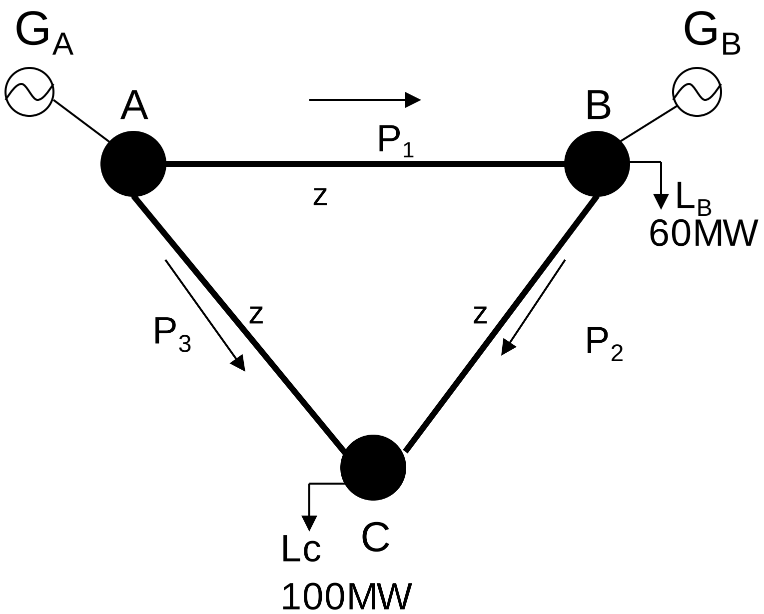


What are the flows on the lines?

$G_A = 120$

$G_B = 40$

Example 6



What are the flows on the lines?

$G_A = 70$

$G_B = 90$

Break

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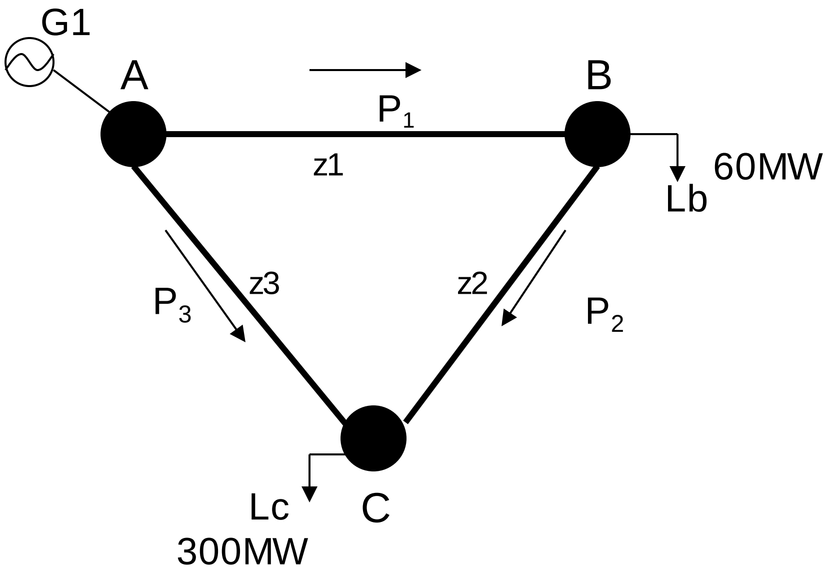
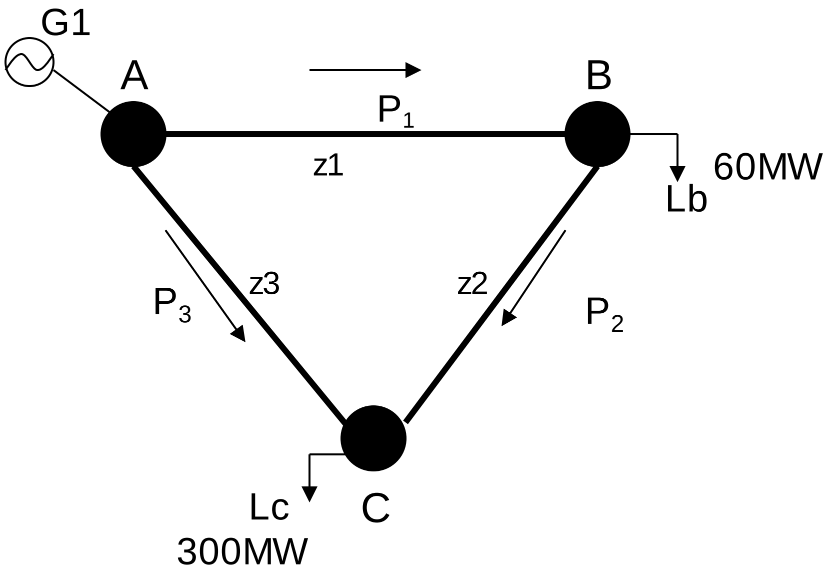
DCOPF Examples 2

Example 7 (new)



- What are the flows on the lines? (no thermal limits)
- Gen 1 sends 60MW to node B; Gen 1 sends 300MW to node C
- We can use superposition
- Calculate the flow on each line for the portion of Gen 1 to bus B and the flow for the portion to bus C

Example 7 continued



- Gen 1 sends 60MW to node B; Gen 1 sends 300MW to node C
- See supplemental notes for derivation and answer
- If Z1 = Z2 = Z3:
- P1 = 140, P2 = 80, P3 = 220

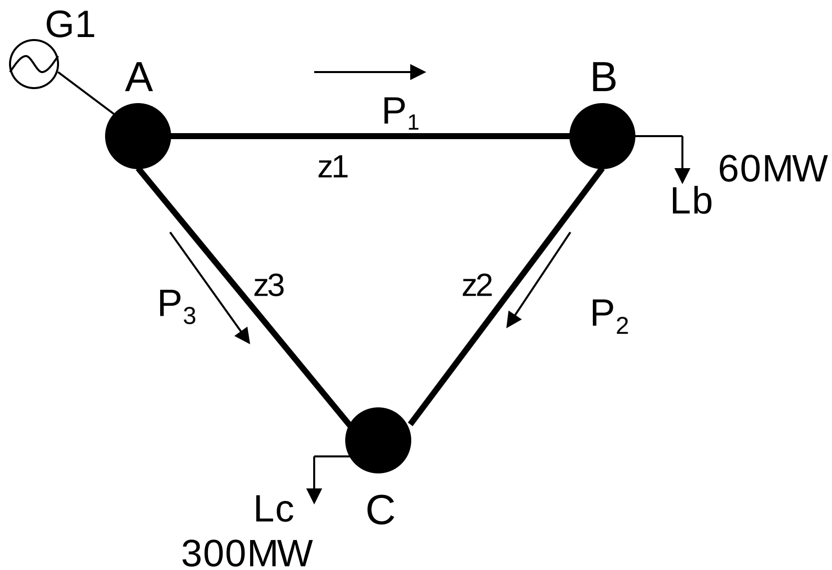
$$P_1 = \frac{2}{3} G_1^B + \frac{1}{3} G_1^C \quad P_2 = -\frac{1}{3} G_1^B + \frac{1}{3} G_1^C \quad P_3 = \frac{1}{3} G_1^B + \frac{2}{3} G_1^C$$

Example 8

- We just solved for the flows when Gen 1 sends 60MW to Node B and 300MW to Node C (example 7)

- How will the line flows change if Gen 1 sends 360MW to Node C and Node C sends 60MW to Node B? (example 8)

Example 8



- This time: Gen 1 sends **360MW to node C**; **Node C sends 60MW** to **node B** (Let G_C^B denote the injection at node C that is sent to B)
  - See supplemental notes for derivation and answer
- With Z1 = Z2 = Z3:
  - P1 = 140, P2 = 80, P3 = 220
- **SAME ANSWER!!!!**

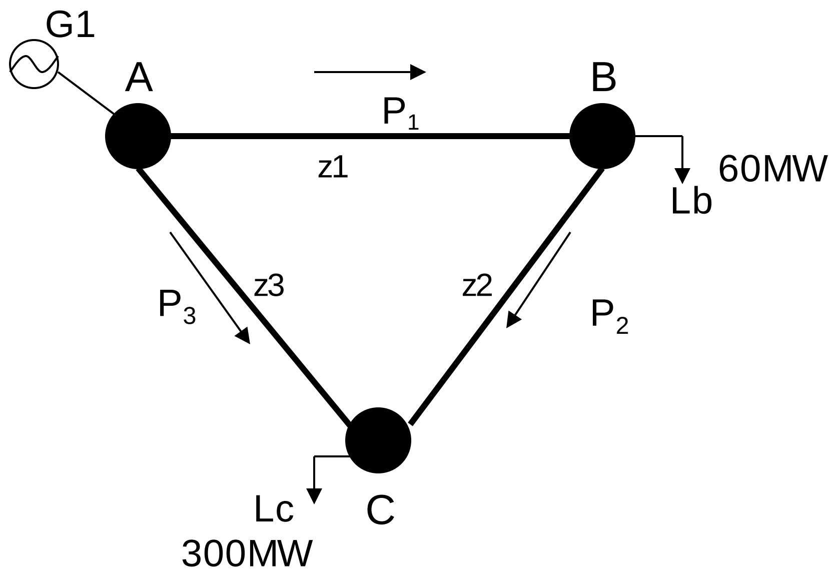
$$P_1 = \frac{1}{3} G_1^C + \frac{1}{3} G_C^B \qquad P_2 = \frac{1}{3} G_1^C - \frac{2}{3} G_C^B \qquad P_3 = \frac{2}{3} G_1^C - \frac{1}{3} G_C^B$$

Example 9

- We just solved for the flows when Gen 1 sends 360MW to Node C and Node C sends 60MW to Node B (example 8)

- How will the line flows change if Gen 1 sends 360MW to Node C and Node B sends **–60MW** to Node C? (example 9)

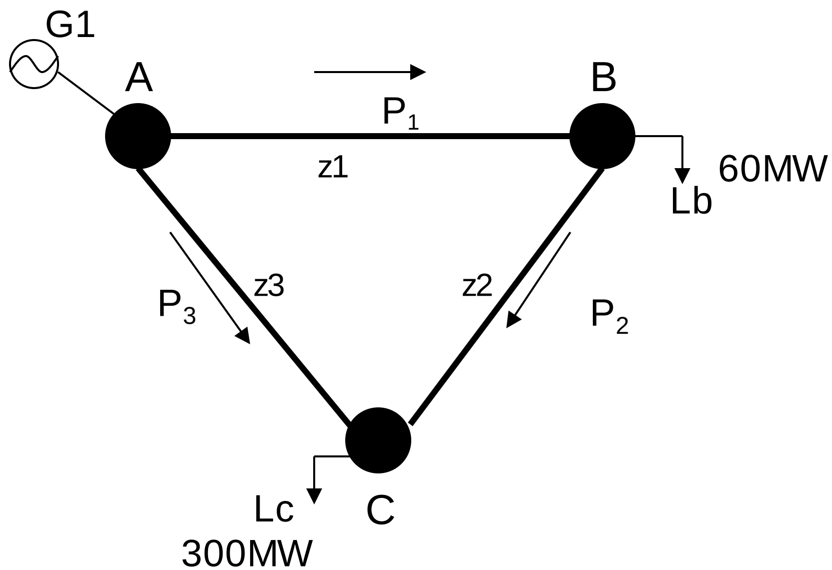
Example 9



- This time: Gen 1 sends 360MW to node C; **Node B sends –60MW to node C** (Let G_B^C denote the **injection** at node B that is sent to C)
  - See supplemental notes for generic answer
- With Z1 = Z2 = Z3:
  - P1 = 140, P2 = 80, P3 = 220
- **SAME ANSWER!!!!**

$$P_1 = \frac{1}{3}G_1^C - \frac{1}{3}G_B^C \qquad P_2 = \frac{1}{3}G_1^C + \frac{2}{3}G_B^C \qquad P_3 = \frac{2}{3}G_1^C + \frac{1}{3}G_B^C$$

Example 9



- What are these coefficients called?

$$P_3 = \left(\frac{2}{3}\right) G_1^C + \left(\frac{1}{3}\right) G_B^C$$

$$P_1 = \left(\frac{1}{3}\right) G_1^C + \left(-\frac{1}{3}\right) G_B^C \qquad P_2 = \left(\frac{1}{3}\right) G_1^C + \left(\frac{2}{3}\right) G_B^C$$

- Can you think of a name for Node C in terms of how we are using Node C?
  - Why?

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Shift Factors

PTDFs and a Reference Bus

- The coefficients are Power Transfer Distribution Factors
  - PTDFs: If you inject 1MW at **bus *i*** and send it to the reference **bus *R***, what is the resulting flow on **line *ℓ***
- Bus C is acting as a reference bus. Each bus in the network is sending its *net injection* to the reference bus.
  - We must be consistent: treat all as net injections or net withdrawals
    - A negative net injection is an actual withdrawal

PTDF

$PTDF_{l,i}^{R}$: ΔFlow on line l for a 1MW injection at bus i and a 1MW withdrawal at bus R (reference bus)

Various names:

- Power Transfer Distribution Factor
- Shift Factor
- Power Distribution Coefficients

PTDF

$PTDF_{l,i}^{R}$: ΔFlow on line l for a 1MW injection at bus i and a 1MW withdrawal at bus R (reference bus)

- Do not forget that this is a linear sensitivity when the grid is a nonlinear system
- Often used with a linearized OPF (e.g., DCOPF)

PTDF

$PTDF_{l,i}^{R}$: ΔFlow on line l for a 1MW injection at bus i and a 1MW withdrawal at bus R (reference bus)

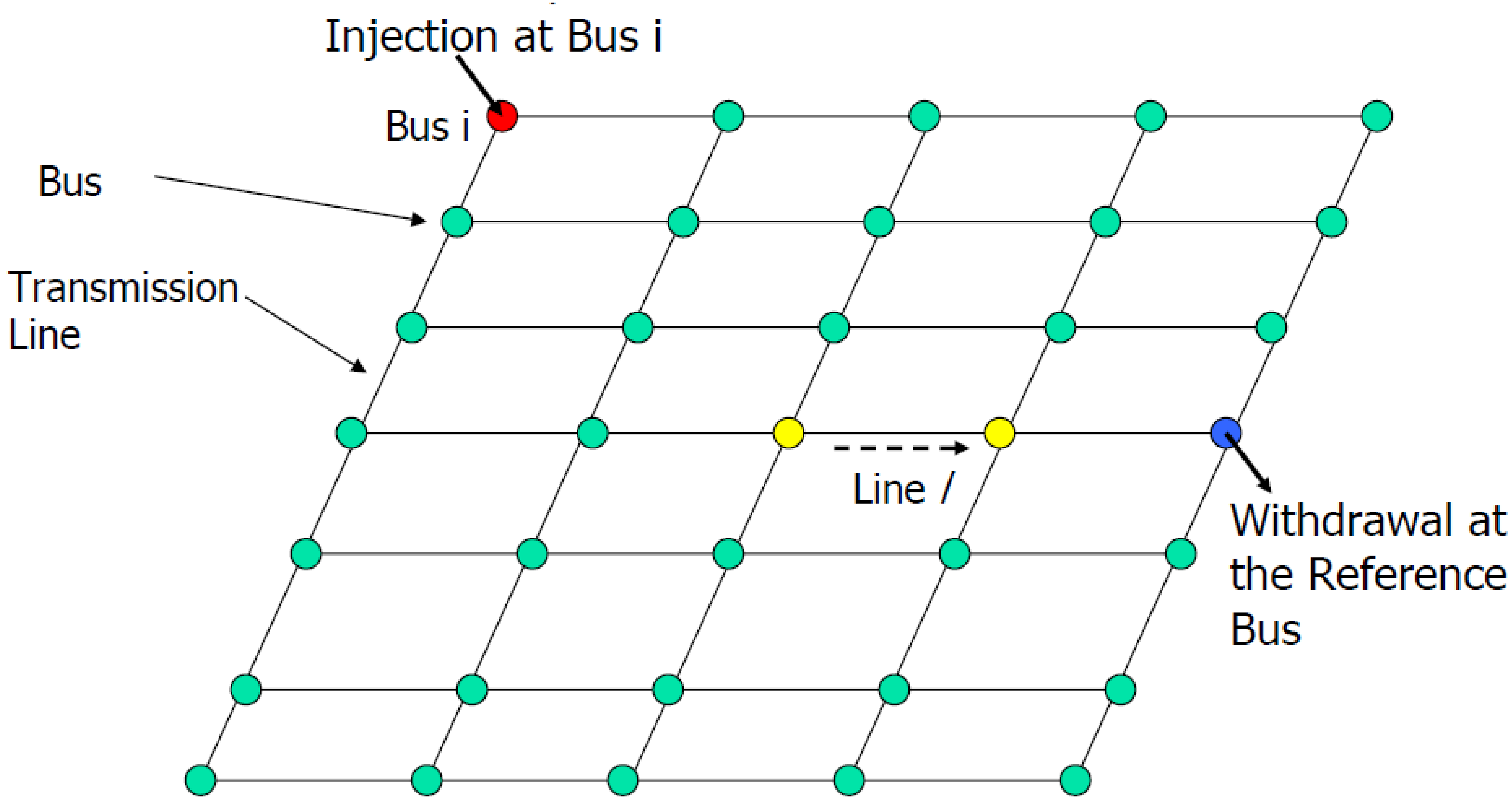
4 Attributes:

- A particular line (with an assumed reference direction)
- A particular injection location
- Withdrawal at the reference bus
- A value of the shift factor

$$\Delta line\ flow\ /\ \Delta bus\ i\ injection$$

PTDF

$PTDF_{l,i}^{R}$: ΔFlow on line l for a 1MW injection at bus i and a 1MW withdrawal at bus R (reference bus)



A DCOPF Formulation with PTDFs – Another way to write it

$$Min: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_\ell^{min} \le \sum_{\forall i} PTDF_{\ell,i}^R \Big[\overbrace{\Big(\sum_{\forall g \epsilon g(i)} P_g\Big) - d_i}^{\text{Net injection at bus } i}\Big] \le P_\ell^{max} \qquad \forall \ell$$

$$P_g^{min} \le P_g \le P_g^{max} \qquad \forall g$$

$$\sum_{\forall g} P_g = \sum_{\forall i} d_i$$

P_g Generator *g*'s real power production

$PTDF_{\ell,i}^R$ Power transfer distribution factor for a net injection at bus *i* sent to reference bus *R*, the resulting flow on line ℓ

$g(i)$ Set of generators *at bus i*

Complete Nomenclature

$$\underset{P_g}{Min}: \sum_{\forall g} c_g P_g$$

Subject to:

$$P_\ell^{min} \leq \sum_{\forall i} PTDF_{\ell,i}^R \left[\left(\sum_{\forall g \epsilon g(i)} P_g \right) - d_i \right] \leq P_\ell^{max}, \forall \ell$$

$$P_g^{min} \leq P_g \leq P_g^{max}, \forall g$$

$$\sum_{\forall g} P_g = \sum_{\forall i} d_i$$

P_g	Generator *g*'s real power production (**only variable**)
$PTDF_{\ell,i}^R$	Power transfer distribution factor for a net injection at bus *i* sent to reference bus *R*, the resulting flow on line ℓ
$g(i)$	Set of generators *at bus i*
g, ℓ	Indices for generators, lines
c_g	Cost (linear) for generator g
c_g	Load (real power) at bus i
$P_\ell^{min}, P_\ell^{max}$	Min and max line flow (rating)
P_g^{min}, P_g^{max}	Min and max gen g real power capacity

You can define the PTDF *to reference bus R* or *from reference bus R*; it does not change the solution; it simply changes the sign of the PTDF – but you need to be consistent

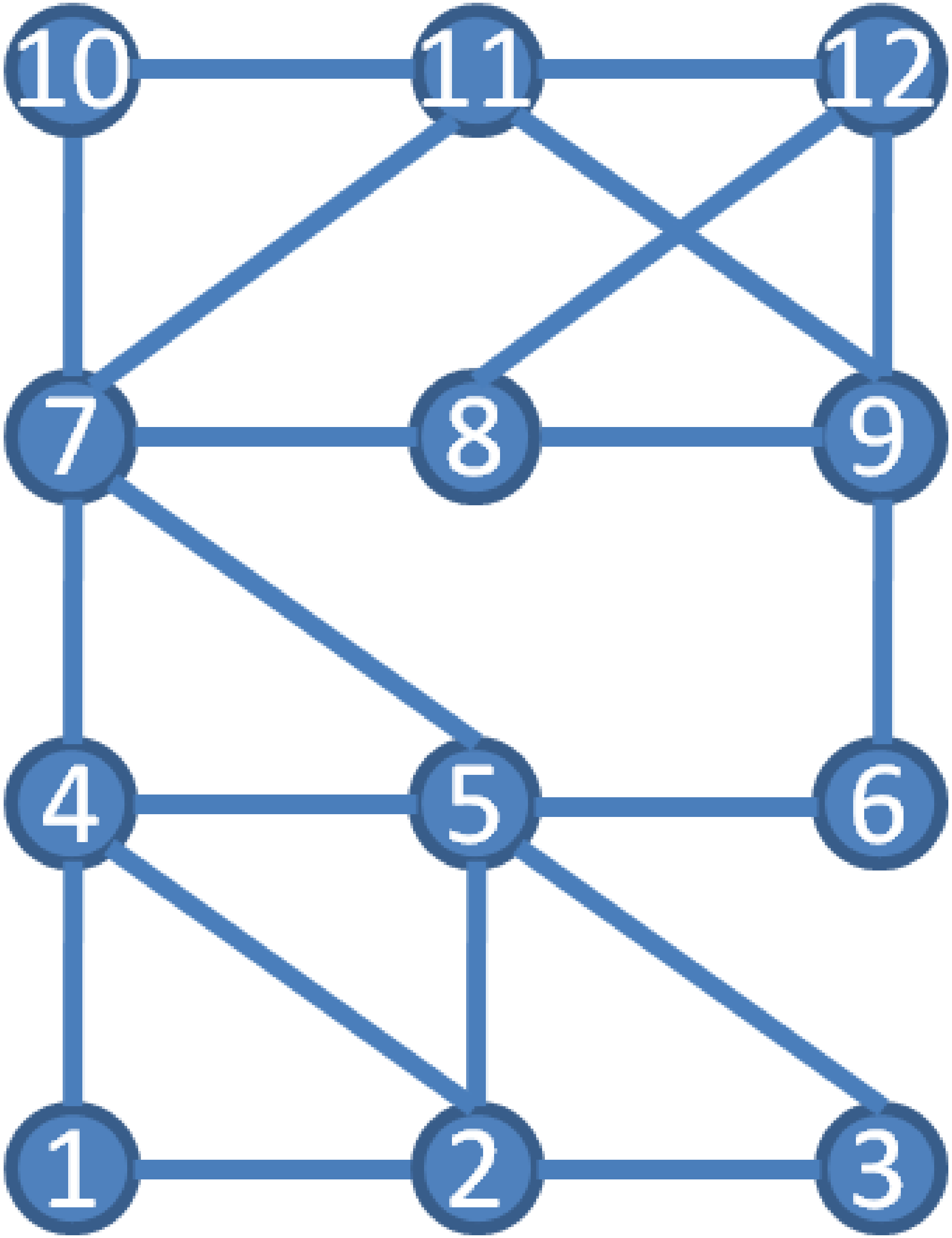
Break

ALFRED P. SLOAN FOUNDATION

Power Flow Review

More on Power Transfer Distribution Factors

12-Bus Example
B Matrix
B’ Matrix
H Matrix
Shift Factors



	From Bus	To Bus	x
Line 1	1	2	0.1
Line 2	1	4	0.15
Line 3	2	3	0.2
Line 4	2	4	0.05
Line 5	2	5	0.25
Line 6	3	5	0.1
Line 7	4	5	0.12
Line 8	4	7	0.25
Line 9	5	6	0.08
Line 10	5	7	0.16
Line 11	6	9	0.1
Line 12	7	8	0.15
Line 13	7	10	0.05
Line 14	7	11	0.12
Line 15	8	9	0.2
Line 16	8	12	0.15
Line 17	9	11	0.16
Line 18	9	12	0.12
Line 19	10	11	0.1
Line 20	11	12	0.15

B Matrix

Power Generation, Operation, and Control by Wood, Wollenberg, and Sheble

B matrix (with the DC approximations applied)

$$B_{ii} = \sum\nolimits_{\forall j \text{ connected to bus } i} \frac{1}{x_{ij}}$$

$$B_{ij} = -\frac{1}{x_{ij}}$$

B Matrix

B:	1	2	3	4	5	6	7	8	9	10	11	12
1	16.67	-10	0	-6.67	0	0	0	0	0	0	0	0
2	-10	39	-5	-20	-4	0	0	0	0	0	0	0
3	0	-5	15	0	-10	0	0	0	0	0	0	0
4	-6.67	-20	0	39	-8.33	0	-4	0	0	0	0	0
5	0	-4	-10	-8.33	41.08	-12.5	-6.25	0	0	0	0	0
6	0	0	0	0	-12.5	22.5	0	0	-10	0	0	0
7	0	0	0	-4	-6.25	0	45.25	-6.67	0	-20	-8.33	0
8	0	0	0	0	0	0	-6.67	18.33	-5	0	0	-6.67
9	0	0	0	0	0	-10	0	-5	29.58	0	-6.25	-8.33
10	0	0	0	0	0	0	-20	0	0	30	-10	0
11	0	0	0	0	0	0	-8.333	0	-6.25	-10	31.25	-6.67
12	0	0	0	0	0	0	0	-6.67	-8.33	0	-6.67	21.67

B’ Matrix

B'	2	3	4	5	6	7	8	9	10	11	12
2	39	-5	-20	-4	0	0	0	0	0	0	0
3	-5	15	0	-10	0	0	0	0	0	0	0
4	-20	0	39	-8.33	0	-4	0	0	0	0	0
5	-4	-10	-8.33	41.08	-12.5	-6.25	0	0	0	0	0
6	0	0	0	-12.5	22.5	0	0	-10	0	0	0
7	0	0	-4	-6.25	0	45.25	-6.67	0	-20	-8.33	0
8	0	0	0	0	0	-6.67	18.33	-5	0	0	-6.67
9	0	0	0	0	-10	0	-5	29.58	0	-6.25	-8.33
10	0	0	0	0	0	-20	0	0	30	-10	0
11	0	0	0	0	0	-8.33	0	-6.25	-10	31.25	-6.67
12	0	0	0	0	0	0	-6.67	-8.33	0	-6.67	21.67

Bus 1 will be the reference bus

B’ is the same as B, just without row 1 and column 1 since bus 1 is the ref

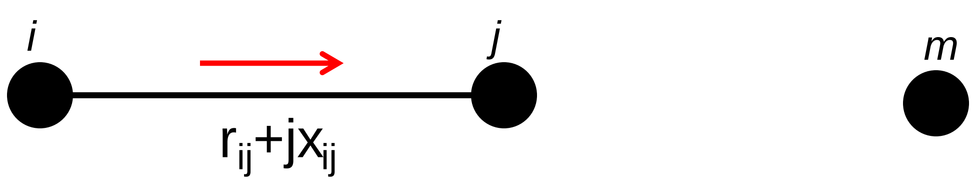
B'Inv	2	3	4	5	6	7	8	9	10	11	12
2	0.0656	0.0600	0.0516	0.0571	0.0567	0.0555	0.0558	0.0561	0.0556	0.0557	0.0559
3	0.0600	0.1507	0.0601	0.0961	0.0932	0.0855	0.0876	0.0895	0.0861	0.0871	0.0882
4	0.0516	0.0601	0.0727	0.0643	0.0650	0.0667	0.0663	0.0658	0.0666	0.0664	0.0661
5	0.0571	0.0961	0.0643	0.1155	0.1114	0.1006	0.1035	0.1062	0.1013	0.1027	0.1043
6	0.0567	0.0932	0.0650	0.1114	0.1711	0.1180	0.1322	0.1457	0.1215	0.1286	0.1363
7	0.0555	0.0855	0.0667	0.1006	0.1180	0.1637	0.1514	0.1398	0.1606	0.1546	0.1479
8	0.0558	0.0876	0.0663	0.1035	0.1322	0.1514	0.2221	0.1682	0.1552	0.1627	0.1831
9	0.0561	0.0895	0.0658	0.1062	0.1457	0.1398	0.1682	0.1952	0.1468	0.1609	0.1763
10	0.0556	0.0861	0.0666	0.1013	0.1215	0.1606	0.1552	0.1468	0.1965	0.1684	0.1560
11	0.0557	0.0871	0.0664	0.1027	0.1286	0.1546	0.1627	0.1609	0.1684	0.1960	0.1723
12	0.0559	0.0882	0.0661	0.1043	0.1363	0.1479	0.1831	0.1763	0.1560	0.1723	0.2233

Inverse [B’]

H Matrix

$$PTDF = HB'^{-1}$$

- You have already learned the B' matrix
- Recall: $P_{ij} = b_{ij}(\theta_j - \theta_i) = \frac{1}{x_{ij}}(\theta_i - \theta_j)$
- Another way to write it: $\boldsymbol{P_{Lines}} = [\boldsymbol{H}]\boldsymbol{\theta}'$
- For line *k* with: bus *i* (from bus), bus *j* (to bus), and it is not connected to bus *m* you get:
- $H(k,i) = \frac{1}{x_{ij}}, H(k,j) = \frac{-1}{x_{ij}}, H(k,m) = 0$

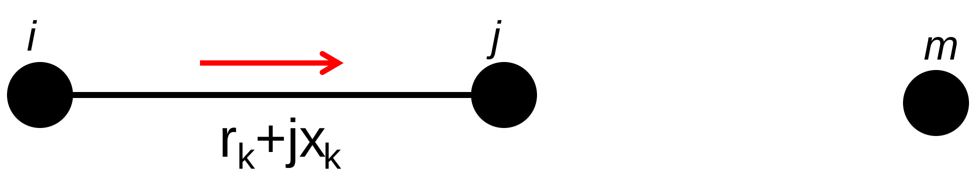


H Matrix

Only difference for this slide is that I referred to the impedance of the line by $r_k + jx_k$ instead of using (ij)

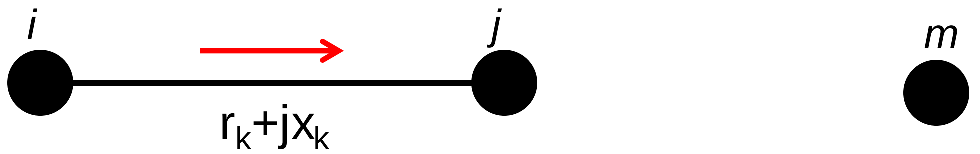
$$PTDF = HB'^{-1}$$

- You have already learned the B' matrix
- Recall: $P_k = b_k(\theta_j - \theta_i) = \frac{1}{x_k}(\theta_i - \theta_j)$
- Another way to write it: $\boldsymbol{P_{Lines}} = [\boldsymbol{H}]\boldsymbol{\theta'}$
- For line *k* with: bus *i* (from bus), bus *j* (to bus), and it is not connected to bus *m* you get:
- $H(k,i) = \frac{1}{x_k}, H(k,j) = \frac{-1}{x_k}, H(k,m) = 0$

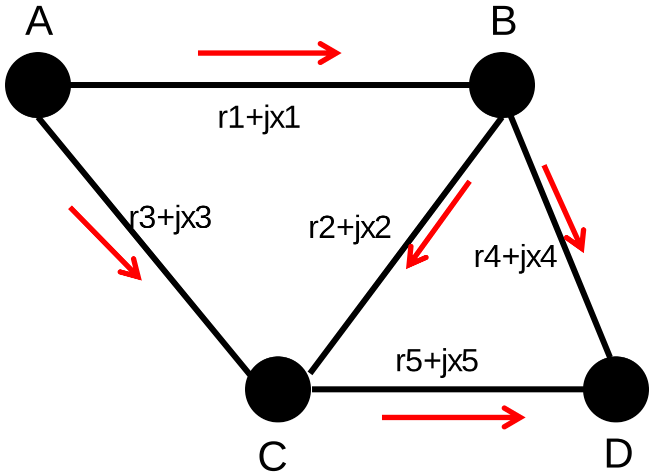


H Matrix Example

- For line *k* with: bus *i* (from bus), bus *j* (to bus), and it is not connected to bus *m* you get:
- $H(k,i) = \frac{1}{x_k}, H(k,j) = \frac{-1}{x_k}, H(k,m) = 0$

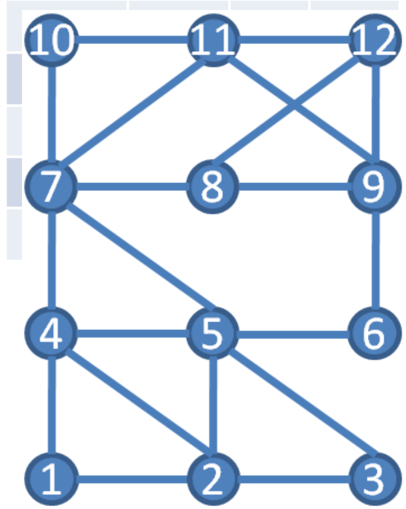


H	Bus B	Bus C	Bus D
Line 1	-1/x1	0	0
Line 2	1/x2	-1/x2	0
Line 3	0	-1/x3	0
Line 4	1/x4	0	-1/x4
Line 5	0	1/x5	-1/x5



H:	Bus 2	3	4	5	6	7	8	9	10	11	12
Line 1	-10	0	0	0	0	0	0	0	0	0	0
Line 2	0	0	-6.67	0	0	0	0	0	0	0	0
Line 3	5	-5	0	0	0	0	0	0	0	0	0
Line 4	20	0	-20	0	0	0	0	0	0	0	0
Line 5	4	0	0	-4	0	0	0	0	0	0	0
Line 6	0	10	0	-10	0	0	0	0	0	0	0
Line 7	0	0	8.33	-8.33	0	0	0	0	0	0	0
Line 8	0	0	4	0	0	-4	0	0	0	0	0
Line 9	0	0	0	12.5	-12.5	0	0	0	0	0	0
Line 10	0	0	0	6.25	0	-6.25	0	0	0	0	0
Line 11	0	0	0	0	10	0	0	-10	0	0	0
Line 12	0	0	0	0	0	6.67	-6.67	0	0	0	0
Line 13	0	0	0	0	0	20	0	0	-20	0	0
Line 14	0	0	0	0	0	8.33	0	0	0	-8.33	0
Line 15	0	0	0	0	0	0	5	-5	0	0	0
Line 16	0	0	0	0	0	0	6.67	0	0	0	-6.67
Line 17	0	0	0	0	0	0	0	6.25	0	-6.25	0
Line 18	0	0	0	0	0	0	0	8.33	0	0	-8.33
Line 19	0	0	0	0	0	0	0	0	10	-10	0
Line 20	0	0	0	0	0	0	0	0	0	6.67	-6.67

	From Bus	To Bus	x
Line 1	1	2	0.1
Line 2	1	4	0.15
Line 3	2	3	0.2
Line 4	2	4	0.05
Line 5	2	5	0.25
Line 6	3	5	0.1
Line 7	4	5	0.12
Line 8	4	7	0.25
Line 9	5	6	0.08
Line 10	5	7	0.16
Line 11	6	9	0.1
Line 12	7	8	0.15
Line 13	7	10	0.05
Line 14	7	11	0.12
Line 15	8	9	0.2



PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 1	0	-0.6563	-0.5997	-0.5156	-0.5714	-0.5669	-0.5551	-0.5582	-0.5612	-0.5559	-0.5574	-0.5591
Line 2	0	-0.3437	-0.4003	-0.4844	-0.4286	-0.4331	-0.4449	-0.4418	-0.4388	-0.4441	-0.4426	-0.4409
Line 3	0	0.0283	-0.4536	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613
Line 4	0	0.2814	-0.0017	-0.4221	-0.1432	-0.1657	-0.2246	-0.2089	-0.1939	-0.2207	-0.2129	-0.2043
Line 5	0	0.0340	-0.1444	-0.0510	-0.2335	-0.2188	-0.1802	-0.1906	-0.2004	-0.1828	-0.1879	-0.1935
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613
Line 7	0	-0.0465	-0.3000	0.0697	-0.4268	-0.3868	-0.2819	-0.3100	-0.3367	-0.2889	-0.3028	-0.3180
Line 8	0	-0.0158	-0.1020	0.0237	-0.1451	-0.2121	-0.3877	-0.3407	-0.2960	-0.3760	-0.3527	-0.3272
Line 9	0	0.0056	0.0363	-0.0084	0.0516	-0.7465	-0.2180	-0.3595	-0.4941	-0.2531	-0.3233	-0.4002
Line 10	0	0.0102	0.0657	-0.0153	0.0934	-0.0414	-0.3943	-0.2998	-0.2099	-0.3709	-0.3240	-0.2727
Line 11	0	0.0056	0.0363	-0.0084	0.0516	0.2535	-0.2180	-0.3595	-0.4941	-0.2531	-0.3233	-0.4002
Line 12	0	-0.0021	-0.0136	0.0032	-0.0193	-0.0948	0.0815	-0.4708	-0.1892	0.0362	-0.0545	-0.2344
Line 13	0	-0.0016	-0.0101	0.0023	-0.0144	-0.0705	0.0607	-0.0754	-0.1408	-0.7184	-0.2765	-0.1624
Line 14	0	-0.0020	-0.0126	0.0029	-0.0180	-0.0882	0.0758	-0.0942	-0.1759	-0.0647	-0.3456	-0.2030
Line 15	0	-0.0015	-0.0097	0.0022	-0.0138	-0.0676	0.0581	0.2694	-0.1349	0.0418	0.0091	0.0338
Line 16	0	-0.0006	-0.0039	0.0009	-0.0055	-0.0272	0.0234	0.2598	-0.0543	-0.0056	-0.0636	-0.2682
Line 17	0	0.0024	0.0154	-0.0036	0.0218	0.1073	-0.0922	0.0340	0.2140	-0.1346	-0.2195	0.0252
Line 18	0	0.0017	0.0113							-0.0767	-0.0947	-0.3916
Line 19	0	-0.0016	-0.0101							0.2816	-0.2765	-0.1624
Line 20	0	-0.0011	-0.0074							0.0823	0.1584	-0.3402

$$PTDF = H[B']^{-1}$$

Break

ALFRED P. SLOAN FOUNDATION

Power Flow Review

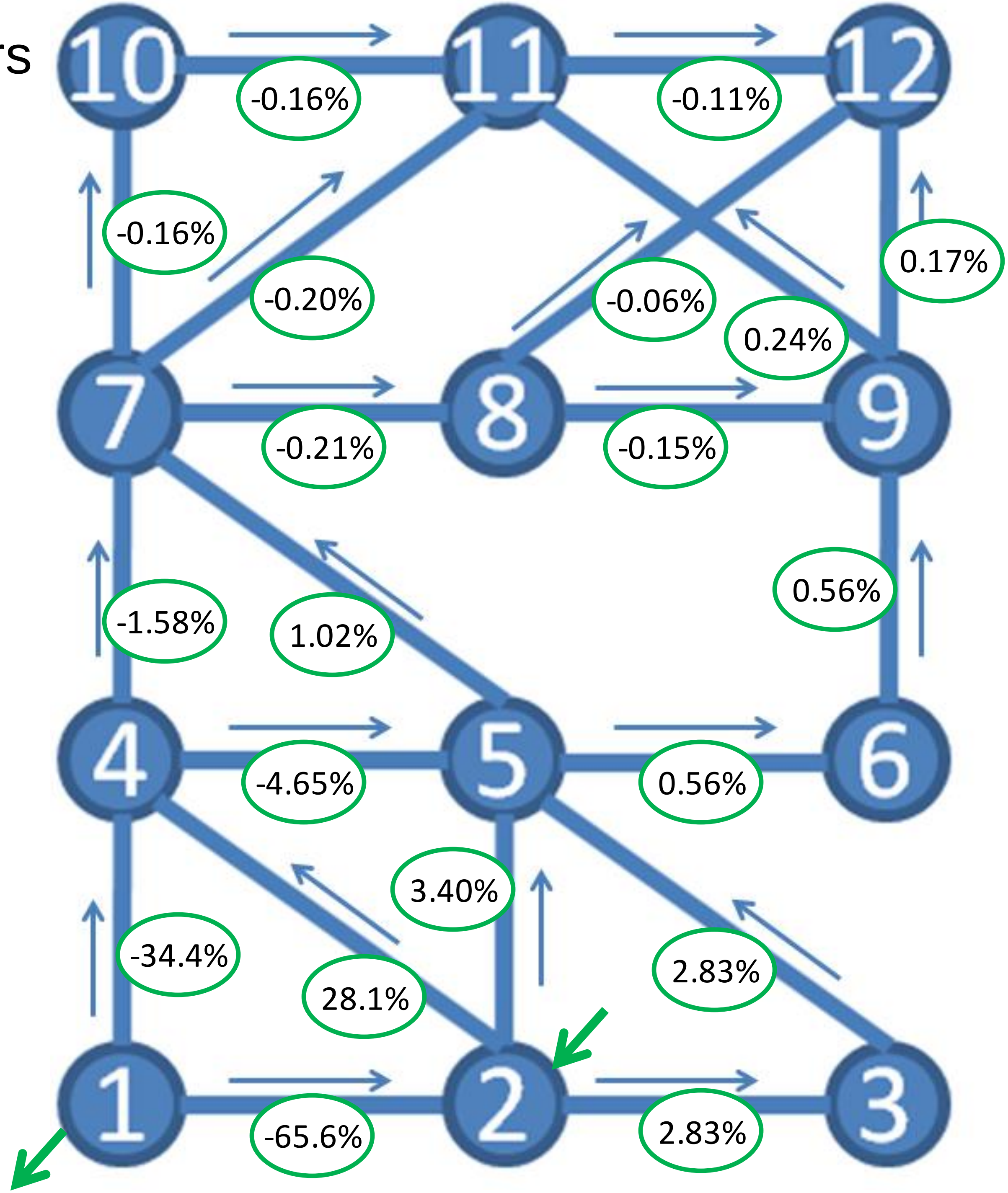
PSERC

PSERC

More on Power Transfer Distribution Factors Continued

Examine the shift factors for an injection at bus 2

	From Bus	To Bus	x	PTDF Inject Bus 2
Line 1	1	2	0.1	-0.6563
Line 2	1	4	0.15	-0.3437
Line 3	2	3	0.2	0.0283
Line 4	2	4	0.05	0.2814
Line 5	2	5	0.25	0.0340
Line 6	3	5	0.1	0.0283
Line 7	4	5	0.12	-0.0465
Line 8	4	7	0.25	-0.0158
Line 9	5	6	0.08	0.0056
Line 10	5	7	0.16	0.0102
Line 11	6	9	0.1	0.0056
Line 12	7	8	0.15	-0.0021
Line 13	7	10	0.05	-0.0016
Line 14	7	11	0.12	-0.0020
Line 15	8	9	0.2	-0.0015
Line 16	8	12	0.15	-0.0006
Line 17	9	11	0.16	0.0024
Line 18	9	12	0.12	0.0017
Line 19	10	11	0.1	-0.0016
Line 20	11	12	0.15	-0.0011



PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 1	0	-0.6563	-0.5997	-0.5156	-0.5714	-0.5669	-0.5551	-0.5582	-0.5612	-0.5559	-0.5574	-0.5591
Line 2	0	-0.3437	-0.4003	-0.4844	-0.4286	-0.4331	-0.4449	-0.4418	-0.4388	-0.4441	-0.4426	-0.4409
Line 3	0	0.0283	-0.4536	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613
Line 4	0	0.2814	-0.0017	-0.4221	-0.1432	-0.1657	-0.2246	-0.2089	-0.1939	-0.2207	-0.2129	-0.2043
Line 5	0	0.0340	-0.1444	-0.0510	-0.2335	-0.2188	-0.1802	-0.1906	-0.2004	-0.1828	-0.1879	-0.1935
Line 6	0	0.0283	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.1613
Line 7	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.3180
Line 8	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.3272
Line 9	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.4002
Line 10	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.2727
Line 11	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.4002
Line 12	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0.2344
Line 13	0	-0.0016	-0.0101	0.0023	-0.0144	-0.0705	0.0607	-0.0754	-0.1408	-0.7184	-0.2765	-0.1624
Line 14	0	-0.0020	-0.0126	0.0029	-0.0180	-0.0882	0.0758	-0.0942	-0.1759	-0.0647	-0.3456	-0.2030
Line 15	0	-0.0015	-0.0097	0.0022	-0.0138	-0.0676	0.0581	0.2694	-0.1349	0.0418	0.0091	0.0338
Line 16	0	-0.0006	-0.0039	0.0009	-0.0055	-0.0272	0.0234	0.2598	-0.0543	-0.0056	-0.0636	-0.2682
Line 17	0	0.0024	0.0154	-0.0036	0.0218	0.1073	-0.0922	0.0340	0.2140	-0.1346	-0.2195	0.0252
Line 18	0	0.0017	0.0113	-0.0026	0.0160	0.0787	-0.0676	-0.1242	0.1570	-0.0767	-0.0947	-0.3916
Line 19	0	-0.0016	-0.0101	0.0023	-0.0144	-0.0705	0.0607	-0.0754	-0.1408	0.2816	-0.2765	-0.1624
Line 20	0	-0.0011	-0.0074	0.0017	-0.0105	-0.0515	0.0443	-0.1356	-0.1027	0.0823	0.1584	-0.3402

For complex optimization problems (*and for reasons other than complexity*), industry will round PTDFs that are close to zero

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 1	0	-0.6563	-0.5997	-0.5156	-0.5714	-0.5669	-0.5551	-0.5582	-0.5612	-0.5559	-0.5574	-0.5591
Line 2	0	-0.3437	-0.4003	-0.4844	-0.4286	-0.4331	-0.4449	-0.4418	-0.4388	-0.4441	-0.4426	-0.4409
Line 3	0	0.0283	-0.4536	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613
Line 4	0	0.2814	0	-0.4221	-0.1432	-0.1657	-0.2246	-0.2089	-0.1939	-0.2207	-0.2129	-0.2043
Line 5	0	0.0340	-0.1444	-0.0510	-0.2335	-0.2188	-0.1802	-0.1906	[illegible]	[illegible]	[illegible]	[illegible]
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	[illegible]	[illegible]	[illegible]	[illegible]
Line 7	0	-0.0465	-0.3000	0.0697	-0.4268	-0.3868	-0.2819	-0.3100	[illegible]	[illegible]	[illegible]	[illegible]
Line 8	0	0	-0.1020	0.0237	-0.1451	-0.2121	-0.3877	-0.3407	-0.2960	-0.3760	-0.3527	-0.3272
Line 9	0	0	0.0363	0	0.0516	-0.7465	-0.2180	-0.3595	-0.4941	-0.2531	-0.3233	-0.4002
Line 10	0	0	0.0657	0	0.0934	-0.0414	-0.3943	-0.2998	-0.2099	-0.3709	-0.3240	-0.2727
Line 11	0	0	0.0363	0	0.0516	0.2535	-0.2180	-0.3595	-0.4941	-0.2531	-0.3233	-0.4002
Line 12	0	0	0	0	0	-0.0948	0.0815	-0.4708	-0.1892	0.0362	-0.0545	-0.2344
Line 13	0	0	0	0	0	-0.0705	0.0607	-0.0754	-0.1408	-0.7184	-0.2765	-0.1624
Line 14	0	0	0	0	0	-0.0882	0.0758	-0.0942	-0.1759	-0.0647	-0.3456	-0.2030
Line 15	0	0	0	0	0	-0.0676	0.0581	0.2694	-0.1349	0.0418	0	0.0338
Line 16	0	0	0	0	0	-0.0272	0.0234	0.2598	-0.0543	0	-0.0636	-0.2682
Line 17	0	0	0	0	0.0218	0.1073	-0.0922	0.0340	0.2140	-0.1346	-0.2195	0.0252
Line 18	0	0	0	0	0	0.0787	-0.0676	-0.1242	0.1570	-0.0767	-0.0947	-0.3916
Line 19	0	0	0	0	0	-0.0705	0.0607	-0.0754	-0.1408	0.2816	-0.2765	-0.1624
Line 20	0	0	0	0	0	-0.0515	0.0443	-0.1356	-0.1027	0.0823	0.1584	-0.3402

ABS(0.02)
rounding

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 1	0	-0.6563	-0.5997	-0.5156	-0.5714	-0.5669	-0.5551	-0.5582	-0.5612	-0.5559	-0.5574	-0.5591
Line 2	0	-0.3437	-0.4003	-0.4844	-0.4286	-0.4331	-0.4449	-0.4418	-0.4388	-0.4441	-0.4426	-0.4409
Line 3	0	0	-0.4536	0	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613
Line 4	0	0.2814	0	-0.4221	-0.1432	-0.1657	-0.2246	-0.2089	-0.1939	-0.2207	-0.2129	-0.2043
Line 5	0	0	-0.1444	-0.0510	-0.2335	-0.2188	-0.1802	-0.1906	-0	[illegible]	[illegible]	935
Line 6	0	0	0.5464	0	-0.1946	-0.1823	-0.1502	-0.1588	-0	[illegible]	[illegible]	513
Line 7	0	0	-0.3000	0.0697	-0.4268	-0.3868	-0.2819	-0.3100	-0	[illegible]	[illegible]	180
Line 8	0	0	-0.1020	0	-0.1451	-0.2121	-0.3877	-0.3407	-0	[illegible]	[illegible]	272
Line 9	0	0	0	0	0.0516	-0.7465	-0.2180	-0.3595	-0	[illegible]	[illegible]	002
Line 10	0	0	0.0657	0	0.0934	0	-0.3943	-0.2998	-0	[illegible]	[illegible]	727
Line 11	0	0	0	0	0.0516	0.2535	-0.2180	-0.3595	-0.4941	-0.2531	-0.3233	-0.4002
Line 12	0	0	0	0	0	-0.0948	0.0815	-0.4708	-0.1892	0	-0.0545	-0.2344
Line 13	0	0	0	0	0	-0.0705	0.0607	-0.0754	-0.1408	-0.7184	-0.2765	-0.1624
Line 14	0	0	0	0	0	-0.0882	0.0758	-0.0942	-0.1759	-0.0647	-0.3456	-0.2030
Line 15	0	0	0	0	0	-0.0676	0.0581	0.2694	-0.1349	0	0	0
Line 16	0	0	0	0	0	0	0	0.2598	-0.0543	0	-0.0636	-0.2682
Line 17	0	0	0	0	0	0.1073	-0.0922	0	0.2140	-0.1346	-0.2195	0
Line 18	0	0	0	0	0	0.0787	-0.0676	-0.1242	0.1570	-0.0767	-0.0947	-0.3916
Line 19	0	0	0	0	0	-0.0705	0.0607	-0.0754	-0.1408	0.2816	-0.2765	-0.1624
Line 20	0	0	0	0	0	-0.0515	0	-0.1356	-0.1027	0.0823	0.1584	-0.3402

ABS(0.05) rounding

Many more zeros for large-scale systems

Break

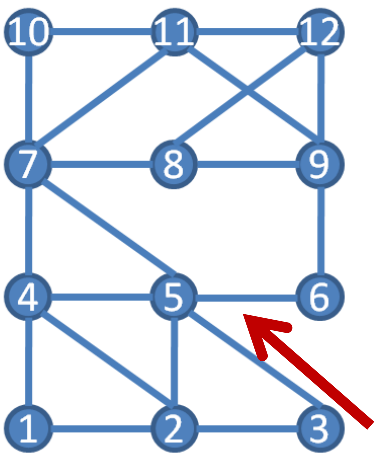
ALFRED P. SLOAN FOUNDATION

Power Flow Review

Redispatch Exercise 1

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 10



- Suppose there is an overload on line 6 (in the direction of bus 3 to 5)
- You need to decrease the flow on line 6 by 100MW
- The only generators that can quickly change their outputs are at bus 2 and bus 3
- What do you do?
  - Assume no other overload will occur based on your action
  - Assume that you wish to move generation the least amount possible (stay as close to existing dispatch schedule)
  - Load hasn't changed so your proposed net gen change = 0

Answer the questions:

1) Should the generator at bus 3 increase or decrease production?

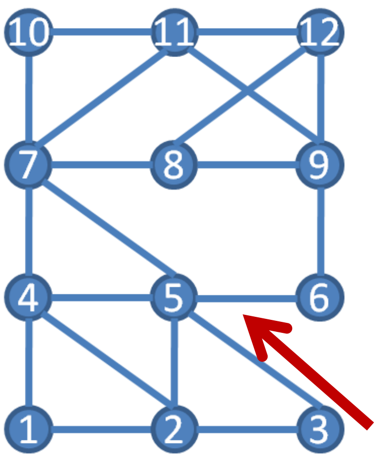
2) What should the generator at bus 2 do (increase or decrease)?

3) What is the change in output for the two generators?

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 10

- Suppose there is an overload on line 6 (in the direction of bus 3 to 5)
- You need to decrease the flow on line 6 by 100MW
- What if there is an available generator at each bus?
- What do you do?
  - Assume no other overload will occur based on your action
  - Assume that you wish to move generation the least amount possible (stay as close to existing dispatch schedule)
  - Load hasn't changed so net gen change = 0



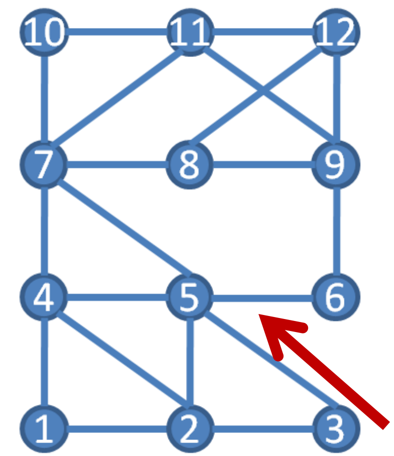
	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option 1	5 (-0.1946)	3 (0.5464)	-0.741	
Option 2	6 (-0.1823)	3 (0.5464)	-0.7287	
Option 3	2 (0.0283)	3 (0.5464)	-0.5181	
Option 4	1 (0)	2 (0.0283)	-0.0283	
Option 5	12 (-0.1613)	11 (-0.1566)	-0.0047	

Determine the amount you must move the generation, in absolute MW

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 10

- Suppose there is an overload on line 6 (in the direction of bus 3 to 5)
- You need to decrease the flow on line 6 by 100MW
- What if there is an available generator at each bus?
- What do you do?
  - Assume no other overload will occur based on your action
  - Assume that you wish to move generation the least amount possible (stay as close to existing dispatch schedule)
  - Load hasn't changed so net gen change = 0



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option 1	5 (-0.1946) (135)*(-0.1946) = -26.27	3 (0.5464) (-135)*(0.5464) = -73.73	-0.741	270 135 up G5 135 dn G3
Option 2	6 (-0.1823)	3 (0.5464)	-0.7287	275
Option 3	2 (0.0283) 193*0.0283= +5.46	3 (0.5464) (-193)*0.5464 = -105.46	-0.5181	386
Option 4	1 (0)	2 (0.0283)	-0.0283	>7000
Option 5	12 (-0.1613)	11 (-0.1566)	-0.0047	>42000

Break

ALFRED P. SLOAN FOUNDATION

Power Flow Review

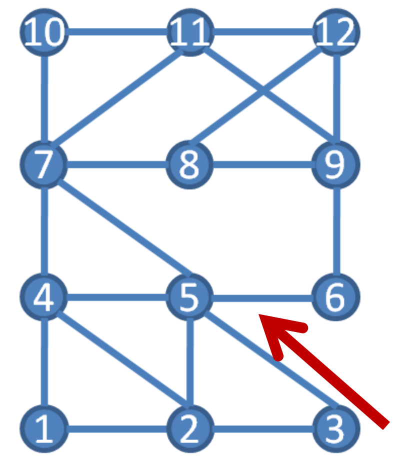
PSERC

PSERC

Redispatch Exercise 2

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

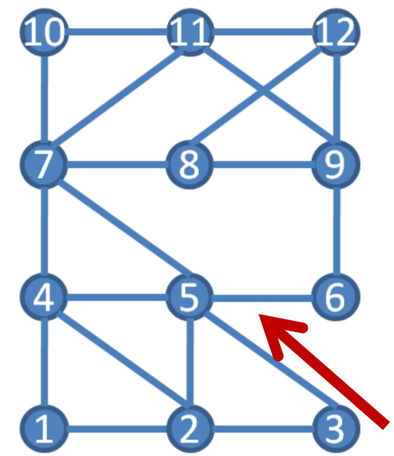
Example 11: Bus 3 has a wind farm. It increases by 10MW at the same time the load at the slack bus increases by 10MW: line 6 overloads by 5.464MW
You have 2 gas units at bus 2 and bus 5 that can respond *(right next to it!)*. **How much do you have to move the gas units at bus 2 and bus 5?**



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	5 (-0.1946)	2 (0.0283)	-0.2229	

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 11: Bus 3 has a wind farm. It increases by 10MW at the same time the load at the slack bus increases by 10MW: line 6 overloads by 5.464MW
You have 2 gas units at bus 2 and bus 5 that can respond *(right next to it!)*. **How much do you have to move the gas units at bus 2 and bus 5?**



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	5 (-0.1946)	2 (0.0283)	-0.2229	49

Increase gen at bus 5: 24.5MW (4.77MW reduction on line 3-5)
Decrease gen at bus 2: 24.5MW (0.693MW reduction on line 3-5)

$G_2 + G_5 = 0 \qquad 0.0283G_2 - 0.1946G_5 = -5.464 \Rightarrow G_2 = -24.5, G_5 = 24.5$

A 10MW movement by the wind farm caused 49MW of movement for the natural gas units that are one bus away! (and the load movement nearby as well)

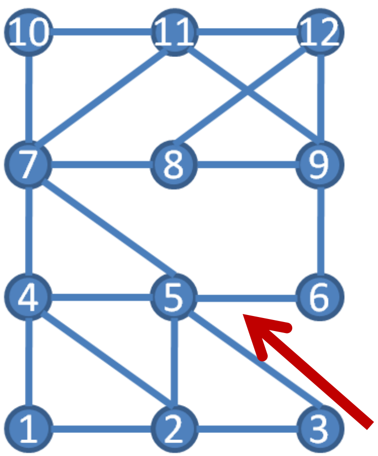
PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 12: Bus 3 has a wind farm. It increases by 10MW: line 6 overloads by 5.464MW

You believe that you should *only move one generator down to compensate for the increase in production and then move the slack bus (1) up to have a net supply change of -10MW*.

You have a natural gas unit at each location except bus 3.

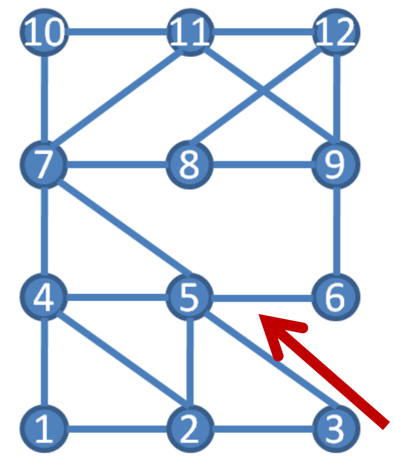
Which generator do you move down? How much?



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	1 (0)	2 (0.0283)	-0.0283	

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 12: Bus 3 has a wind farm. It increases by 10MW: line 6 overloads by 5.464MW
You believe that you should *only move one generator down to compensate for the increase in production and then move the slack bus (1) up to have a net supply change of -10MW*.
You have a natural gas unit at each location except bus 3.
Which generator do you move down? How much?



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	1 (0)	2 (0.0283)	-0.0283	376!

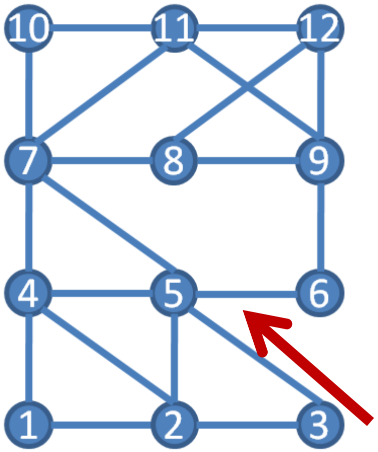
$$G_1 + G_2 = -10 \qquad 0G_1 + 0.0283G_2 = -5.464 \quad \Rightarrow \quad G_1 = 183, G_2 = -193$$

Bus 2 has the largest positive valued PTDF (except bus 3), which means decreasing bus 2 will have the largest decrease on the line's flow.
But you are compensating by increasing the slack bus generator, which is farther away

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 13: Bus 3 has a wind farm. It increases by 10MW: line 6 overloads by 5.464MW

This time you are looking for any 2 generators to increase and decrease to get rid of the flow (except bus 3)
Which generators do you choose? How much change?



	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	5 (-0.1946)	2 (0.0283)	-0.2229	

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 13: Bus 3 has a wind farm. It increases by 10MW: line 6 overloads by 5.464MW

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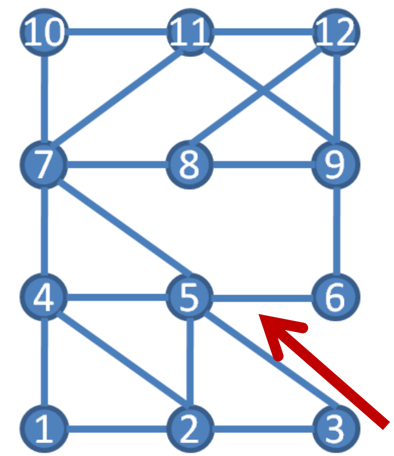
Which generators do you choose? How much change?



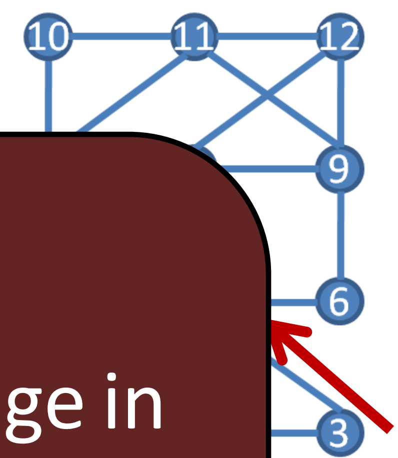
	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option	5 (-0.1946)	2 (0.0283)	-0.2229	56!

$$G_2 + G_5 = -10 \quad 0.0283G_2 - 0.1946G_5 = -5.464 \Rightarrow G_2 = -33.2, G_5 = 23.2$$

You choose gen bus 2 as it has the largest positive value (except bus 3) and you choose gen bus 5 as it has the largest negative (magnitude) value. They are both one bus away from bus 3 and still it is 56!

PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

Example 13: Bus 3 has a wind farm. It increases by 10MW: line 6 overloads by 5.464MW



Key Takeaway:

Many people assume that 1MW change in renewable production only requires 1MW change in another resource

Grid complexities can compound the required change

Op

G_2

3)

and you choose gen bus 5 as it has the largest negative (magnitude) value. They are both one bus away from bus 3 and still it is 56!

Break

ALFRED P. SLOAN FOUNDATION

Power Flow Review

PSERC

PSERC

Redispatch Exercise 3

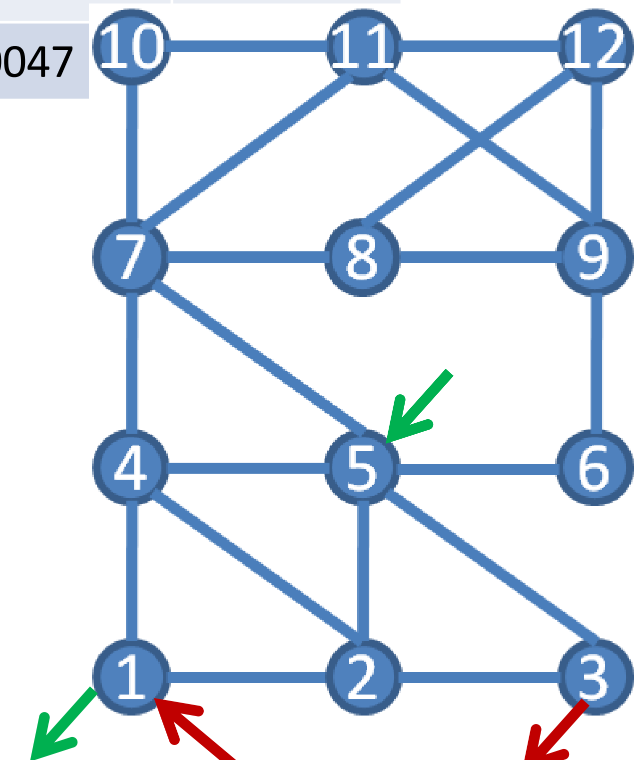
PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

What does the net change value really represent?

	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option 1	5 (-0.1946)	3 (0.5464)	-0.741	270
Option 2	6 (-0.1823)	3 (0.5464)	-0.7287	275
Option 3	2 (0.0283)	3 (0.5464)	-0.5181	386
Option 4	1 (0)	2 (0.0283)	-0.0283	>7000
Option 5	12 (-0.1613)	11 (-0.1566)	-0.0047	

What does each PTDF represent?

Inject at Bus 5 + **Withdraw at Bus 3**



PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

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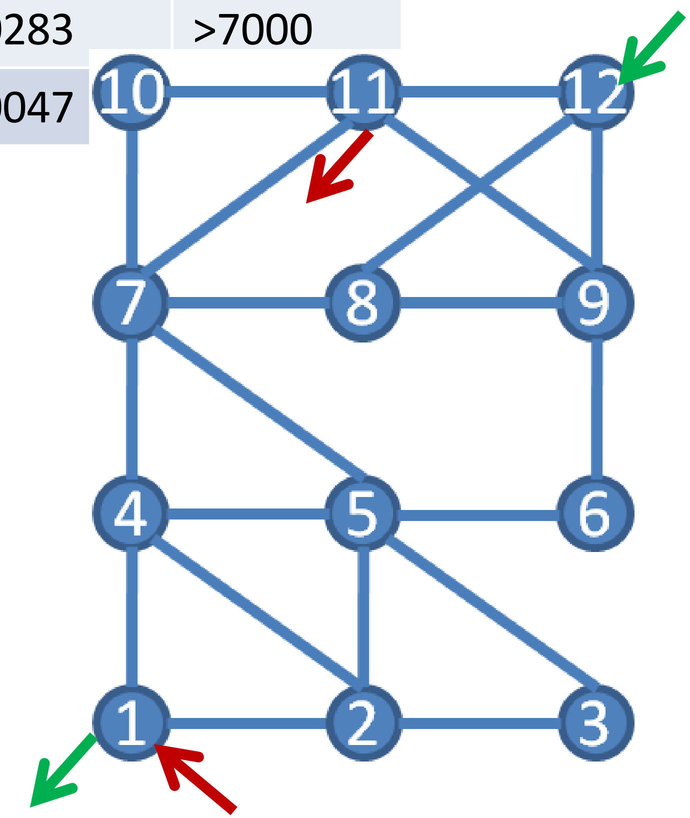
Option 4	1 (0)	2 (0.0283)	-0.0283	>7000
Option 5	12 (-0.1613)	11 (-0.1566)	-0.0047	

What does each PTDF represent?

Inject at Bus 12 + **Withdraw at Bus 11**



PTDF	1	2	3	4	5	6	7	8	9	10	11	12
Line 6	0	0.0283	0.5464	-0.0425	-0.1946	-0.1823	-0.1502	-0.1588	-0.1670	-0.1523	-0.1566	-0.1613

What does the net change value really represent?

	Increase Bus (PTDF):	Decrease Bus (PTDF):	Net Change:	ABS ΔMW
Option 1	5 (-0.1946)	3 (0.5464)	-0.741	270
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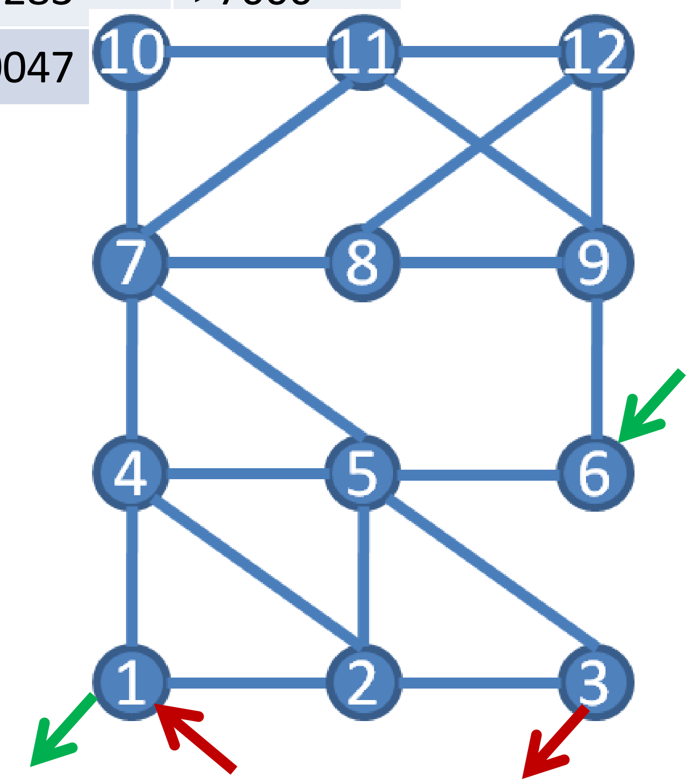
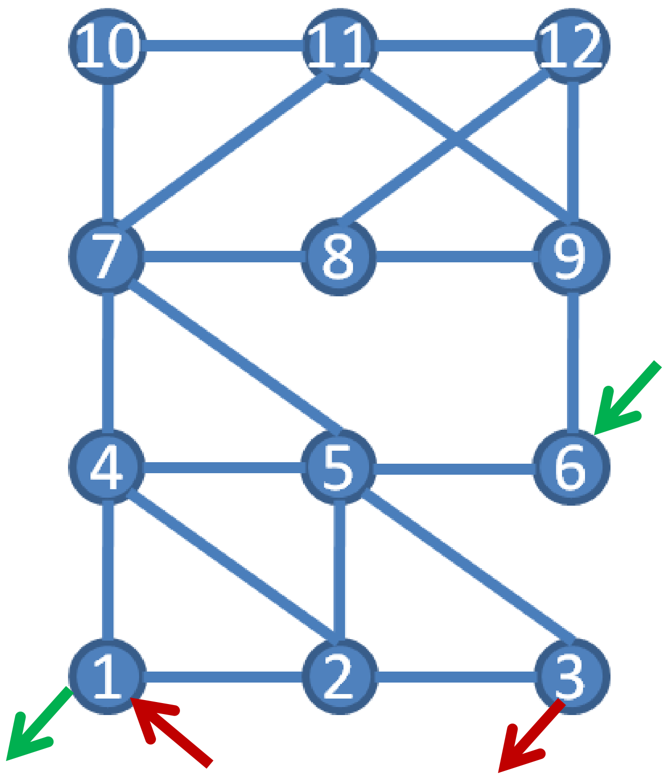
Option 3	2 (0.0283)	3 (0.5464)	-0.5181	386
Option 4	1 (0)	2 (0.0283)	-0.0283	>7000
Option 5	12 (-0.1613)	11 (-0.1566)	-0.0047	

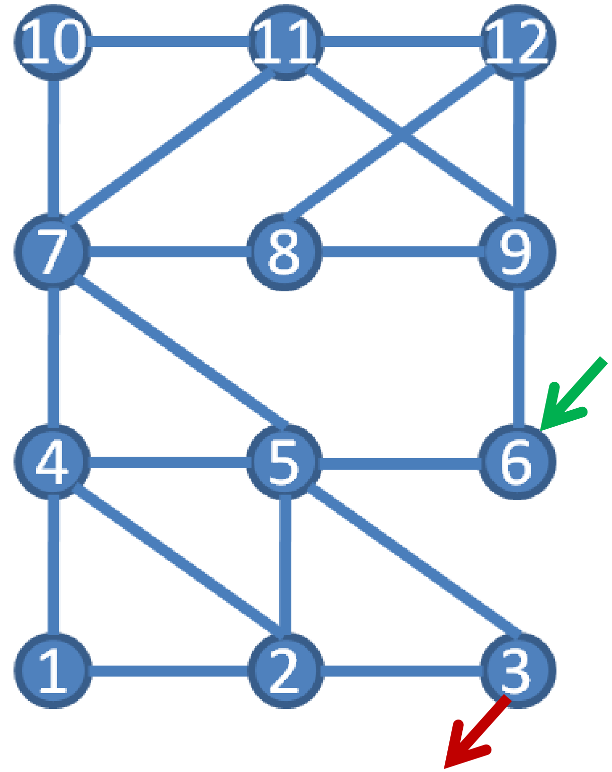
What does each PTDF represent?

Inject at Bus 6 + **Withdraw at Bus 3**

- The PTDFs were calculated with Bus 1 as the reference bus (a sink)
- Injection point: source
- Withdrawal point: sink
- Why not just recalculate the PTDFs for a sink defined as bus 3?
- Would there be a difference?







- The resulting flows from the two examples are identical:
- 1MW injected at 6 & withdrawn at 1 + 1MW injected at 1 & withdrawn at 3
- = 1MW injected at 6 & withdrawn at 3
- = 1MW injected at 6 & withdrawn at 1 + (-1MW) injected at 3 & withdrawn at 1 = $PTDF_{l,6}^{1} + \left(-PTDF_{l,3}^{1}\right) = PTDF_{l,6}^{3}$

PTDF

$PTDF_{l,i}^{R}$: ΔFlow on line l for a 1MW injection at bus i and a 1MW withdrawal at bus R (reference bus)

- What if you want a transfer from (inject) bus i to (withdrawal) bus j?
- $PTDF_{l,i}^{j} = PTDF_{l,i}^{R} - PTDF_{l,j}^{R}$
- There is no need to have to recalculate PTDFs for different injection and withdraw points
- Do not forget that this is a linear sensitivity when the grid is a non-linear system

PTDF and Injection Shift Factors

$$PTDF_{l,i}^{j} = PTDF_{l,i}^{R} - PTDF_{l,j}^{R}$$

- Let's reconsider these terms above
- Some people prefer to refer to the terms on the right as injection shift factors
- Really, this is playing with terms
- The only difference is that the reference bus assignment changes
- What is important is to know where these terms come from and what they represent

Break

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Power Flow Review

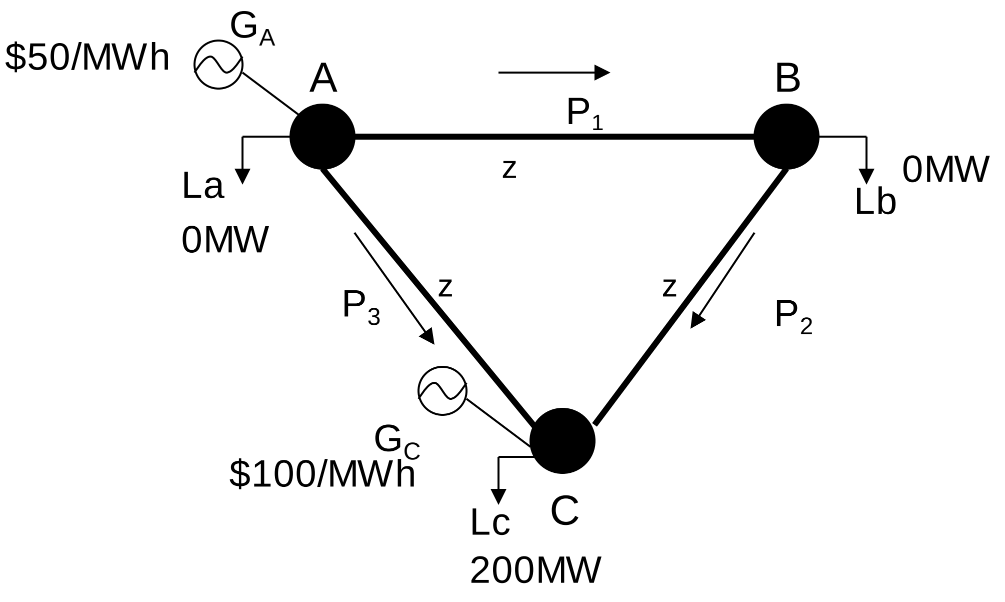
PSERC

PSERC

Optimal Dispatch Exercise

Optimal Dispatch Exercise

- DCOPF Exercise 14
- Determine the optimal dispatch when:
  - P1 Limit: 50MW, P2 Limit: 50MW, P3 Limit: 50MW

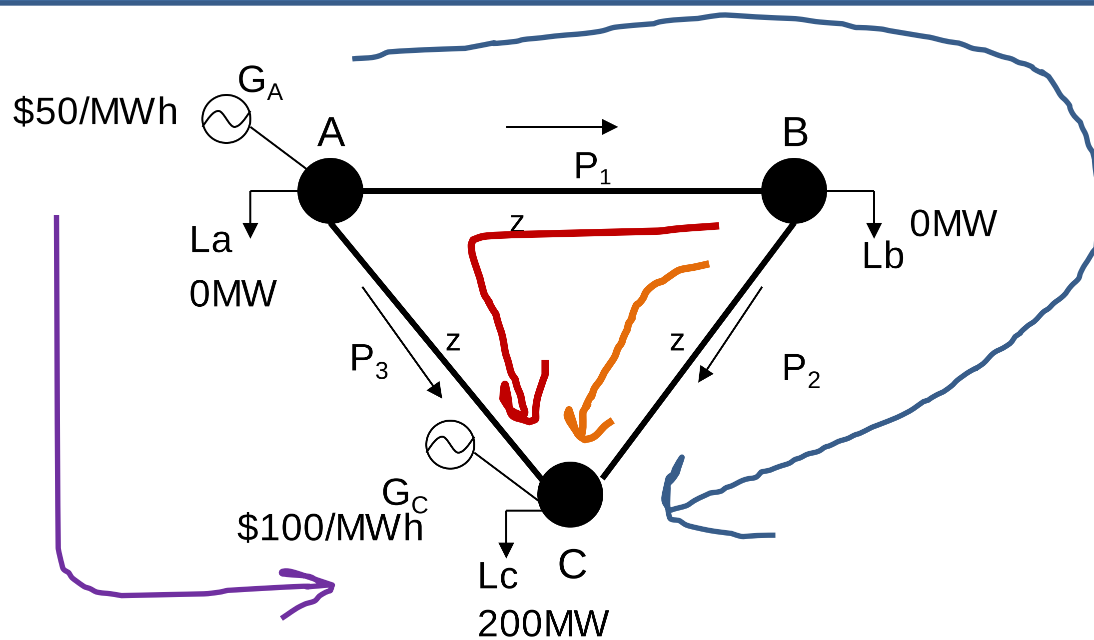


$$P_1 = \left(\frac{1}{3}\right)(G_A - L_A) + \left(\frac{-1}{3}\right)(G_B - L_B), -50 \le P_1 \le 50 \qquad P_1 = \left(\frac{1}{3}\right)(G_A)$$

$$P_2 = \left(\frac{1}{3}\right)(G_A - L_A) + \left(\frac{2}{3}\right)(G_B - L_B), -50 \le P_2 \le 50 \qquad P_2 = \left(\frac{1}{3}\right)(G_A)$$

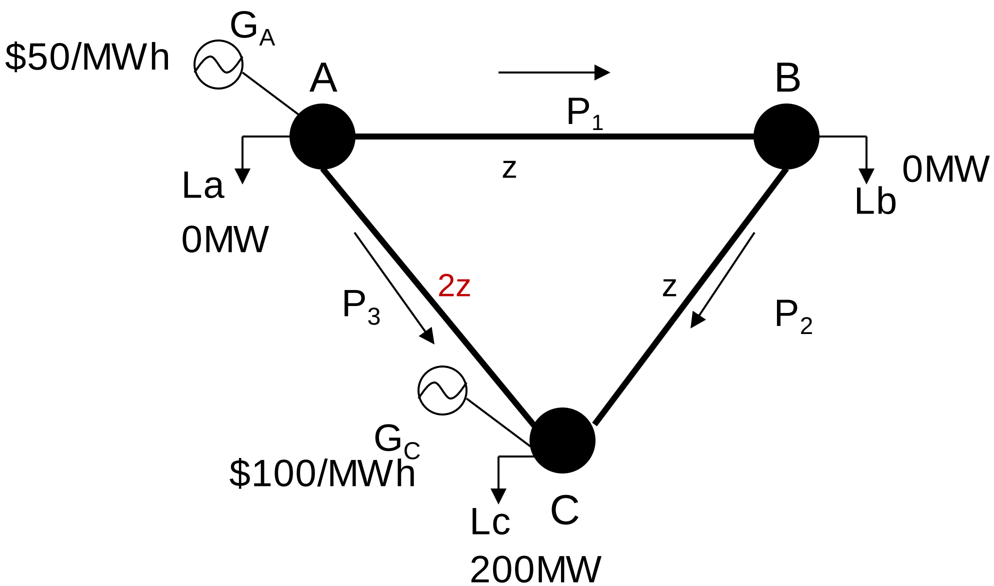
$$P_3 = \left(\frac{2}{3}\right)(G_A - L_A) + \left(\frac{1}{3}\right)(G_B - L_B), -50 \le P_3 \le 50 \qquad P_3 = \left(\frac{2}{3}\right)(G_A)$$

By inspection, Ga is the cheapest and since line limits are the only thing preventing Ga from serving the entire demand, the line that binds first will set a limit on how much we can get from Ga and the remaining demand must be met by Gc. This is a very simple example since there are only 2 generators and one load. P3 binds first limiting Ga to 75MW.
Ga = 75MW, Gc = 125MW, P1 = 25MW, P2 = 25MW, P3 = 50MW.



Optimal Dispatch Exercise

- DCOPF Exercise 15
- Determine the optimal dispatch when:
  - P1 Limit: 50MW, P2 Limit: 50MW, P3 Limit: 50MW
  - Note the change in impedance for P3



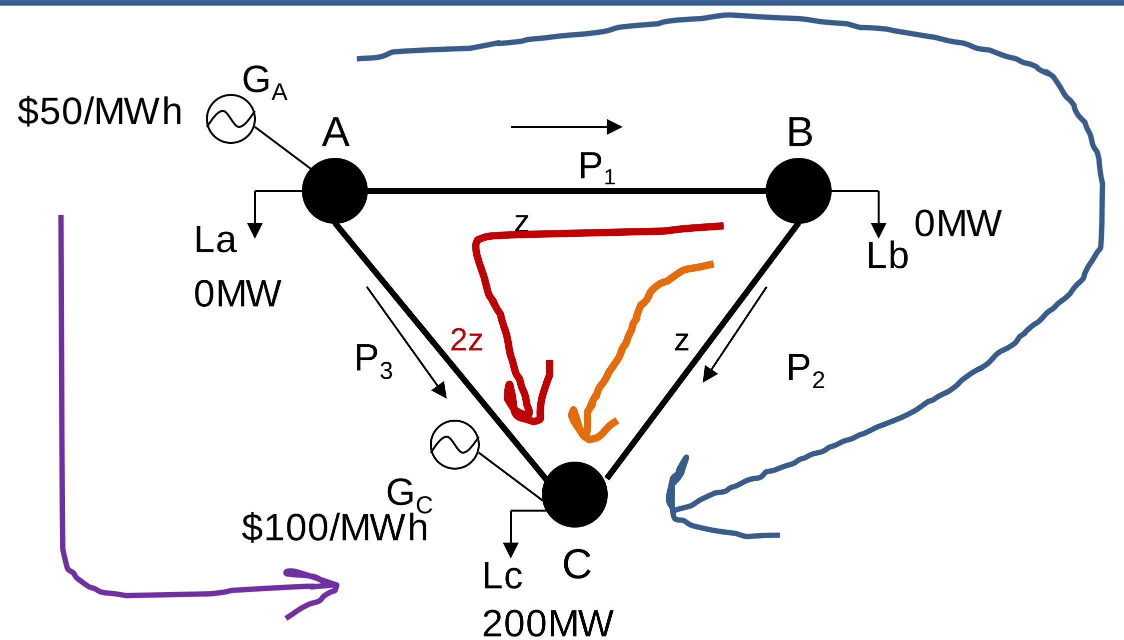
$$P_1 = \left(\frac{1}{2}\right)(G_A - L_A) + \left(\frac{-1}{4}\right)(G_B - L_B), -50 \le P_1 \le 50 \qquad P_1 = \left(\frac{1}{2}\right)(G_A)$$

$$P_2 = \left(\frac{1}{2}\right)(G_A - L_A) + \left(\frac{3}{4}\right)(G_B - L_B), -50 \le P_2 \le 50 \qquad P_2 = \left(\frac{1}{2}\right)(G_A)$$

$$P_3 = \left(\frac{1}{2}\right)(G_A - L_A) + \left(\frac{1}{4}\right)(G_B - L_B), -50 \le P_3 \le 50 \qquad P_3 = \left(\frac{1}{2}\right)(G_A)$$

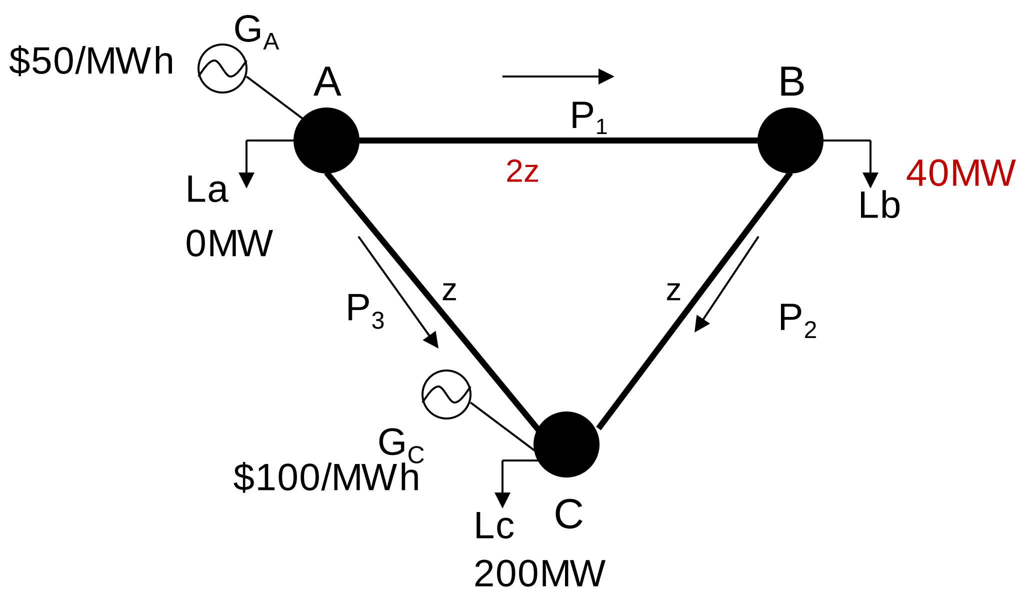
Similar situation, just account for the change in impedance at P3.

Most that Ga can produce without an overload is 100MW.
Ga = 100MW, Gc = 100MW, P1 = 50MW, P2 = 50MW, P3 = 50MW



Optimal Dispatch Exercise

- DCOPF Exercise 16
- Determine the optimal dispatch when:
  - P1 Limit: 50MW, P2 Limit: 50MW, P3 Limit: 50MW
  - Note the change to the impedance on line 1 and the load at B



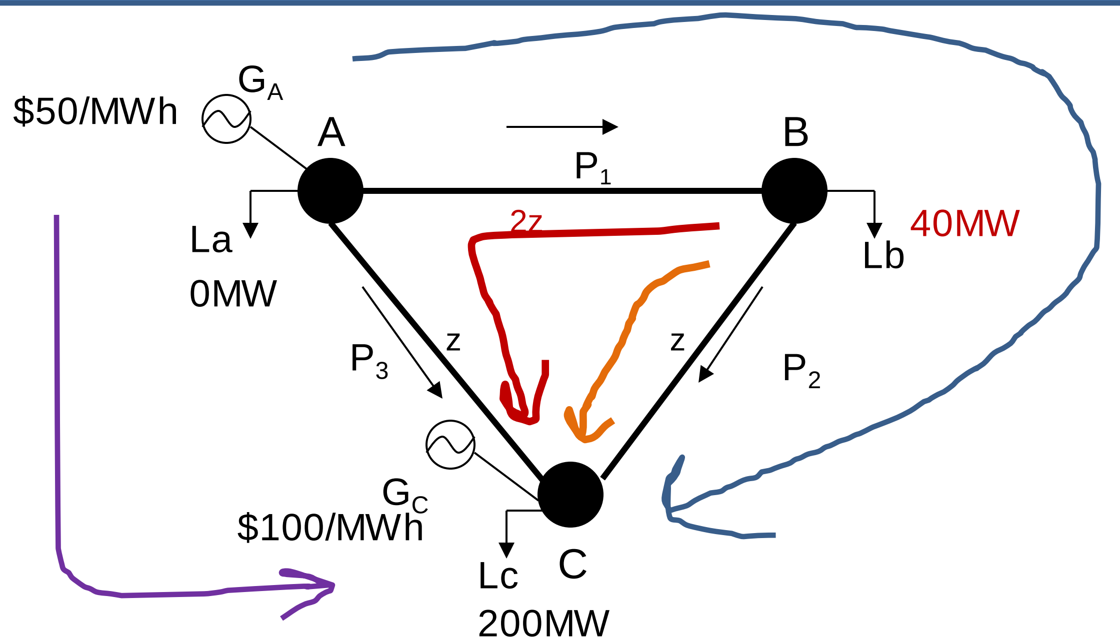
$$P_1 = \left(\frac{1}{4}\right)(G_A - L_A) + \left(\frac{-1}{4}\right)(G_B - L_B), -50 \le P_1 \le 50 \quad P_1 = \left(\frac{1}{4}\right)(G_A) + \left(\frac{-1}{4}\right)(-40)$$

$$P_2 = \left(\frac{1}{4}\right)(G_A - L_A) + \left(\frac{3}{4}\right)(G_B - L_B), -50 \le P_2 \le 50 \quad P_2 = \left(\frac{1}{4}\right)(G_A) + \left(\frac{3}{4}\right)(-40)$$

$$P_3 = \left(\frac{3}{4}\right)(G_A - L_A) + \left(\frac{1}{4}\right)(G_B - L_B), -50 \le P_3 \le 50 \quad P_3 = \left(\frac{3}{4}\right)(G_A) + \left(\frac{1}{4}\right)(-40)$$

Similar situation, just account for the new load at b and the change in impedance at P1.

Most that Ga can produce without an overload is 80MW.
Ga = 80MW, Gc = 160MW, P1 = 30MW, P2 = -10MW, P3 = 50MW



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Power Flow Review

PSERC

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Additional Information on Shift Factors

DC PF Based PTDFs

- For the DC Power Flow:
- Do PTDFs ever change or are they always the same?
- Do they change based on what generator is committed?
- What causes a change in the PTDFs?
- For the traditional DC Power Flow, the PTDFs do not change for a particular topology of the grid
- For a static topology (and no change in other items like switchable shunts), the PTDFs do not change

AC PF Based PTDFs

- For the AC Power Flow:
- Do PTDFs ever change or are they always the same?
- Do they change based on what generator is committed?
- What causes a change in the PTDFs?
- For the traditional AC Power Flow, the PTDFs **CHANGE** based on the operating condition of the system
- Give an example as to why…

AC PF Based PTDFs

- AC based PTDFs are not constant; they are a linear sensitivity to the existing operating state of the system and if the system conditions change, that linear sensitivity changes
- Simple reason: they are a linear approximation of a non-linear system
- AC PF based PTDFs would change based on changing voltage

Comparison of the PTDF based DCOPF and the B-θ DCOPF

PTDF Based DCOPF

- Note that there are no longer any bus voltage angle variables
- Note that there are no longer any line flow variables
- Shift factors can be calculated offline.

Industry DCOPF Formulations

- Most DCOPF formulations for commercial grade software use a PTDF formulation
- The earlier formulation is called the B-θ formulation as it relies on the Susceptance and the voltage angles
- PTDF formulations are easier to solve
- PTDF formulations allow you to ignore transmission lines that you know (or assume) will not be congested
  - With the B-θ formulation, you must have variables for all bus voltage angles and line flows, making it harder to reduce the problem size even if you know that you do not have to model all transmission lines

Industry Use of DCOPF

What products include a DCOPF?

- Most production cost software
  - ABB Gridview, PLEXOS, ABB PROMOD, GE MAPS
    - Note that these tools may be updated with more advanced features; the general standard though is to use a linearized OPF (generally a DCOPF) whenever solving an optimization-based problem related to economic operations
- All market optimization software in the USA (market based SCUC tools; market based SCED tools)
  - No market environment in the USA uses a full ACOPF formulation
  - Some market environments run an iterative algorithm: solve a DCOPF (with linear sensitivities); check AC feasibility (run AC PF); update linear sensitivities and rerun optimization (SCUC or SCED) model