Risk-Sensitive Market Design for Electric Power Systems

Subhonmesh Bose

Electrical and Computer Engineering

University of Illinois at Urbana-Champaign

Collaborators: Avinash N. Madavan (UIUC), Mariola Ndrio (UIUC), Ye Guo (Tsinghua-Berkeley-Shenzen), Lang Tong (Cornell)

The economic dispatch (ED) problem

Minimize power procurement costs from controllable grid assets within their capabilities to meet the needs of those that are uncontrollable, respecting the engineering constraints of the underlying power network.

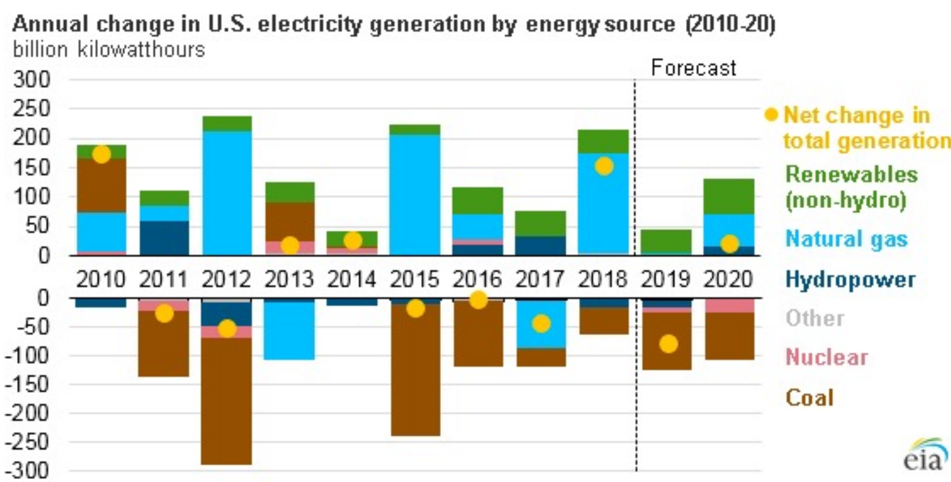
$$\underset{\mathbf{g}}{\text{minimize}} \quad \mathbf{c}^T \mathbf{g},$$
$$\text{subject to} \quad \mathbf{g} \in \mathbb{G}, \; \mathbf{g} - \mathbf{d} \in \mathbb{P}.$$

Discrete Uncertainty: A collection of "what if" component failure scenarios to guard against

Hines '09

Year	MW Lost	People Affected
1984	7,901	3,159,559
1993	7,130	2,142,000
1996	12,500	7,500,000
2003	57,669	15,300,850

Continuous Uncertainty: Account for large forecast errors in wind and solar power output.



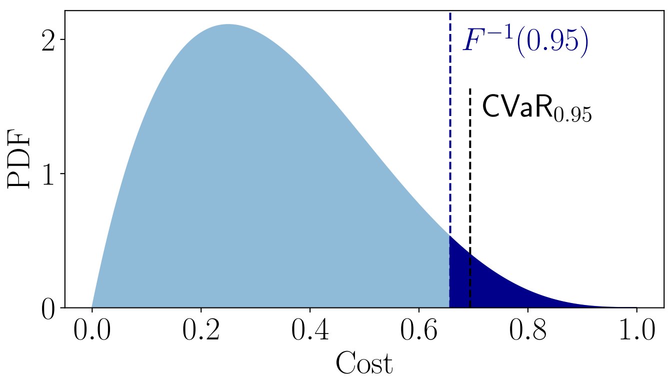
Assume linear network constraints

$$\mathbb{P} := \{\mathbf{x} : \mathbf{H}\mathbf{x} \leq \mathbf{f}, \; \mathbb{1}^T \mathbf{x} = 0\}$$

…sidesteps difficulties due to nonconvexity of power flow equations

Account for Uncertainty via Conditional Value at Risk (CVaR)

For losses with continuous distributions, conditional value at risk with parameter α measures the expected tail loss in the worst $(1-\alpha)$ fraction of the outcomes



$$\mathrm{CVaR}_\alpha(\chi) := \mathbb{E}[\chi \mid \chi \geq F^{-1}(\alpha)]$$

α = 0 ≡ expected value
$\alpha \uparrow 1$ ≡ worst-case (ess-sup)

For losses with general distributions, CVaR is characterized by

$$\mathrm{CVaR}_\alpha(\chi) := \min_z \left\{ z + \frac{1}{1-\alpha} \mathbb{E}\left[\chi - z\right]^+ \right\}.$$

Rockafellar and Uryasev 2000, 2002

Approaches to solve CVaR-sensitive optimization

- Reformulate into a deterministic optimization problem
- Stochastic approximation with data
- Sample average approximation with data

Handling Discrete Uncertainty due to Security Constraints

Add preventive security (P-SCED)

$$\begin{aligned} \underset{\mathbf{g}}{\text{minimize}} \quad & \mathbf{c}^T \mathbf{g}, \\ \text{subject to} \quad & \mathbf{g} \in \mathbb{G},\ \mathbf{g} - \mathbf{d} \in \mathbb{P},\ \mathbf{g} - \mathbf{d} \in \mathbb{P}^k, \\ & \text{for each } k = 1, \ldots, K. \end{aligned}$$

Alsac and Stott 1974,
Capitanescu et al. 2007

…does not model corrective actions or dynamic line rating

Model corrective actions (C-SCED)

$$\begin{aligned} \underset{\mathbf{g}, \delta\mathbf{g}}{\text{minimize}} \quad & \mathbf{c}^T \mathbf{g}, \\ \text{subject to} \quad & \mathbf{g} \in \mathbb{G},\ \mathbf{g} - \mathbf{d} \in \mathbb{P},\ \mathbf{g} - \mathbf{d} \in \mathbb{P}^k_{\mathsf{DA}}, \\ & \mathbf{g} + \delta\mathbf{g}^k \in \mathbb{G},\ |\delta\mathbf{g}^k| \le \mathbf{\Delta}_g, \\ & \mathbf{g} + \delta\mathbf{g}^{\mathbf{k}} - \mathbf{d} \in \mathbb{P}^k_{\mathsf{SE}}, \\ & \text{for each } k = 1, \ldots, K. \end{aligned}$$

Monticelli, Pereira, and Granville 1987,
Capitanescu and Wehenkel 2007,
Liu, Ferris and Zhao 2015

…does not model load shedding and recourse costs (some formulations do)

Risk-Sensitive SCED Formulation

$$\underset{\boldsymbol{g},\underline{\boldsymbol{r}},\overline{\boldsymbol{r}},\boldsymbol{\delta g},\boldsymbol{\delta d}}{\text{minimize}} \quad \boldsymbol{c}^\top \boldsymbol{g} + \underline{\boldsymbol{c}}_r^\top \underline{\boldsymbol{r}} + \overline{\boldsymbol{c}}_r^\top \overline{\boldsymbol{r}} + \text{CVaR}_\alpha \left[\mathcal{C}(\boldsymbol{\delta d})\right],$$

$$\text{subject to} \quad \boldsymbol{g} \in \mathbb{G},$$

$$\boldsymbol{\lambda} \;\ldots\ldots\; \mathbb{1}^\top (\boldsymbol{g} - \boldsymbol{d}) = 0,$$

$$\boldsymbol{\mu} \;\ldots\ldots\; \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d}) \leq \boldsymbol{f},$$

$$\boldsymbol{\mu}_k^{\text{DA}} \;\ldots\ldots\; \boldsymbol{H}_k (\boldsymbol{g} - \boldsymbol{d}) \leq \boldsymbol{f}_k^{\text{DA}},$$

$$\boldsymbol{g} + \boldsymbol{\delta g}_k \in \mathbb{G}, \; \mathbb{1}^\top (\boldsymbol{\delta g}_k + \boldsymbol{\delta d}_k) = 0,$$

$$\boldsymbol{\mu}_k^{\text{SE}} \;\ldots\ldots\; \boldsymbol{H}_k (\boldsymbol{g} + \boldsymbol{\delta g}_k - \boldsymbol{d} + \boldsymbol{\delta d}_k) \leq \boldsymbol{f}_k^{\text{SE}}.$$

$$\overline{\boldsymbol{\rho}}_k, \underline{\boldsymbol{\rho}}_k \;\ldots\; -\underline{\boldsymbol{r}} \leq \boldsymbol{\delta g}_k \leq \overline{\boldsymbol{r}},$$

$$0 \leq \overline{\boldsymbol{r}}, \underline{\boldsymbol{r}} \leq \boldsymbol{R},$$

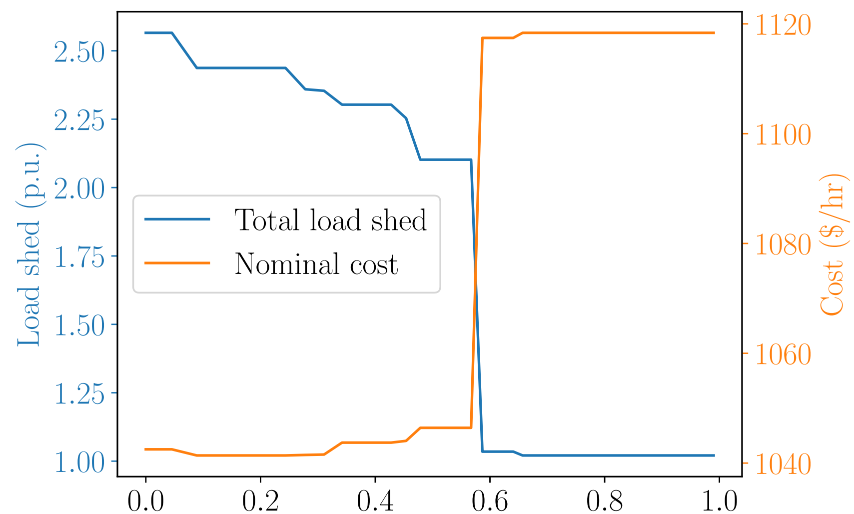
$$0 \leq \boldsymbol{\delta d}_k \leq \boldsymbol{\Delta},$$

$$\text{for } k = 1, \ldots, K.$$

Load shed penalized through nodal values of lost load

α is a tunable parameter for the system operator to explore the tradeoff between power procurement costs and reliability of power delivery.

On an IEEE 24-bus test system



Risk neutral α Risk averse

Risk-Sensitive Security-Constrained Economic Dispatch via Critical Region Exploration,
A. N. Madavan, S. Bose, Y. Guo, and L. Tong.
Presented at the IEEE PES General Meeting 2019.

Pricing Risk-Sensitive SCED

Candidate locational marginal prices:

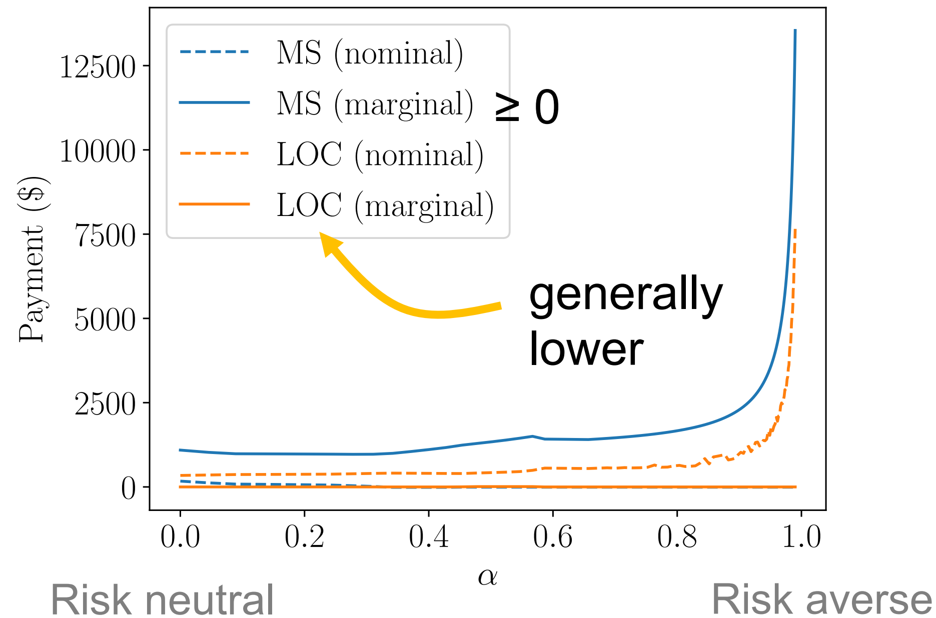
$$\pi_{\text{nom}} = \lambda \mathbb{1} - \boldsymbol{H}^\top \boldsymbol{\mu}$$ *……………………………based on the nominal dispatch alone*

$$\pi_{\text{mar}} = \lambda \mathbb{1} - \boldsymbol{H}^\top \boldsymbol{\mu} - \sum_{k=1}^{K} \boldsymbol{H}_k^\top \left(\boldsymbol{\mu}^{\text{DA}} + \boldsymbol{\mu}^{\text{SE}}\right)$$ *………marginal sensitivity to demand*

Payments for reserve: $\sum_{k=1}^{K} \overline{\boldsymbol{\rho}}_k^\top \overline{\boldsymbol{r}} + \sum_{k=1}^{K} \underline{\boldsymbol{\rho}}_k^\top \underline{\boldsymbol{r}}$

Revenue Adequacy: If the rents payable to the generators can be covered by the rents collected from demand.

Lost Opportunity Costs: Profit with dispatch and price prescribed by the system operator and the maximum profit attainable with the price offered.



The Scalability Challenge

$$
\begin{aligned}
\underset{\boldsymbol{g},\underline{\boldsymbol{r}},\overline{\boldsymbol{r}},\boldsymbol{\delta g},\boldsymbol{\delta d}}{\text{minimize}} \quad & \boldsymbol{c}^\top \boldsymbol{g} + \underline{\boldsymbol{c}}_r^\top \underline{\boldsymbol{r}} + \overline{\boldsymbol{c}}_r^\top \overline{\boldsymbol{r}} + \mathrm{CVaR}_\alpha \left[\mathcal{C}(\boldsymbol{\delta d})\right], \\
\text{subject to} \quad & \boldsymbol{g} \in \mathbb{G}, \\
& \mathbb{1}^\top (\boldsymbol{g} - \boldsymbol{d}) = 0, \\
& \boldsymbol{H} (\boldsymbol{g} - \boldsymbol{d}) \leq \boldsymbol{f}, \\
& \boldsymbol{H}_k (\boldsymbol{g} - \boldsymbol{d}) \leq \boldsymbol{f}_k^{\mathsf{DA}}, \\
& \boldsymbol{g} + \boldsymbol{\delta g}_k \in \mathbb{G}, \; \mathbb{1}^\top (\boldsymbol{\delta g}_k + \boldsymbol{\delta d}_k) = 0, \\
& \boldsymbol{H}_k (\boldsymbol{g} + \boldsymbol{\delta g}_k - \boldsymbol{d} + \boldsymbol{\delta d}_k) \leq \boldsymbol{f}_k^{\mathsf{SE}}, \\
& -\underline{\boldsymbol{r}} \leq \boldsymbol{\delta g}_k \leq \overline{\boldsymbol{r}}, \\
& 0 \leq \overline{\boldsymbol{r}}, \underline{\boldsymbol{r}} \leq \boldsymbol{R}, \\
& 0 \leq \boldsymbol{\delta d}_k \leq \boldsymbol{\Delta}, \\
& \text{for } k = 1, \ldots, K.
\end{aligned}
$$

Rewrite it as a linear program

$$
\begin{aligned}
\underset{\mathbf{x}^0,\mathbf{x}^1,\ldots,\mathbf{x}^K}{\text{minimize}} \quad & \left[\mathbf{c}^0\right]^T \mathbf{x}^0 + \alpha' \sum_{k=1}^{K} \left[\mathbf{c}^k\right]^T \mathbf{x}^k, \\
\text{subject to} \quad & \mathbf{A}^0 \mathbf{x}^0 \leq \mathbf{b}^0, \\
& \mathbf{A}^k \mathbf{x}^0 + \mathbf{E}^k \mathbf{x}^k \leq \mathbf{b}^k, \\
& \text{for each } k = 1, \ldots, K.
\end{aligned}
$$

"Resulting linear program (LP) is not solvable by traditional LP methods due to its large size." Liu, Ferris and Zhao 2015

On a 2746-bus Polish network

Formulation	Variables	Constraints
ED	520	7,599
C-SCED	1,706,120	49,827,526
Risk-SCED	10,713,534	67,842,352

- Pre-filter contingencies
- Parallelize computation

…exploit the structure of the SCED problem

Problem Structure of Risk-Sensitive SCED

$$\underset{\mathbf{x}^0}{\text{minimize}} \quad \left[\mathbf{c}^0\right]^T \mathbf{x}^0 + \alpha' \sum_{k=1}^{K} J_*^k(\mathbf{x}^0),$$
$$\text{subject to} \quad \underbrace{\mathbf{A}^0\mathbf{x}^0 \leq \mathbf{b}^0,}_{\mathbb{X}^0}$$

$$\underset{\mathbf{x}^0,\mathbf{x}^1,\ldots,\mathbf{x}^K}{\text{minimize}} \quad \left[\mathbf{c}^0\right]^T \mathbf{x}^0 + \alpha' \sum_{k=1}^{K} \left[\mathbf{c}^k\right]^T \mathbf{x}^k,$$
$$\text{subject to} \quad \mathbf{A}^0\mathbf{x}^0 \leq \mathbf{b}^0,$$
$$\mathbf{A}^k\mathbf{x}^0 + \mathbf{E}^k\mathbf{x}^k \leq \mathbf{b}^k,$$
$$\text{for each } k = 1, \ldots, K.$$

$$J_*^k(\mathbf{x}^0) := \underset{\mathbf{x}^k}{\text{minimize}} \quad \left[\mathbf{c}^k\right]^T \mathbf{x}^k,$$
$$\text{subject to} \quad \mathbf{A}^k\mathbf{x}^0 + \mathbf{E}^k\mathbf{x}^k \leq \mathbf{b}^k.$$

Multi-parametric linear program, linearly parameterized in $\mathbf{x}^0$

Borrelli 2011

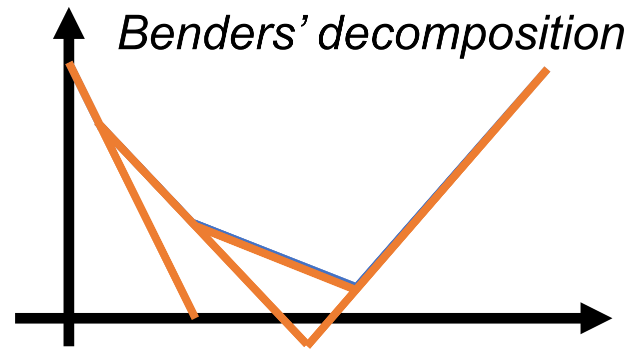
This parametric optimal cost is piecewise affine and convex in $\mathbf{x}_0$. Moreover, the pieces over which the cost is affine defines a polyhedral partition of $\mathbb{X}^0$.

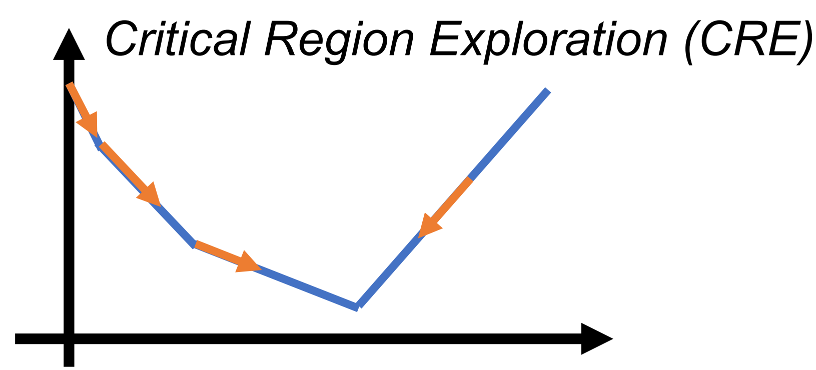
…critical region

Algorithms for Solving Risk-Sensitive SCED

Table of Run-Times (in s)

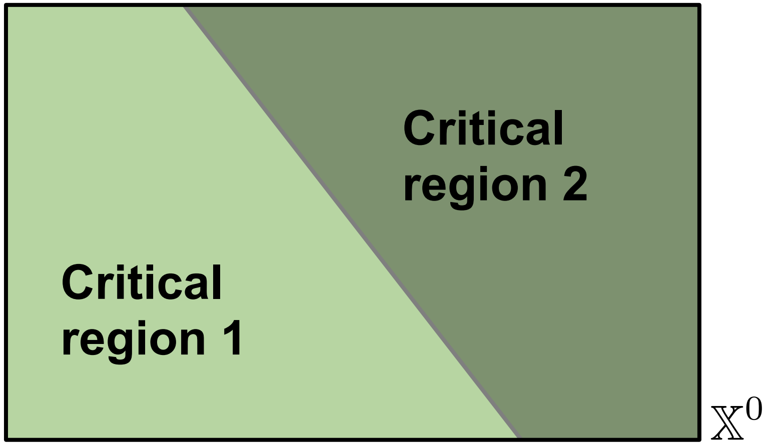
Case Name	$\alpha = 0$ Gurobi	$\alpha = 0$ Benders'	$\alpha = 0.9$ Gurobi	$\alpha = 0.9$ Benders'
case3_lmbd	0.0016	0.0092	0.0006	0.0010
case3_lmbd__api	0.0007	0.0008	0.0005	0.0016
case3_lmbd__sad	0.0009	0.0013	0.0007	0.0010
case5_pjm	0.0023	0.0024	0.0021	0.0024
case5_pjm__api	0.0021	0.0022	0.0022	0.0029
case5_pjm__sad	0.0022	0.0034	0.0023	0.0020
case14_ieee	0.0078	0.0122	0.0078	0.0083
case14_ieee__api	0.0080	0.0091	0.0085	0.0087
case14_ieee__sad	0.0073	0.0082	0.0073	0.0081
case24_ieee_rts	0.2078	0.0473	0.1935	0.0445
case24_ieee_rts__api	0.2527	0.0990	0.2445	0.0986
case24_ieee_rts__sad	0.1931	0.0456	0.1909	0.0484
case30_as	0.0755	0.0407	0.0742	0.0422
case30_ieee	0.0465	0.0381	0.0526	0.0393
case30_as__api	0.0913	0.0641	0.0894	0.7291
case30_ieee__api	0.0489	0.0442	0.0487	0.0443
case30_as__sad	0.0697	0.0387	0.0687	0.0358
case30_ieee__sad	0.0407	0.0399	0.0419	0.0400
case39_epri	0.1436	0.0658	0.1341	0.0611
case39_epri__api	0.1586	0.0806	0.1586	0.0870
case39_epri__sad	0.1263	0.0607	0.1266	0.0619
case57_ieee	0.3670	0.2721	0.3643	0.2631
case57_ieee__api	0.3904	0.2048	0.3827	0.2124
case57_ieee__sad	0.3700	0.2610	0.3753	0.2609
case73_ieee_rts	6.3827	1.8741	7.2492	1.2606
case73_ieee_rts__api	10.4875	2.0926	13.5974	2.4620
case73_ieee_rts__sad	6.8249	1.2365	6.5114	1.2421
case118_ieee	17.0832	4.7362	15.7959	4.8882
case118_ieee__api	17.0718	5.0415	17.2200	4.9935
case118_ieee__sad	14.7307	3.7659	14.7360	3.7275
case200_activ	17.4605	4.7793	18.4642	4.5208
case200_activ__api	28.8419	9.7479	28.5164	9.3644
case200_activ__sad	17.1610	4.7703	16.7447	4.3801





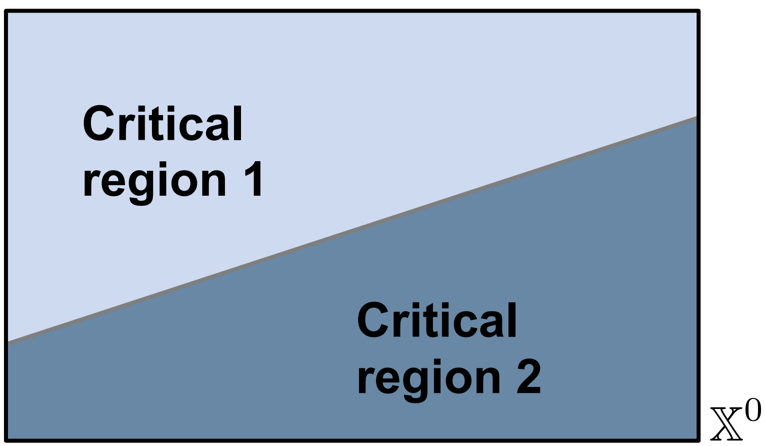
The CRE Algorithm

Optimize over a critical region, check for optimality, systematically move to a neighboring critical region, and keep exploring till optimality can be certified.



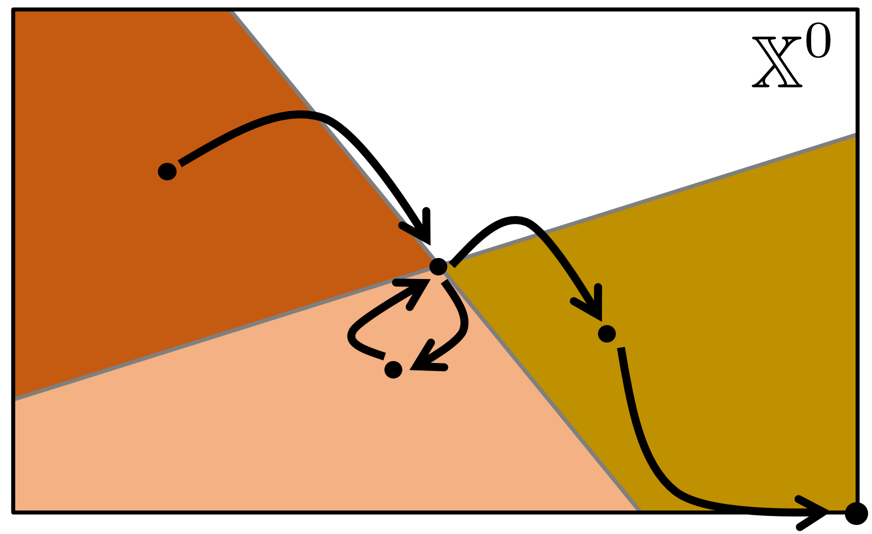
Critical regions induced by J_*^1

$\cap$



Critical regions induced by J_*^2

$=$

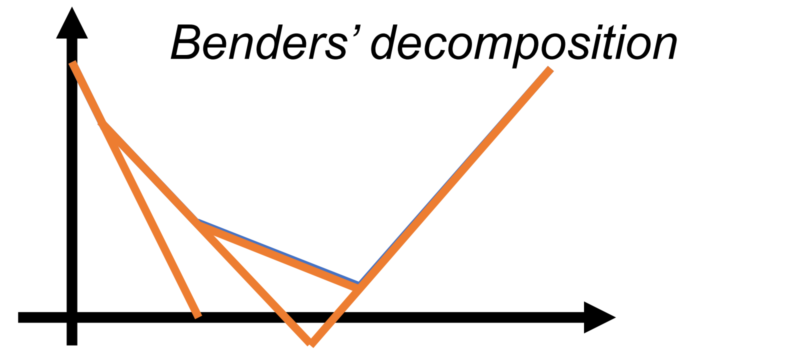


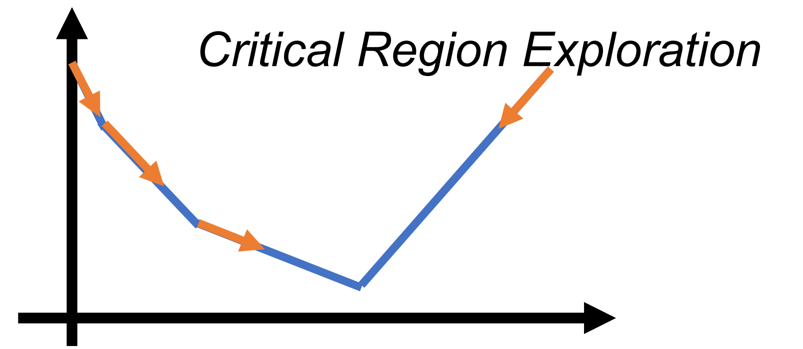
Critical regions induced by the aggregate cost.

Theorem: The CRE algorithm converges to an optimal solution of the risk-sensitive SCED problem in finitely many iterations.

Algorithm 1 Solving R-SCED via CRE.

1: **Initialize:**
   $\mathbf{x}^0 \in \mathbb{X}^0, J^* \leftarrow \infty, \mathbb{D} \leftarrow$ empty set, $\epsilon \leftarrow$ small positive number
2: **do**
3: Given $\mathbf{x}^0$, compute $\boldsymbol{\rho}^k, \eta^k, \mathbb{C}^k$ for $k = 1, \dots, K$.
4: Minimize $\left[\mathbf{c}^0\right]^\intercal \mathbf{x}^0 + \alpha' \sum_{k=1}^{K} \{\left[\boldsymbol{\rho}^k\right]^\intercal \mathbf{x}^0 + \eta^k\}$ over $\cap_{k=1}^{K} \mathbb{C}^k(\mathbf{x}^0)$.
5: $\left[\mathbf{x}^0\right]^{\text{opt}} \leftarrow$ lexicographically smallest minimizer of step 4.
6: $J^{\text{opt}} \leftarrow$ optimal cost of step 4.
7: **if** $J^{\text{opt}} < J^*$ **then**
8: $\left[\mathbf{x}^0\right]^* \leftarrow \left[\mathbf{x}^0\right]^{\text{opt}}, J^* \leftarrow J^{\text{opt}}, \mathbb{D} \leftarrow \{\mathbf{c}\}$.
9: **else**
10: $\mathbb{D} \leftarrow \mathbb{D} \cup \{\mathbf{c}^0 + \alpha' \sum_{k=1}^{K} \boldsymbol{\rho}^k\}$.
11: **end if**
12: $\mathbf{v}^* \leftarrow \text{argmin}_{\mathbf{v} \in \text{conv}(\mathbb{D}) + N_{\mathbb{X}^0}\left(\left[\mathbf{x}^0\right]^*\right)} \|\mathbf{v}\|^2$.
13: $\mathbf{x}^0 \leftarrow \left[\mathbf{x}^0\right]^{\text{opt}} - \epsilon \mathbf{v}^*$.
14: **while** $v^* \neq 0$





On Robust Tie-line Scheduling in Multi-Area Power Systems, Y. Guo, S. Bose, and L. Tong. IEEE Transactions on Power Systems, vol. 33, no. 4, pp. 4144-4154, 2018.

Risk-Sensitive Economic Dispatch with Uncertain Wind $\omega \in \Omega$

$$\underset{\mathbf{g}}{\text{minimize}} \quad \mathbf{c}^T \mathbf{g},$$
$$\text{subject to} \quad \mathbf{g} \in \mathbb{G}, \;\; \mathbf{g} - \mathbf{d} \in \mathbb{P}.$$

Regulation via affine feedback $\mathbf{g} - \mathbf{G}\underbrace{(\omega - \mathbb{E}[\omega])}_{\Delta\omega}$

Net demand (without wind spillage) $\mathbf{d} - \omega$

A box set $[\underline{\mathbf{G}}, \overline{\mathbf{G}}]$

A CVaR-penalized CVaR-constrained formulation

$$\begin{aligned}
\underset{\mathbf{g},\mathbf{G}}{\text{minimize}} \quad & \text{CVaR}_{\alpha}\left[\mathbf{c}^T\left(\mathbf{g} - \mathbf{G}\Delta\omega\right)\right], \\
\text{subject to} \quad & \mathbb{1}^T(\mathbf{g} - \mathbf{G}\Delta\omega - \mathbf{d} + \omega) = 0 \text{ a.s.}, \\
& \text{CVaR}_{\beta^{\mathsf{F}}}\left[\mathbf{H}\left(\mathbf{g} - \mathbf{G}\Delta\omega - \mathbf{d} + \omega\right)\right] \le \mathbf{f}, \\
& \text{CVaR}_{\beta^{\mathsf{G}}}\left[\mathbf{g} - \mathbf{G}\Delta\omega\right] \le \overline{\mathbf{G}}, \\
& \text{CVaR}_{\beta^{\mathsf{G}}}\left[-\mathbf{g} + \mathbf{G}\Delta\omega\right] \le -\underline{\mathbf{G}}.
\end{aligned}$$

Summers et al. 2015

Risk-Sensitive Energy Procurement with Uncertain Wind, A. N. Madavan, and S. Bose. Proceedings of the IEEE GlobalSIP 2019.

CVaR-Aware Optimization $\mathcal{P}_{\mathrm{CVaR}}$

$$\mathcal{P}_{\mathrm{CVaR}}: \begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \mathsf{CVaR}_{\alpha}\left[F_{\omega}(\mathbf{x})\right], \\ \text{subject to} & \mathsf{CVaR}_{\beta^i}\left[G^i_{\omega}(\mathbf{x})\right] \leq 0, \; i = 1, \ldots, I. \end{array}$$

α expresses the modeler's tolerance to large uncertain costs

$\boldsymbol{\beta} = (\beta^1, \ldots, \beta^I)$ expresses the modeler's tolerance to the extent of constraint violation

…varies between average to robust and encompasses a wide variety of optimization problems under uncertainty

Relation to chance constraints

$$\mathsf{CVaR}_{1-\varepsilon}[G^i_{\omega}(\mathbf{x})] \leq 0 \implies \mathbf{P}\{G^i_{\omega}(\mathbf{x}) \leq 0\} \geq 1 - \varepsilon$$

…CVaR-based constraint enforcement defines a convex inner approximation of a chance constraint

Primal-Dual Stochastic Approximation to Solve $\mathcal{P}_{\mathbb{E}} = \mathcal{P}_{\mathrm{CVaR}}(\alpha = 0, \boldsymbol{\beta} = 0)$

$$
\mathcal{P}_{\mathbb{E}} : \underset{\mathbf{x}}{\text{minimize}} \quad \mathbb{E}\left[F_\omega(\mathbf{x})\right], \qquad \text{subject to} \quad \mathbb{E}\left[G^i_\omega(\mathbf{x})\right] \le 0, \ i = 1, \ldots, I.
$$

Initialization: Choose $\boldsymbol{x}_1 \in \mathbb{X}$, $\boldsymbol{z}_1 = 0$, and a positive sequence $\boldsymbol{\gamma}$.

for $k \geq 1$ **do**

Sample $\omega_k \in \Omega$. Update $\boldsymbol{x}$ as

$$
\boldsymbol{x}_{k+1} \leftarrow \underset{\boldsymbol{x} \in \mathbb{X}}{\operatorname{argmin}} \left\langle \nabla F_{\omega_k}(\boldsymbol{x}_k) + \sum_{i=1}^{m} z^i_k \nabla G^i_{\omega_k}(\boldsymbol{x}_k), \boldsymbol{x} - \boldsymbol{x}_k \right\rangle + \frac{1}{2\gamma_k} \left\| \boldsymbol{x} - \boldsymbol{x}_k \right\|^2 . \tag{1}
$$

Sample $\omega_{k+1/2} \in \Omega$. For each $i = 1, \ldots, m$, update z^i as

$$
z^i_{k+1} \leftarrow \underset{z^i \in \mathbb{R}_+}{\operatorname{argmax}} \left\langle G^i_{\omega_{k+1/2}}(\boldsymbol{x}_{k+1}), z^i - z^i_k \right\rangle - \frac{1}{2\gamma_k} \| z^i - z^i_k \|^2. \tag{2}
$$

…sample and update the iterates

...generalize to the CVaR-aware problem using the variational characterization of CVaR

Theorem. Convergence Result for $\mathcal{P}_{\mathrm{CVaR}}$

The iterates generated by the stochastic primal-dual algorithm on the reformulation of $\mathcal{P}_{\mathrm{CVaR}}$ using the variational characterization of CVaR with parameters $\alpha, \boldsymbol{\beta}$ satisfy

$$\mathbb{E}[\mathrm{CVaR}_\alpha(f_\omega(\bar{\boldsymbol{x}}_{K+1}))] - p_\star \leq \frac{\eta(\alpha, \boldsymbol{\beta})}{\sqrt{K}},$$

$$\mathbb{E}[\mathrm{CVaR}_{\beta^i}(g^i_\omega(\bar{\boldsymbol{x}}_{K+1}))] \leq \frac{\eta(\alpha, \boldsymbol{\beta})}{\sqrt{K}}$$

…$1/\sqrt{K}$ rate for CVaR-constrained CVaR-penalized optimization

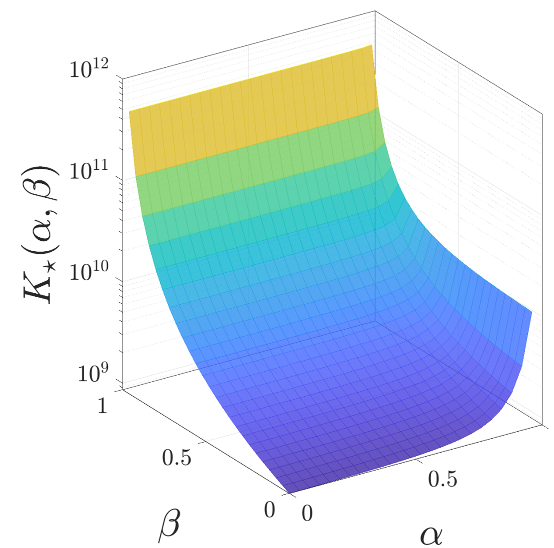
for $i = 1, \ldots, m$ with step sizes $\gamma_k = \gamma/\sqrt{K}$ for $k = 1, \ldots, K$ with $0 < \gamma < P_3^{-1/2}(\alpha, \boldsymbol{\beta})$, where $\eta(\alpha, \boldsymbol{\beta}) = \frac{P_1 + \gamma^2 P_2(\alpha, \boldsymbol{\beta})}{4\gamma(1 - \gamma^2 P_3(\alpha, \boldsymbol{\beta}))}$ and

$$P_2(\alpha, \boldsymbol{\beta}) = \frac{16(C_F^2 + 1)}{(1-\alpha)^2} + 2\left\|\mathrm{diag}(\mathbb{1} + \boldsymbol{\beta})\mathrm{diag}(\mathbb{1} - \boldsymbol{\beta})^{-1}\boldsymbol{D}_G\right\|^2,$$

$$P_3(\alpha, \boldsymbol{\beta}) = 16m\left\|\begin{pmatrix}\mathrm{diag}(\mathbb{1} - \boldsymbol{\beta})^{-1}\boldsymbol{C}_G \\ \mathrm{diag}(\mathbb{1} - \boldsymbol{\beta})^{-1}\mathbb{1}\end{pmatrix}\right\|^2.$$

P_1 …depends on initial conditions

…computational cost of the preference for robustness



A Stochastic Primal-Dual Method for Optimization with Conditional Value at Risk Constraints. A. N. Madavan and S. Bose. Journal of Optimization Theory and Applications, vol. 190, pp. 428-460, 2021.

Pricing CVaR-Based Economic Dispatch

$$
\begin{aligned}
\underset{\mathbf{g},\mathbf{G}}{\text{minimize}} \quad & \mathrm{CVaR}_\alpha \left[\mathbf{c}^T\left(\mathbf{g} - \mathbf{G}\Delta\omega\right)\right], \\
\text{subject to} \quad & \mathbb{1}^T(\mathbf{g} - \mathbf{G}\Delta\omega - \mathbf{d} + \omega) = 0 \text{ a.s.}, \\
& \mathrm{CVaR}_{\beta^{\mathsf{F}}}\left[\mathbf{H}\left(\mathbf{g} - \mathbf{G}\Delta\omega - \mathbf{d} + \omega\right)\right] \le \mathbf{f}, \\
& \mathrm{CVaR}_{\beta^{\mathsf{G}}}\left[\mathbf{g} - \mathbf{G}\Delta\omega\right] \le \overline{\mathbf{G}}, \\
& \mathrm{CVaR}_{\beta^{\mathsf{G}}}\left[-\mathbf{g} + \mathbf{G}\Delta\omega\right] \le -\underline{\mathbf{G}}.
\end{aligned}
$$

…expected objective $\alpha = 0$

…nominal dispatch clears with expected wind $\mathbf{G}^T\mathbb{1} = \mathbb{1}$

... using the variational characterization of CVaR

$$
\begin{aligned}
\underset{\substack{\mathbf{g_0},\mathbf{G},\mathbf{u}\\ \underline{\mathbf{v}},\overline{\mathbf{v}}}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{g}, \\
\text{subject to} \quad & \mathbb{1}^T\left(\mathbf{g} + \mathbb{E}[\omega] - \mathbf{d}\right) = 0, && \lambda \\
& \mathbf{u} + \frac{1}{1-\beta^{\mathsf{F}}}\mathbb{E}\left[\left(\mathbf{H}(\mathbf{g} - \mathbf{G}\Delta\omega + \omega - \mathbf{d}) - \mathbf{f} - \mathbf{u}\right)^+\right] \le 0, && \mu \\
& \overline{\mathbf{v}} + \frac{1}{1-\beta^{\mathsf{G}}}\mathbb{E}\left[\left(\mathbf{g} - \mathbf{G}\Delta\omega - \overline{\mathbf{G}} - \overline{\mathbf{v}}\right)^+\right] \le 0, \\
& \underline{\mathbf{v}} + \frac{1}{1-\beta^{\mathsf{G}}}\mathbb{E}\left[\left(-\mathbf{g} + \mathbf{G}\Delta\omega + \underline{\mathbf{G}} - \underline{\mathbf{v}}\right)^+\right] \le 0, \\
& \mathbf{G}^T\mathbb{1} = \mathbb{1}. && \nu
\end{aligned}
$$

Lagrange multipliers

CVaR-Sensitive Prices & Payments

Vector of risk-sensitive locational marginal prices (risk-LMPs)

$$\pi = \lambda \mathbb{1} - \mathbf{H}^T \mu$$

…equals marginal sensitivity to nodal demands

…depends on congestion across scenarios through μ

…becomes uniform across the network if no congestion occurs in any scenario

…reduces to classical LMPs if the wind variance disappears

Pricing recourse actions via ν, payments by various parties at bus i are given by:

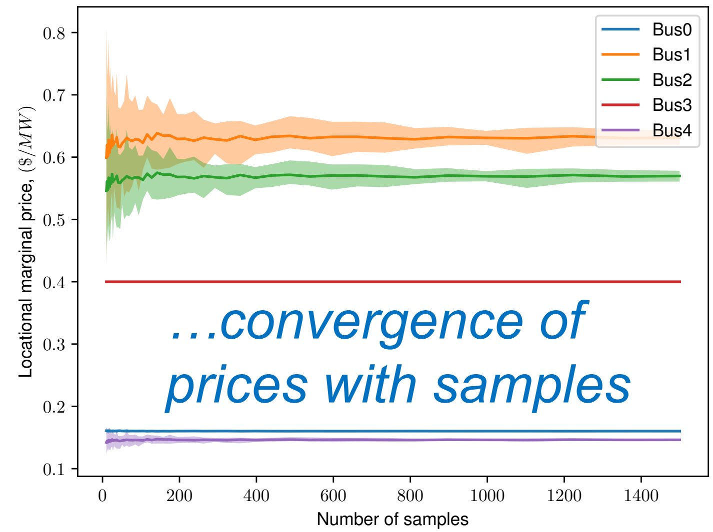
- Demander pays $\pi_i d_i$
- Wind power producer is paid $\pi_i \mathbb{E}\left[\omega_i\right] - \nu_i^\star$
- Dispatchable generator is paid $\pi_i g_i + \sum_{k=1}^{n} G_{ik}^\star \nu_k^\star$

$$\text{Merchandising surplus} \geq \mu^T \mathbf{f} + \text{risk-dependent term}$$

Computation via Sample Average Approximation

Vector of risk-sensitive locational marginal prices (risk-LMPs) from sample average approximation

$$\hat{\pi} := \hat{\lambda}_N \mathbb{1} - \mathbf{H}^T \hat{\mu}_N$$



…convergence of prices with samples

…replace expectation with empirical mean over N samples

$$
\begin{aligned}
\underset{\substack{\mathbf{g_0}, \mathbf{G}, \mathbf{u} \\ \underline{\mathbf{v}}, \overline{\mathbf{v}}}}{\text{minimize}} \quad & \mathbf{c}^T \mathbf{g}, \\
\text{subject to} \quad & \mathbb{1}^T (\mathbf{g} + \mathbb{E}[\omega] - \mathbf{d}) = 0, && \hat{\lambda}_N \\
& \mathbf{u} + \frac{1}{1-\beta^{\mathsf{F}}} \mathbb{E}\left[(\mathbf{H}(\mathbf{g} - \mathbf{G}\Delta\omega + \omega - \mathbf{d}) - \mathbf{f} - \mathbf{u})^+ \right] \leq 0, && \hat{\mu}_N \\
& \overline{\mathbf{v}} + \frac{1}{1-\beta^{\mathsf{G}}} \mathbb{E}\left[(\mathbf{g} - \mathbf{G}\Delta\omega - \overline{\mathbf{G}} - \overline{\mathbf{v}})^+ \right] \leq 0, \\
& \underline{\mathbf{v}} + \frac{1}{1-\beta^{\mathsf{G}}} \mathbb{E}\left[(-\mathbf{g} + \mathbf{G}\Delta\omega + \underline{\mathbf{G}} - \underline{\mathbf{v}})^+ \right] \leq 0, \\
& \mathbf{G}^T \mathbb{1} = \mathbb{1}. && \hat{\nu}_N
\end{aligned}
$$

Lagrange multipliers

Pricing Conditional Value at Risk-Sensitive Economic Dispatch, M. Ndrio, A. N. Madavan, and S. Bose, Presented at the IEEE PES General Meeting 2021.

Future directions

- CRE-enhanced Benders' implementation
- Speed up stochastic approx. for CVaR-sensitive dispatch
- Complete analysis of CVaR-sensitive prices

	Algorithm	Pricing
Discrete	✓	✓
Continuous	✓	✓
Disc. + Cont.	?	?
+ nonconvexity	?	?




Questions?

Website: http://boses.ece.illinois.edu/