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Joint work with C. Chen (Cornell), A. N. Madavan (UIUC),
N. Dahlin (U. Albany), L. Tong (Cornell)
The Emerging Landscape of Distributed Energy Resources (DERs)

Distributed solar, storage, EV infrastructure, demand management, etc.


A complex marketplace with DER owner-operators, DER aggregators (DERAs), distribution utility, possibly a distribution system operator (DSO) and the transmission system operator (TSO).
Ensuring Distribution Network Security

Voltage limits
Power flow limits

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Wholesale electricity market (ISO)

- DER aggregator (DERA)
- Distribution utility and LSE (DSO)
- DER aggregator (DERA)

DERA1
DERA2
DERA3
DERA4

DSO

DSO customers
DERA customers

POI
Substation
PCC

Increasing levels of DSO’s control

Type I
- Limit Hosting Capacity
Type II
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- Command and Control

Power flow (kW)
Financial flow ($)
The Hosting Capacity Problem

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Risk-Based Hosting Capacity Analysis in Distribution Systems
A. N. Madavan, N. Dahlin, S. Bose, and L. Tong
The Hosting Capacity Problem

\[ \Omega \] = scenarios of solar irradiance and loads

\[ \psi \] = solar capacity configuration
The Hosting Capacity Problem

\[ \Omega = \text{scenarios of solar irradiance and loads} \]

\[ \psi = \text{solar capacity configuration} \]

Can the distribution network host a solar capacity configuration, given the scenarios of solar irradiance and loads? Are the risks of constraint violation acceptable?

Q1. Does a capacity configuration have acceptable risks of constraint violation?

Q2. Maximize total installed capacity among acceptable configurations.
...the risk measure: Conditional Value at Risk (CVaR)

For losses with continuous distributions, conditional value at risk with parameter $\delta$ measures the expected tail loss in the worst $1 - \delta$ fraction of the outcomes.

\[
\text{CVaR}_\delta(\zeta) := \min_z \left\{ z + \frac{1}{1-\delta} \mathbb{E}[\zeta - z]^+ \right\}.
\]

...retains convexity of an underlying deterministic optimization problem
...allows the use of off-the-shelf solvers with convergence guarantees

Rockafellar and Uryasev 2000, 2002
The Questions

\[ \Omega = \text{scenarios of solar irradiance and loads} \]

\[ \psi = \text{solar capacity configuration} \]

Encode a constraint for a \textit{particular} scenario 
\[ \omega \in \Omega \] in inequality constraints

Encode a constraint \textit{across} scenarios in \[ \Omega \] via a risk-sensitive inequality constraint

\[ g_i(\psi; \omega) \leq 0 \]

\[ \text{CVaR}_{\delta_i} \left[ g_i(\psi; \Omega) \right] \leq 0 \]

Q1. Does a capacity configuration have acceptable risks of constraint violation?
Q2. Maximize total installed capacity among acceptable configurations.
The Feasibility and the Optimization Problems

\[ \alpha(\omega) \odot \psi - p_D(\omega) = B^T P(\omega) + R \odot L(\omega) \text{ a.s.,} \]
\[ \eta_G \odot \alpha(\omega) \odot \psi - q_D(\omega) = B^T Q(\omega) + X \odot L(\omega) \text{ a.s.,} \]
\[ BW(\omega) = 2 [R \odot P(\omega) + X \odot Q(\omega)] - (R^2 + X^2) \odot L(\omega) \text{ a.s.,} \]
\[ [B_+ W(\omega)] \odot L(\omega) \geq [P(\omega)]^2 + [Q(\omega)]^2 \text{ a.s.,} \]

CVaR-based constraint enforcement ensures probabilistic guarantees...

Increasing number of scenarios reduces variability...

...but increases problem dimension and runtime.

- CVaR-based formulation retains convexity
- Scenario approximations can be cast as a second-order cone program

An SOCP relaxation

CVaR-based risk assessment
Build on Past Knowledge in the Feasibility Problem

...convexity of the set allows sequential inner and outer approximations

<table>
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<tr>
<th>Test</th>
<th>Mean Runtime (s)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi \in \Psi_{in}$ test</td>
<td>$9.678 \times 10^{-4}$</td>
<td>451</td>
</tr>
<tr>
<td>$\psi \notin \Psi_{out}$ test</td>
<td>$1.195 \times 10^{-6}$</td>
<td>489</td>
</tr>
<tr>
<td>Solving opt when $\psi \in \Psi_{in}$</td>
<td>46.288</td>
<td>47</td>
</tr>
<tr>
<td>Solving opt when $\psi \notin \Psi_{out}$</td>
<td>133.107</td>
<td>13</td>
</tr>
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As you test more configurations for acceptability, the faster the algorithm becomes in certifying acceptability of new test configurations.
Auctioning Network Access Allocation

Wholesale Market Participation of DERAs: DSO-DERA-ISO Coordination
C. Chen, S. Bose, T. D. Mount, and L. Tong
IEEE Transactions on Power Systems (Accepted, 2024)
Maximize induced social welfare while allocating access limits subject to network constraints under all DSO and DERA net-injections.

\[
\begin{align*}
\text{maximize} & \quad C.C.P.\overline{P}
\text{subject to} & \quad \sum_{k=1}^{K} \varphi_k(C_k, \overline{C}_k) - J(\overline{P}, \overline{P}),
\text{Benefit of DERA} \quad \text{Cost of DSO}
\text{Clearing injection access} & \quad \overline{P} = \sum_{k=1}^{K} C_k + \overline{p}_0,
\text{Clearing withdrawal access} & \quad P = \sum_{k=1}^{K} C_k + p_0,
\text{Benefit of DERA} & \quad \overline{b} \leq A \left( \sum_{k=1}^{K} p_k + p_0 \right) \leq \overline{b},
\text{Cost of DSO} & \quad \text{for all } p_k \in [-C_k, \overline{C}_k], p_0 \in [-\overline{p}_0, \overline{p}_0],
\text{for } k = 1, \ldots, K.
\end{align*}
\]

...does not require the DSO to intervene in real-time DERA-ISO interactions.
Reformulate Robust Constraints

\[
\text{maximize} \quad \sum_{k=1}^{K} \varphi_k(C_k^*, C_k^*) - J(P, P),
\]

subject to

Profit of DERA

\[
P = \sum_{k=1}^{K} C_k^* + \bar{p}_0, \quad \text{clearing injection access}
\]

Cost of DSO

\[
P = \sum_{k=1}^{K} C_k^* + p_0, \quad \text{clearing withdrawal access}
\]

\[
b \leq A \left( \sum_{k=1}^{K} p_k + p_0 \right) \leq \bar{b},
\]

for all \( p_k \in [-C_k^*, \bar{C}_k^*], \ p_0 \in [-\bar{p}_0, \bar{p}_0], \) for \( k = 1, \ldots, K. \)

Payment by DERA \( k \):

\[
\Pi_k^{\text{DERA}} := \varphi_k(C_k^*, C_k^*) - \mathcal{P}_k(C_k^*, C_k^*)
\]

Surplus of DERA \( k \):

\[
\Pi_k^{\text{DSO}} := \sum_{k=1}^{K} \mathcal{P}_k(C_k^*, C_k^*) - \left( J(P^*, P^*) - J(P_0, P_0) \right)
\]

Locational Marginal Access Price (LMAP)

Maximize...

\[
\begin{align*}
\bar{\lambda} : & \quad \bar{P} = \sum_{k=1}^{K} \bar{C}_k + \bar{p}_0, \\
\lambda : & \quad P = \sum_{k=1}^{K} C_k + p_0, \\
\bar{\mu} : & \quad A_+P + A_-P \leq \bar{b}, \\
\mu : & \quad b \leq -A_+P - A_-P.
\end{align*}
\]
Properties of the Auction Outcome

Payment by DERA $k$:
$$\mathcal{P}_k(\overline{C}_k^*, \overline{C}_k^*) = \overline{X}^* \mathbf{T} \overline{C}_k^* + \lambda^* \mathbf{T} \overline{C}_k^*$$

Surplus of DERA $k$:
$$\Pi_k^{\text{DERA}} = \varphi_k(\overline{C}_k^*, \overline{C}_k^*) - \mathcal{P}_k(\overline{C}_k^*, \overline{C}_k^*)$$

Surplus of DSO:
$$\Pi_k^{\text{DSO}} = \sum_{k=1}^{K} \mathcal{P}_k(\overline{C}_k^*, \overline{C}_k^*) - \left( J(\overline{P}^*, P^*) - J(\overline{P}_0, P_0) \right)$$

LMAPs are nodally uniform and reflect the DSO’s marginal costs, network congestion, and pre-defined access limits.

Under reasonable assumptions, DERAs and the DSO have non-negative surpluses.

Under reasonable assumptions, LMAPs increase along the distribution feeder.
Robust Access Allocation

\[
\text{maximize} \quad \sum_{k=1}^{K} \varphi_k(C_k, \overline{C}_k) - J(\overline{P}, \underline{P}),
\]

subject to

\[
\begin{align*}
\overline{P} &= \sum_{k=1}^{K} C_k + \overline{p}_0, \\
\underline{P} &= \sum_{k=1}^{K} C_k + p_0,
\end{align*}
\]

Benefit of DERA

Cost of DSO

Clearing injection access

Clearing withdrawal access

\[
\bar{b} \leq A \left( \sum_{k=1}^{K} p_k + p_0 \right) \leq \overline{b},
\]

for all \( p_k \in [-C_k, \overline{C}_k] \), \( p_0 \in [-\overline{p}_0, \overline{p}_0] \),

for \( k = 1, \ldots, K \).

Stochastic Access Allocation

Locational Marginal Access Price (LMAP)

\[
\bar{\Lambda} : \quad \overline{P} = \sum_{k=1}^{K} \overline{C}_k + p_0,
\]

\[
\underline{\Lambda} : \quad \underline{P} = \sum_{k=1}^{K} C_k - p_0,
\]

\[
\text{CVaR}_{\delta}[A(\sum_{k=1}^{K} p_k + p_0)] \leq \overline{b},
\]

for all \( p_k \in [-C_k, \overline{C}_k] \),

for \( k = 1, \ldots, K \).

...robust CVaR Constraints
The Data-Driven Counterpart

...with \( S \) scenarios of DSO customers’ demands

\[
\begin{align*}
\text{maximize} \quad & \sum_{k=1}^{K} \varphi_k(C_k, \overline{C}_k) - \frac{1}{S} \sum_{s=1}^{S} J(\overline{P}[s], P[s]), \\
\text{subject to} \quad & \overline{\lambda}[s] : \quad \overline{P}[s] = \sum_{k=1}^{K} \overline{C}_k + p_0[s], \\
& \lambda[s] : \quad P[s] = \sum_{k=1}^{K} C_k - p_0[s], \\
& \beta[s] : \quad A_+ \overline{P}[s] + A_- P[s] - \overline{b} - \overline{t} \leq \gamma[s], \\
& \theta[s] : \quad A_- \overline{P}[s] + A_+ P[s] + b - t \leq \gamma[s], \\
& \alpha[s] : \quad 0 \leq \gamma[s], \\
& \mu : \quad \overline{t} + \frac{1}{1 - \delta} \frac{1}{S} \sum_{s=1}^{S} \gamma[s] \leq 0, \\
& \alpha[s] : \quad 0 \leq \gamma[s], \\
& \mu : \quad t + \frac{1}{1 - \delta} \frac{1}{S} \sum_{s=1}^{S} \gamma[s] \leq 0.
\end{align*}
\]

Stochastic Access Allocation

Locational Marginal Access Price (LMAP)

\[
\begin{align*}
\bar{\Lambda} : \quad & \bar{P} = \sum_{k=1}^{K} \overline{C}_k + p_0, \\
\Lambda : \quad & P = \sum_{k=1}^{K} C_k - p_0, \\
& \text{CVaR}_\delta[A(\sum_{k=1}^{K} p_k + p_0)] \leq \overline{b}, \\
& \text{CVaR}_\delta[-A(\sum_{k=1}^{K} p_k + p_0)] \leq -b, \quad \text{for all } p_k \in [-C_k, \overline{C}_k], \\
& \text{for } k = 1, \ldots, K.
\end{align*}
\]

...robust CVaR Constraints
The Data-Driven Counterpart
...with $S$ scenarios of DSO customers’ demands

maximize $\sum_{k=1}^{K} \varphi_k (C_k, \overline{C}_k) - \frac{1}{S} \sum_{s=1}^{S} J(\overline{P}[s], \overline{P}[s]),$

\begin{align*}
\overline{\lambda}[s] : & \quad \overline{P}[s] = \sum_{k=1}^{K} \overline{C}_k + p_0[s], \\
\lambda[s] : & \quad P[s] = \sum_{k=1}^{K} C_k - p_0[s], \\
\overline{\beta}[s] : & \quad A_+ \overline{P}[s] + A_- P[s] - \overline{b} - \overline{t} \leq \overline{\gamma}[s], \\
\beta[s] : & \quad A_- P[s] + A_+ P[s] + b - t \leq \gamma[s], \\
\overline{\alpha}[s] : & \quad 0 \leq \overline{\gamma}[s], \\
\mu : & \quad \overline{t} + \frac{1}{1 - \delta} \frac{1}{S} \sum_{s=1}^{S} \overline{\gamma}[s] \leq 0, \\
\alpha[s] : & \quad 0 \leq \gamma[s], \\
\mu : & \quad t + \frac{1}{1 - \delta} \frac{1}{S} \sum_{s=1}^{S} \gamma[s] \leq 0.
\end{align*}

20% gain in surplus by tolerating less than 1% constraint violation risk
### Retail Market Design

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#### Design supply offers/demand bids

A Scalar-Parameterized Mechanism For Two-Sided Markets


#### Design and analyze price formation

Pricing Economic Dispatch with AC Power Flow via Local Multipliers and Conic Relaxation

M. Ndrio, A. Winnicki, and S. Bose, IEEE Transactions on Control of Network Systems (Accepted, 2022)
Optimal “Dispatch” of DERs

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<tr>
<td>Capacity</td>
<td>Access</td>
<td>Market</td>
<td>Control</td>
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...perhaps a fully distributed solution architecture will be too slow

Distributed Dual Subgradient Methods with Averaging and Applications to Grid Optimization
Journal of Optimization Theory and Applications (Accepted, 2024)
Distribution Network Security with Distributed Energy Resource Aggregators

Subhonmesh Bose
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http://bozes.ece.Illinois.edu

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The Hosting Capacity Problem

Auctioning Network Access Allocation

Retail Market Design

Optimal “Dispatch” of DERs

Thank you!