A Review of Recent Developments in Nonlinear Optimization of Electric Power Systems

Daniel Molzahn

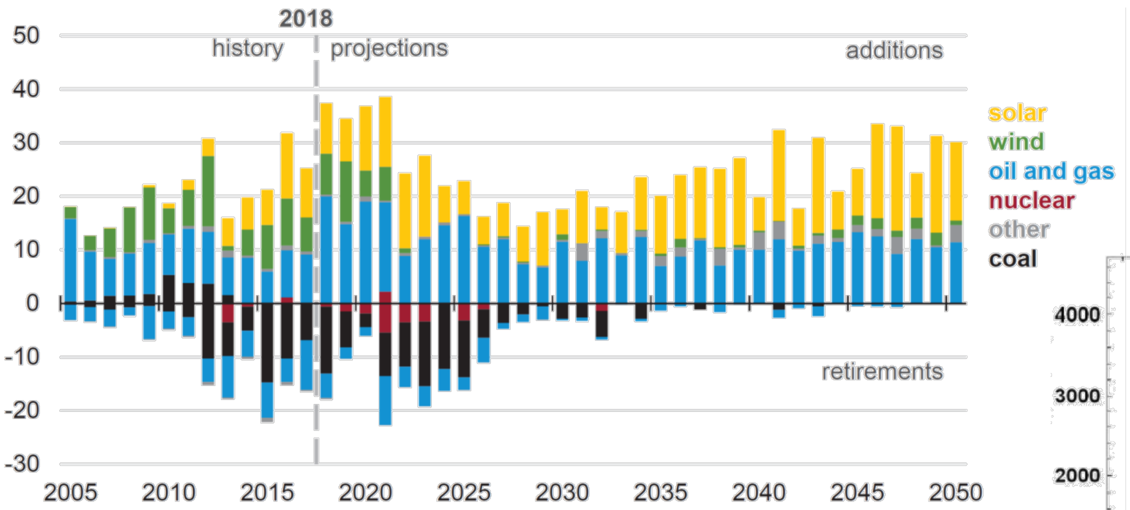
Georgia Institute of Technology

(molzahn@gatech.edu)

PSERC Webinar
March 10, 2015

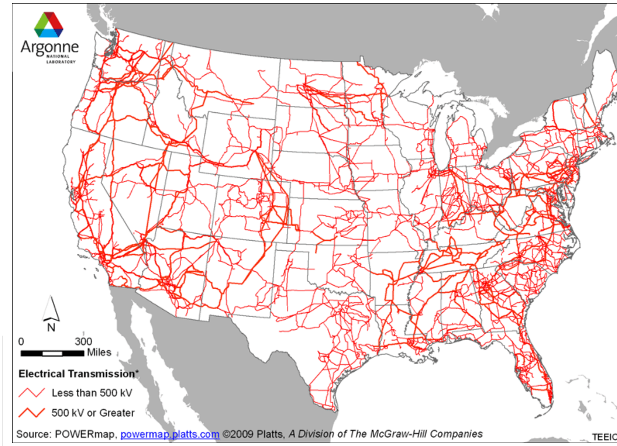
Motivation

US Annual Electricity Generating Capacity Additions and Retirements



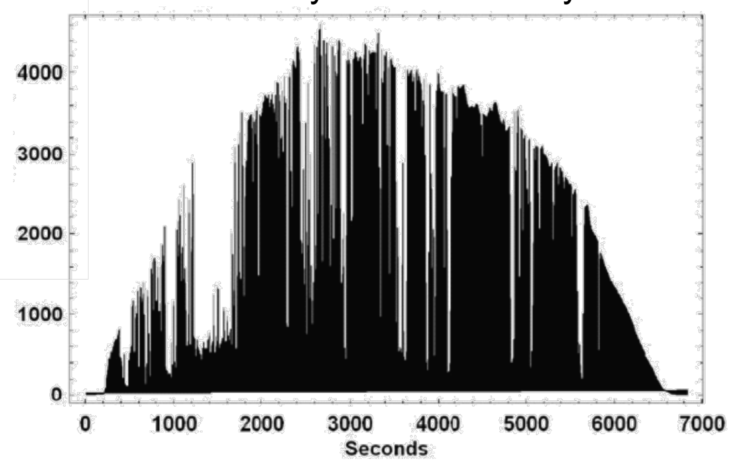
U.S. Energy Information Administration, Annual Energy Outlook 2019.

The Legacy Grid



http://teeic.anl.gov/er/transmission/restech/dist/index.cfm

Variability and Uncertainty



A Typical Day of Solar PV Generation, [Apt and Curtright '08]

New computational tools are needed to economically and reliably operate electric grids with significant renewable generation.

The Power Flow Equations

- Model the relationship between the voltages and the power injections.

AC power flow equations	Voltages: $V_i = \lvert V_i \rvert \angle \theta_i$

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right)\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right)\right)$$

- Central to many power system optimization and control problems.
  - Optimal power flow, unit commitment, voltage stability, contingency analysis, transmission switching, etc.

Optimal Power Flow

$$\min_{|V|,\theta} \quad \sum_{i \in \mathcal{G}} \left(c_{2i} {P_{Gi}}^2 + c_{1i} P_{Gi} + c_{0i} \right)$$

Generation Cost

$$\text{subject to} \qquad P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}$$

$$V_i^{\min} \leq |V_i| \leq V_i^{\max}$$

$$\theta_{ik}^{\min} \leq \theta_i - \theta_k \leq \theta_{ik}^{\max}$$

$$|S_{flow,ik}| \ \leq \ S_{flow,ik}^{\max}$$

Engineering Constraints

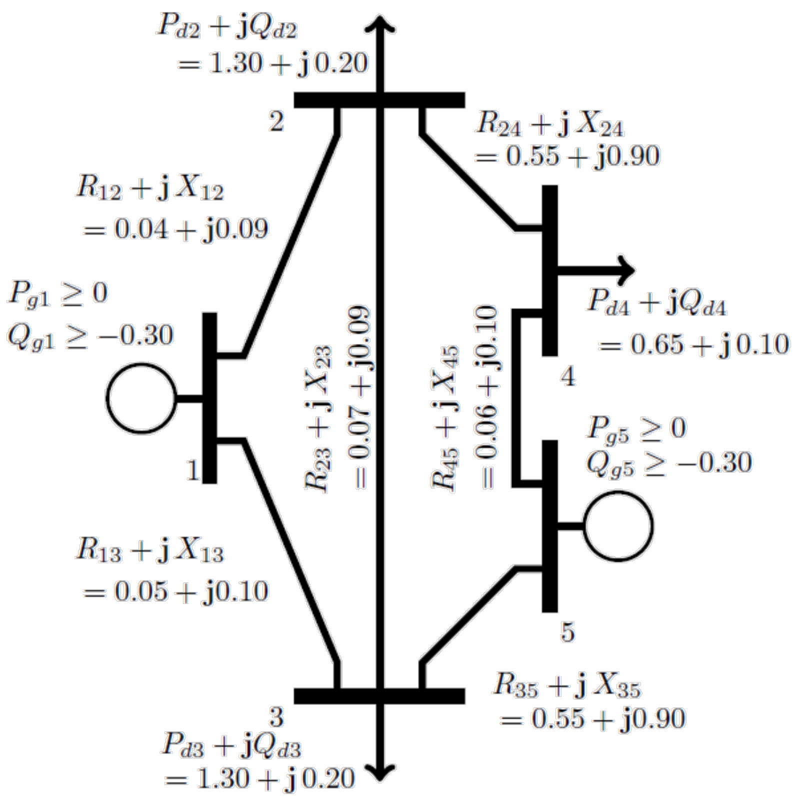
$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \ \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right) \right)$$

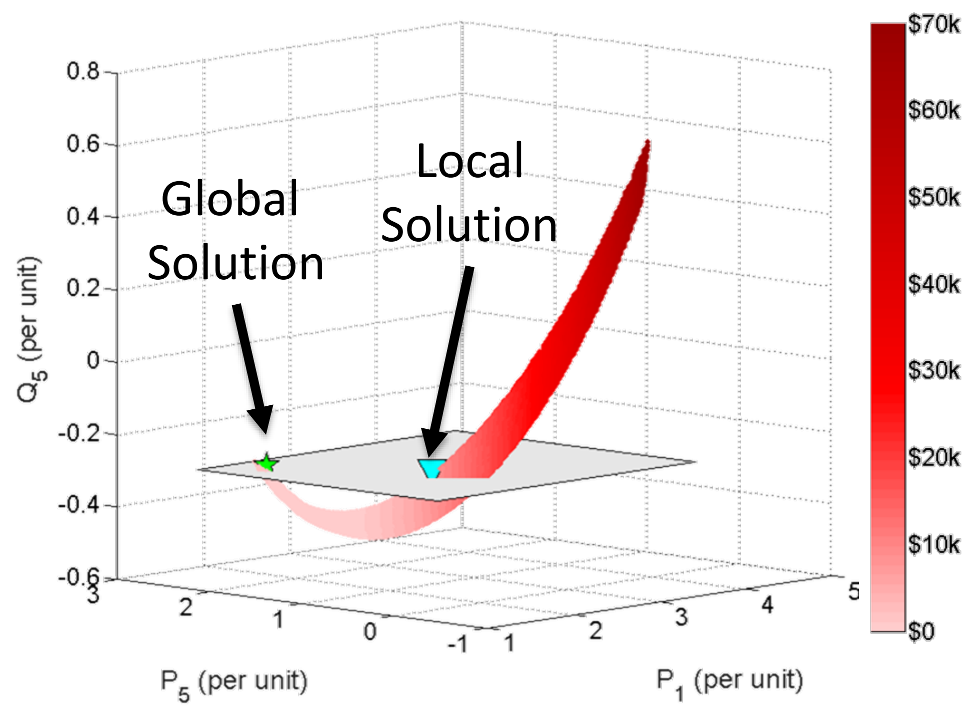
$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \ \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right) \right)$$

Power Flow Equations

A Disconnected Feasible Space

- Five-bus example problem
  [Bukhsh, Grothey, McKinnon, & Trodden '13]

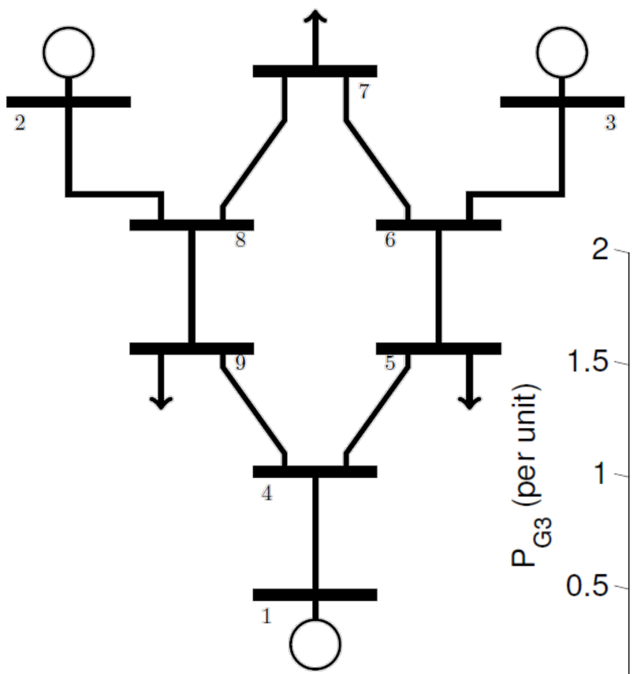


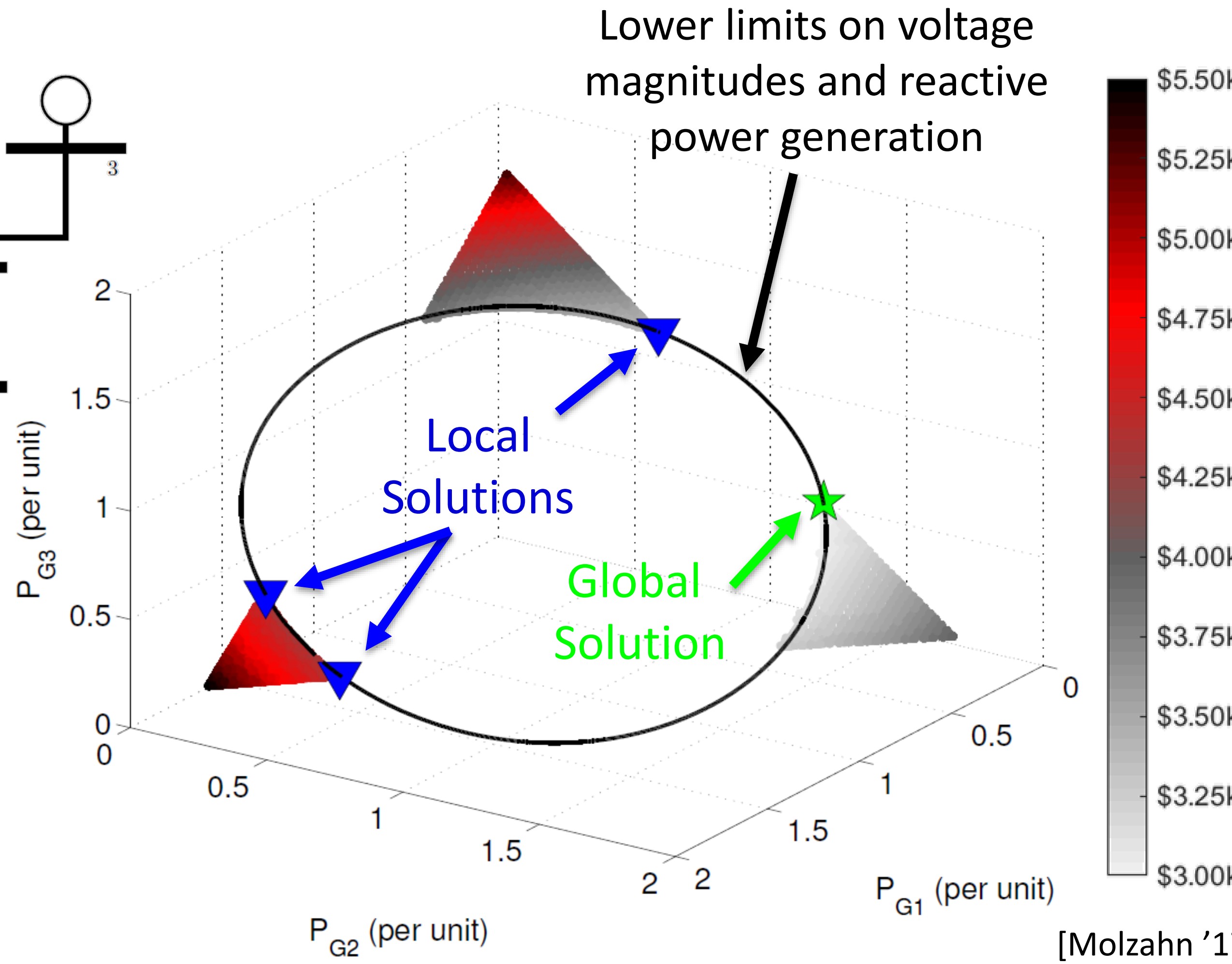


[Molzahn '17]

Another Disconnected Space

- Nine-bus example problem
  [Bukhsh, Grothey, McKinnon, & Trodden '13]





[Molzahn '17]

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \sin(\theta_i - \theta_k)\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin(\theta_i - \theta_k) - \mathbf{B}_{ik} \cos(\theta_i - \theta_k)\right)$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \sin(\theta_i - \theta_k) \right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin(\theta_i - \theta_k) - \mathbf{B}_{ik} \cos(\theta_i - \theta_k) \right)}$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = \cancelto{1}{|V_i|} \sum_{k=1}^{n} \cancelto{1}{|V_k|} \left(\mathbf{G}_{ik} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \sin(\theta_i - \theta_k) \right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin(\theta_i - \theta_k) - \mathbf{B}_{ik} \cos(\theta_i - \theta_k) \right)}$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = \overset{1}{|V_i|} \sum_{k=1}^{n} \overset{1}{|V_k|} \left(\overset{0}{\mathbf{G}_{ik}} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \sin(\theta_i - \theta_k) \right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin(\theta_i - \theta_k) - \mathbf{B}_{ik} \cos(\theta_i - \theta_k) \right)}$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = \overset{1}{|V_i|} \sum_{k=1}^{n} \overset{1}{|V_k|} \left(\overset{0}{\mathbf{G}_{ik}} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \overset{(\theta_i - \theta_k)}{\sin\left(\theta_i - \theta_k\right)} \right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right) \right)}$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

DC Power Flow Approximation

- Linearization of the power flow equations:

$$P_i = \overset{1}{|V_i|} \sum_{k=1}^{n} \overset{1}{|V_k|} \left(\overset{0}{\mathbf{G}_{ik}} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \overset{(\theta_i - \theta_k)}{\sin(\theta_i - \theta_k)} \right)$$

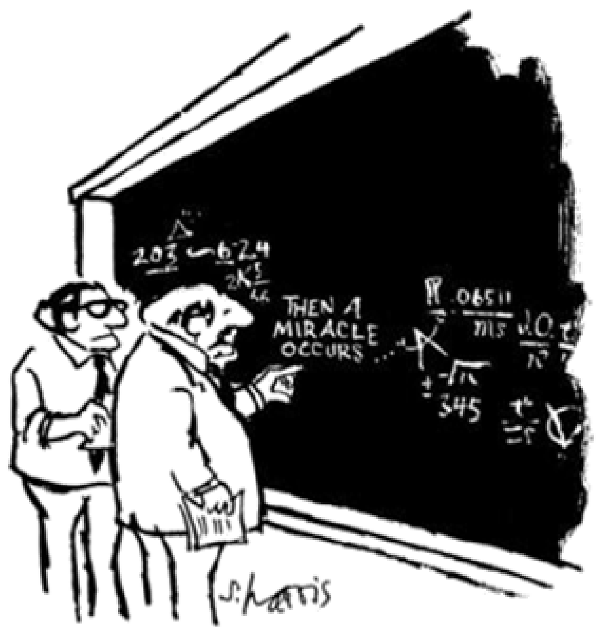
$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin(\theta_i - \theta_k) - \mathbf{B}_{ik} \cos(\theta_i - \theta_k) \right)}$$

$$\Rightarrow \quad P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} (\theta_i - \theta_k)$$

- Advantages:
  - Fast and reliable solution using linear programming.
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power.
  - Approximation error.

Current Practice

Analysis using the DC power flow approximation → Engineering intuition and heuristics → Feasible operating point for the AC power flow equations



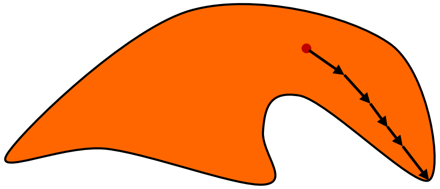
"I think you should be more explicit here in step two."

Sidney Harris,
Science Cartoons Plus

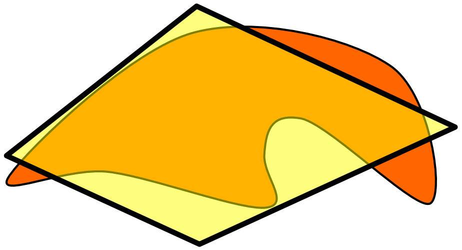
Methods for Handling Nonlinearities

- Local optimization

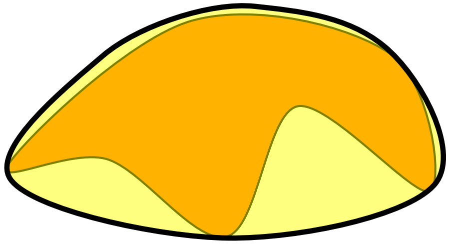
  Recent successes in Dept. of Energy
  ARPA-E Grid Optimization Competition



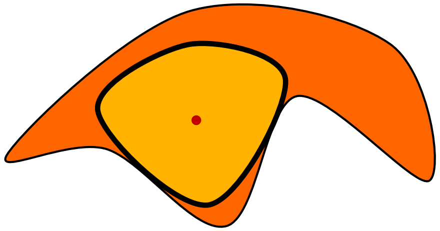
- Approximation



- Convex relaxation



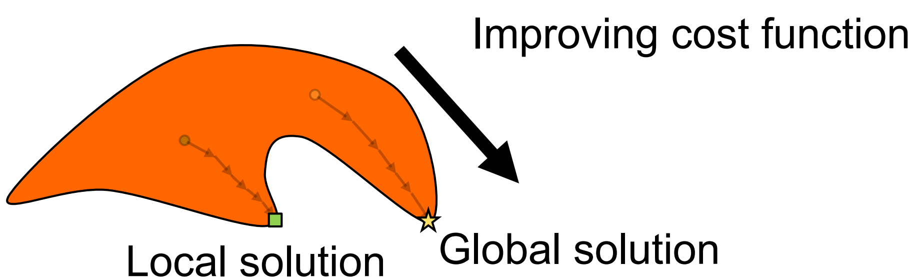
- Convex restriction



Local Optimization

- Seek a "*local solution*" that is superior to all nearby points but possibly inferior to more distant points.

- Dependent on the initialization.



- Two main classes of tools:
  - Interior point methods (e.g., Ipopt).
  - Sequential quadratic programming methods (e.g., SNOPT).

Dept. of Energy ARPA-E Grid Optimization Competition

- Security-Constrained AC Optimal Power Flow problem
  - Jointly optimize the generators' real power outputs and voltage magnitude setpoints.
  - N-1 preventative security requirements.

Dept. of Energy ARPA-E Grid Optimization Competition

- Security-Constrained AC Optimal Power Flow problem

minimize Piecewise Linear Generation Cost

subject to

Base Case

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

P_g^*, V_g^* $\quad$ P_g^*, V_g^* $\quad \cdots \quad$ P_g^*, V_g^*

Contingency 1

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

Contingency 2

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

$\cdots$

Contingency k

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

Dept. of Energy ARPA-E Grid Optimization Competition

- Security-Constrained AC Optimal Power Flow problem

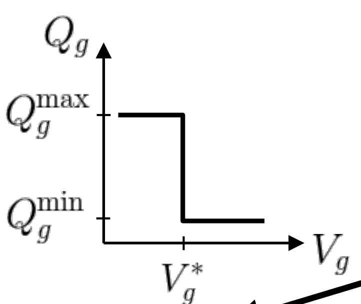
minimize Piecewise Linear Generation Cost

subject to

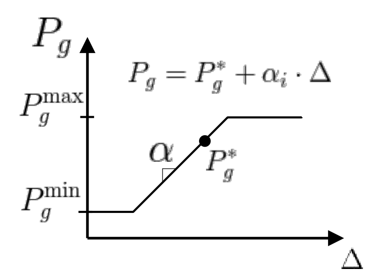
Base Case

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

<u>Reactive Power Limits</u>



<u>Real Power Limits</u>



P_g^*, V_g^* P_g^*, V_g^* $\cdots$ P_g^*, V_g^*

Contingency 1

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

Contingency 2

Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

$\cdots$

Contingency k

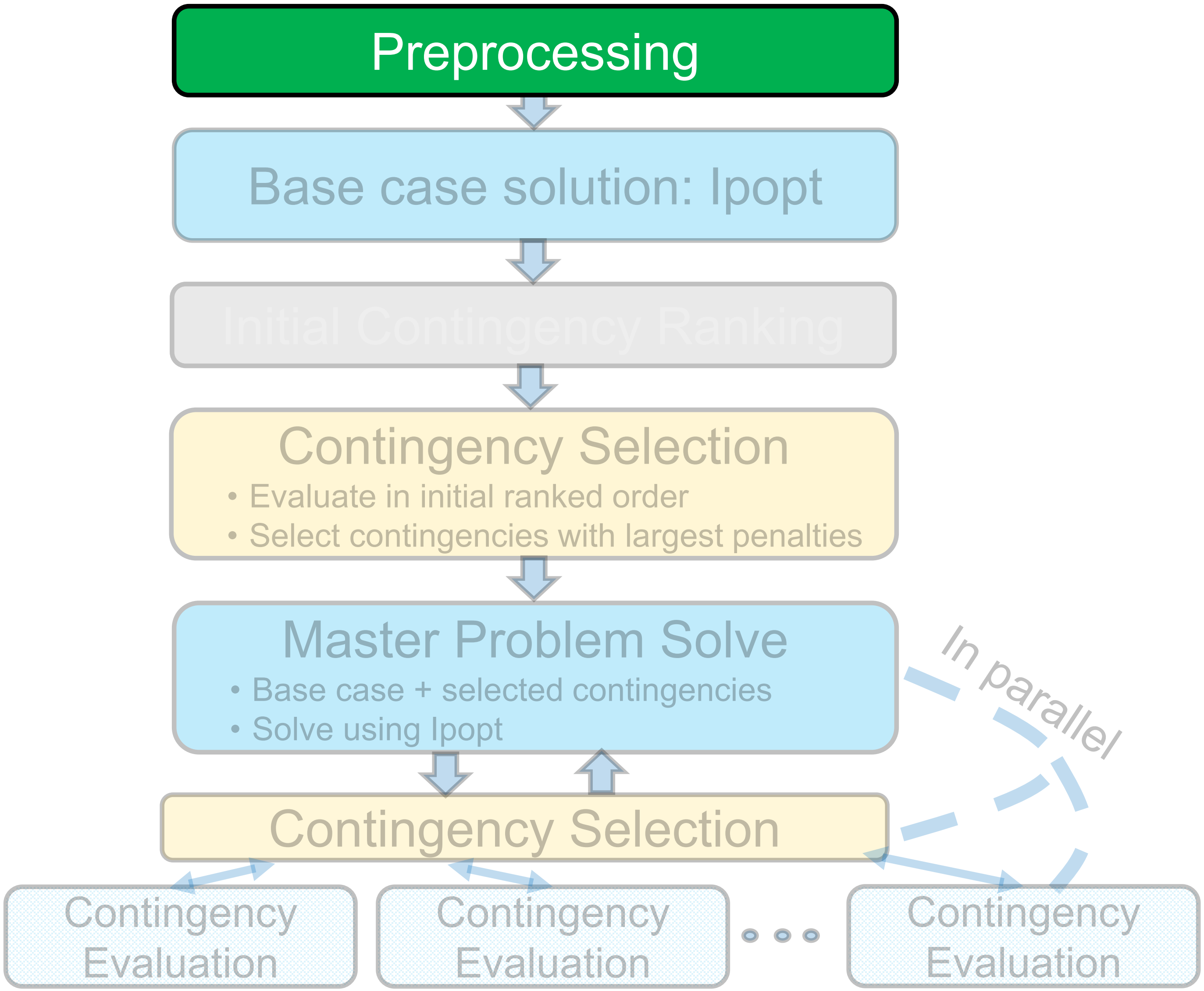
Voltage Magnitude Limits
Line Flow Limits
Real and Reactive Generation Limits
AC Power Flow Equations

Dept. of Energy ARPA-E Grid Optimization Competition

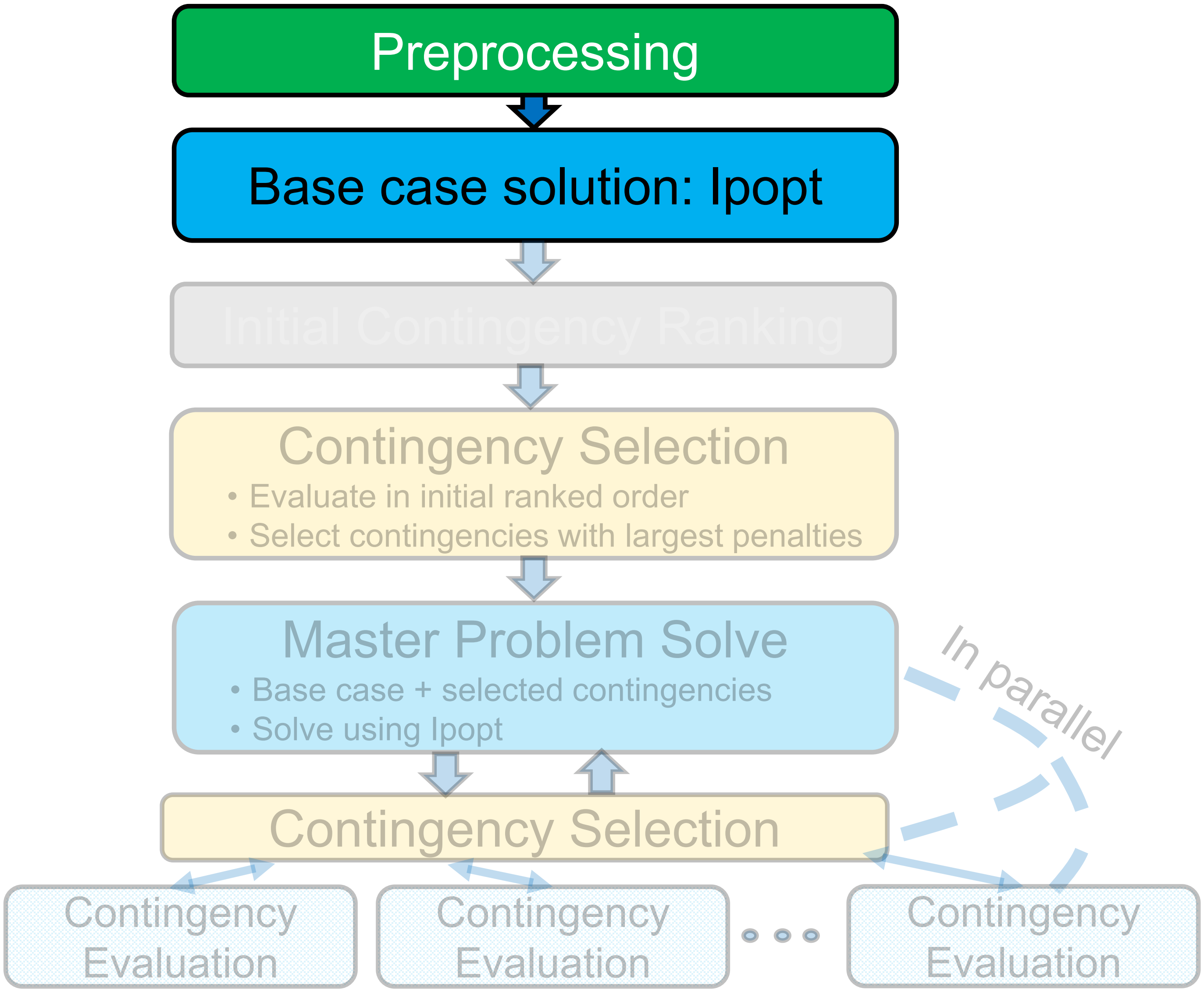
- Security-Constrained AC Optimal Power Flow problem
  - Jointly optimize the generators' real power outputs and voltage magnitude setpoints.
  - N-1 preventative security requirements.

- Key challenges:
  - Problem size: up to 30,000 buses and 10,800 contingencies.
  - Nonlinearity from the AC power flow equations.
  - Complementarity conditions from generator limits.

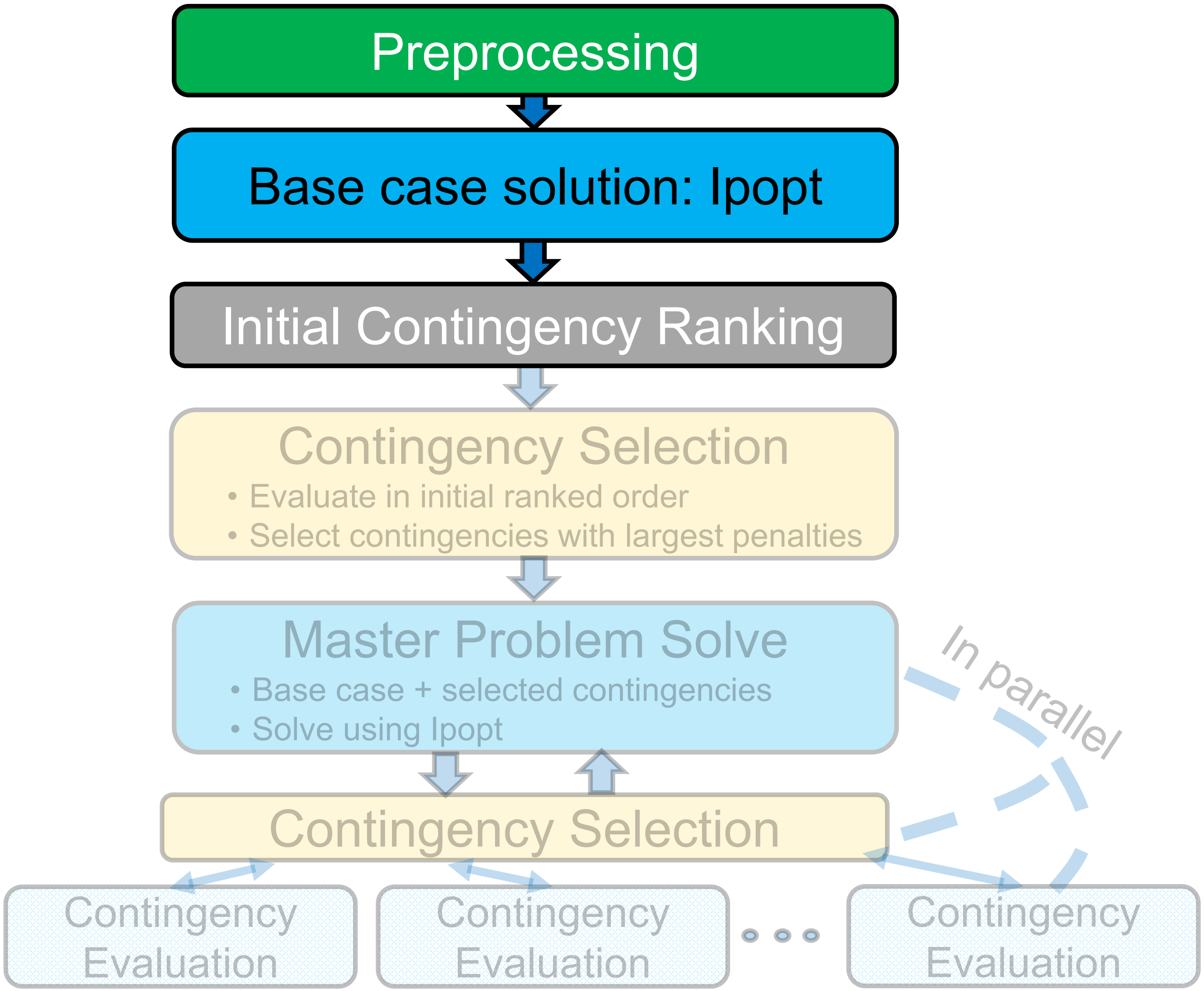
Our Local Optimization Approach



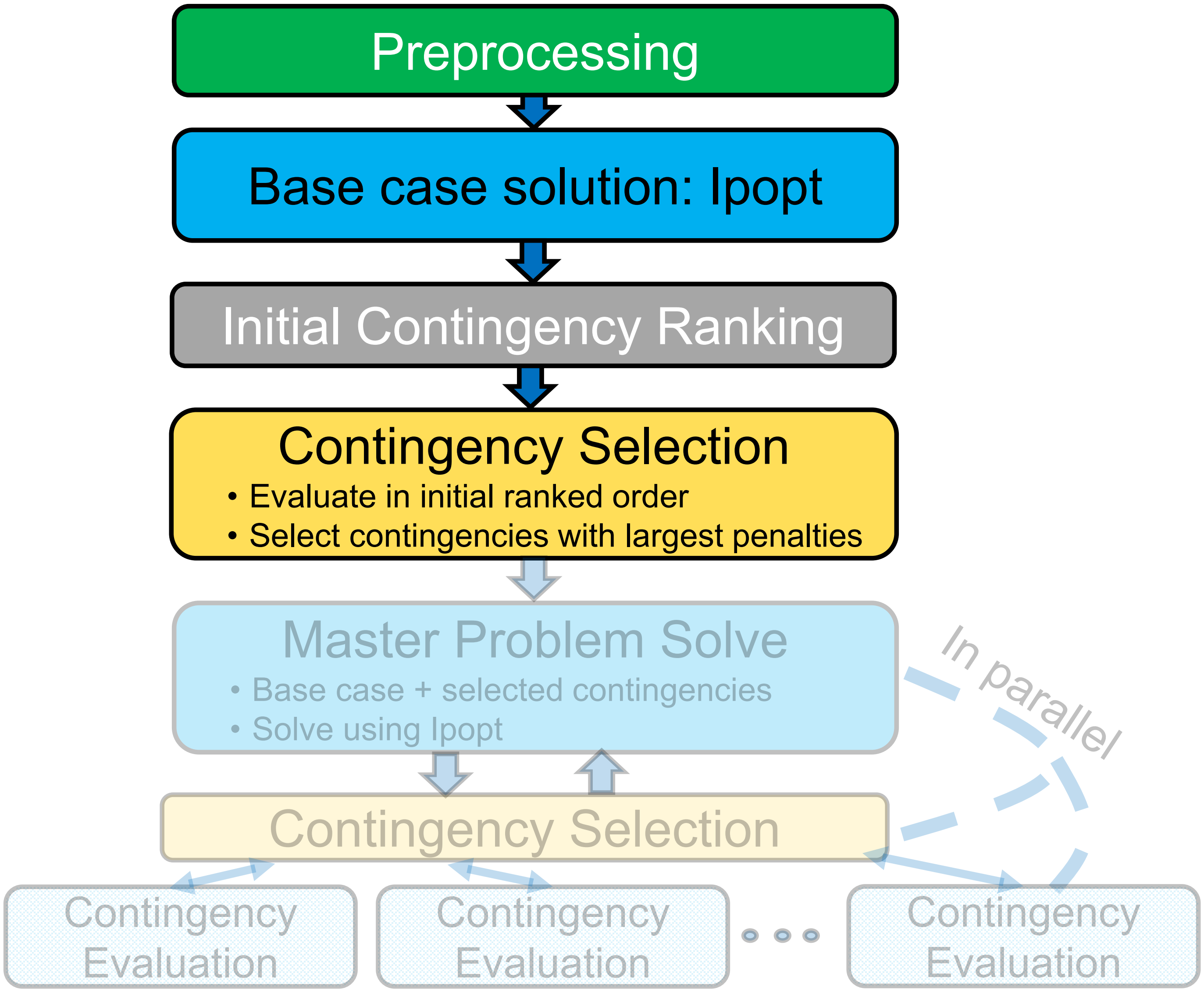
Our Local Optimization Approach



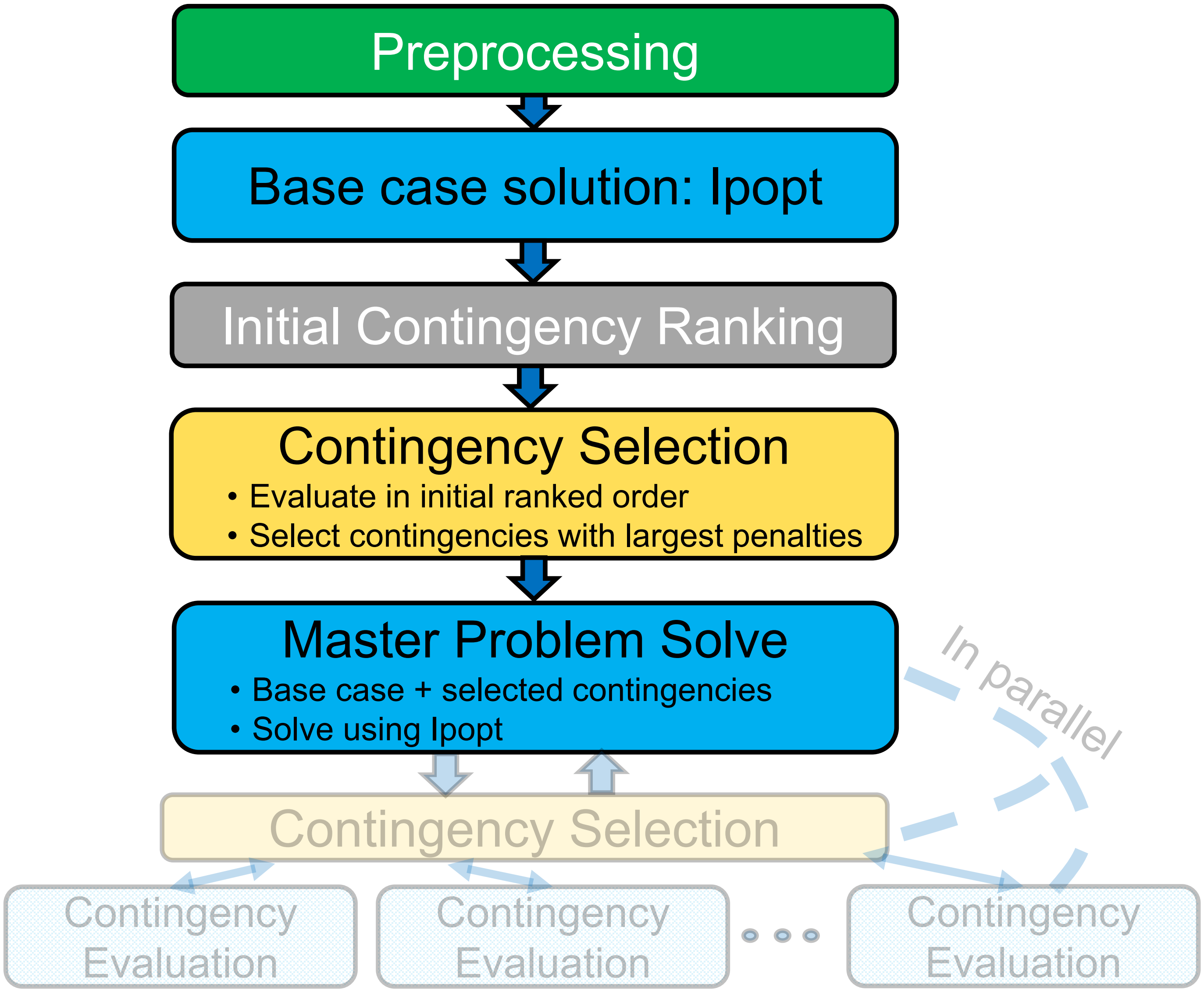
Our Local Optimization Approach



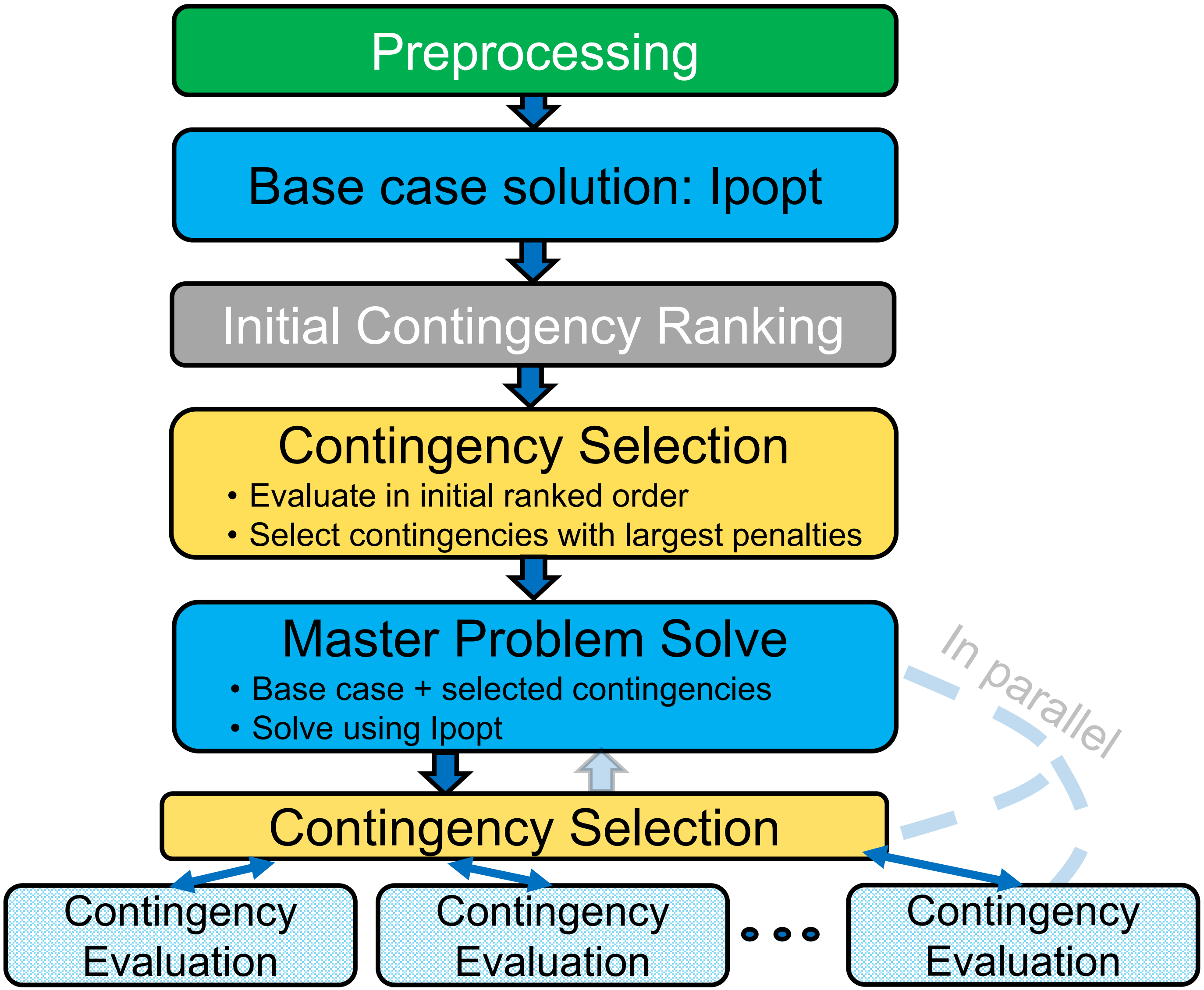
Our Local Optimization Approach



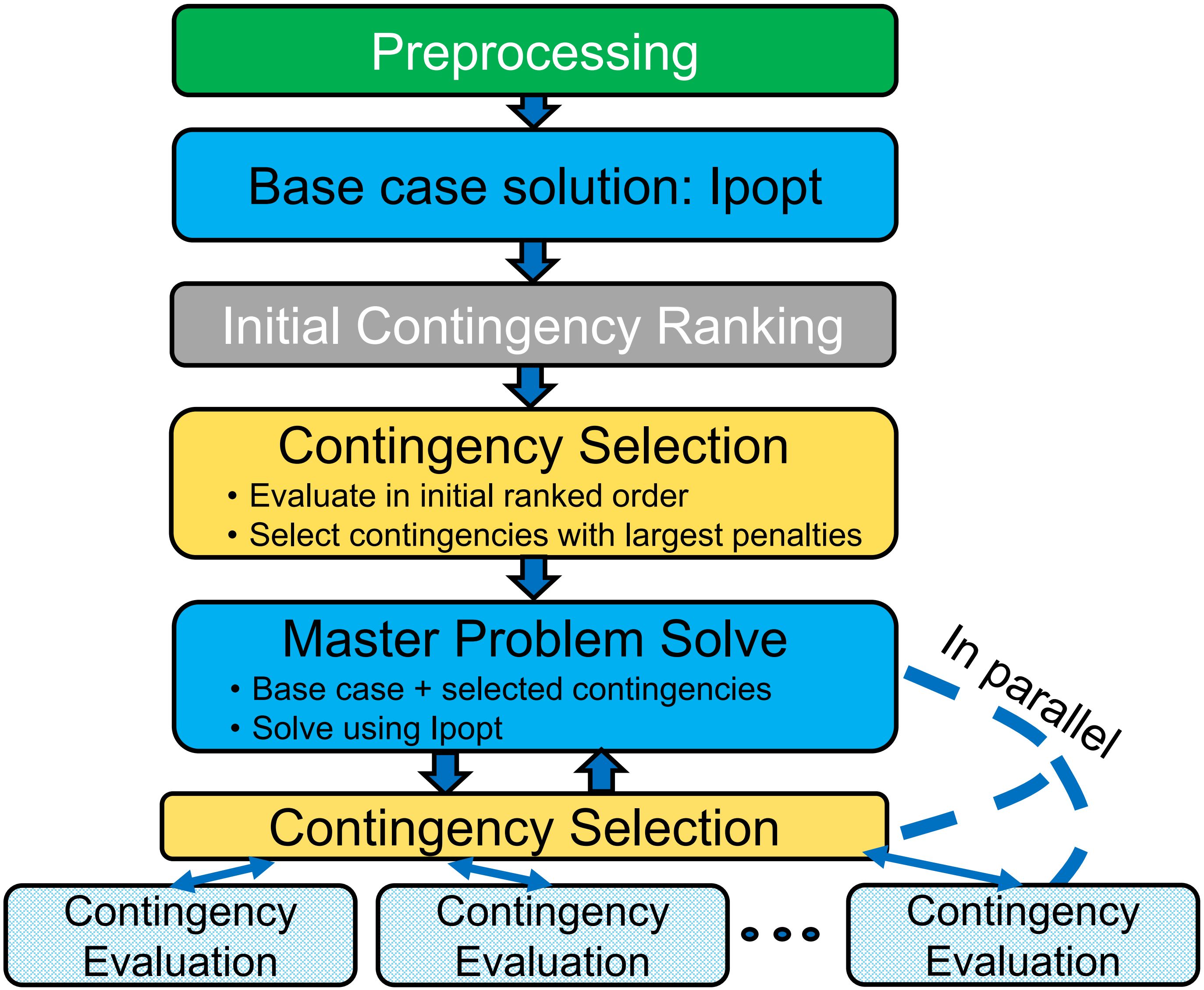
Our Local Optimization Approach



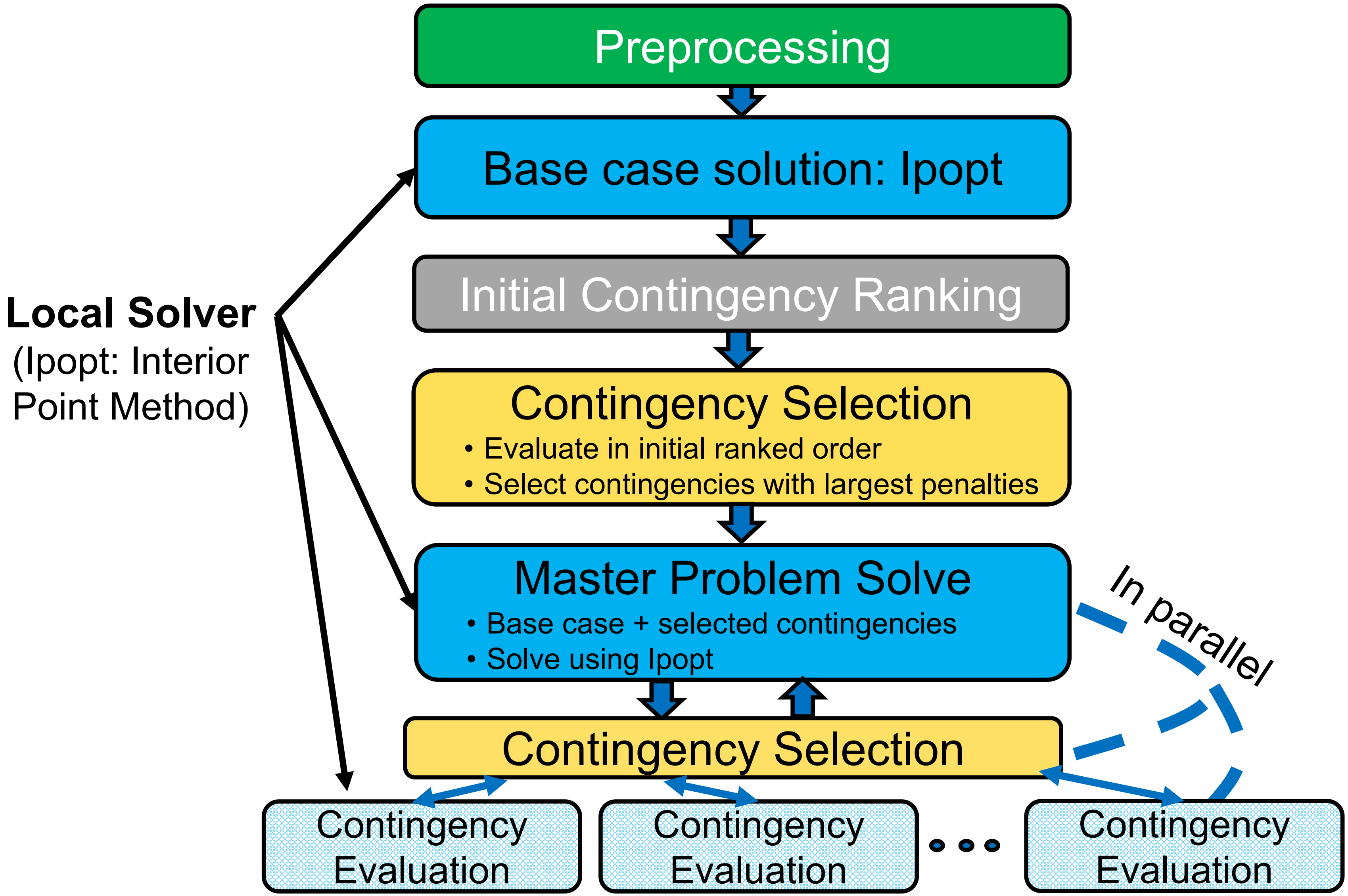
Our Local Optimization Approach



Our Local Optimization Approach



Our Local Optimization Approach



Final Results

- Top teams reliably output solutions for large-scale systems with low generation cost and small constraint violation penalties.

Final Results

Local Solver
(Ipopt used by at least four of the top five teams)

LEADERBOARD - CHALLENGE 1 - FINAL EVENT

Updated: 2/12/2020

*Top 10 Placement		Division 1	Division 2	Division 3	Division 4
Organization:	Lawrence Livermore National Laboratory	1	1	1	1
Team Name:	gollnlp				
Team Lead:	Cosmin G. Petra				
Members:	Omar DeGuchy, Ignacio Andres Aravena Solis, Deepak Rajan				
Organization:	Lehigh University	4	3	2	2
Team Name:	GO-SNIP				
Team Lead:	Frank Edward Curtis				
Members:	Daniel Kenneth Molzahn, Andreas Waechter, Ermin Wei, Elizabeth Wong				
Organization:	Georgia Institute of Technology	2	7	3	3
Team Name:	GMI-GO				
Team Lead:	Xu Sun				
Members:	Santanu Subhas Dey, Amin Gholami, Kaizhao Sun, Shixuan Zhang				
Organization:	individuals	8	2	6	5
Team Name:	BAT				
Team Lead:	Andrew George Telyatnik				
Members:	Oleg Michailovich Strelnikov				
Organization:	individuals	6	6	5	4
Team Name:	gravityx				
Team Lead:	Nathan Lemons				
Members:	Hassan Lionel Hijazi				

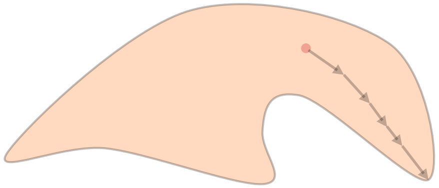
Final Results

- Top teams reliably output solutions for large-scale systems with low generation cost and small constraint violation penalties.

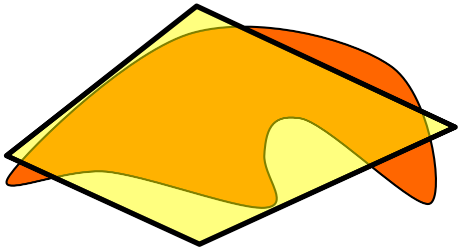
Takeaway: State-of-the-art nonlinear solvers are capable of jointly optimizing real power and voltage magnitude setpoints in security-constrained AC optimal power flow problems!

Methods for Handling Nonlinearities

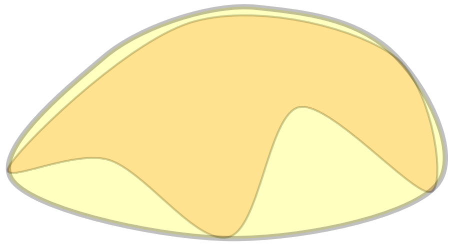
- Local optimization



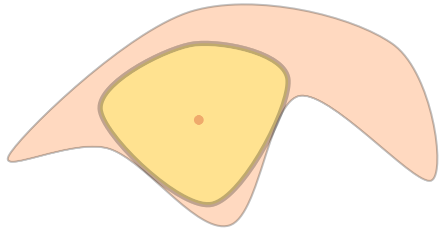
- Approximation



- Convex relaxation



- Convex restriction



Power Flow Approximations

- Approximate nonlinearities using assumptions regarding typical system characteristics.

- DC power flow for transmission systems.

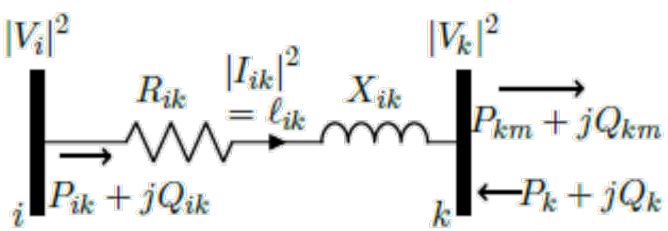
$$P_i = \overset{1}{|V_i|} \sum_{k=1}^{n} \overset{1}{|V_k|} \left(\overset{0}{\mathbf{G}_{ik}} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \overset{(\theta_i - \theta_k)}{\sin\left(\theta_i - \theta_k\right)} \right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right) \right)}$$

$$\Rightarrow \quad P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i - \theta_k\right)$$

Power Flow Approximations

- Approximate nonlinearities using assumptions regarding typical system characteristics.
- DC power flow for transmission systems.
- Linearized DistFlow for distribution systems. [Baran & Wu ‘89]



$$P_{ik} = \cancel{R_{ik}\ell_{ik}} - P_k + \sum_{m:k\to m} P_{km}$$

$$Q_{ik} = \cancel{X_{ik}\ell_{ik}} - Q_k + \sum_{m:k\to m} Q_{km}$$

$$|V_k|^2 = |V_i|^2 - 2\left(R_{ik}P_{ik} + X_{ik}Q_{ik}\right) + \cancel{\left(R_{ik}^2 + X_{ik}^2\right)\ell_{ik}}$$

$$\cancel{\ell_{ik}|V_i|^2 = P_{ik}^2 + Q_{ik}^2}$$

Power Flow Approximations

- Approximate nonlinearities using assumptions regarding typical system characteristics.
- DC power flow for transmission systems.
- Linearized DistFlow for distribution systems. [Baran & Wu '89]
- Many recently proposed alternatives!

Foundations and Trends® in Electric Energy Systems

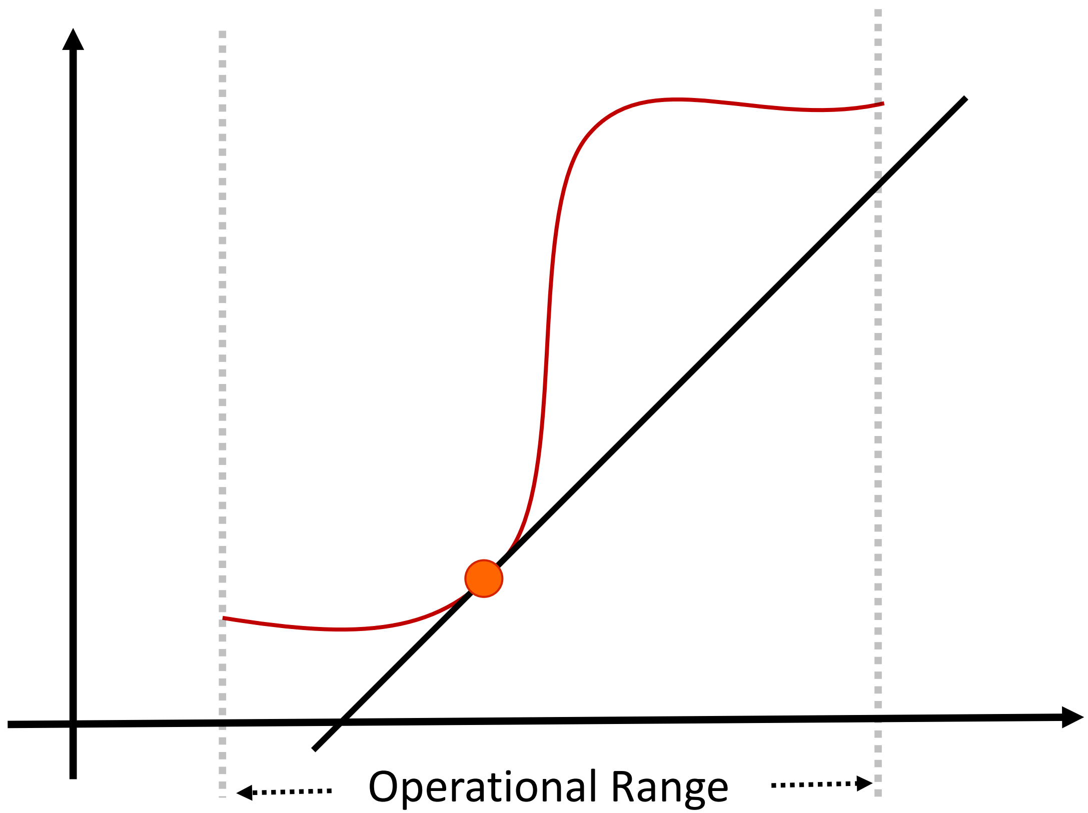
A Survey of Relaxations and Approximations of the Power Flow Equations

Daniel K. Molzahn
Georgia Institute of Technology
molzahn@gatech.edu

Ian A. Hiskens
University of Michigan
hiskens@umich.edu

Optimal Adaptive Approximations

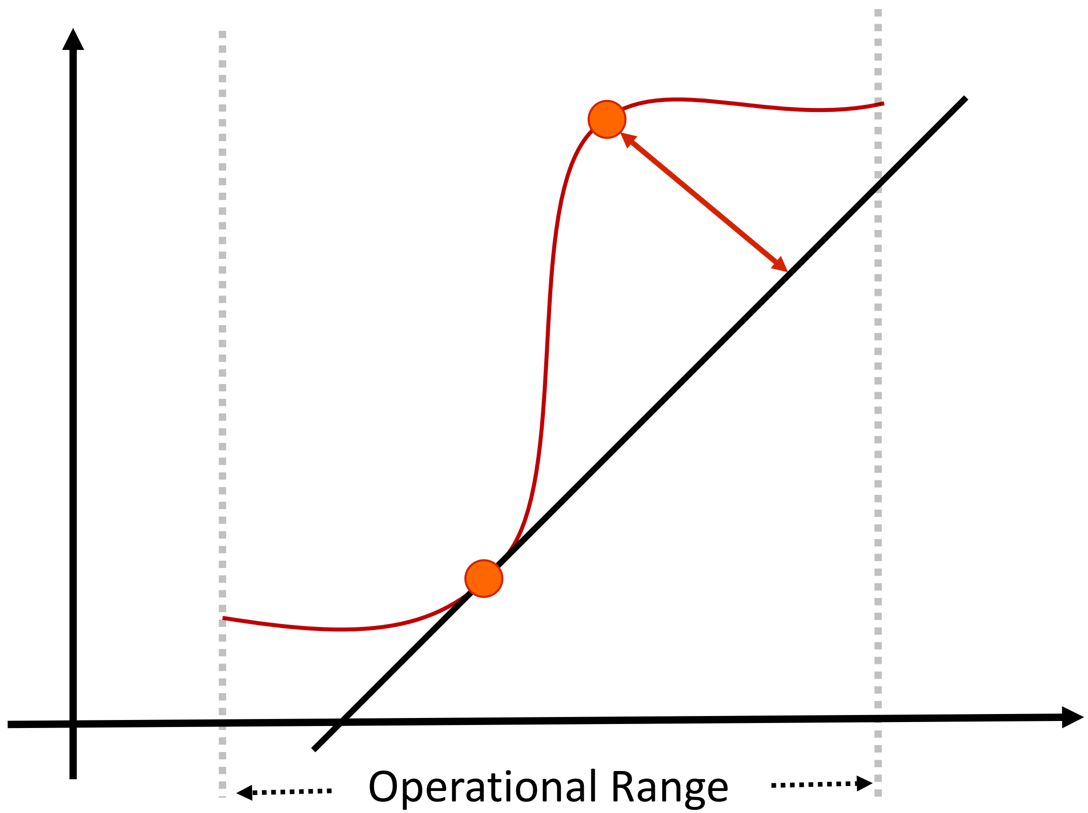
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC‘18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech’19]

Optimal Adaptive Approximations

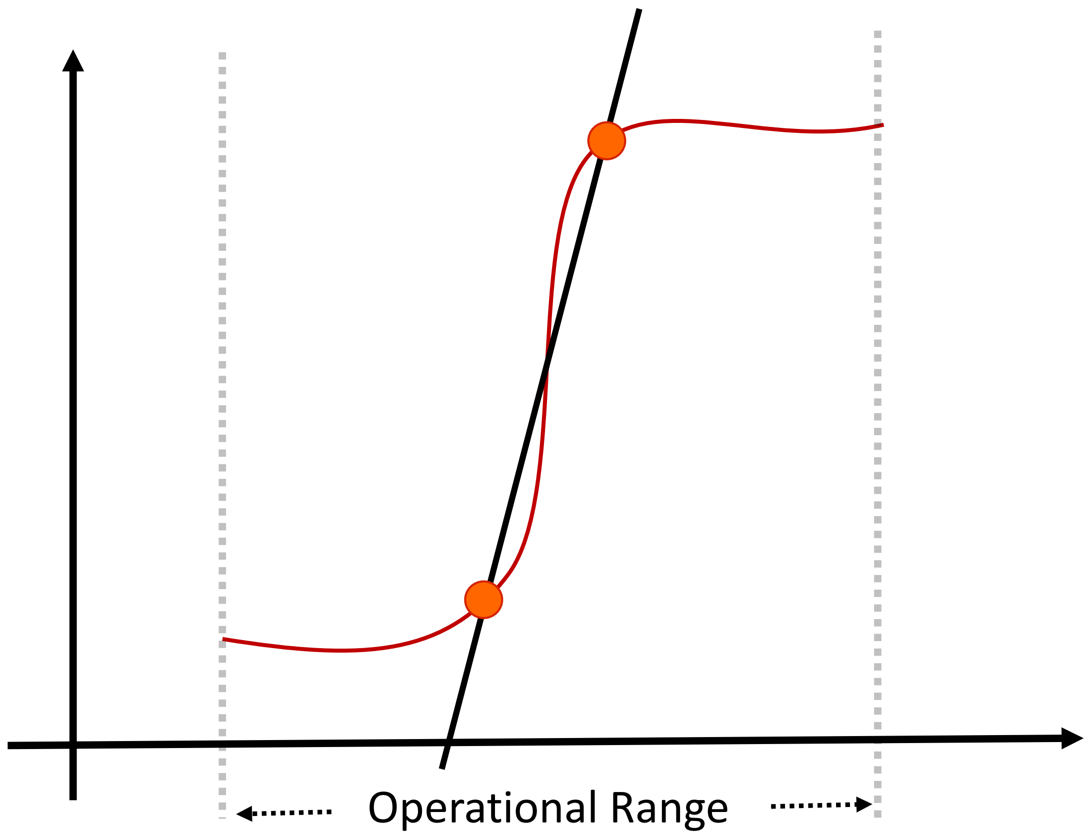
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Optimal Adaptive Approximations

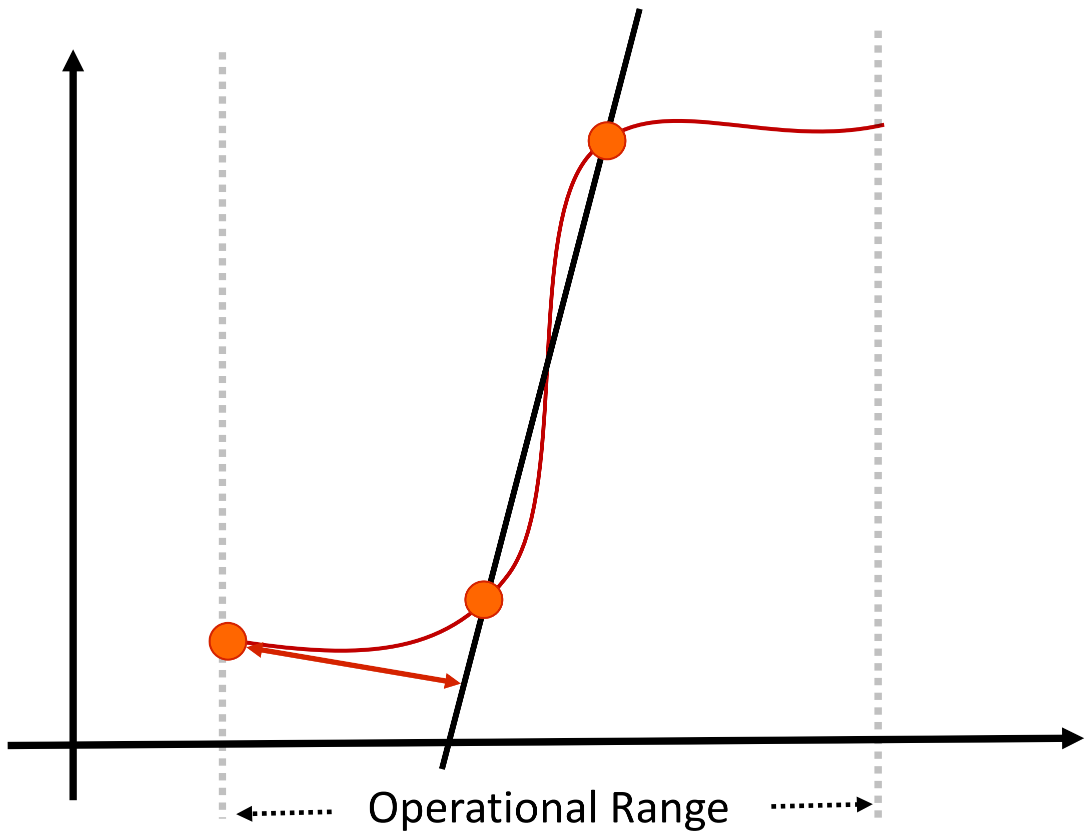
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Optimal Adaptive Approximations

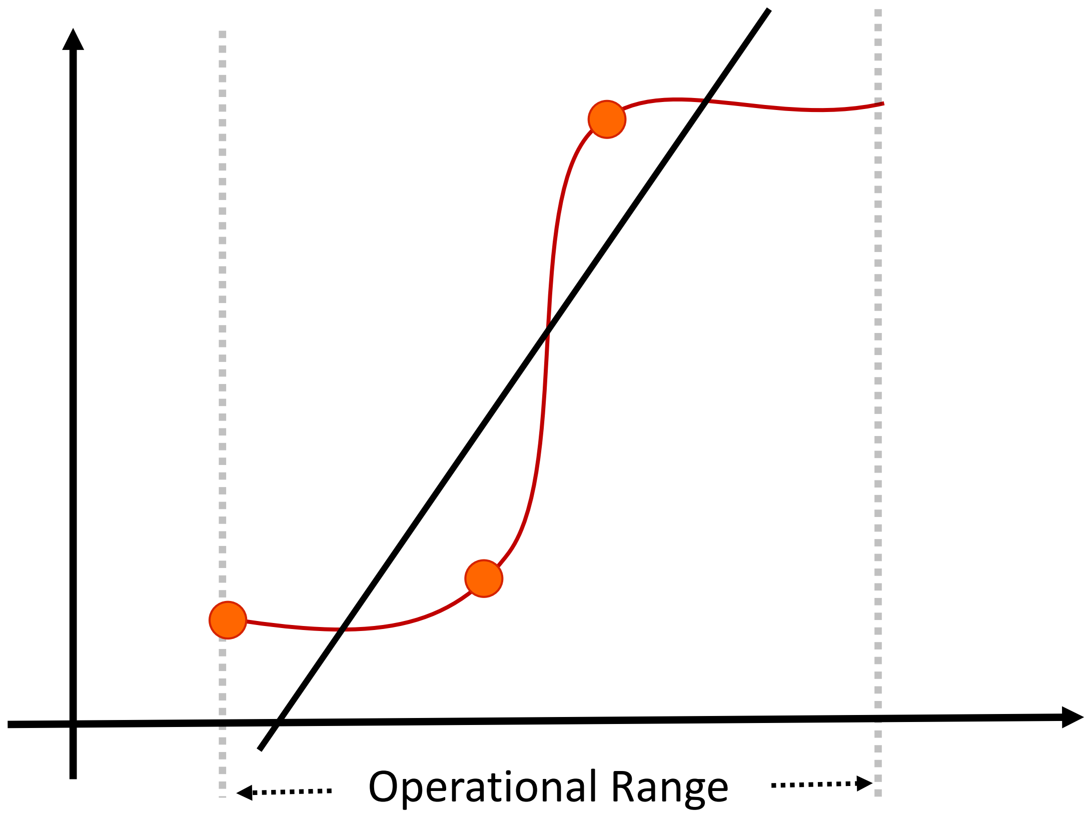
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Optimal Adaptive Approximations

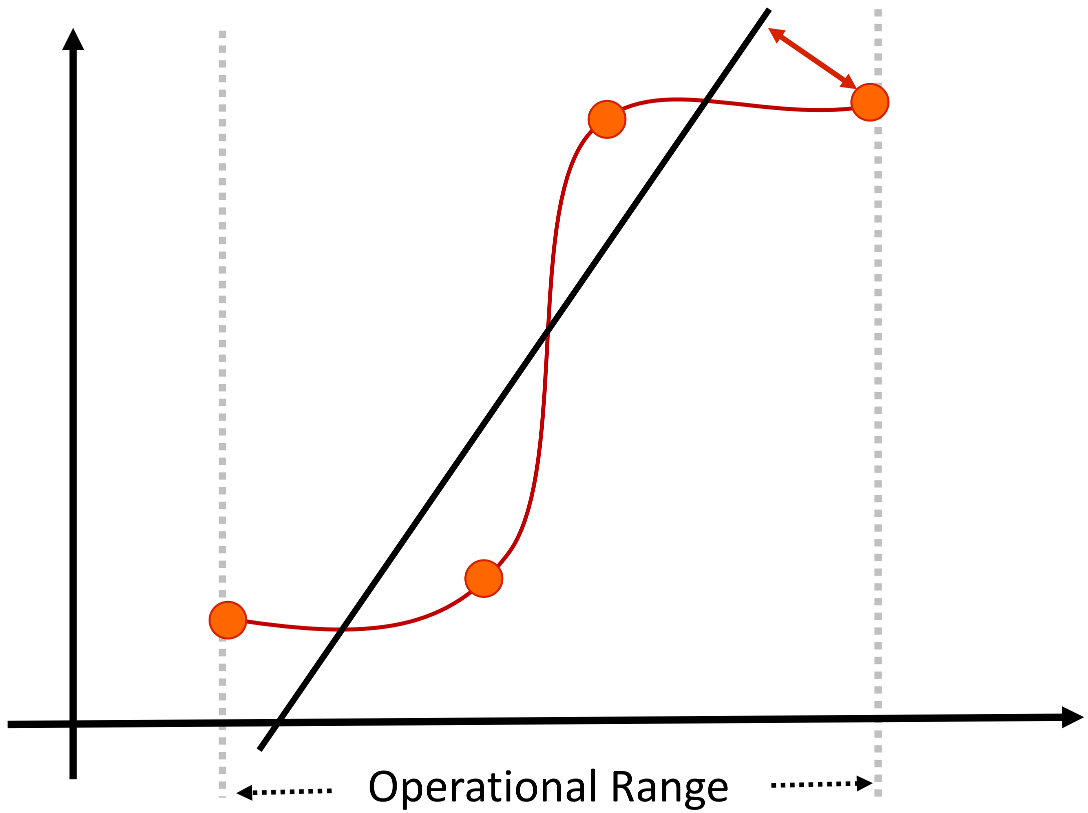
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Optimal Adaptive Approximations

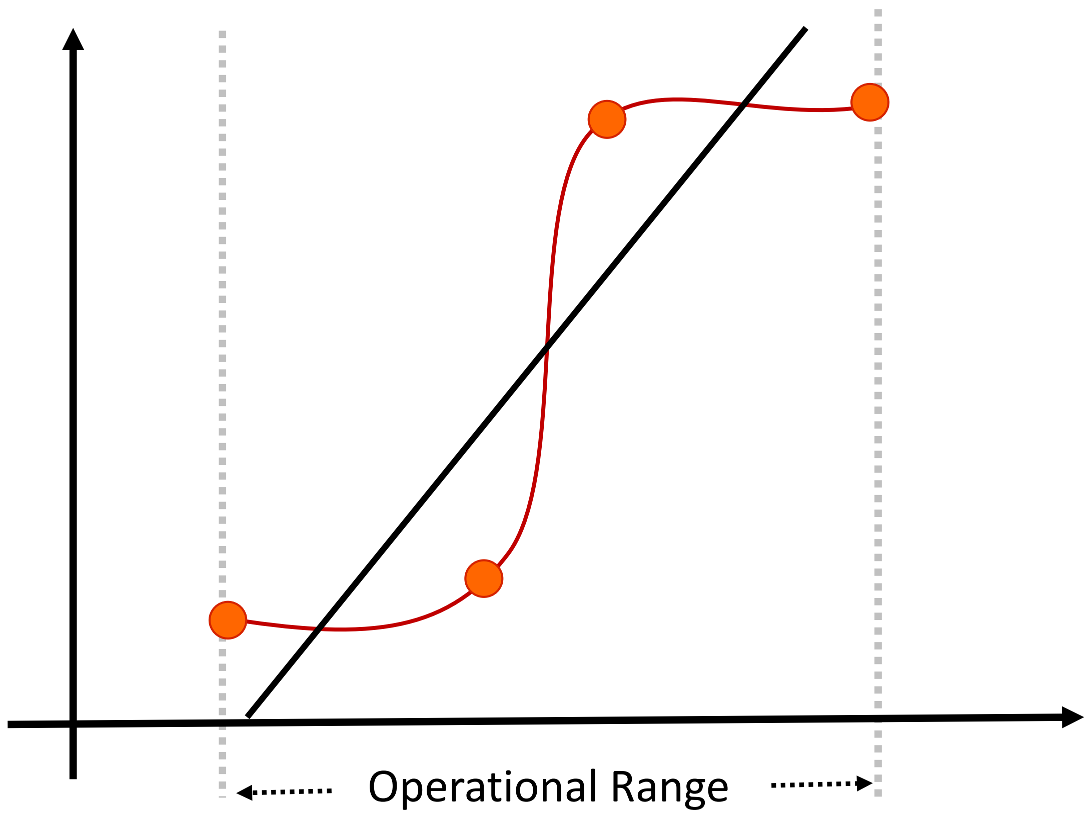
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Optimal Adaptive Approximations

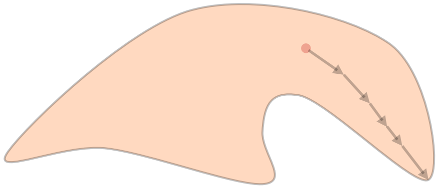
- Compute linear approximations that minimize the worst-case error for a specific system and operating range



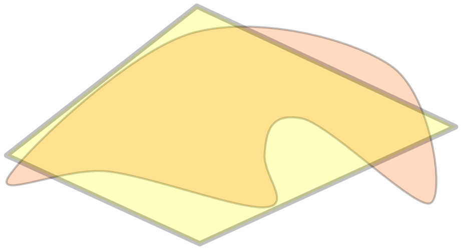
[Misra, Molzahn, & Dvijotham PSCC'18],
[Mühlpfordt, Molzahn, Hagenmeyer, & Misra PowerTech'19]

Methods for Handling Nonlinearities

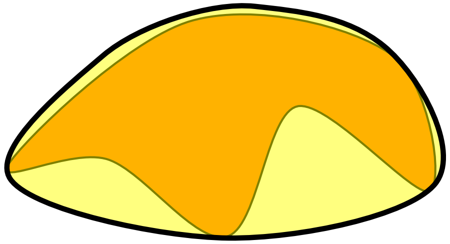
- Local optimization



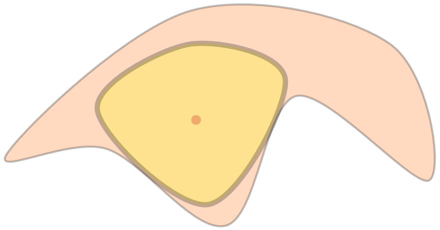
- Approximation



- Convex relaxation

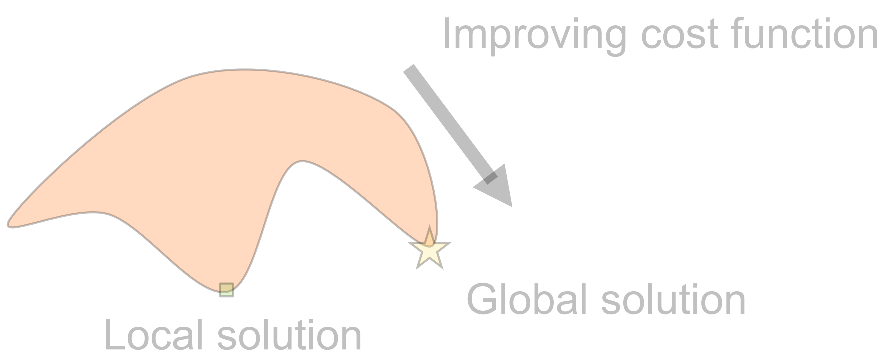


- Convex restriction



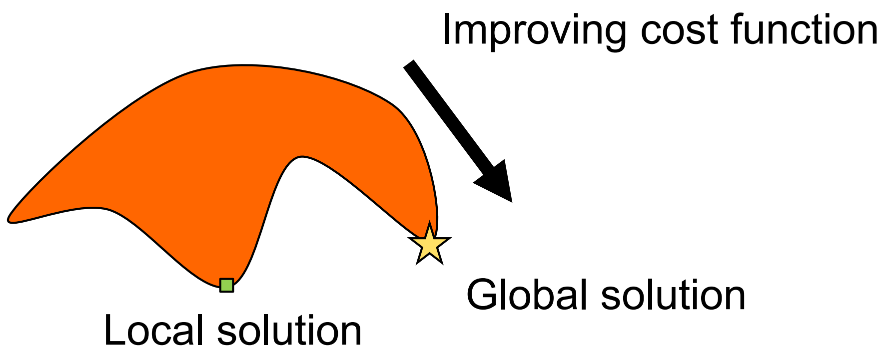
Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.



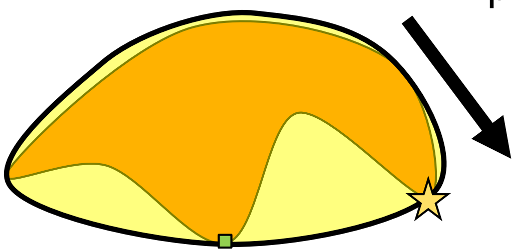
Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.



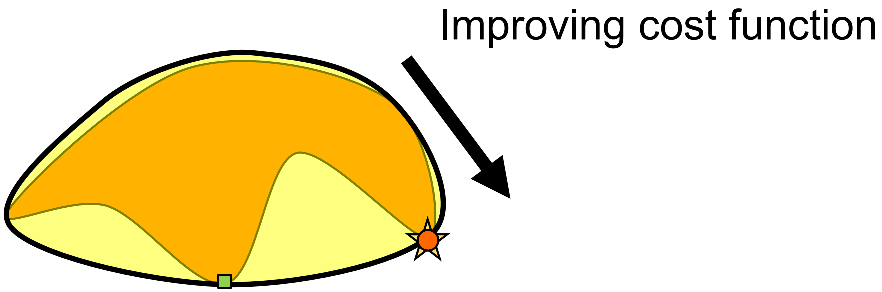
Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.



Power Flow Relaxations

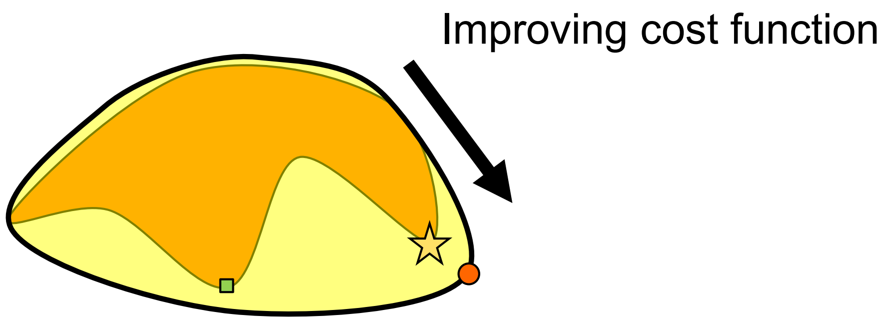
- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.



The solution to the relaxation matches
the solution to the non-convex problem
→ **Zero Relaxation Gap**

Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.



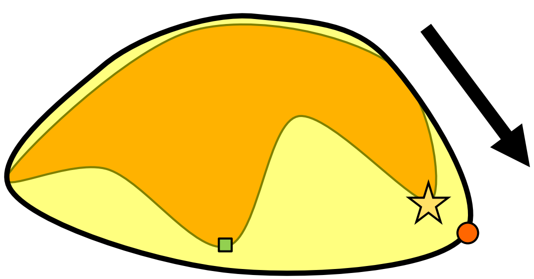
The solution to the relaxation does not match the solution to the non-convex problem
→ **Non-zero Relaxation Gap**

Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.

- Three main advantages over local solution algorithms:

  1. Bounds the optimal objective value.
  2. Provides a sufficient condition for infeasibility.
  3. Solutions which satisfy an easily checkable conditions are guaranteed to be globally optimal.



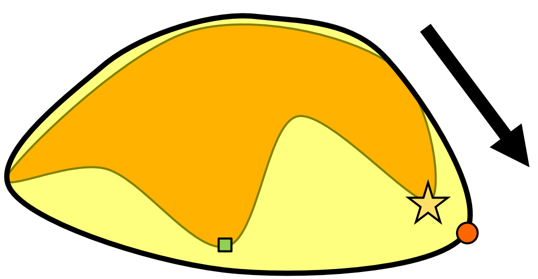
D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.

- Three main advantages over local solution algorithms:

  1. Bounds the optimal objective value.
  2. Provides a sufficient condition for infeasibility.
  3. Solutions which satisfy an easily checkable conditions are guaranteed to be globally optimal.



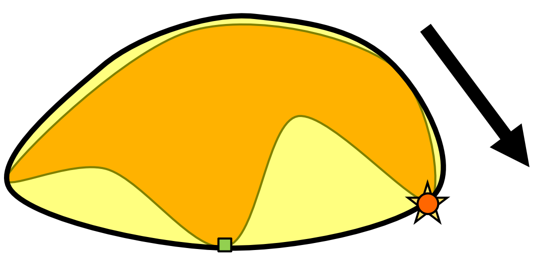
D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.

- Three main advantages over local solution algorithms:

  1. Bounds the optimal objective value.
  2. Provides a sufficient condition for infeasibility.
  3. Solutions which satisfy an easily checkable conditions are guaranteed to be globally optimal.



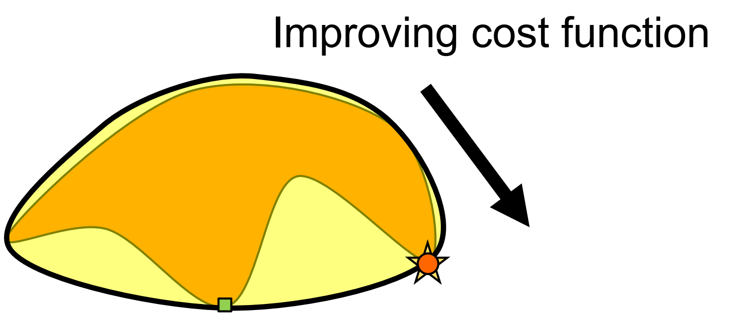
D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

Power Flow Relaxations

- Relax nonlinearities using less stringent conditions in order to enclose the non-convex feasible space within a larger convex space.

- Three main advantages over local solution algorithms:

  1. Bounds the optimal objective value.
  2. Provides a sufficient condition for infeasibility.
  3. Solutions which satisfy an easily checkable conditions are guaranteed to be globally optimal.



D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

Example: The QC Relaxation

- Based on the polar form of the power flow equations:

$$P_{ik} = g_{ik}\left|V_i\right|^2 - g_{ik}\left|V_i\right|\left|V_k\right|\cos\left(\theta_i - \theta_k\right) - b_{ik}\left|V_i\right|\left|V_k\right|\sin\left(\theta_i - \theta_k\right)$$

$$Q_{ik} = -\left(b_{c,ik}/2 + b_{ik}\right)\left|V_i\right|^2 - g_{ik}\left|V_i\right|\left|V_k\right|\sin\left(\theta_i - \theta_k\right) + b_{ik}\left|V_i\right|\left|V_k\right|\cos\left(\theta_i - \theta_k\right)$$

Trilinear monomials in variables representing $|V_i|$, $|V_k|$, and convex envelopes for $\cos\left(\theta_i - \theta_k\right)$ or $\sin\left(\theta_i - \theta_k\right)$:

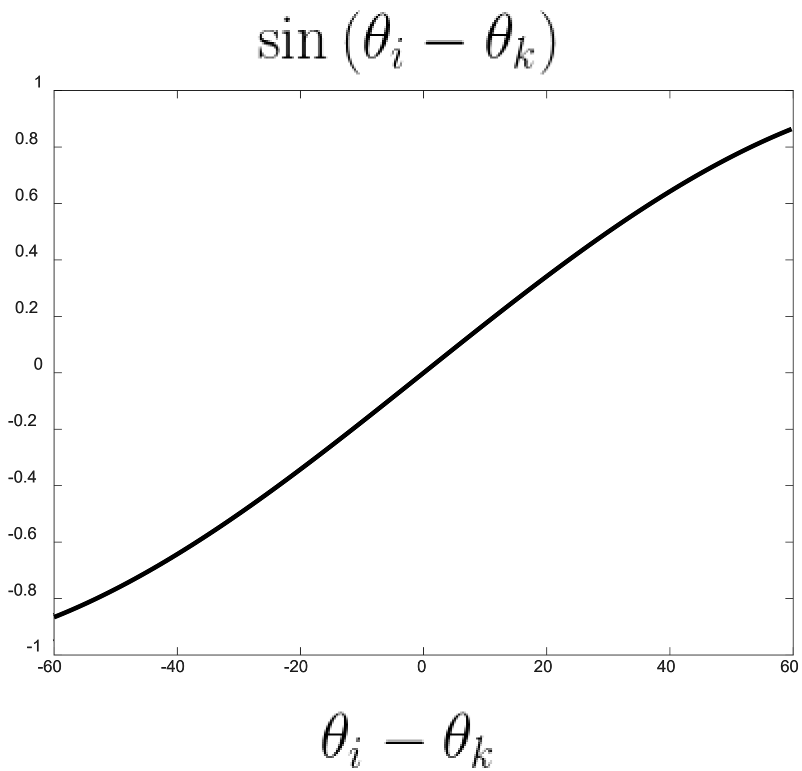
$$\left|V_i\right| \cdot \left|V_k\right| \cdot \cos\left(\theta_i - \theta_k\right),$$

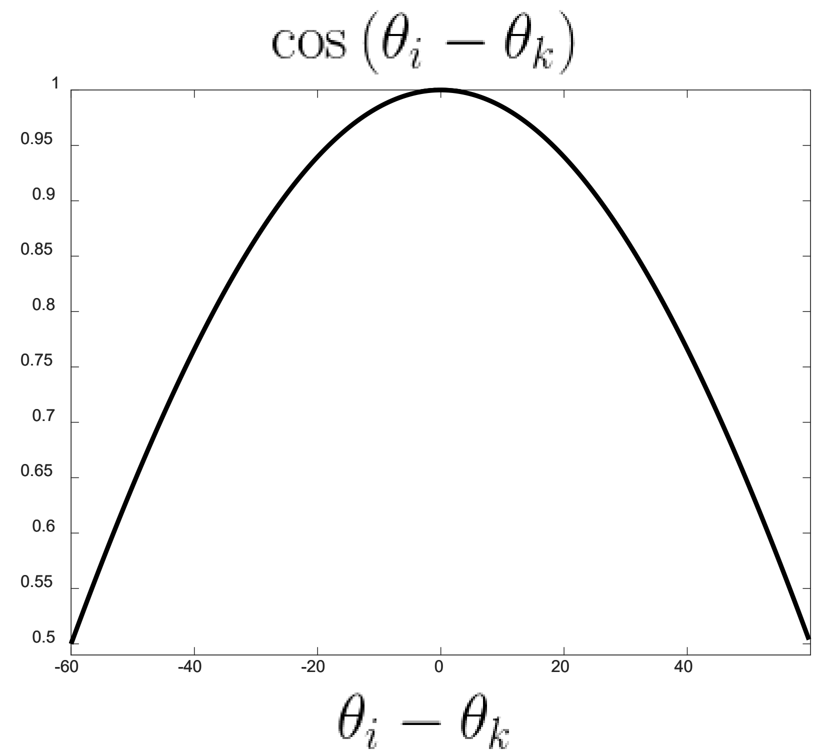
$$\left|V_i\right| \cdot \left|V_k\right| \cdot \sin\left(\theta_i - \theta_k\right).$$

[Coffrin, Hijazi & Van Hentenryck ‘15]

Example: The QC Relaxation

- Constructs convex envelopes around the sine and cosine functions in the power flow equations with polar voltages.

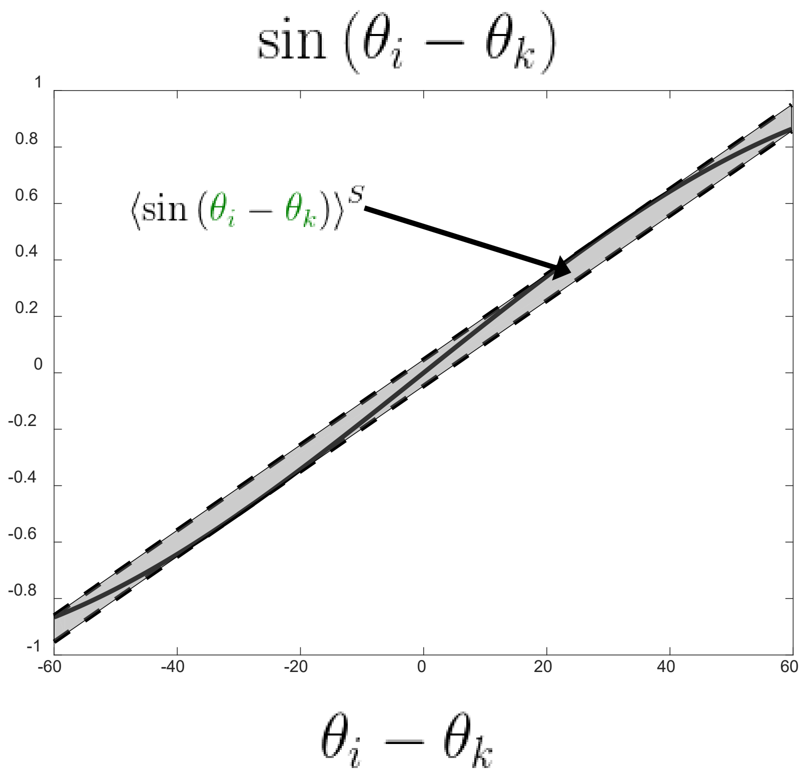


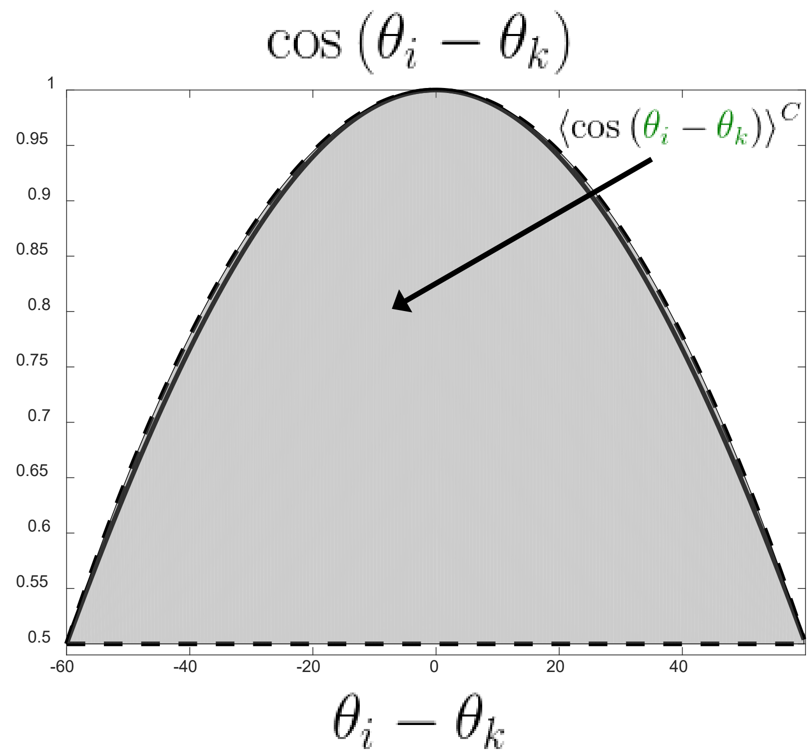


[Coffrin, Hijazi & Van Hentenryck '15]

Example: The QC Relaxation

- Constructs convex envelopes around the sine and cosine functions in the power flow equations with polar voltages.

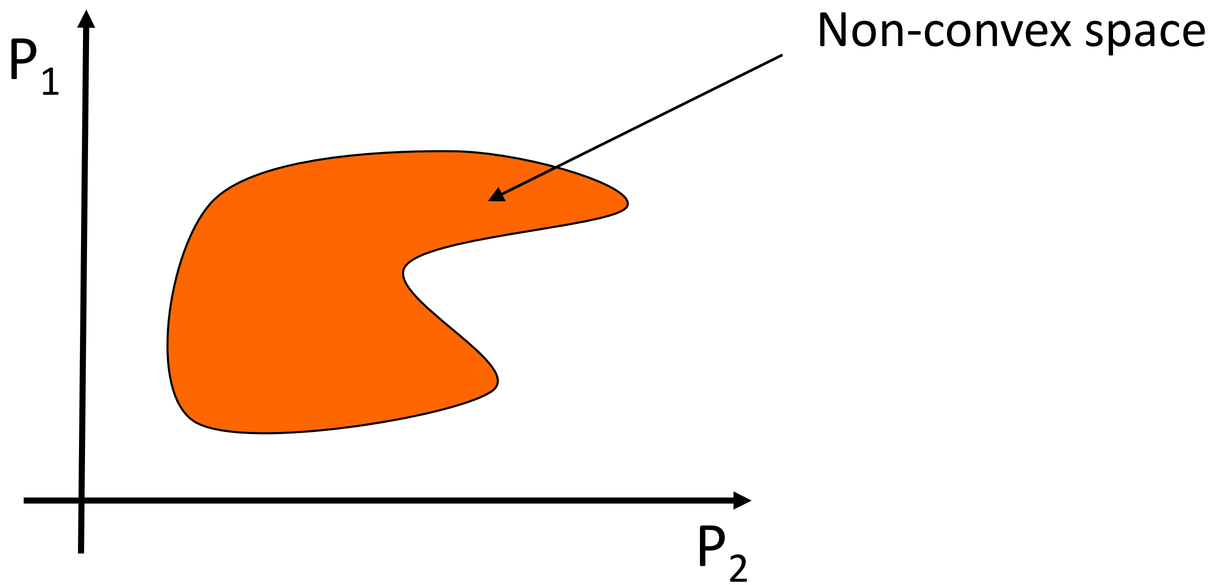




[Coffrin, Hijazi & Van Hentenryck '15]

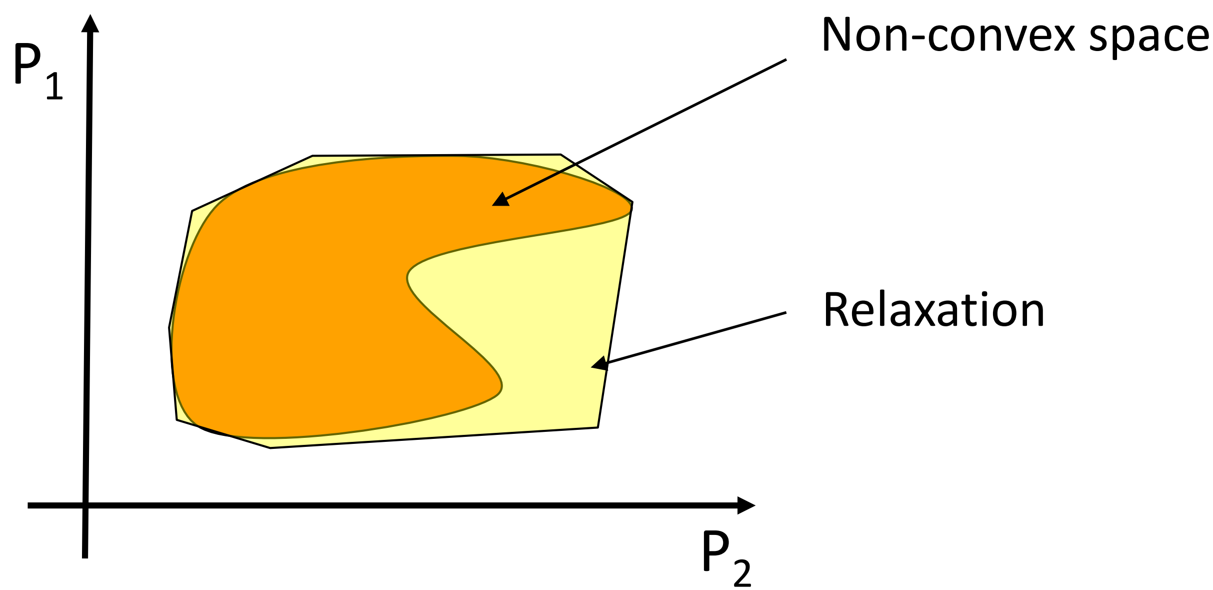
Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms



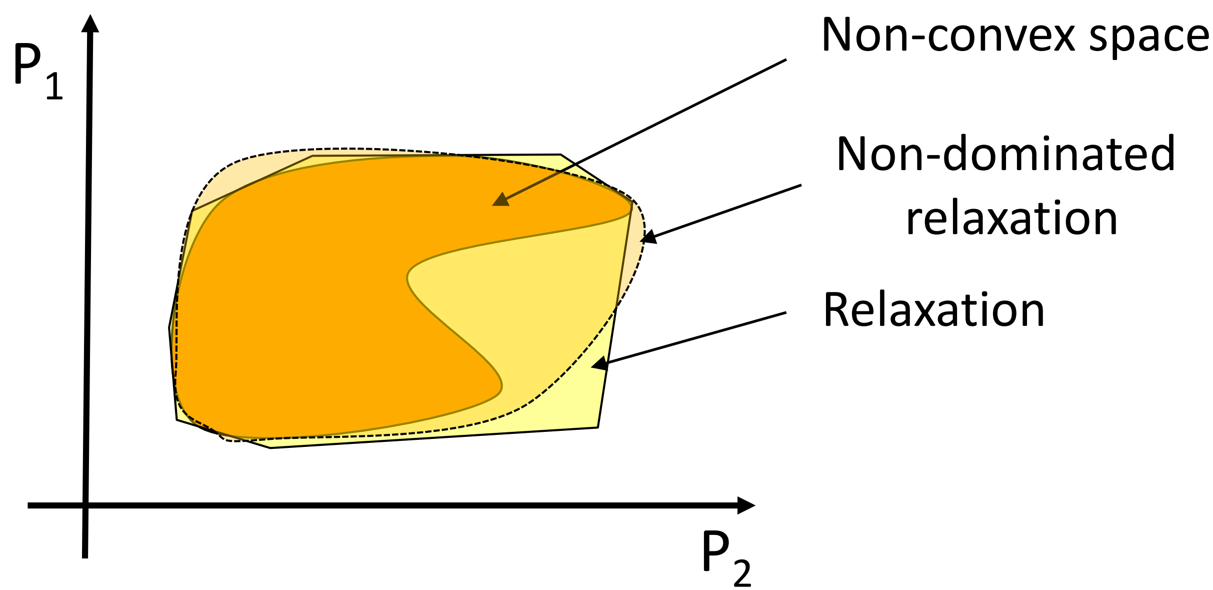
Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms



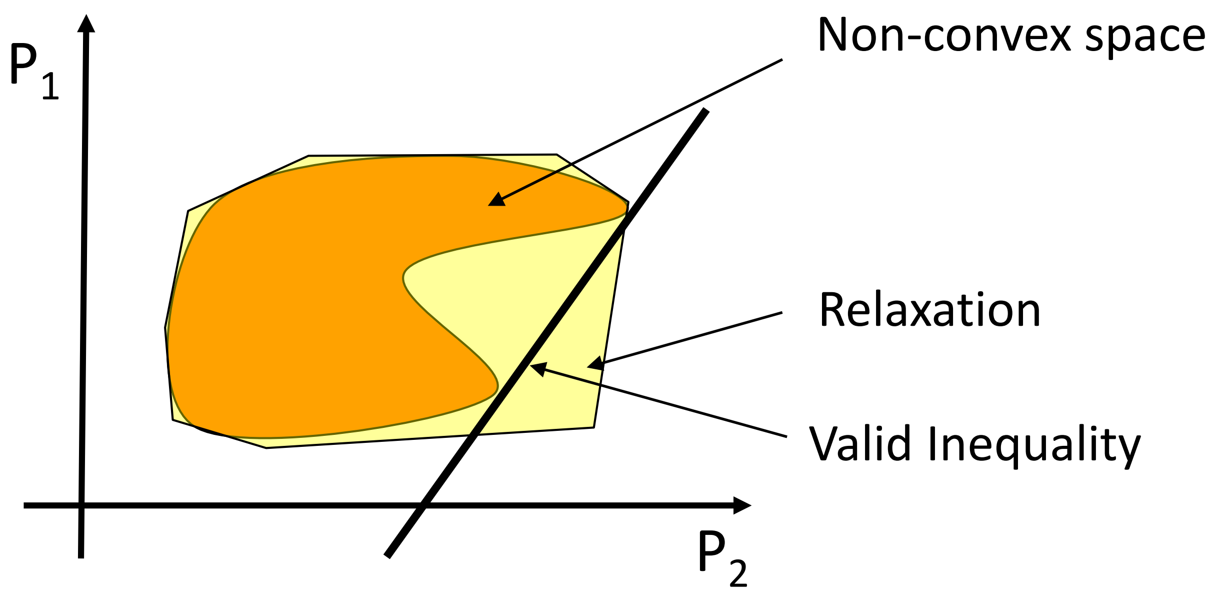
Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms



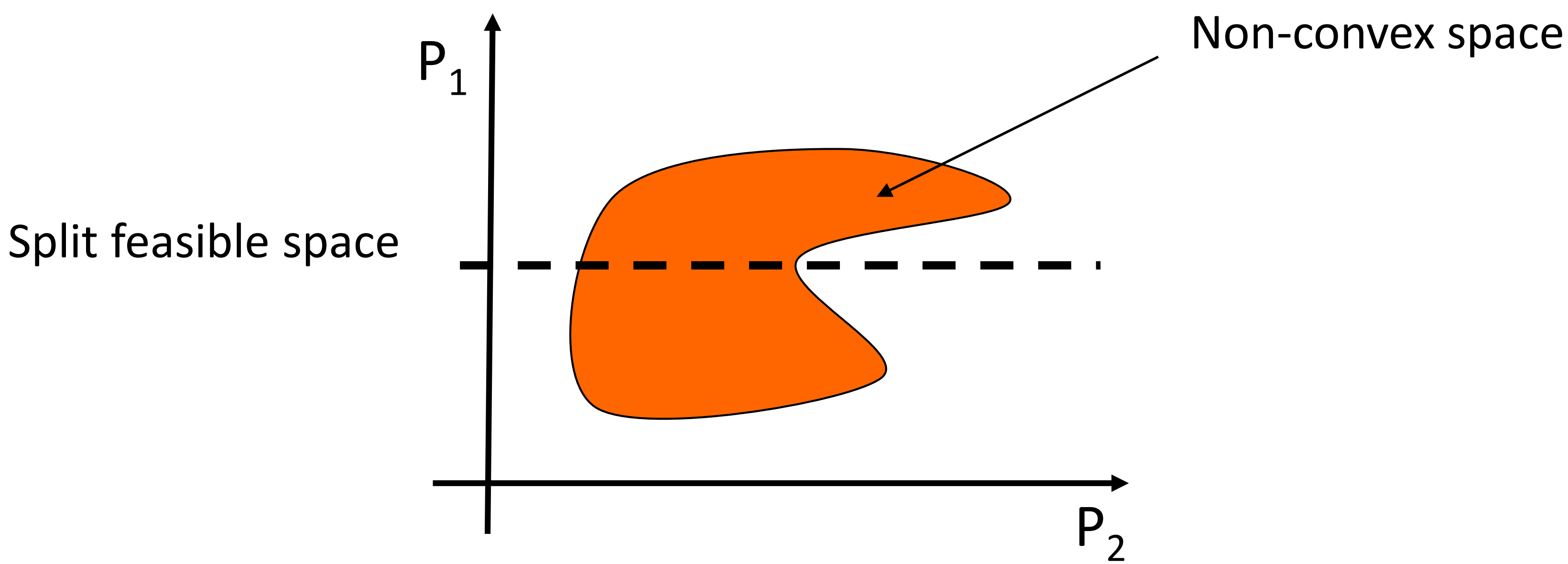
Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms



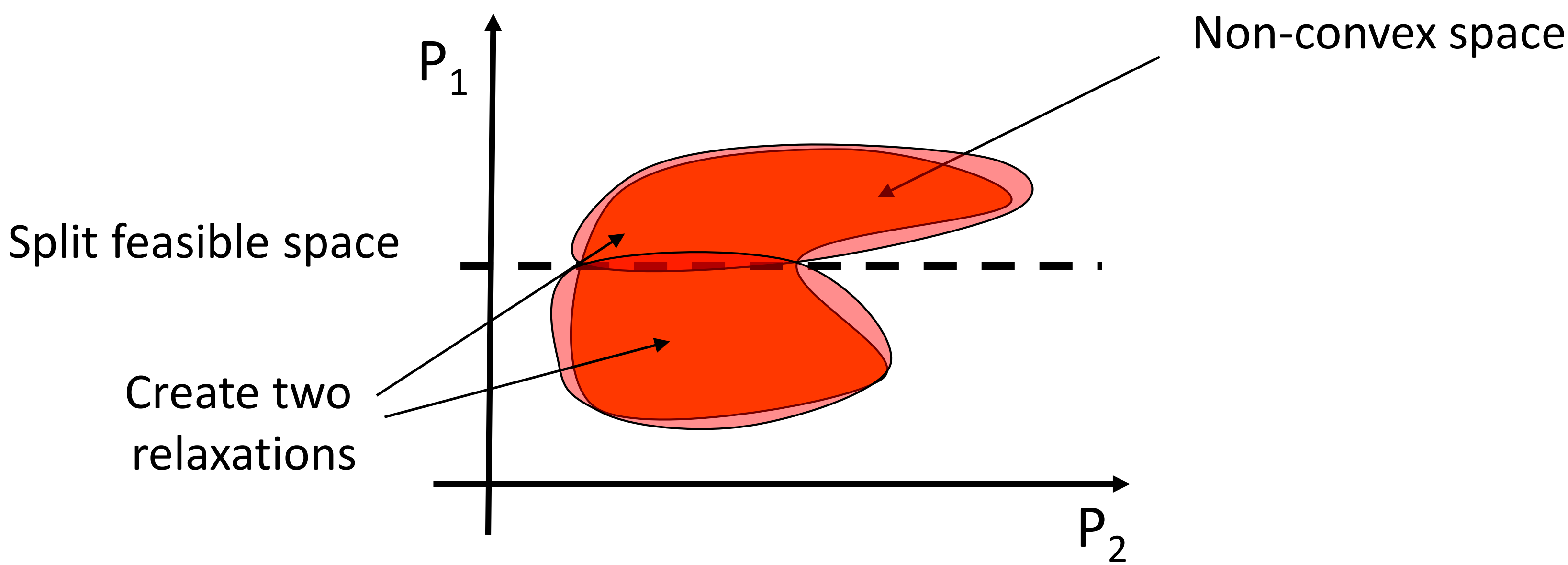
Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms



Methods for Tightening Relaxations

- Combining non-dominated relaxations
- Valid inequalities
- Branch-and-bound algorithms

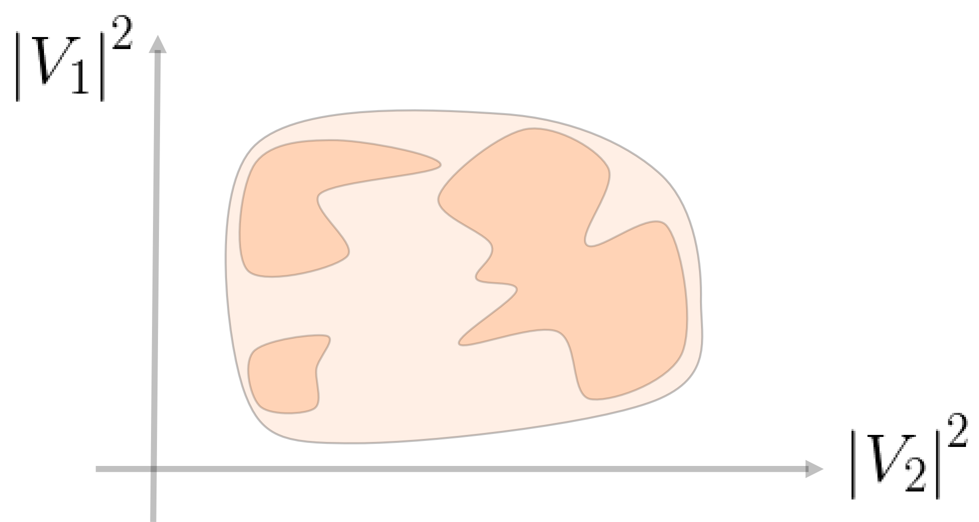


Applications of Convex Relaxations

(Five Examples)

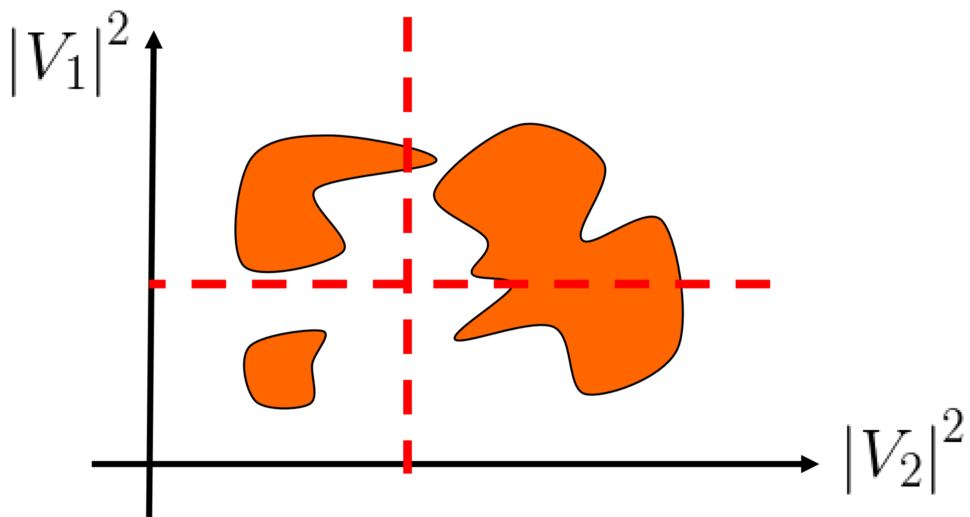
1. Global Solution via Spatial Branch-and-Bound

1. Segment the feasible space into adjoining subregions.
2. In each subregion, compute an upper bound using a local solver and a lower bound using a relaxation.
3. Eliminate subregions whose lower bounds are greater than an upper bound obtained in any other subregion.
4. Iterate until finding an upper bound that is sufficiently close to the least lower bound.



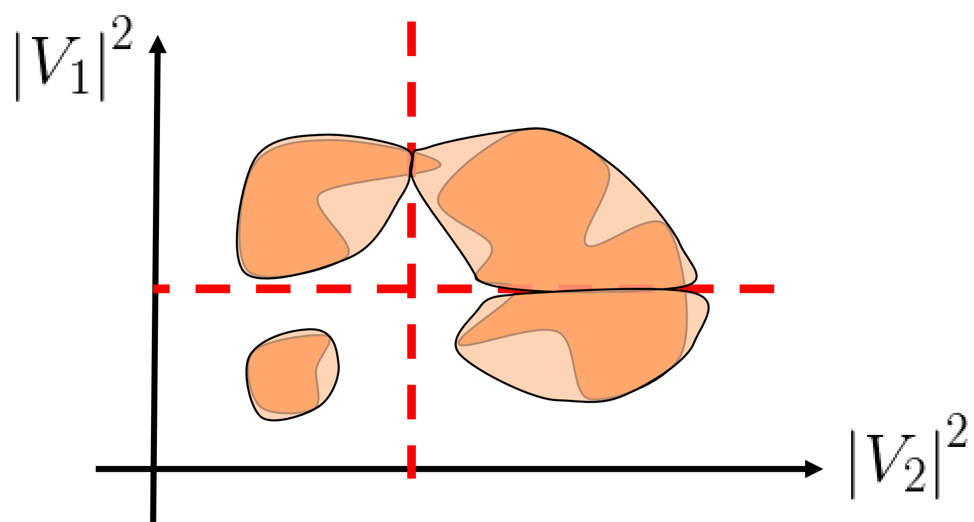
1. Global Solution via Spatial Branch-and-Bound

1. Segment the feasible space into adjoining subregions.
2. In each subregion, compute an upper bound using a local solver and a lower bound using a relaxation.
3. Eliminate subregions whose lower bounds are greater than an upper bound obtained in any other subregion.
4. Iterate until finding an upper bound that is sufficiently close to the least lower bound.



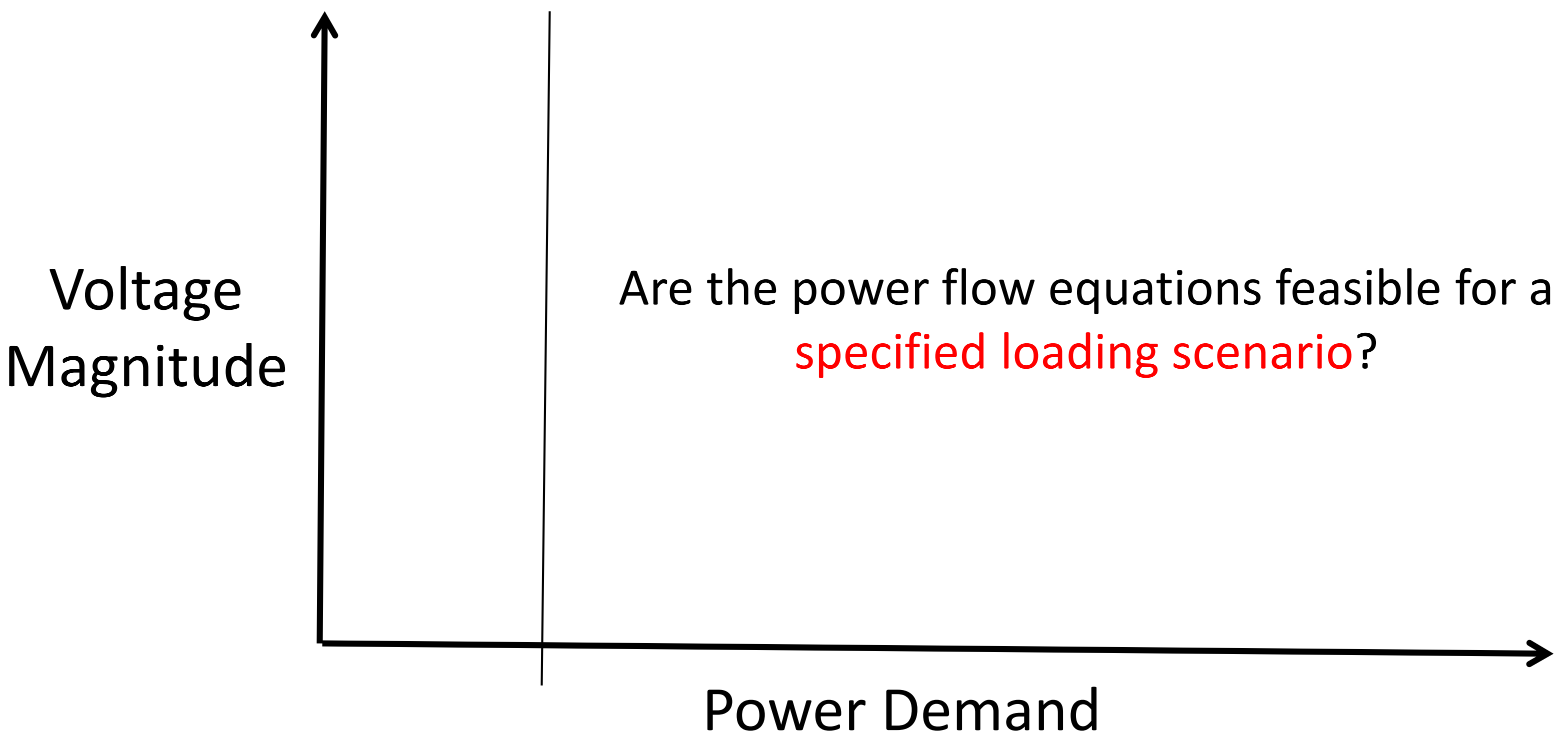
1. Global Solution via Spatial Branch-and-Bound

1. Segment the feasible space into adjoining subregions.
2. In each subregion, compute an upper bound using a local solver and a lower bound using a relaxation.
3. Eliminate subregions whose lower bounds are greater than an upper bound obtained in any other subregion.
4. Iterate until finding an upper bound that is sufficiently close to the least lower bound.



2. Certify Power Flow Insolvability

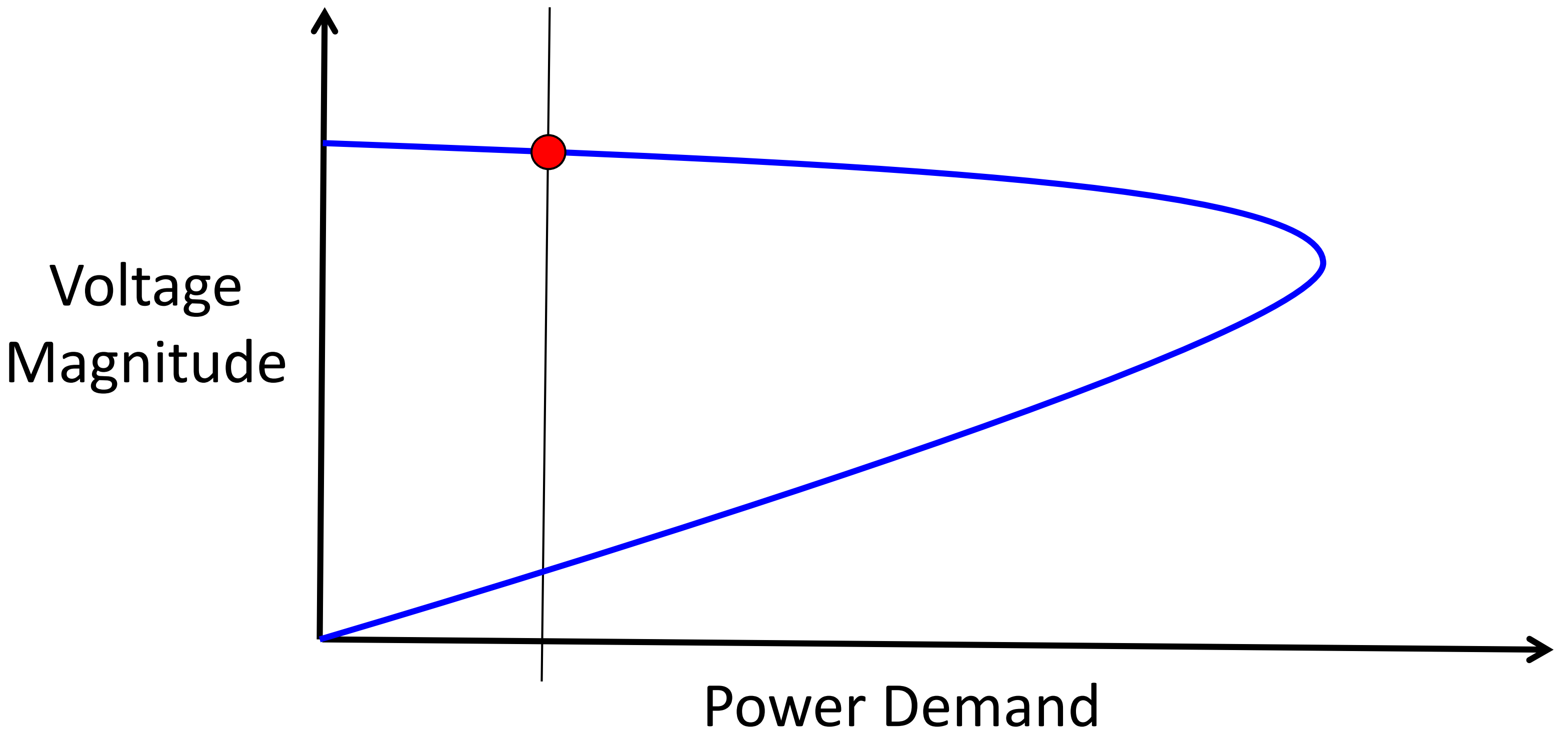
- Compute upper bounds on the maximum achievable loading.



[Molzahn, Lesieutre, & DeMarco ’13]

2. Certify Power Flow Insolvability

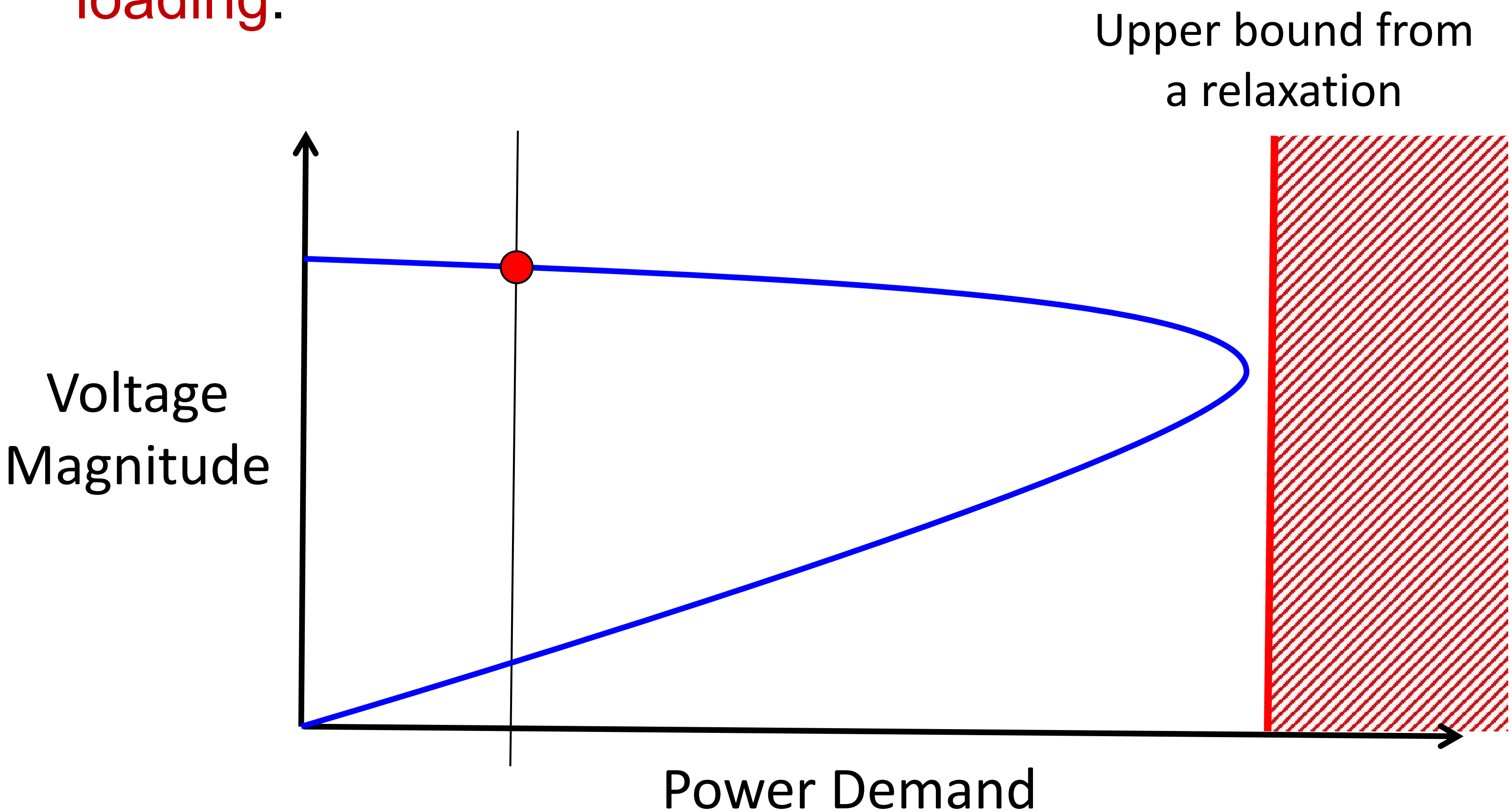
- Compute upper bounds on the maximum achievable loading.



[Molzahn, Lesieutre, & DeMarco '13]

2. Certify Power Flow Insolvability

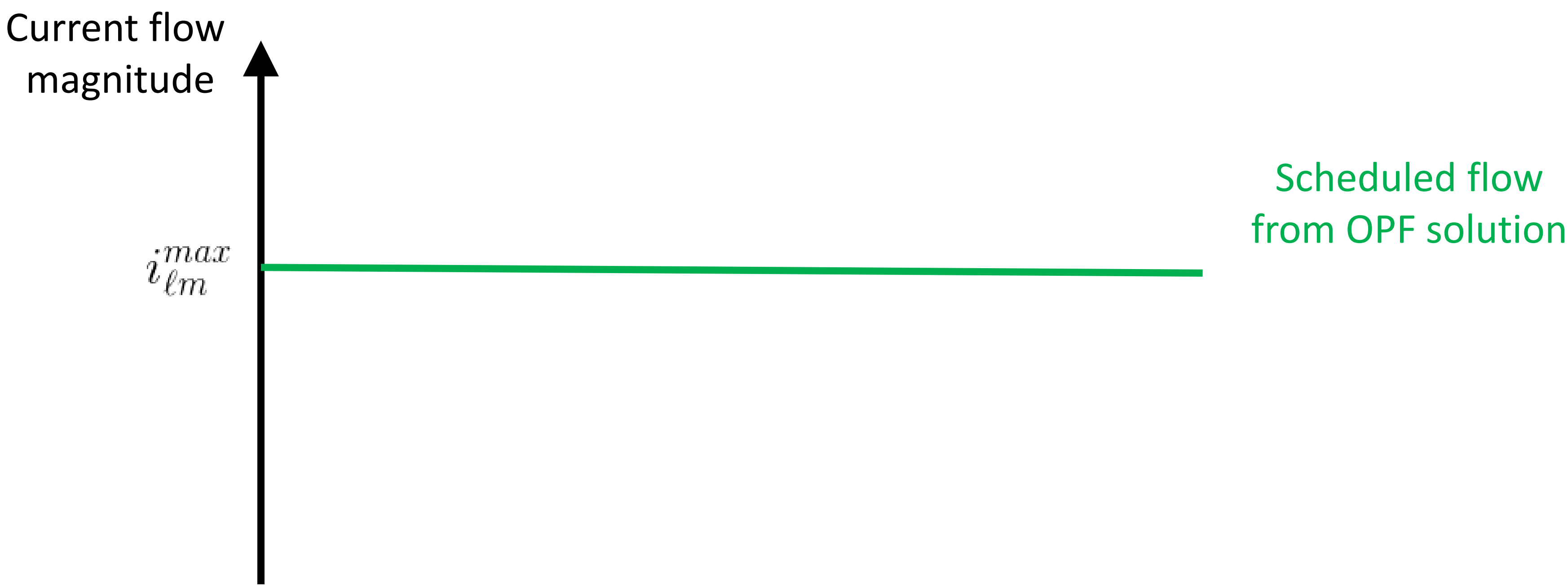
- Compute upper bounds on the maximum achievable loading.



[Molzahn, Lesieutre, & DeMarco ’13]

3. Robust Optimal Power Flow

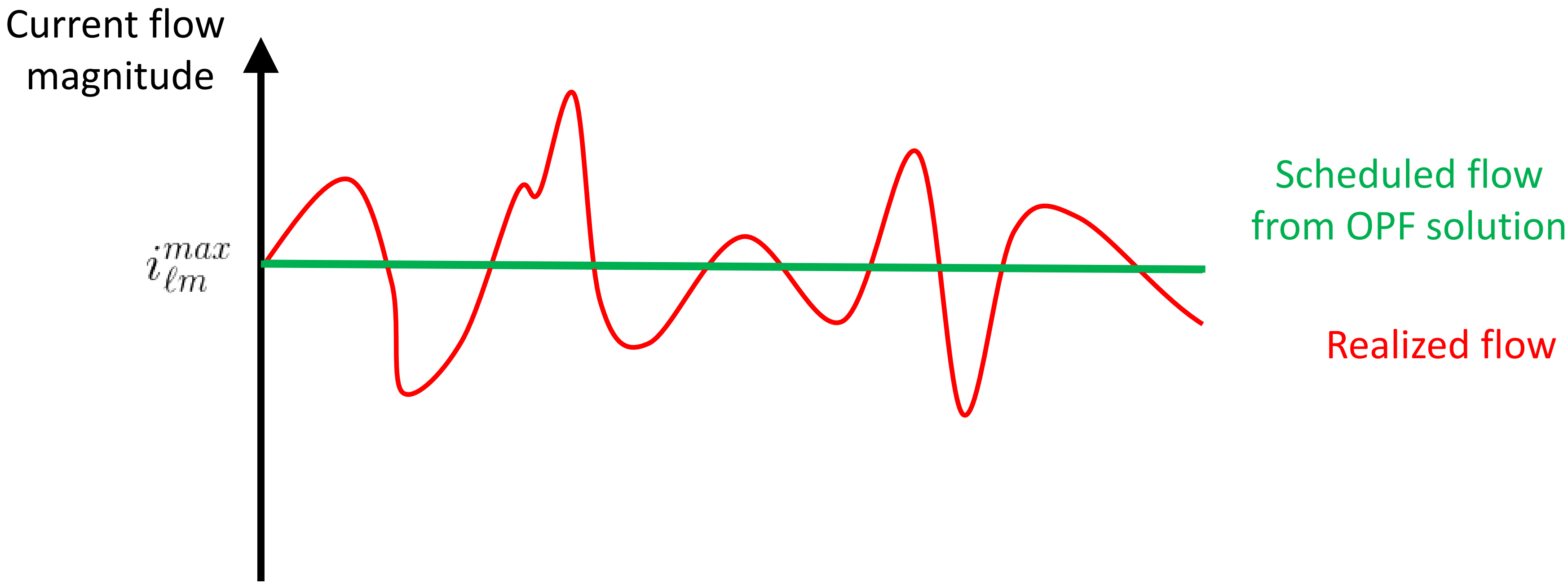
- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.



[Molzahn & Roald ’18]

3. Robust Optimal Power Flow

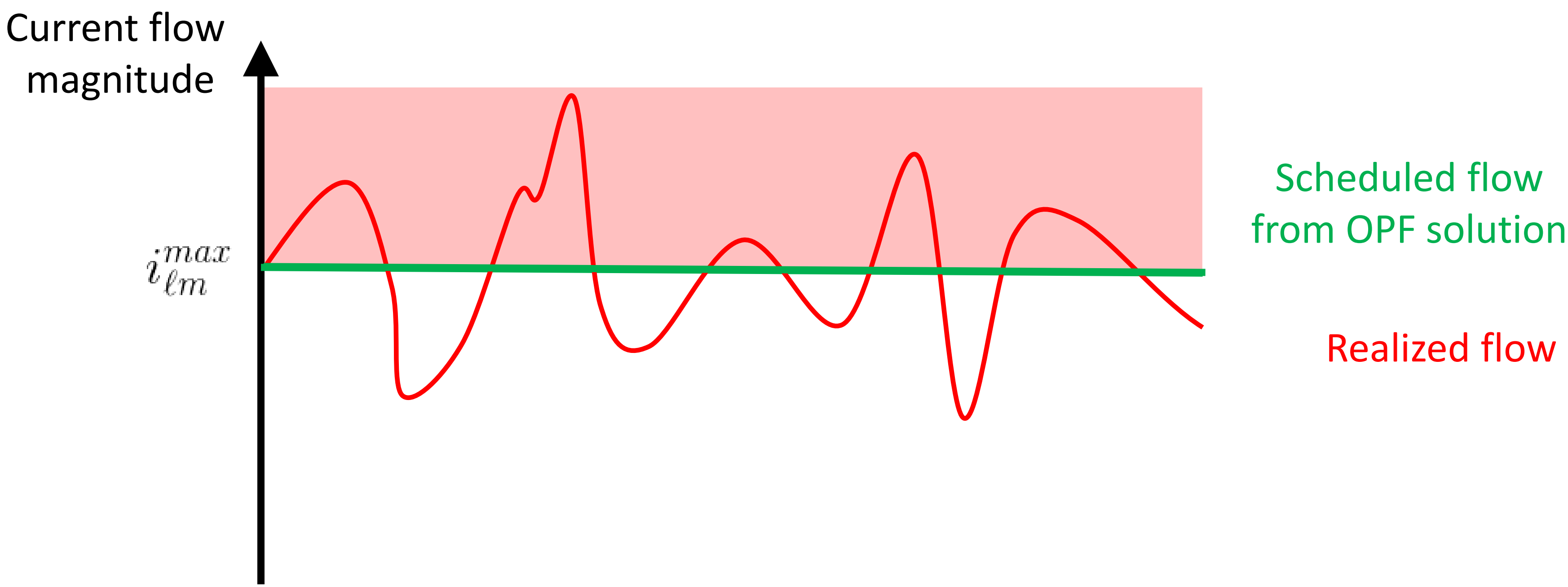
- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.



[Molzahn & Roald '18]

3. Robust Optimal Power Flow

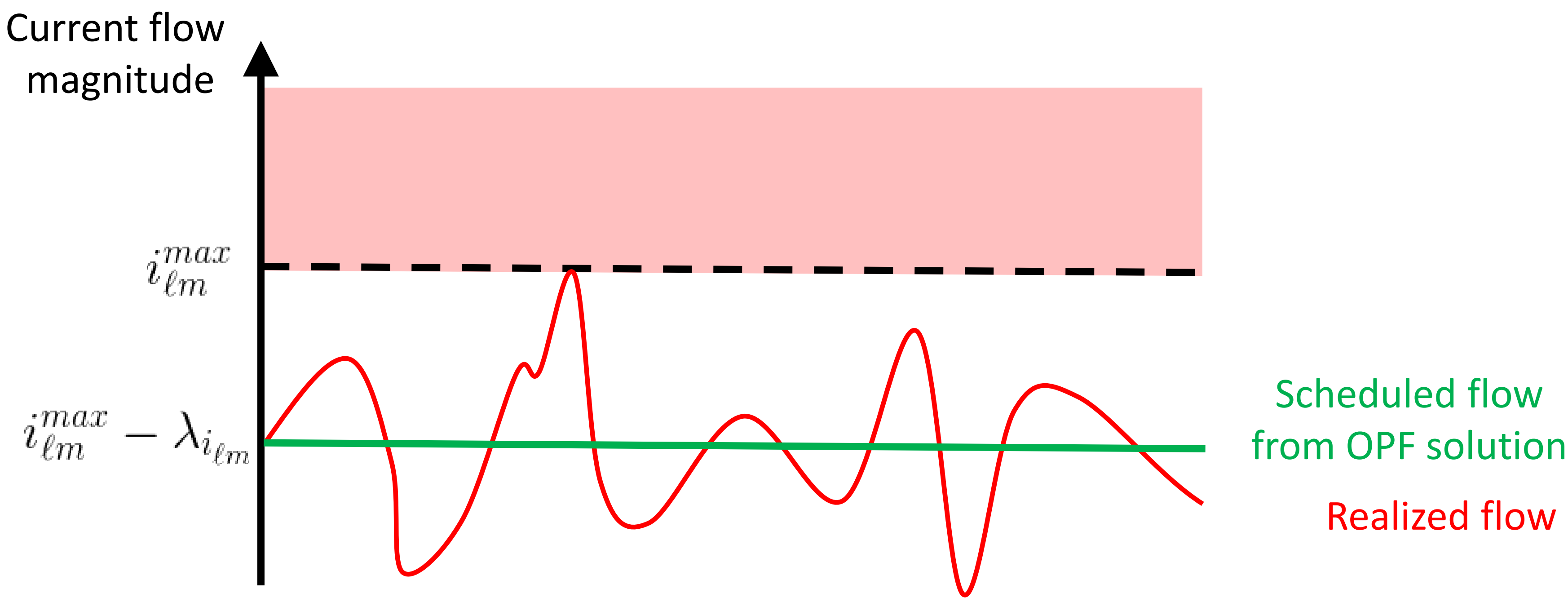
- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.



[Molzahn & Roald ’18]

3. Robust Optimal Power Flow

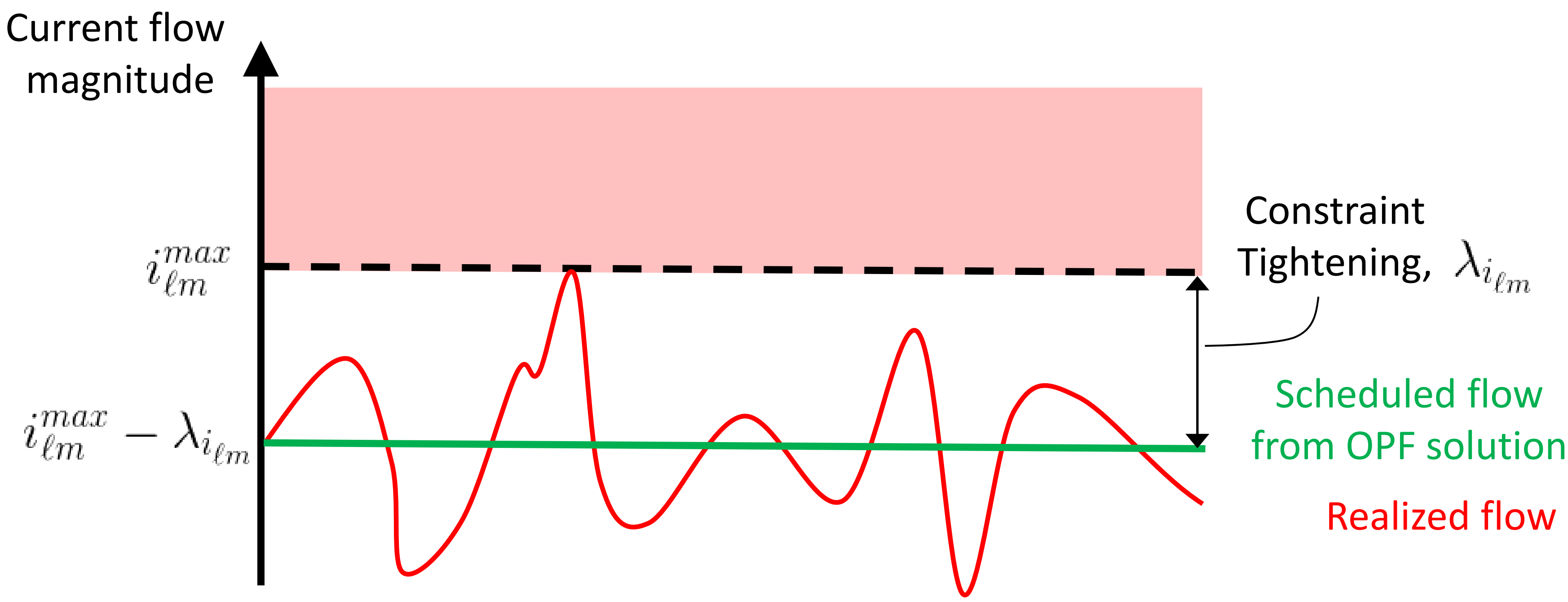
- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.



[Molzahn & Roald ’18]

3. Robust Optimal Power Flow

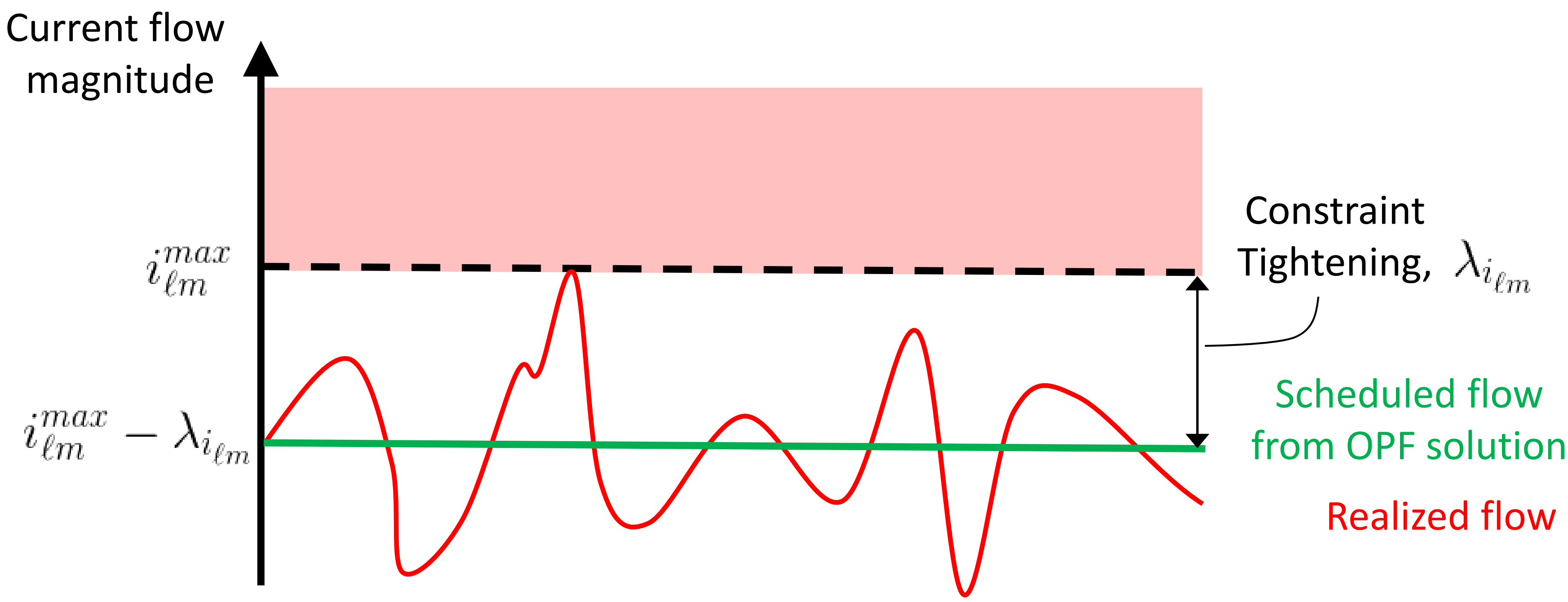
- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.



[Molzahn & Roald ’18]

3. Robust Optimal Power Flow

- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints.

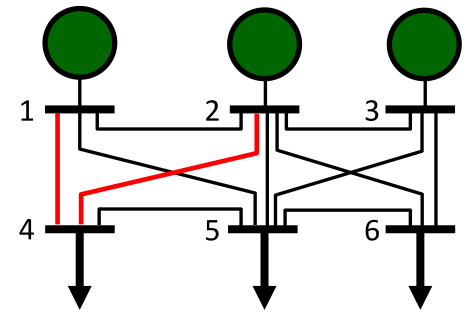


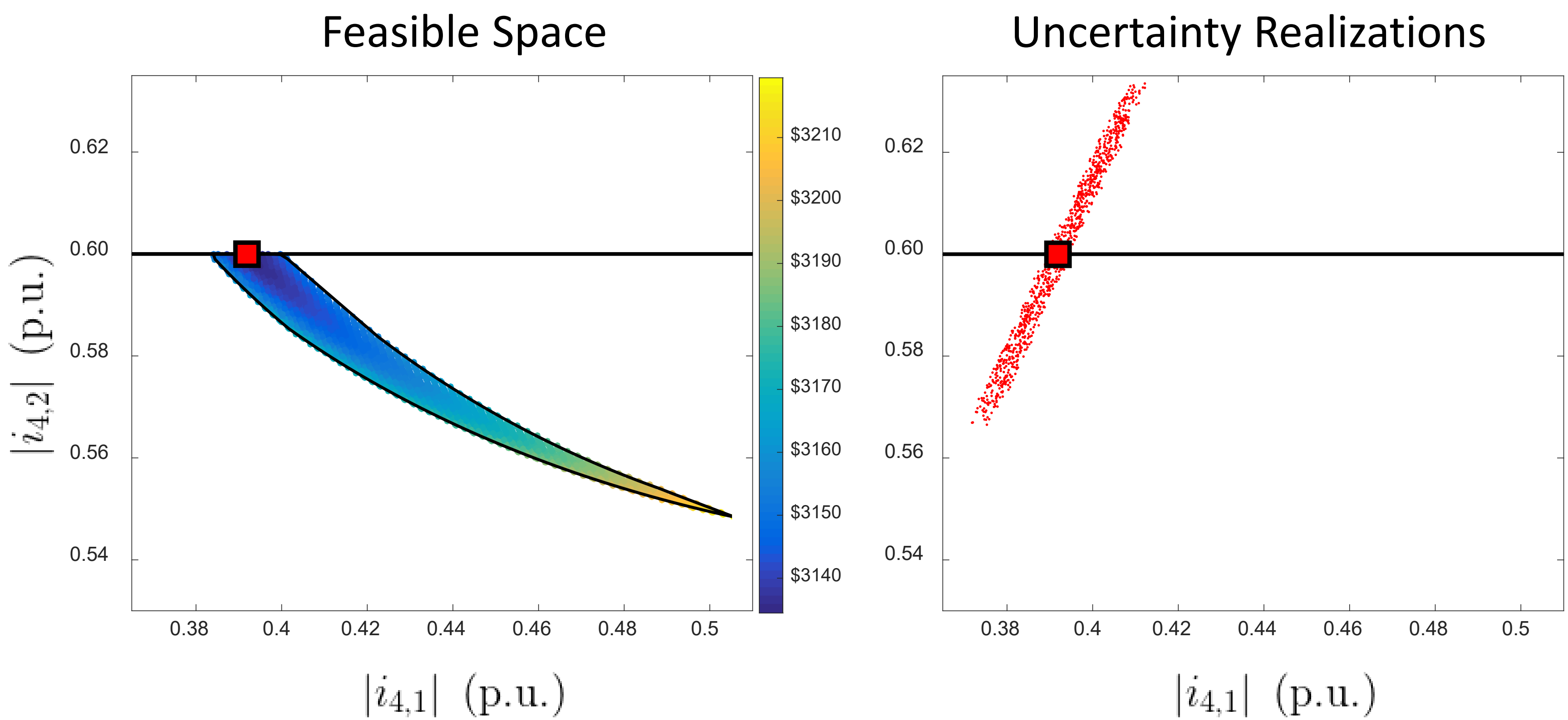
Define appropriate constraint tightenings using convex relaxations.

[Molzahn & Roald '18]

Robust OPF Example

- 6-bus system “case6ww”
  - ±5% uncertainty in each load demand

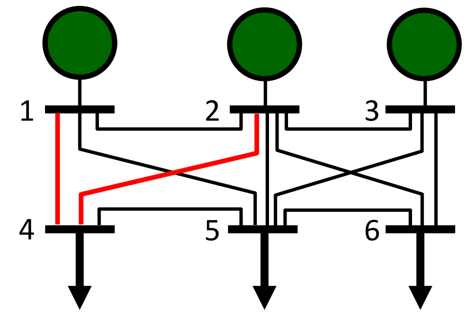


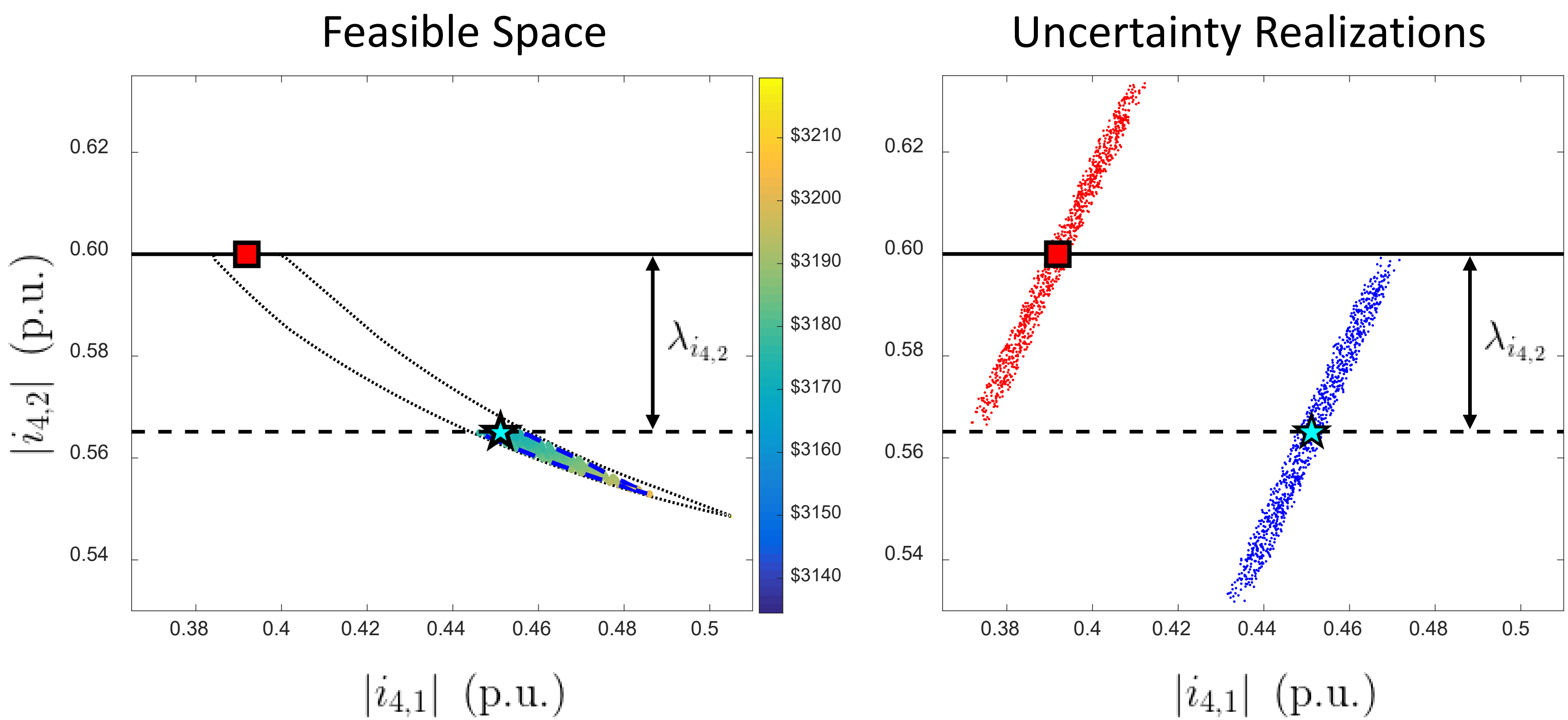


[Molzahn & Roald ’18]

Robust OPF Example

- 6-bus system "case6ww"
  - $\pm$5% uncertainty in each load demand



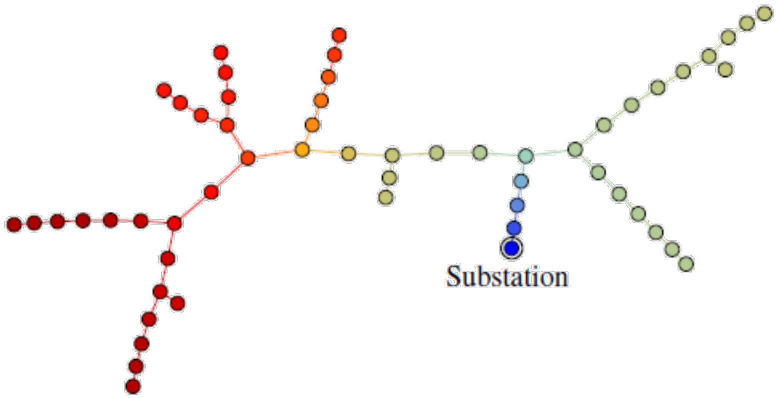


[Molzahn & Roald '18]

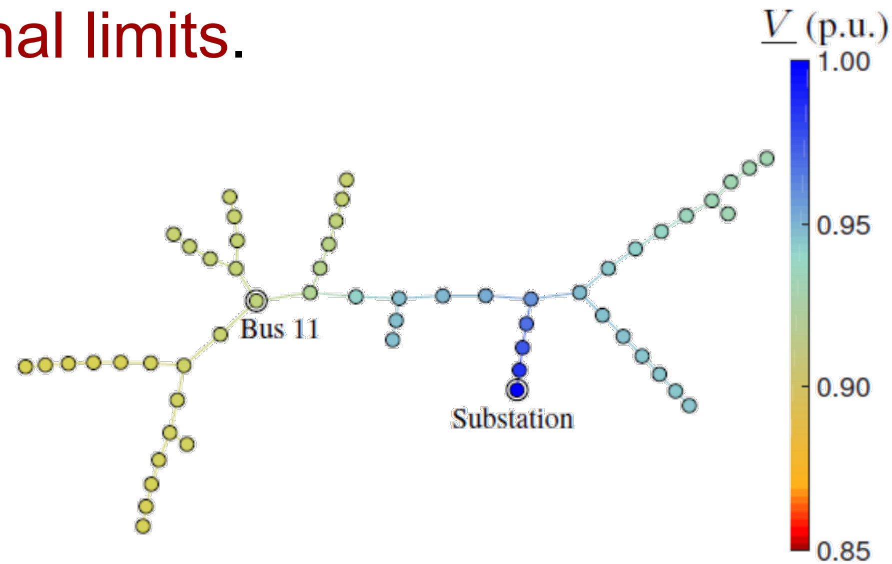
4. Certify Distribution Grid Security

- For specified ranges of variable power injections, use convex relaxations to certify that limited measurements and control are sufficient to preclude constraint violations.

- Certify security if upper bounds on worst-case violations are within desired operational limits.



Insufficient measurement and control to certify security



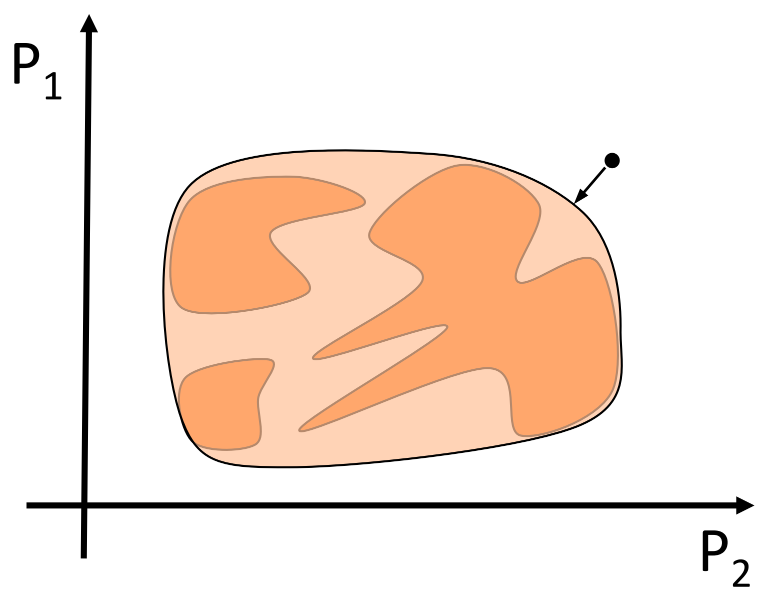
Voltage control at bus 11 is sufficient to certify security

[Molzahn & Roald '19]

Stay Tuned! PSERC Project T-64: Who Controls the DERs?

5. Create Machine Learning Datasets

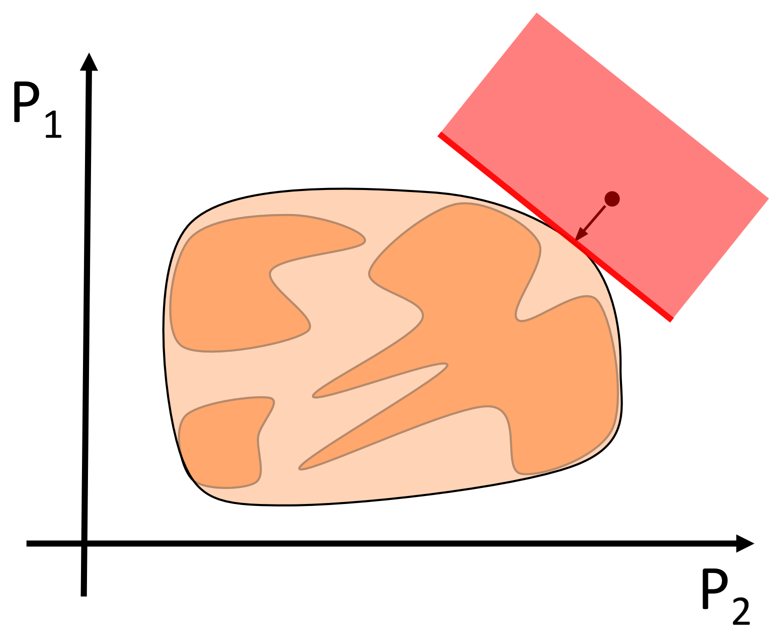
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn ’17], [Venzke, Molzahn, & Chatzivasileiadis ’19]

5. Create Machine Learning Datasets

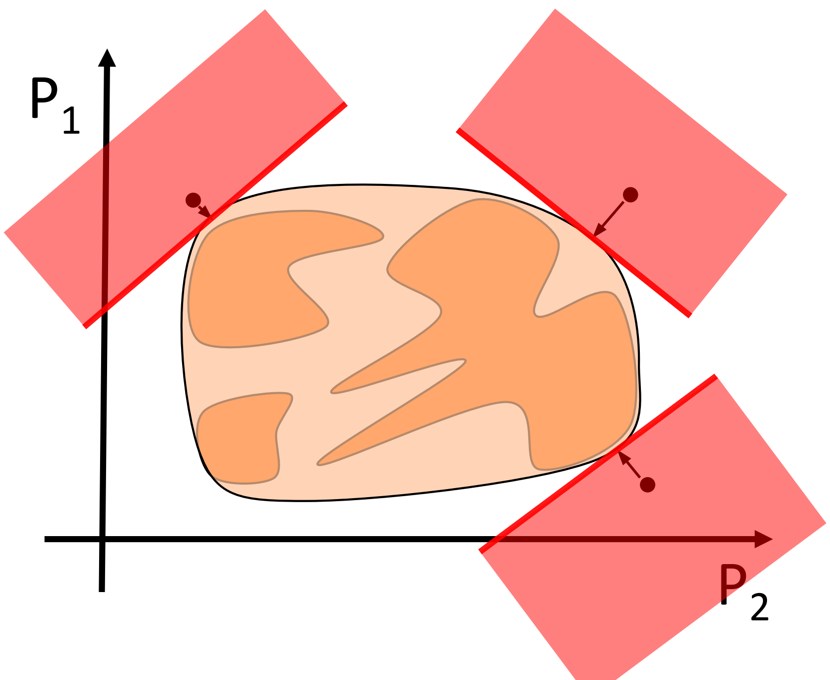
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn '17], [Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

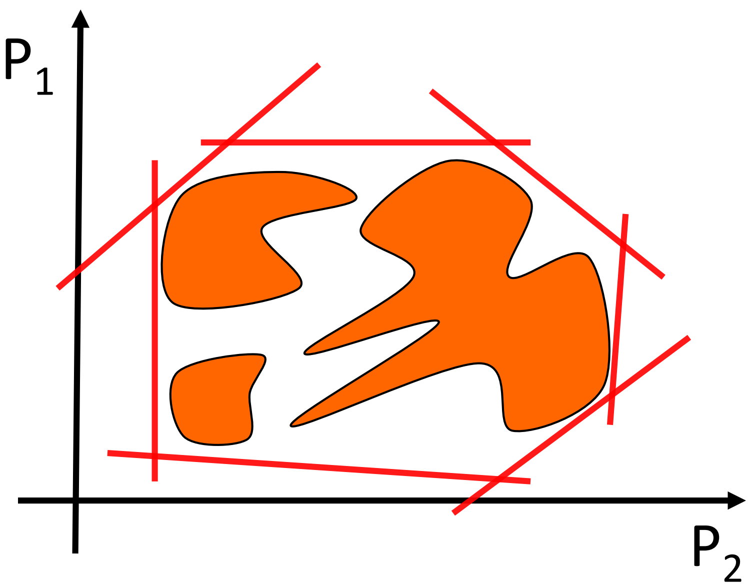
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn '17],
[Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

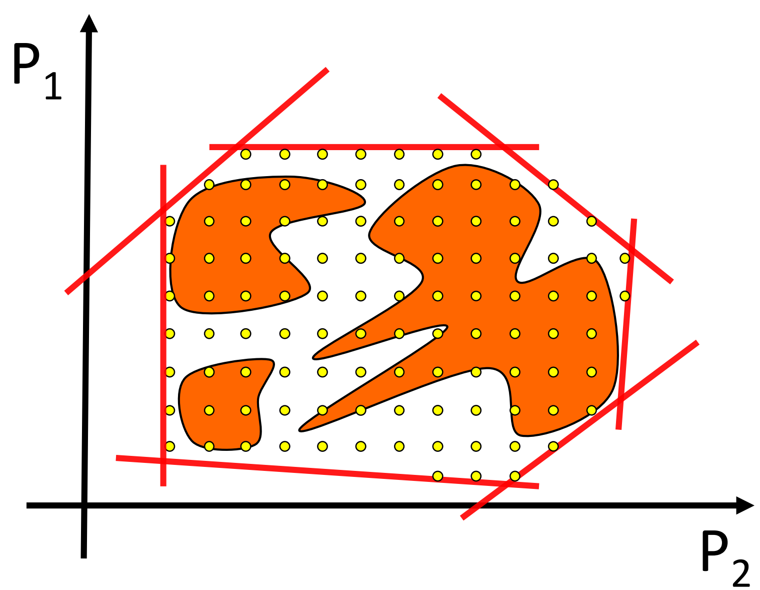
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn '17], [Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

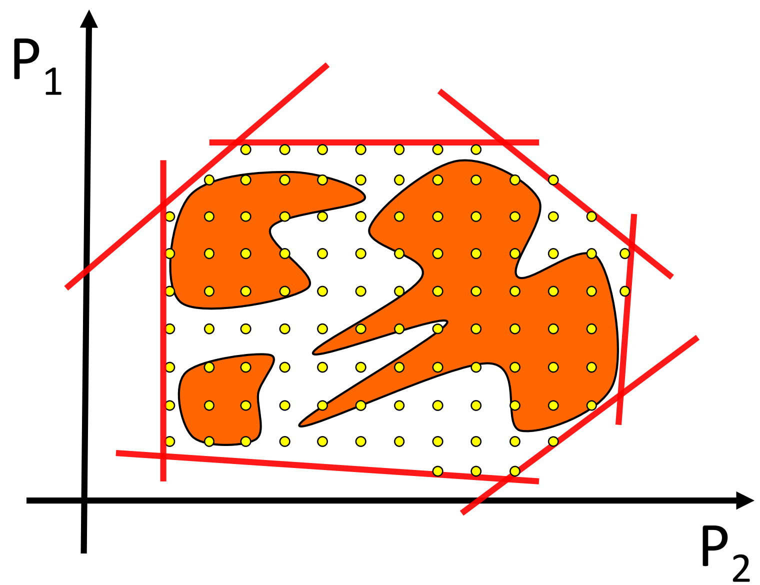
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn '17], [Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

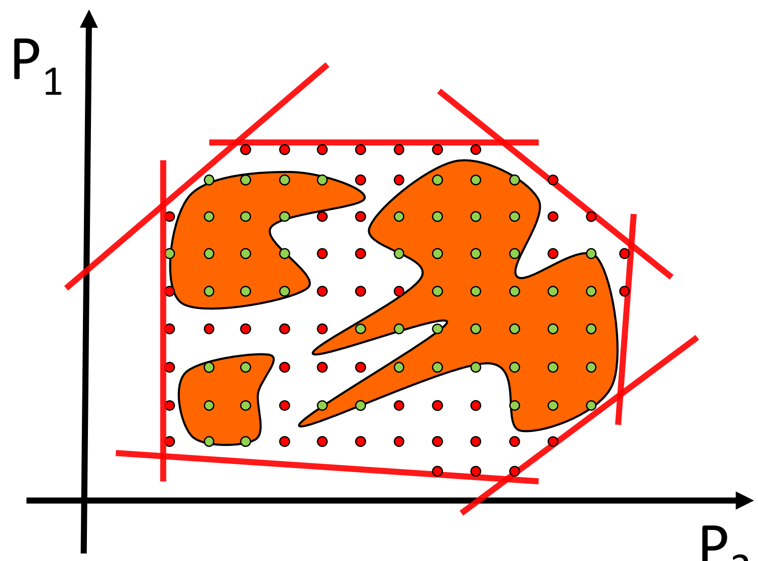
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



[Molzahn '17], [Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

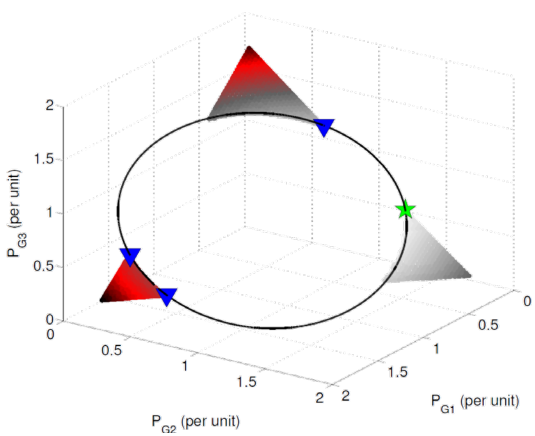
1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.

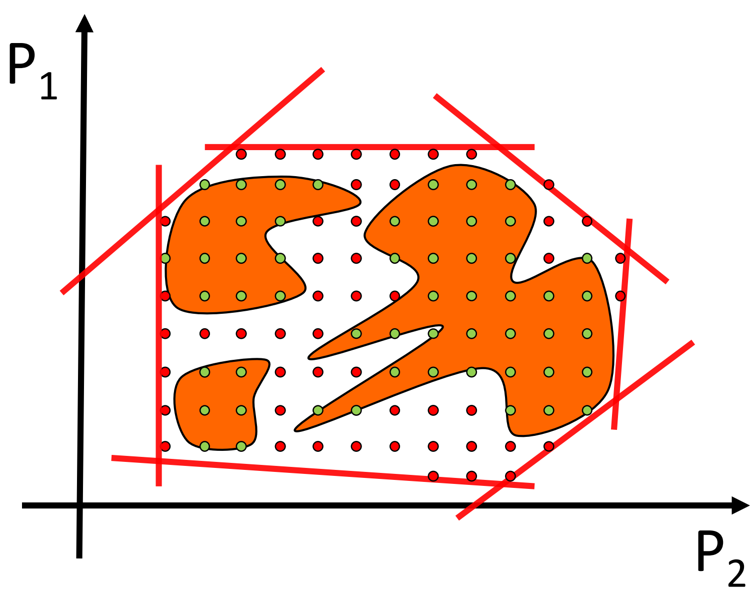


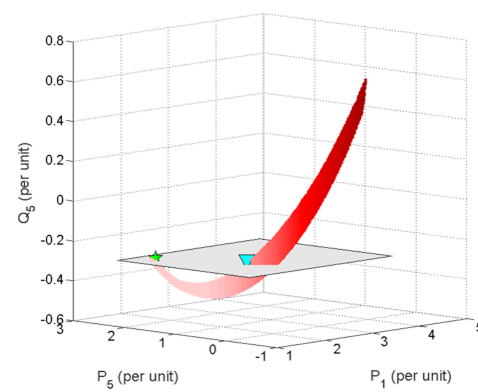
[Molzahn '17],
[Venzke, Molzahn, & Chatzivasileiadis '19]

5. Create Machine Learning Datasets

1. Use convex relaxations to construct an enclosing polytope.
2. Sample points inside the polytope.
3. Calculate the power flow solutions at each sampled point.
4. Classify feasible and infeasible points.



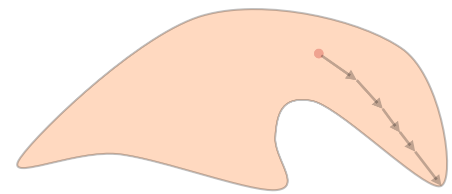




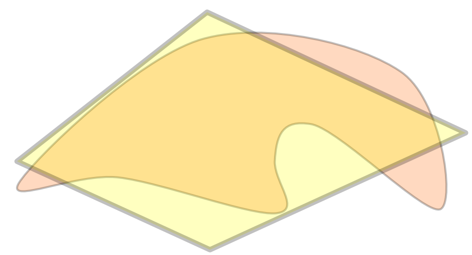
[Molzahn ’17],
[Venzke, Molzahn, & Chatzivasileiadis ’19]

Methods for Handling Nonlinearities

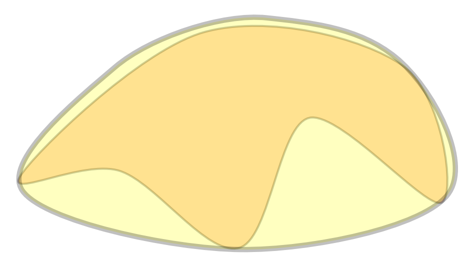
- Local optimization



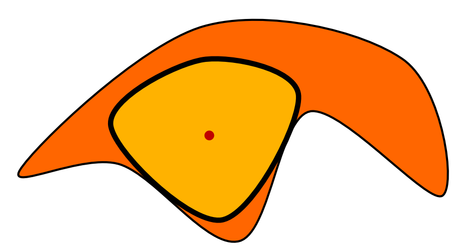
- Approximation



- Convex relaxation

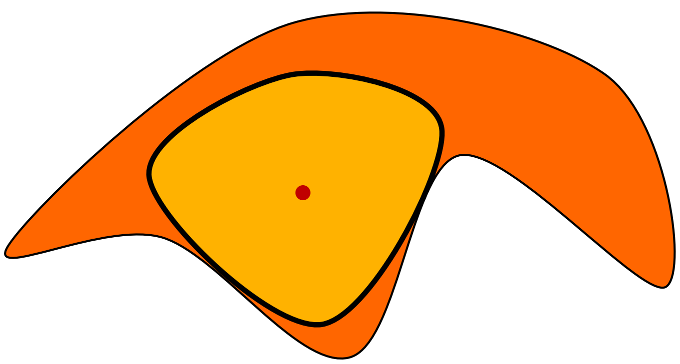


- Convex restriction



Convex Restrictions

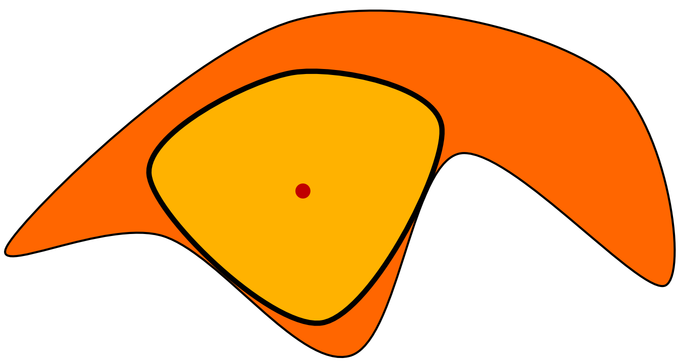
- Compute convex regions that are completely contained within the non-convex feasible space.
- Based on fixed-point theorems.
- Dependent on a base point.



[Cui & Sun ’19], [Lee, Nguyen, Dvijotham, Turitsyn ’19]

Convex Restrictions

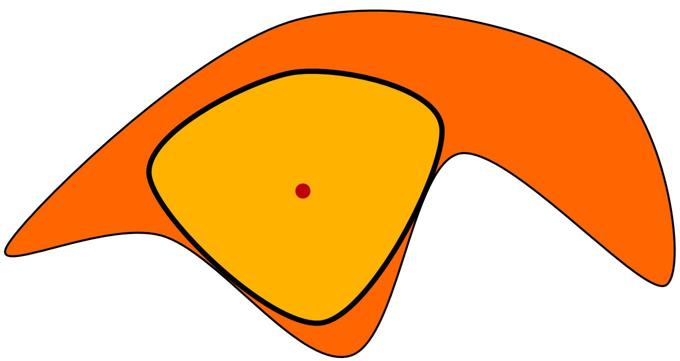
- Compute convex regions that are completely contained within the non-convex feasible space.
- Based on fixed-point theorems.
- Dependent on a base point.



[Cui & Sun ’19], [Lee, Nguyen, Dvijotham, Turitsyn ’19]

Convex Restrictions

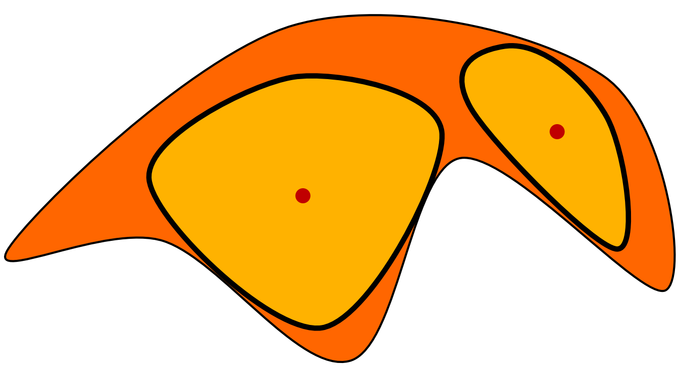
- Compute convex regions that are completely contained within the non-convex feasible space.
- Based on fixed-point theorems.
- Dependent on a base point.



[Cui & Sun ’19], [Lee, Nguyen, Dvijotham, Turitsyn ’19]

Convex Restrictions

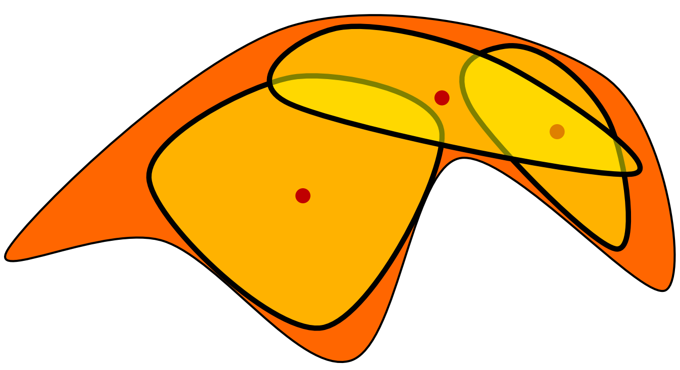
- Compute convex regions that are completely contained within the non-convex feasible space.
- Based on fixed-point theorems.
- Dependent on a base point.



[Cui & Sun ’19], [Lee, Nguyen, Dvijotham, Turitsyn ’19]

Convex Restrictions

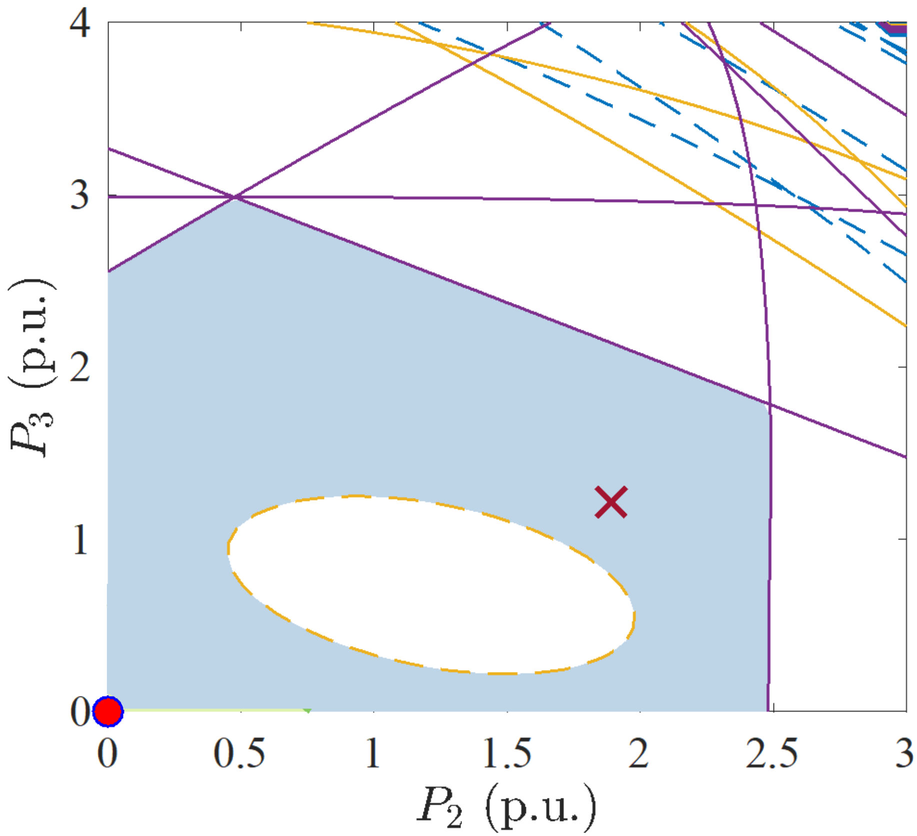
- Compute convex regions that are completely contained within the non-convex feasible space.
- Based on fixed-point theorems.
- Dependent on a base point.



[Cui & Sun '19], [Lee, Nguyen, Dvijotham, Turitsyn '19]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

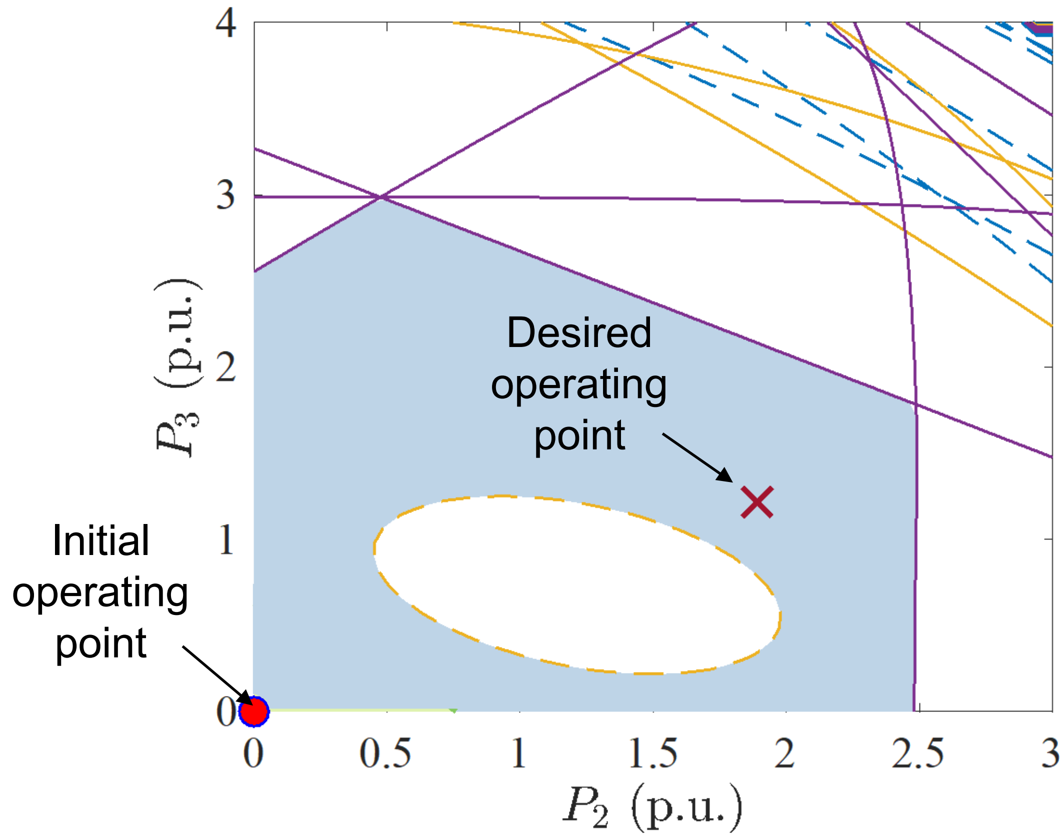


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

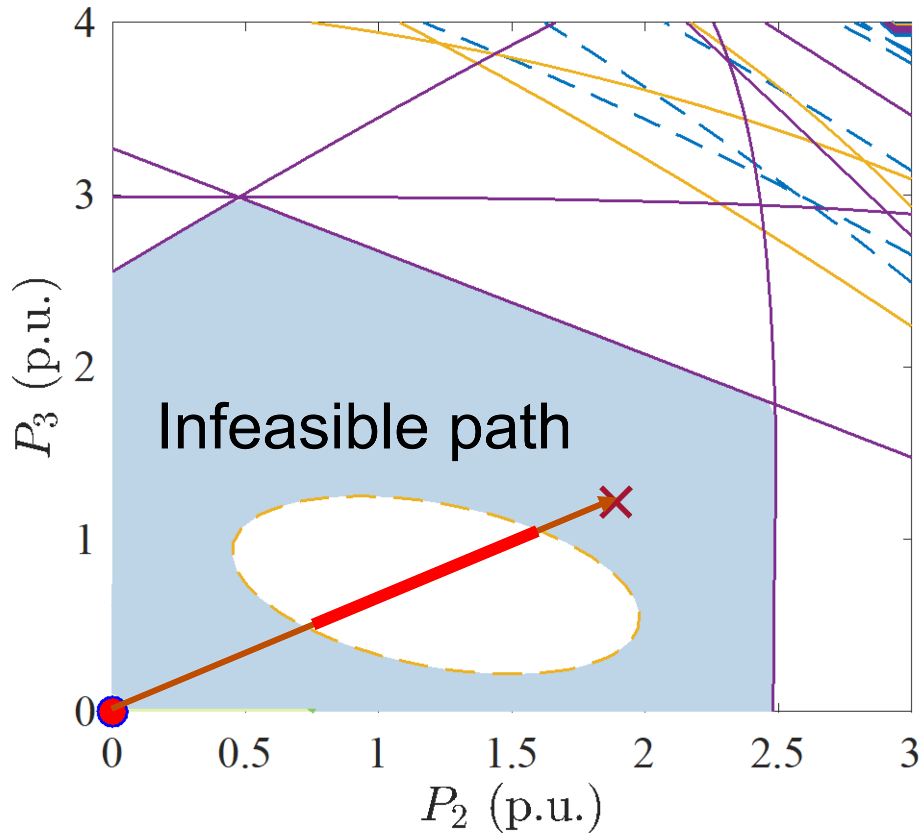


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

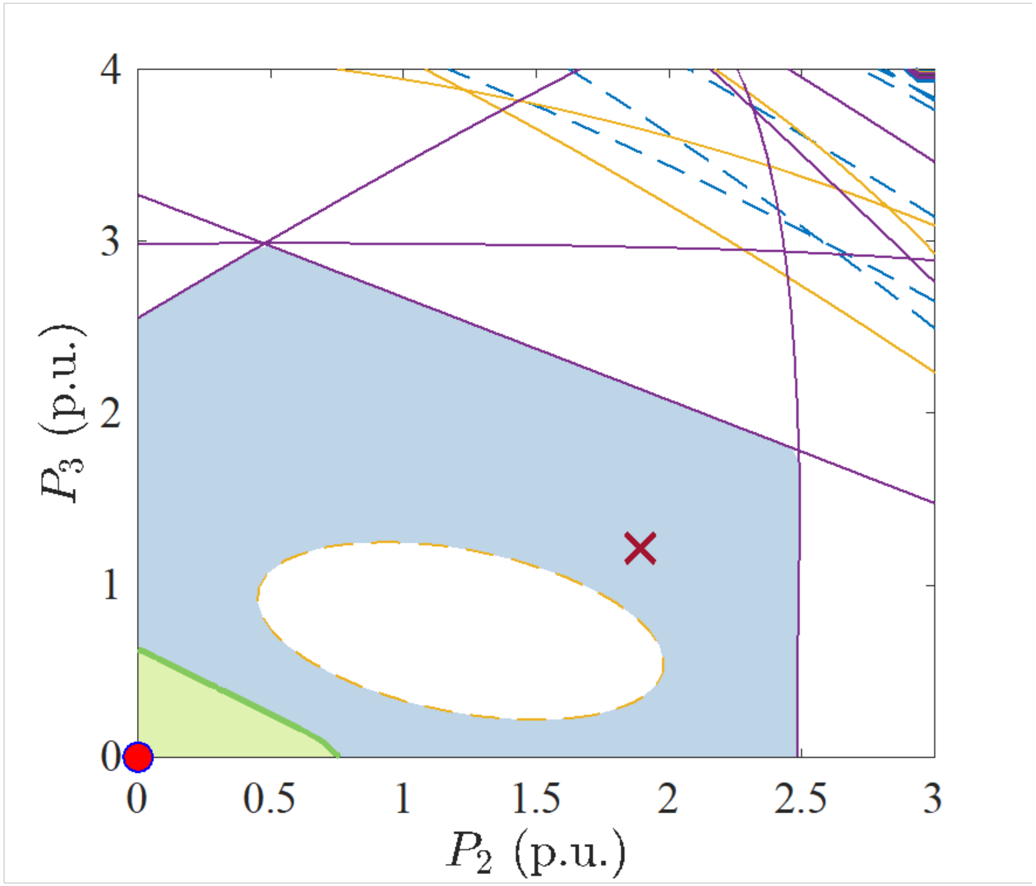


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

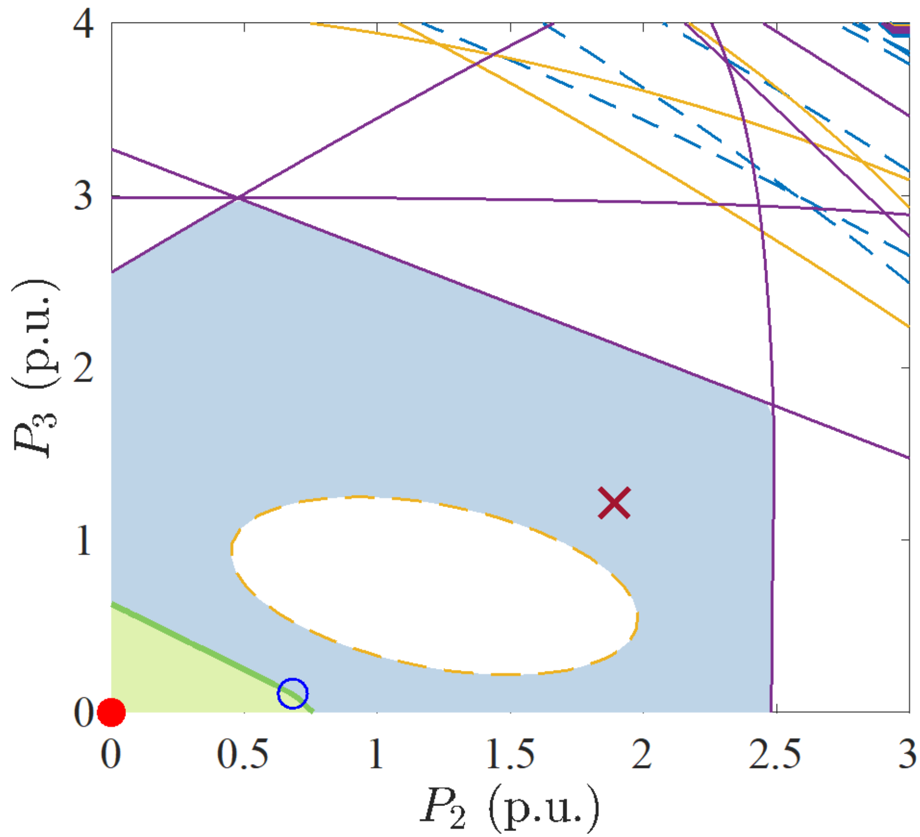


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

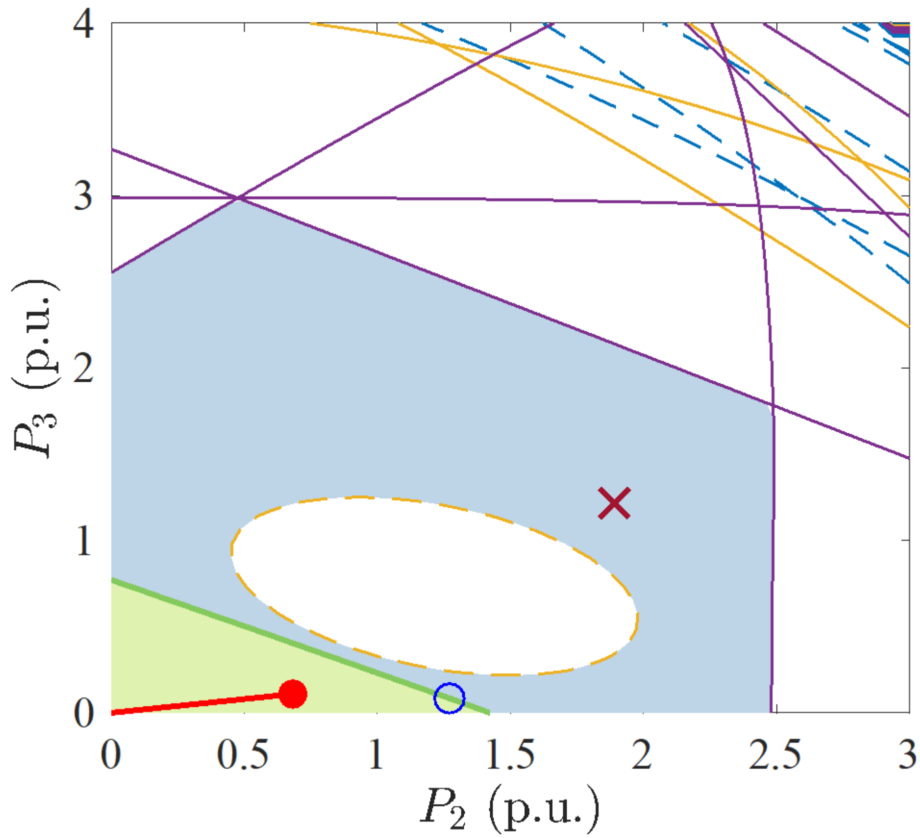


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

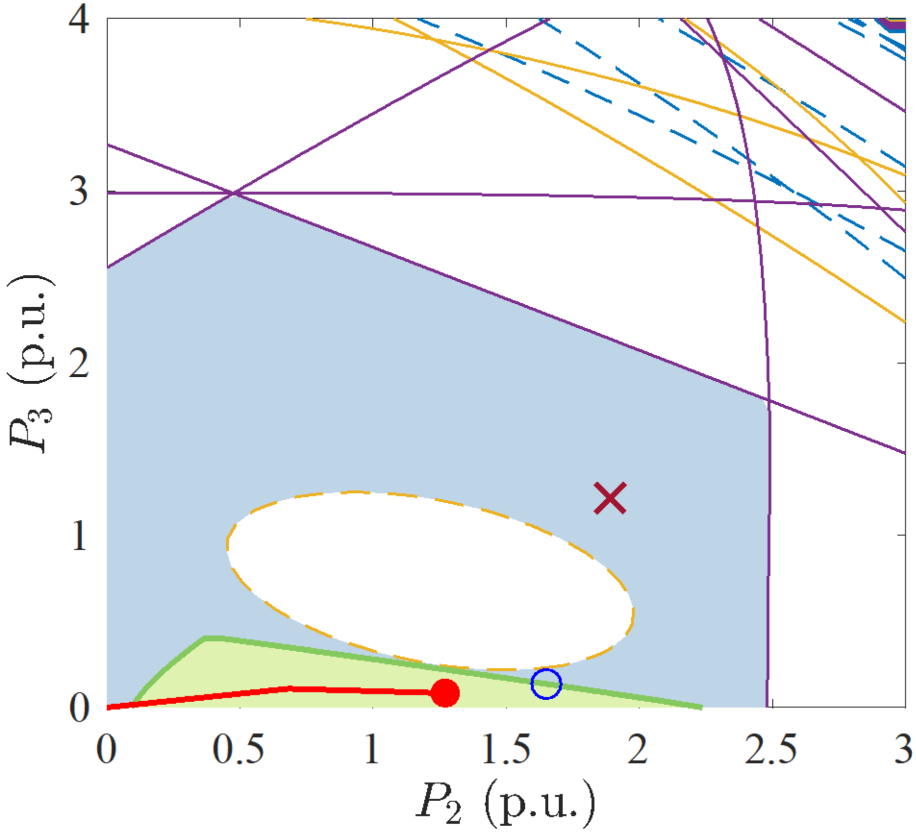


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

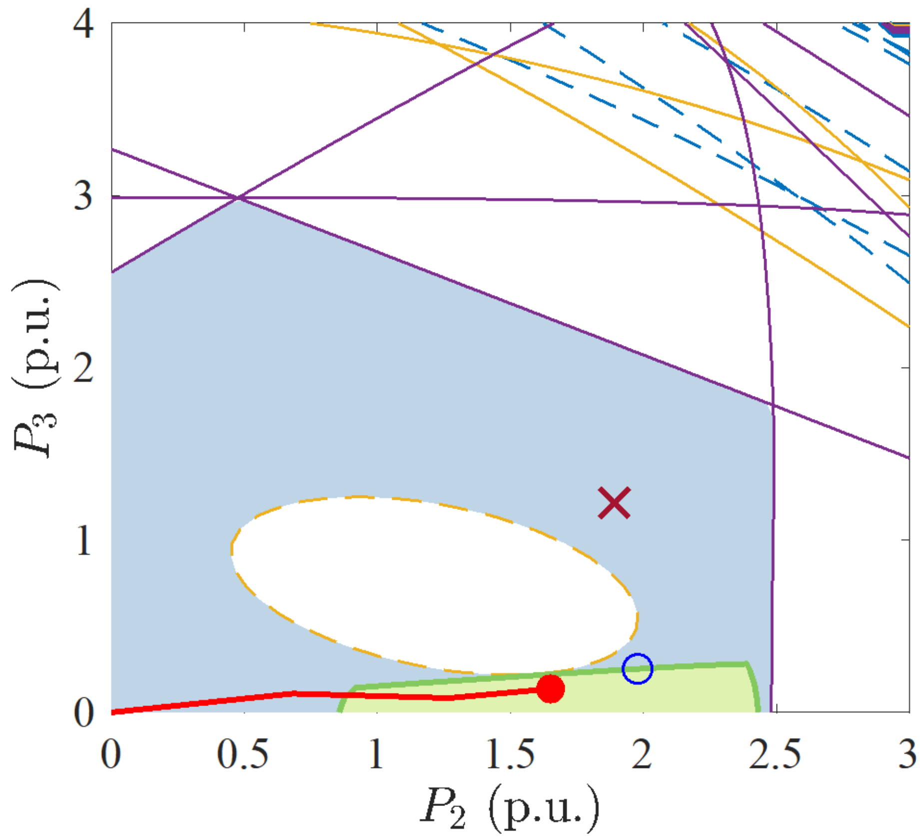


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

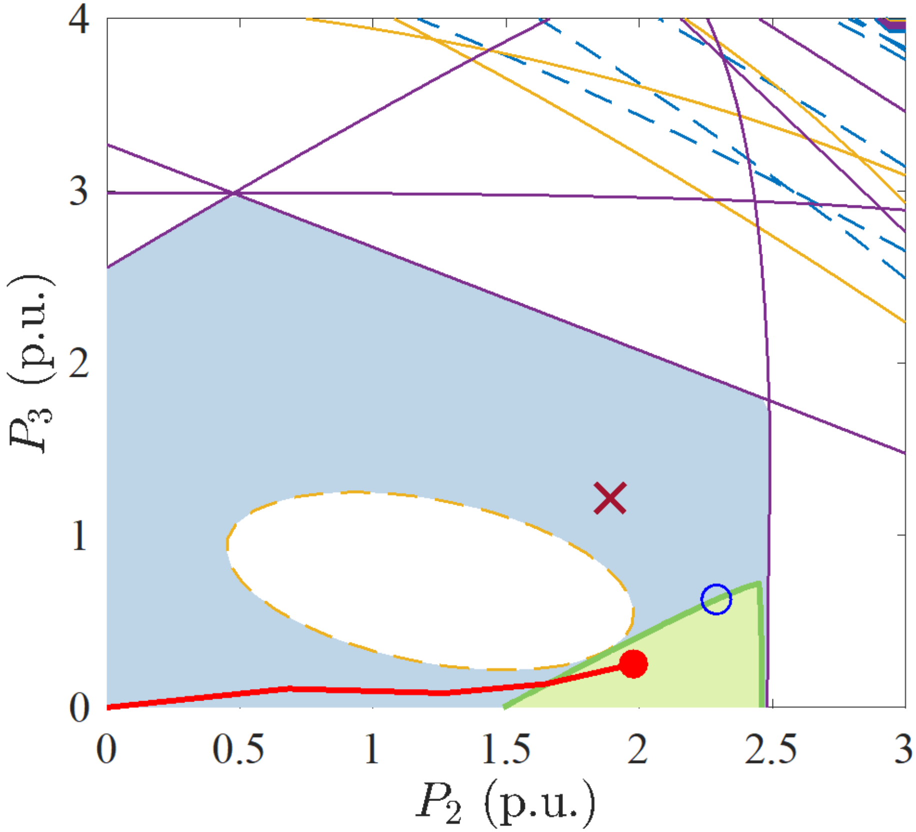


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

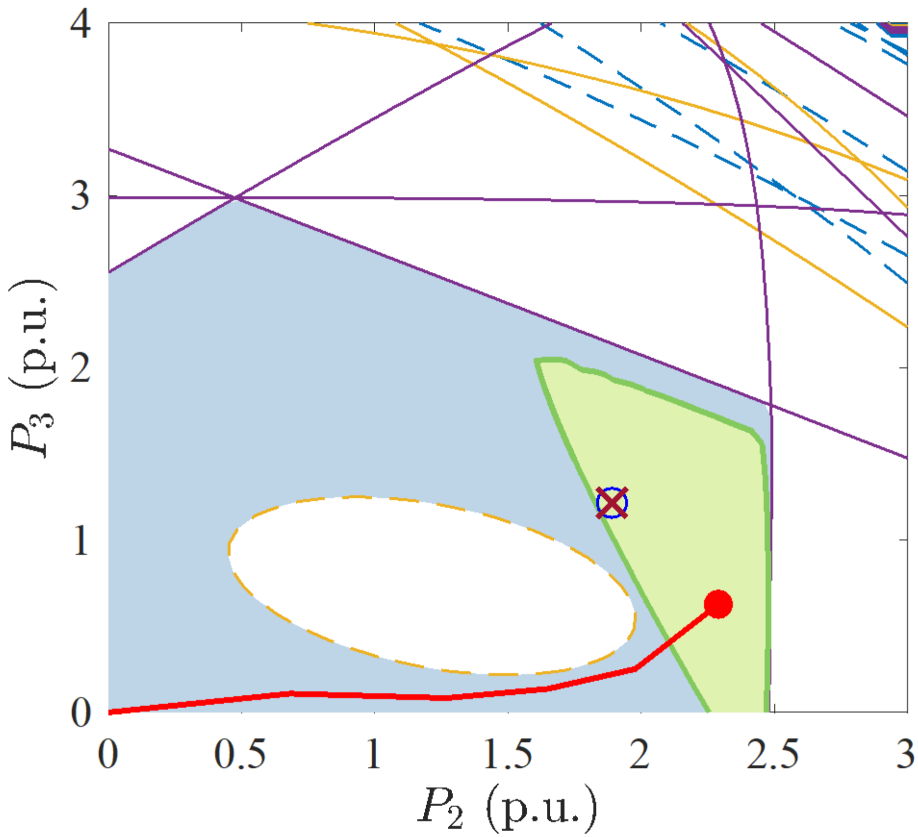


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

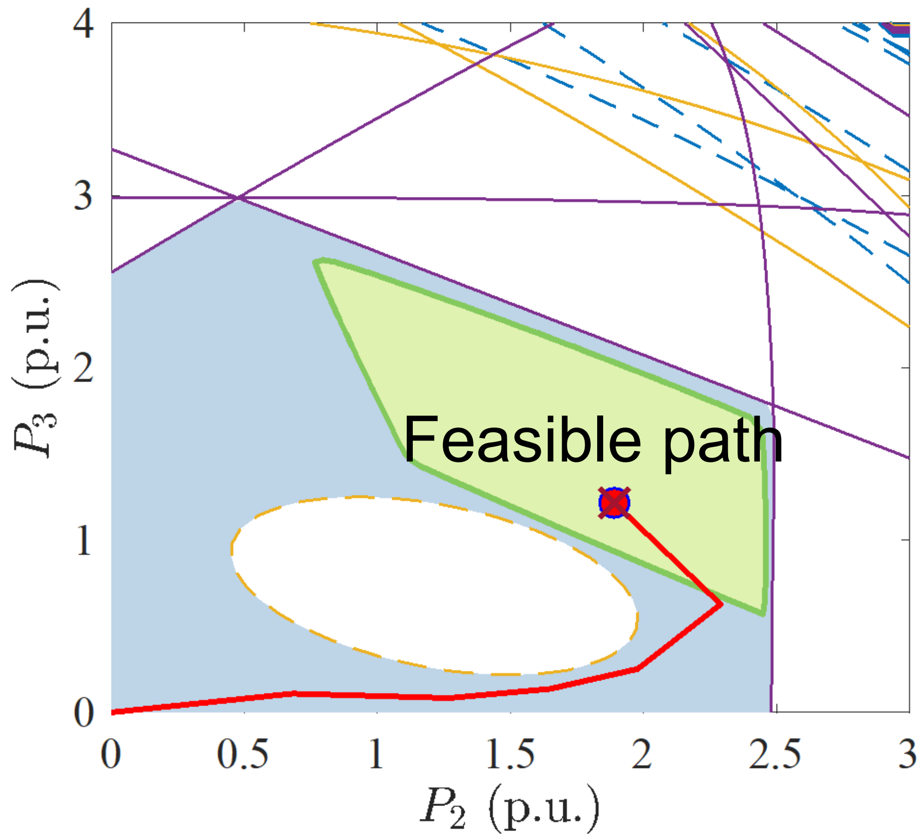


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 1: Feasible Path

- Goal: Compute a path between operating points that is guaranteed to avoid constraint violations.

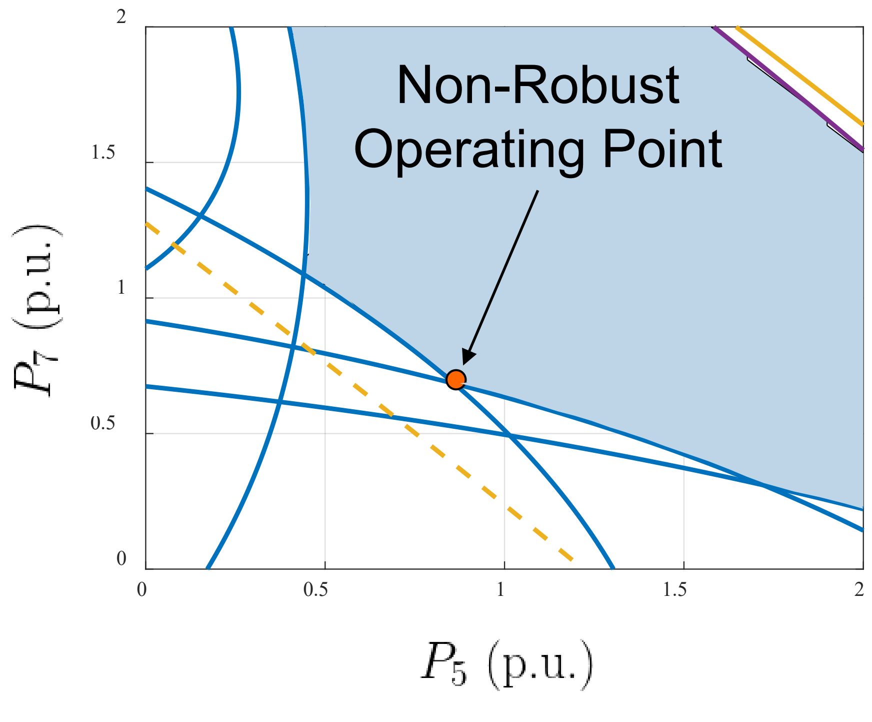


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald '20]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

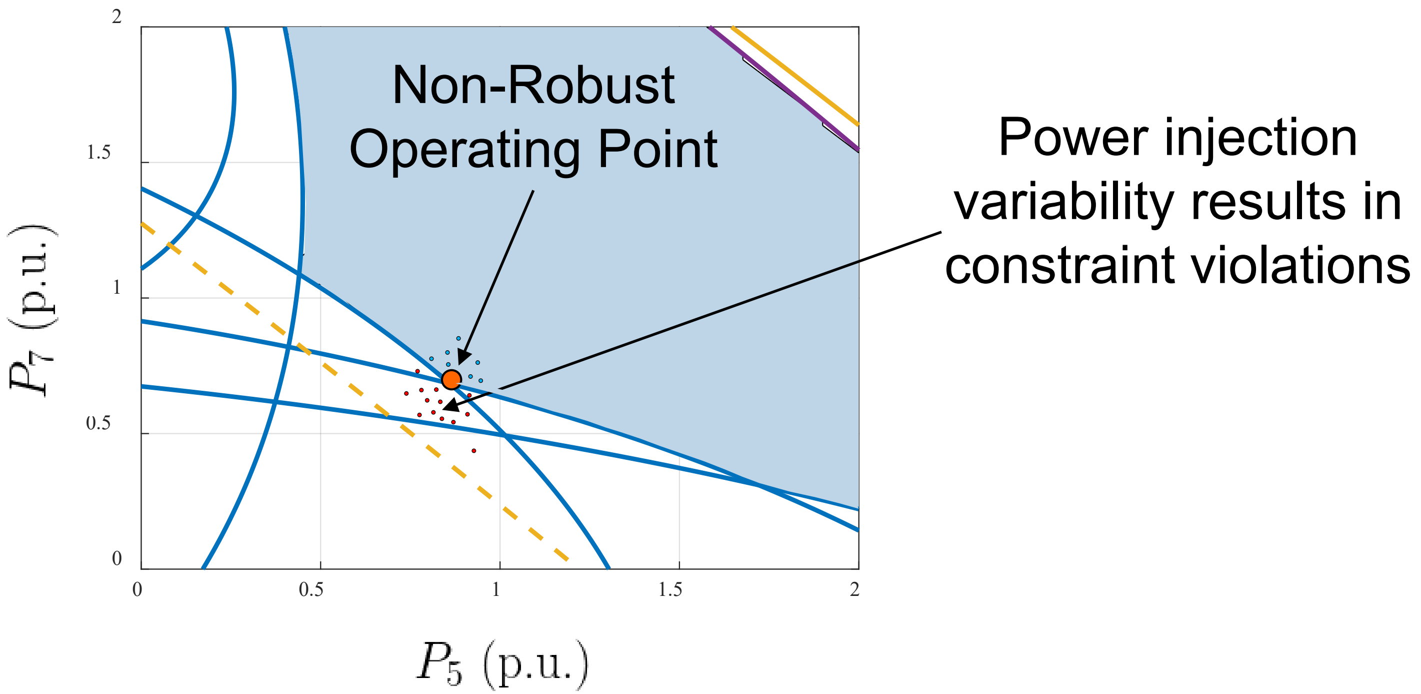


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

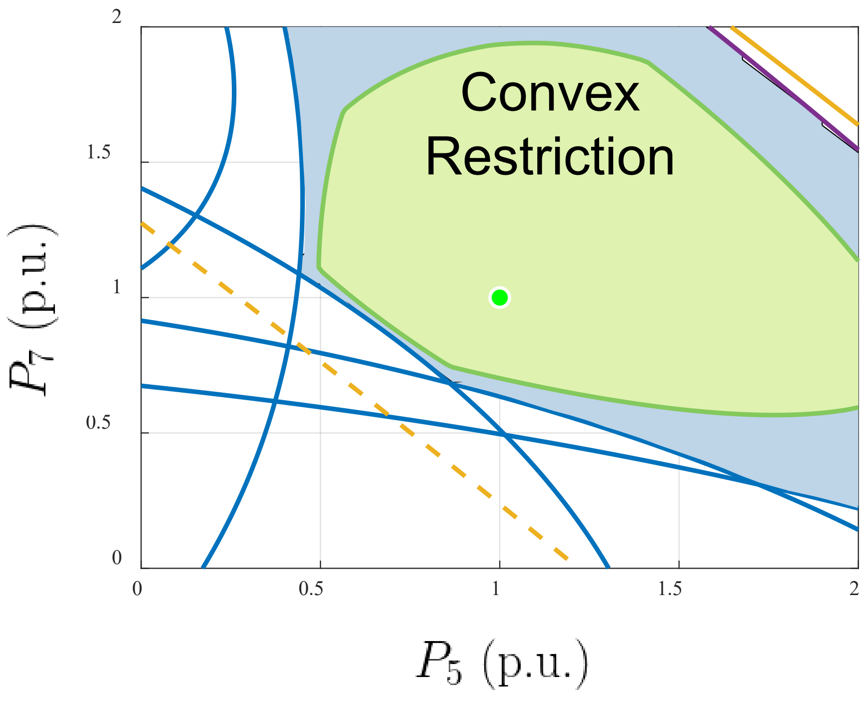


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

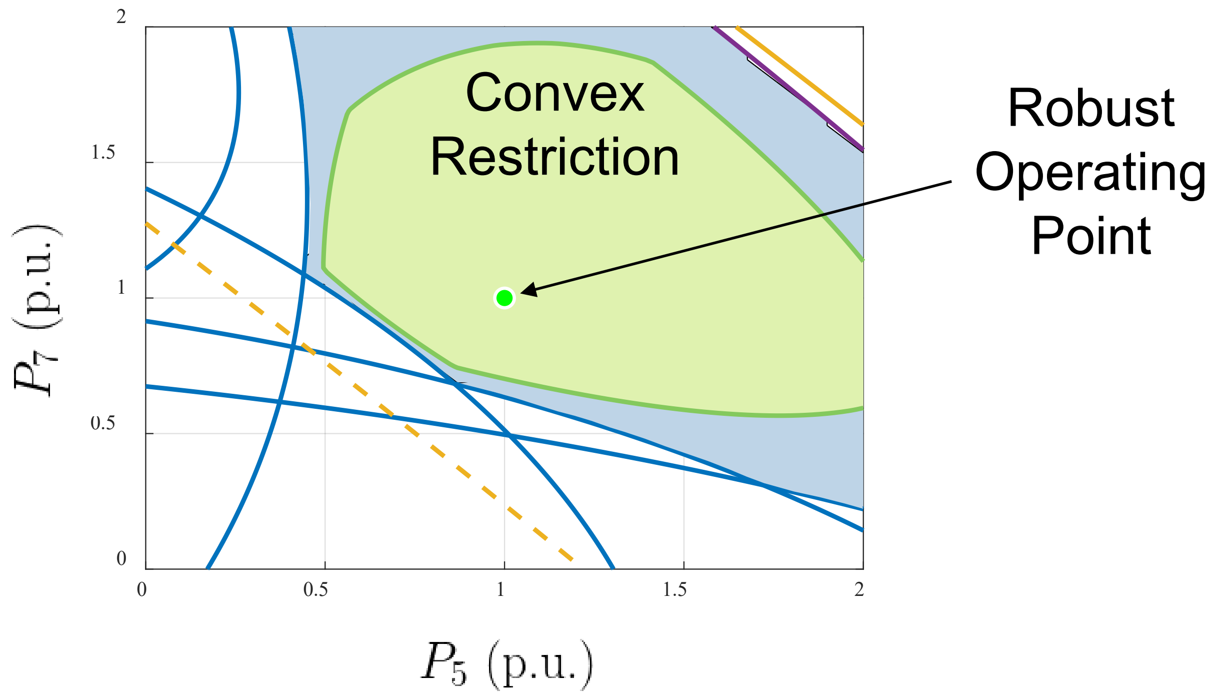


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

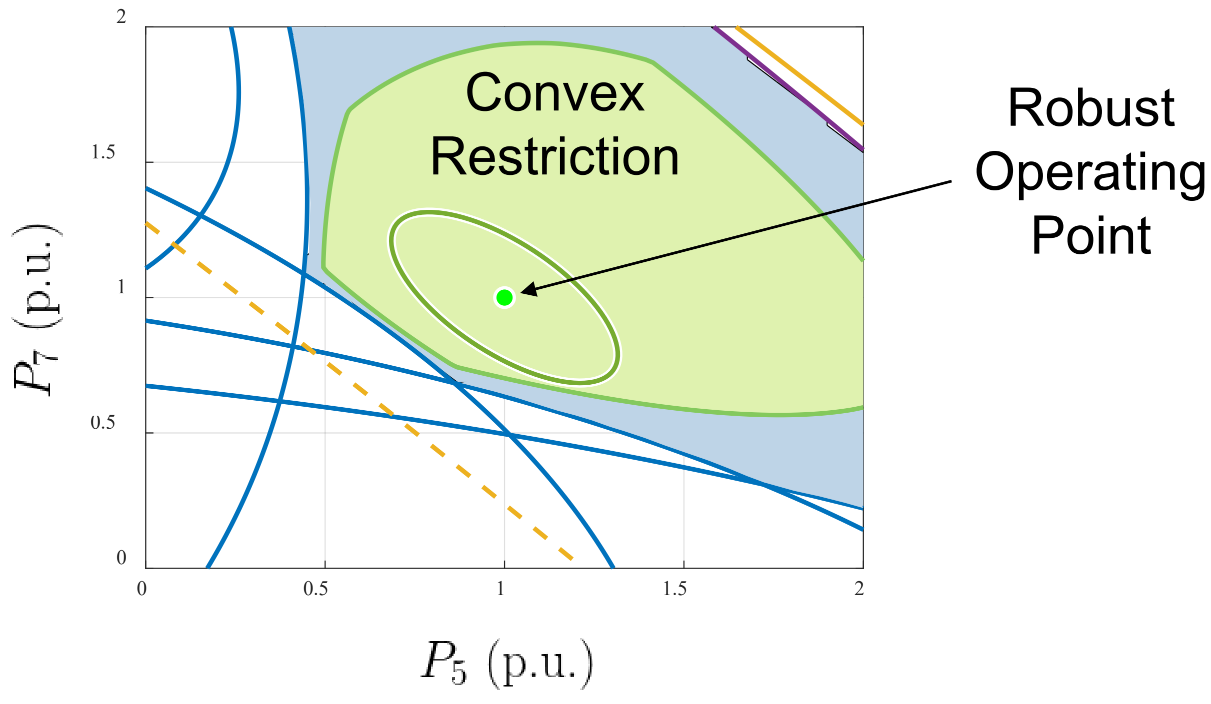


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

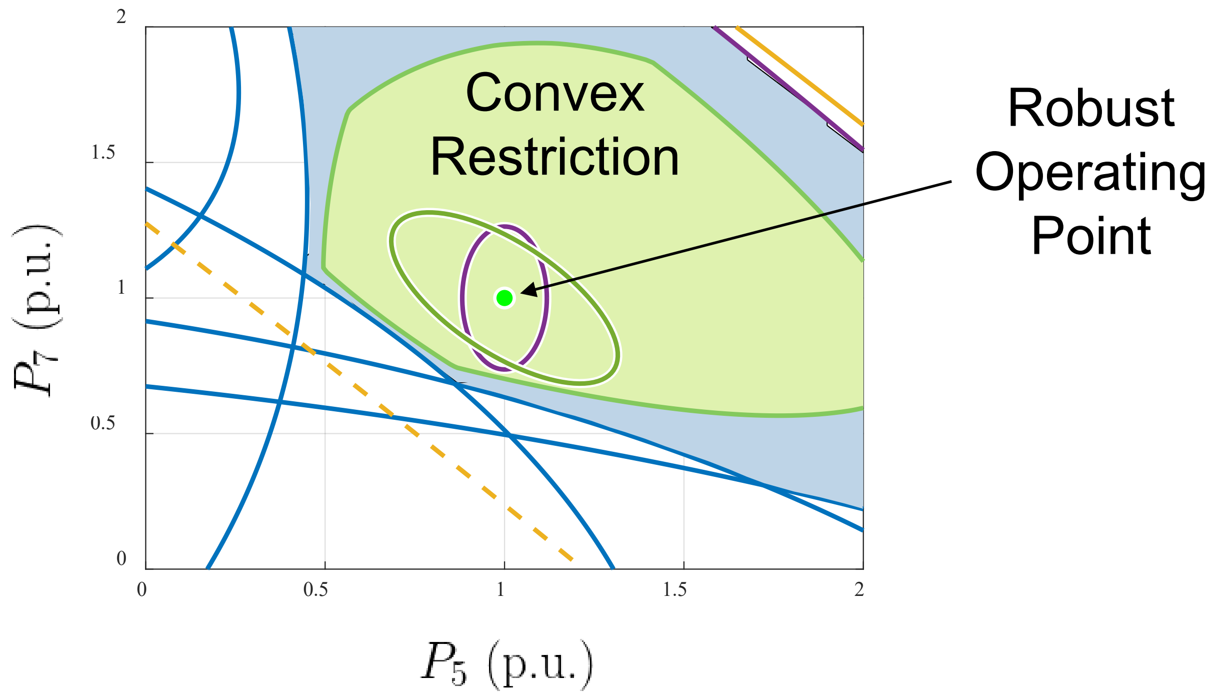


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

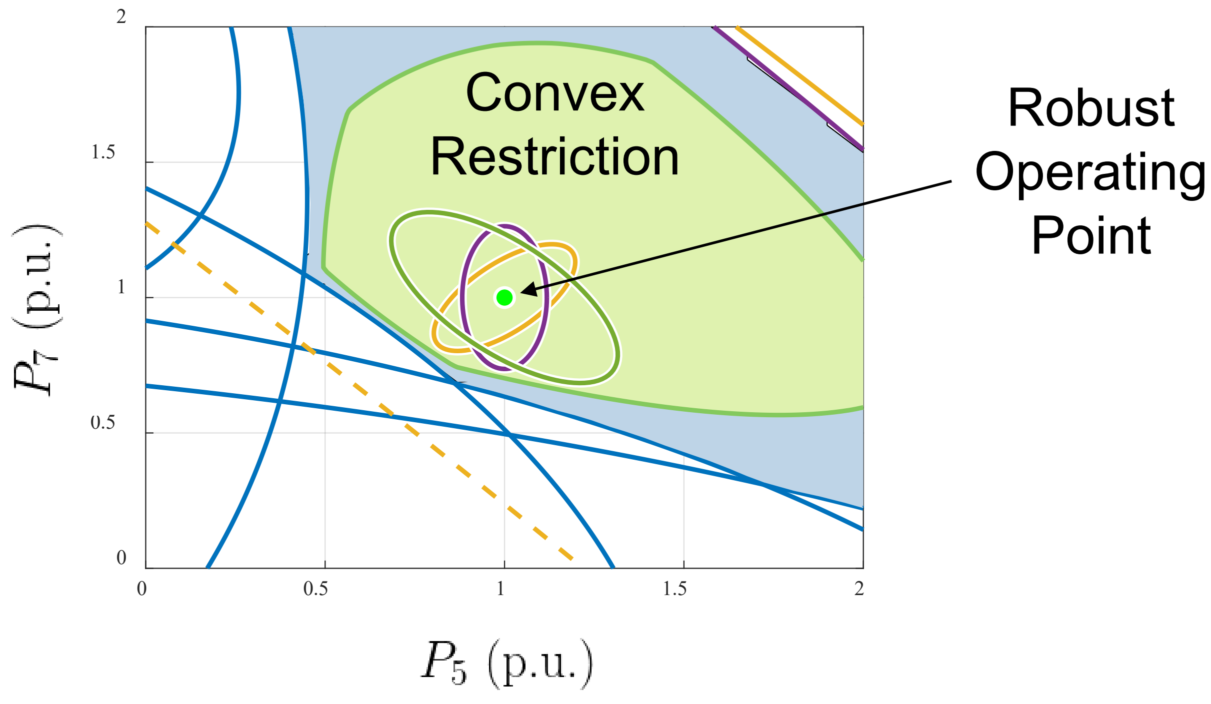


Figures courtesy of Dongchan Lee

[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Application 2: Robust Optimization

- Goal: Compute an operating point that is robust to variations in the net power injections.

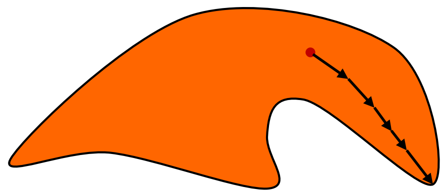


Figures courtesy of Dongchan Lee

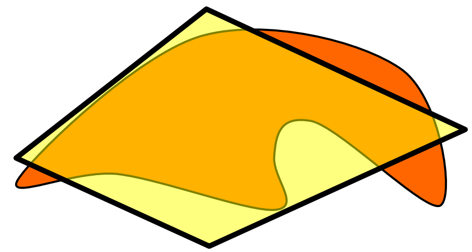
[Lee, Turitsyn, Molzahn, & Roald, to be submitted]

Conclusion

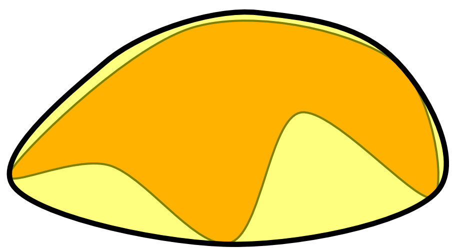
- Four approaches for handling power system nonlinearities:



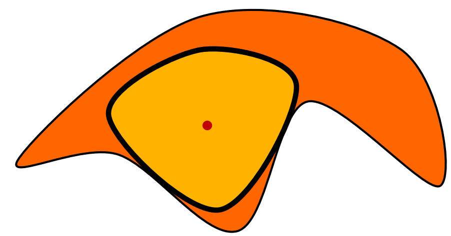
Local Optimization



Approximation



Convex Relaxation

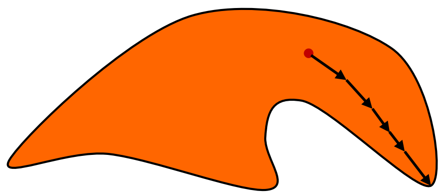


Convex Restriction

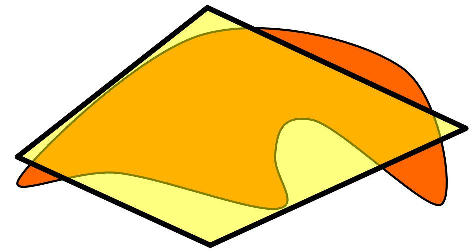
- Capabilities of each are useful for different applications.

D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

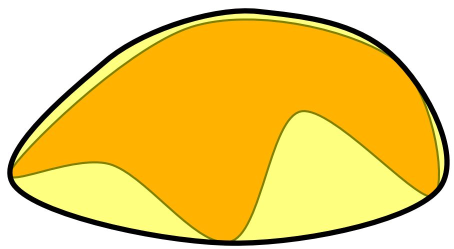
Questions?



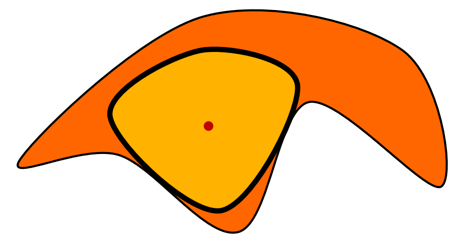
Local Optimization



Approximation



Convex Relaxation



Convex Restriction

D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," *Foundations and Trends in Electric Energy Systems*, vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

Dan Molzahn
molzahn@gatech.edu

References

W.A. Bukhsh, A. Grothey, K.I. McKinnon, and P.A. Trodden, “Local Solutions of the Optimal Power Flow Problem,” *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780-4788, 2013.

C. Chen, A. Atamtürk, and S.S. Oren, “A Spatial Branch-and-Cut Algorithm for Nonconvex QCQP with Bounded Complex Variables,” *Mathematical Programming*, pp. 1–29, 2016.

C. Chen, A. Atamtürk, and S.S. Oren, “Bound Tightening for the Alternating Current Optimal Power Flow Problem,” *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3729–3736, Sept. 2016.

C. Coffrin, H. Hijazi, and P. Van Hentenryck, “Strengthening Convex Relaxations with Bound Tightening for Power Network Optimization,” in *Principles and Practice of Constraint Programming*, ser. Lecture Notes in Computer Science, G. Pesant, Ed. Springer International Publishing, 2015, vol. 9255, pp. 39–57.

C. Coffrin, H. Hijazi, and P. Van Hentenryck, “The QC Relaxation: A Theoretical and Computational Study on Optimal Power Flow,” *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 3008-3018, July 2016.

C. Coffrin, H. Hijazi, and P. Van Hentenryck, “Strengthening the SDP Relaxation of AC Power Flows with Convex Envelopes, Bound Tightening, and Lifted Nonlinear Cuts,” *IEEE Transactions on Power Systems*, vol. 32, no. 5, Sept. 2017.

B. Cui and A.X. Sun, “Solvability of Power Flow Equations through Existence and Uniqueness of Complex Fixed Point,” *arXiv:1904.08855,* April 2019.

B. Kocuk, S.S. Dey, and A.X. Sun, “Strong SOCP Relaxations of the Optimal Power Flow Problem,” *Operations Research*, vol. 64, no. 6, pp. 1177–1196, 2016.

D. Lee, H.D. Nguyen, K. Dvijotham, and K. Turitsyn, “Convex Restriction of Power Flow Feasibility Sets,” *IEEE Transactions on Control of Network Systems,* vol, 6, no. 3, pp. 1235-1245, 2019.

D. Lee, K. Turitsyn, D.K. Molzahn, and L.A. Roald, “Feasible Path Identification with Sequential Convex Restriction,” to appear in *IEEE Transactions on Power Systems*.

D. Lee, K. Turitsyn, D.K. Molzahn, and L.A. Roald, “Robust AC Optimal Power Flow with Convex Restriction,” to be submitted.

G. McCormick, “Computability of Global Solutions to Factorable Non-convex Programs: Part I–Convex Underestimating Problems,” *Mathematical Programming*, vol. 10, no. 1, pp. 147–175, 1976.

C. Meyer and C. Floudas, Trilinear Monomials with Positive or Negative Domains: Facets of the Convex and Concave Envelopes. Boston, MA: Springer US, 2004, pp. 327–352.

——, “Trilinear Monomials with Mixed Sign Domains: Facets of the Convex and Concave Envelopes,” *Journal of Global Optimization*, vol. 29, no. 2, pp.125–155, 2004.

D.K. Molzahn, “Computing the Feasible Spaces of Optimal Power Flow Problems,” *IEEE Transactions on Power Systems*, vol. 32, no. 6, pp. 4762-4763, Nov. 2017.

D.K. Molzahn, B.C. Lesieutre, and C.L. DeMarco, "A Sufficient Condition for Power Flow Insolvability with Applications to Voltage Stability Margins," *IEEE Transactions on Power Systems,* vol. 28, no. 3, pp. 2592-2601, August 2013.

D.K. Molzahn and I.A. Hiskens, “A Survey of Relaxations and Approximations of the Power Flow Equations,” *Foundations and Trends in Electric Power Systems,* vol. 4, no. 1-2, pp. 1-221, Feb. 2019.

D.K. Molzahn and L.A. Roald, "Towards an AC Optimal Power Flow Algorithm with Robust Feasibility Guarantees," *20th Power Systems Computation Conference (PSCC)*, June 2018.

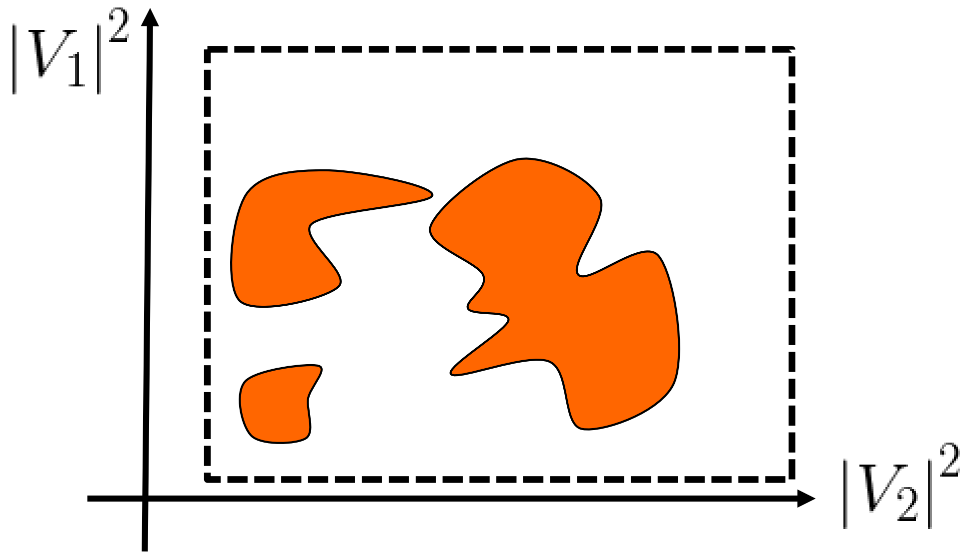
D.K. Molzahn and L.A. Roald, "Grid-Aware versus Grid-Agnostic Distribution System Control: A Method for Certifying Engineering Constraint Satisfaction," *52nd Hawaii International Conference on System Sciences (HICSS)*, Jan. 2019.

A. Venzke, D.K. Molzahn, and S. Chatzivasileiadis, “Efficient Creation of Datasets for Data-Driven Power System Applications,” to appear in *Electric Power Systems Research*, to be presented at *21st Power Systems Computation Conference (PSCC)*, June 29 - July 3, 2020.

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

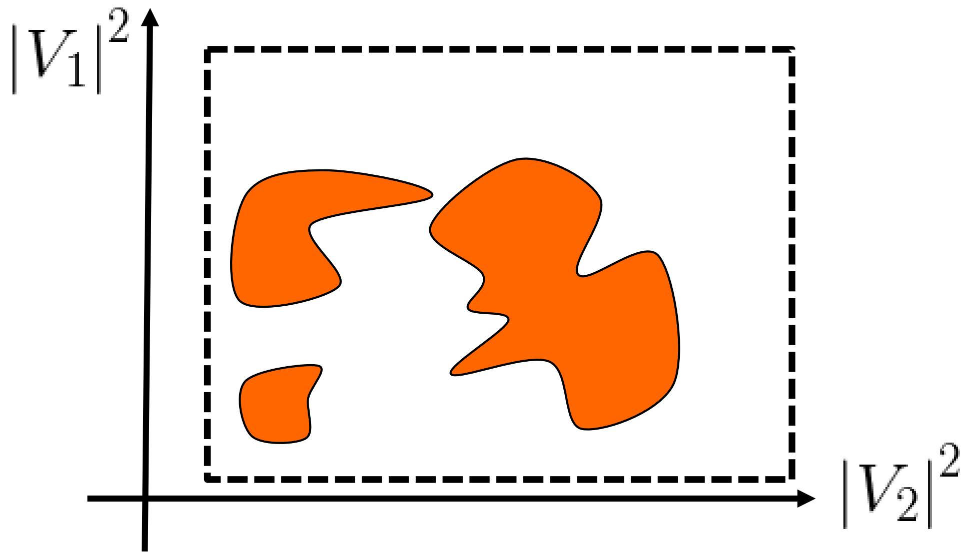


[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

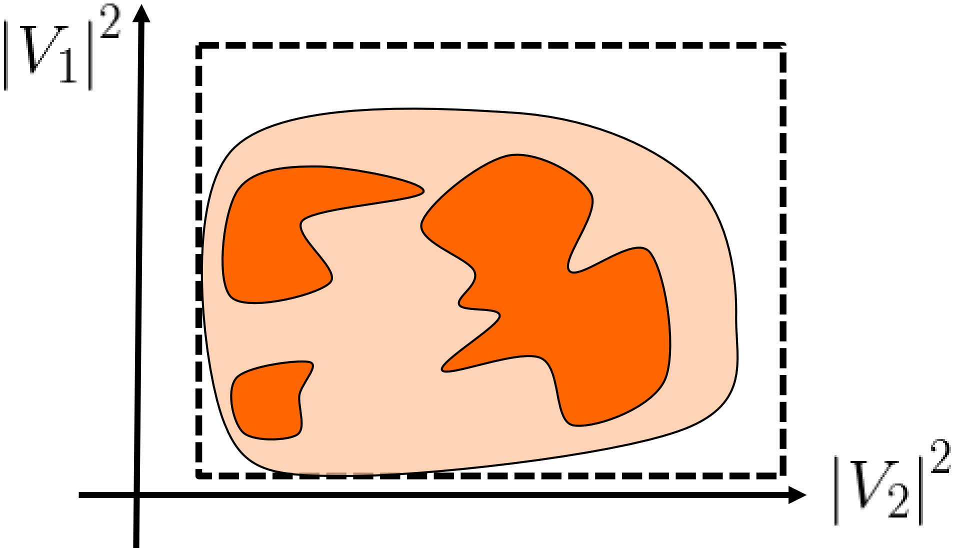


[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:



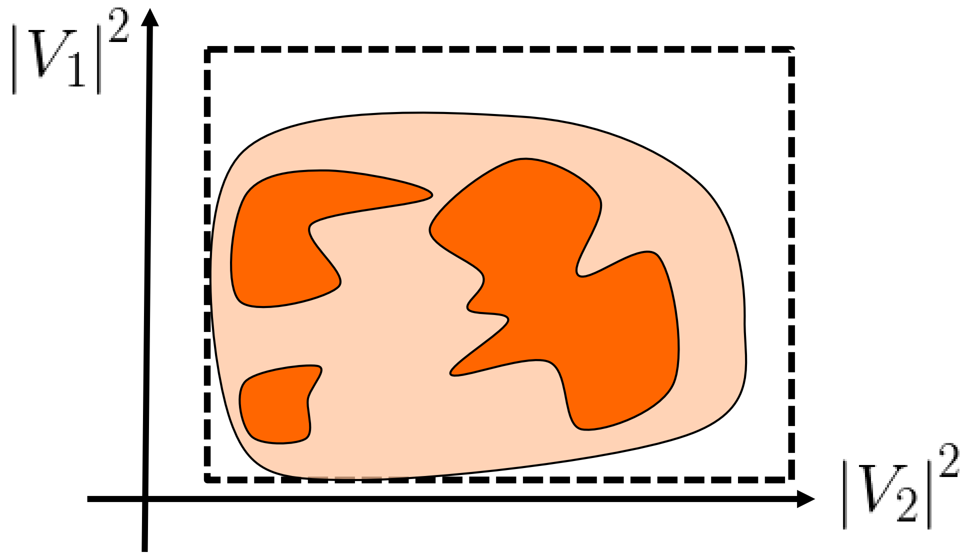
[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

$$\begin{aligned}&\max\ |V_1|^2 \ \text{ subject to}\\&1)\ \text{Operational constraints}\\&2)\ \text{A power flow relaxation}\end{aligned}$$

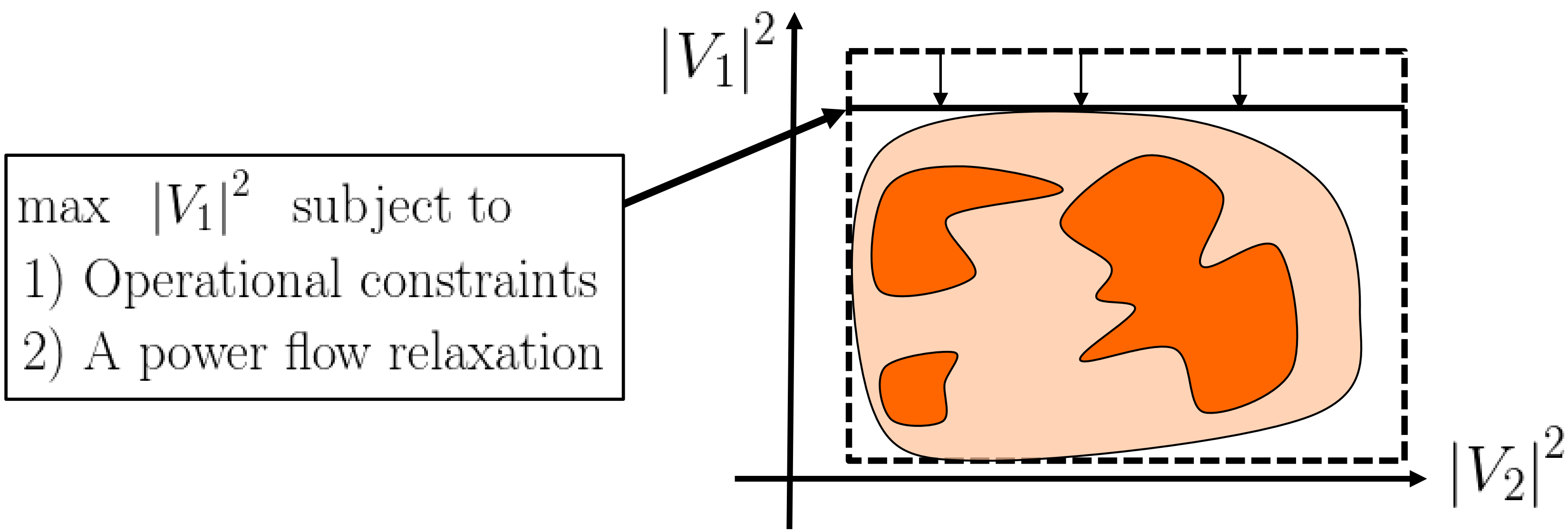


[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:



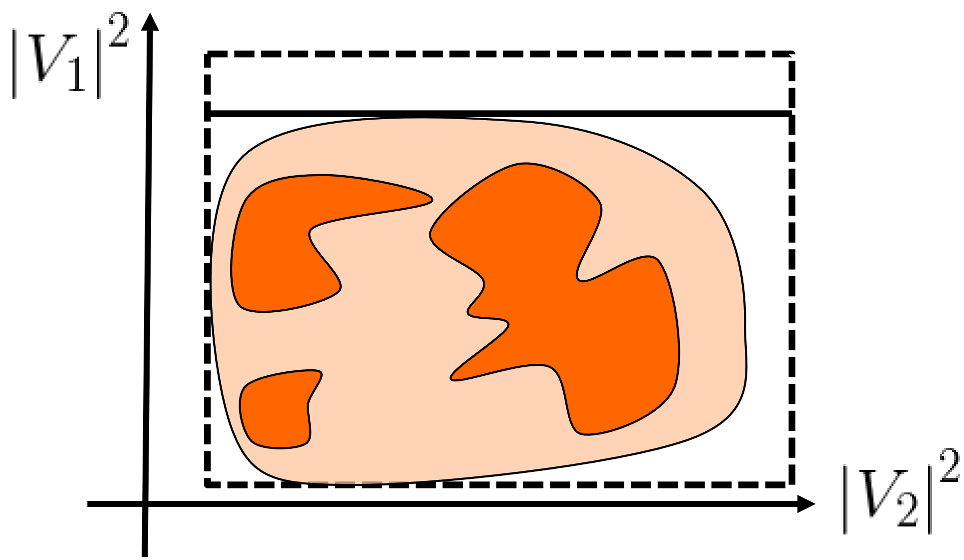
[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_1|^2$ subject to
1) Operational constraints
2) A power flow relaxation



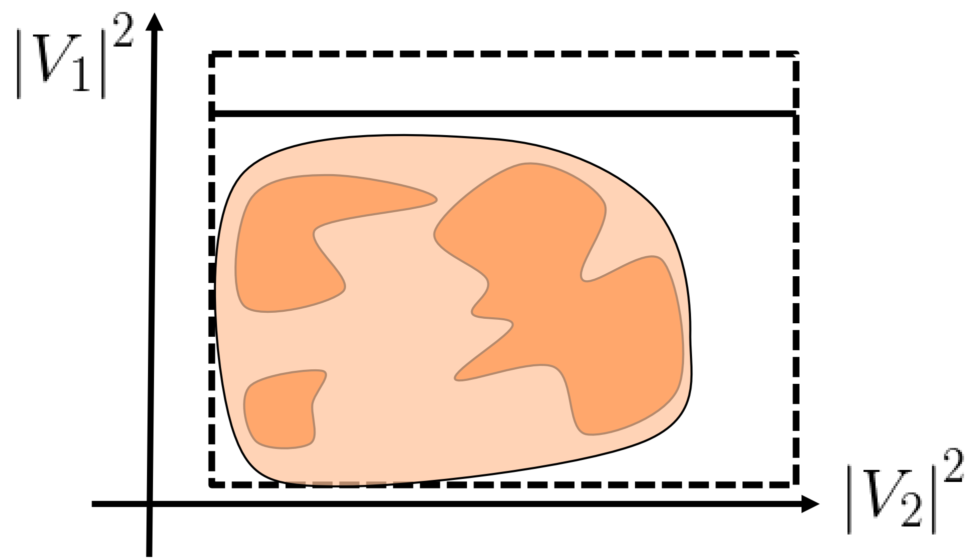
[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_1|^2$ subject to
1) Operational constraints
2) A power flow relaxation



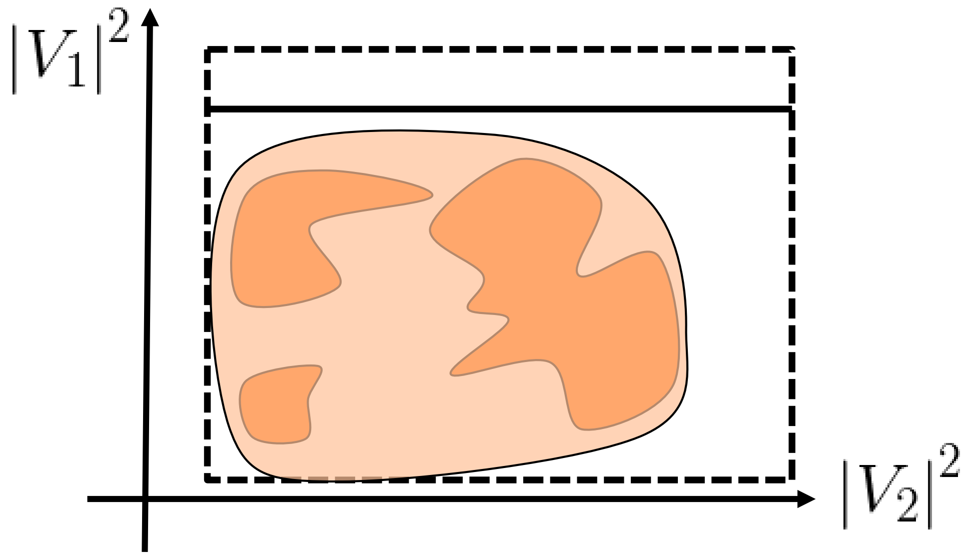
[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_1|^2$ subject to
1) Operational constraints
2) A power flow relaxation

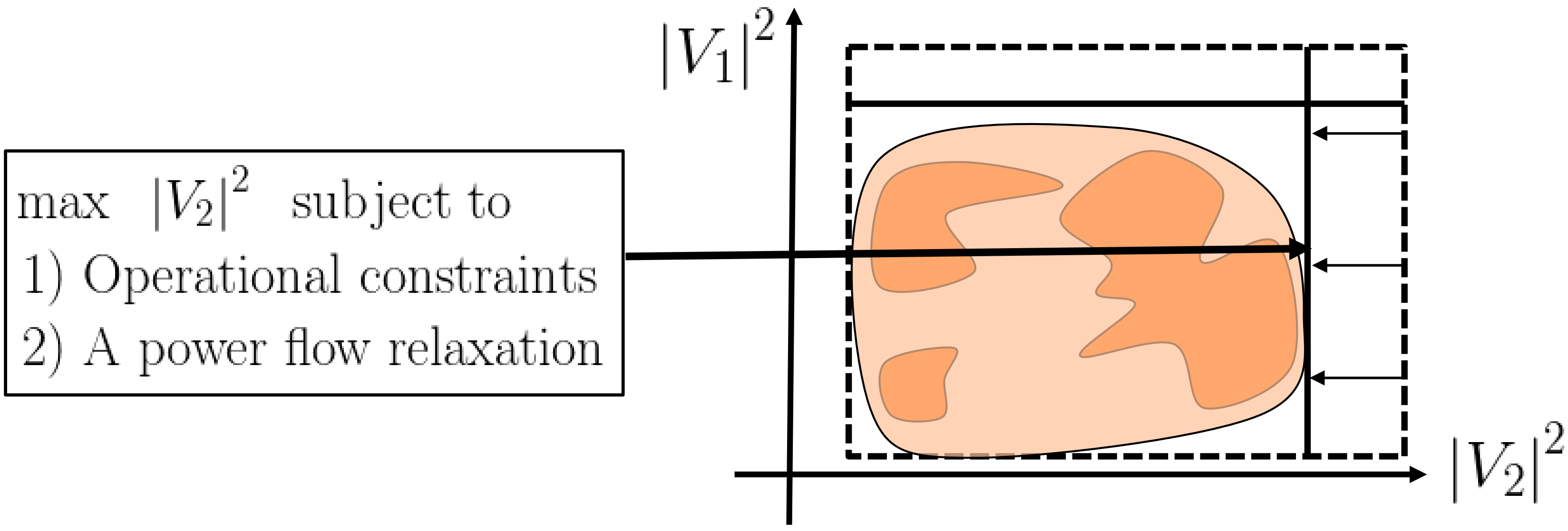


[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:



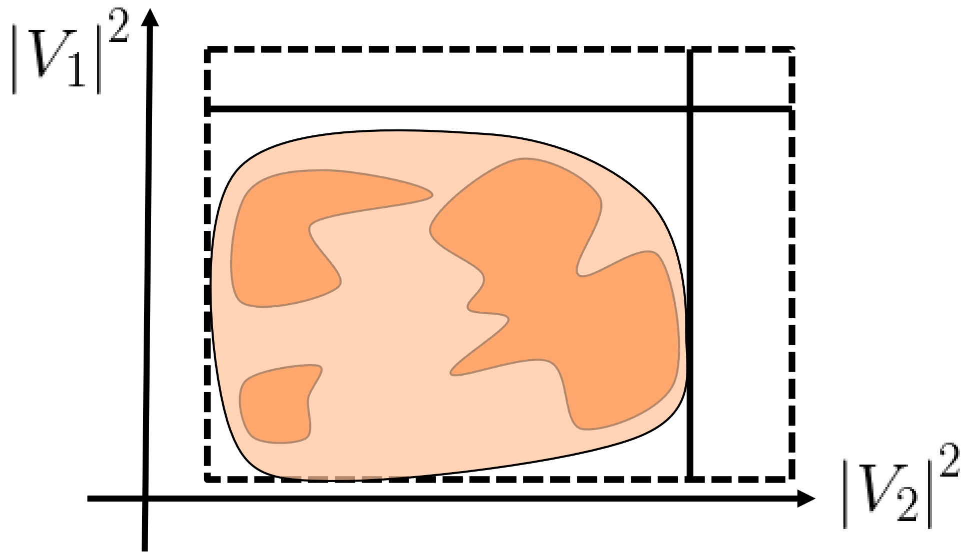
[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_2|^2$ subject to
1) Operational constraints
2) A power flow relaxation



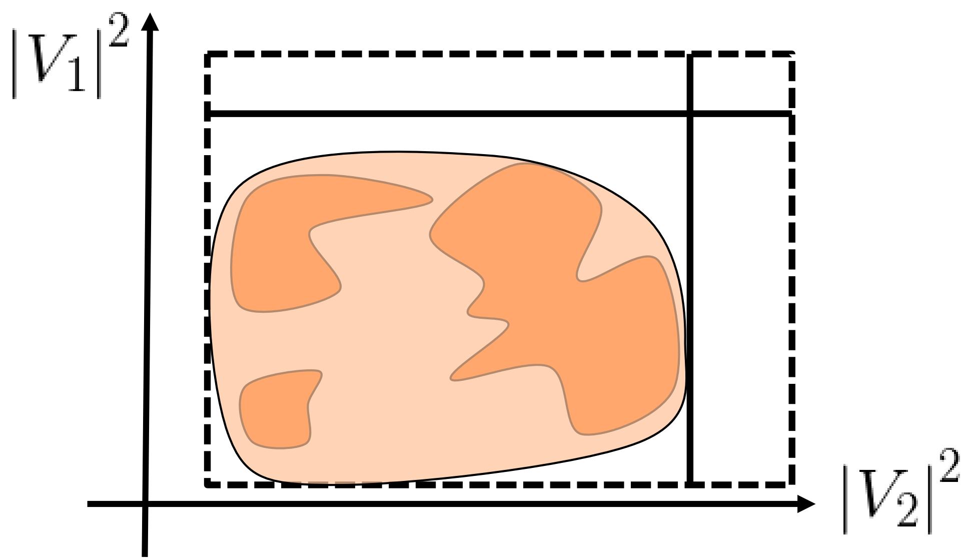
[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_2|^2$ subject to
1) Operational constraints
2) A power flow relaxation



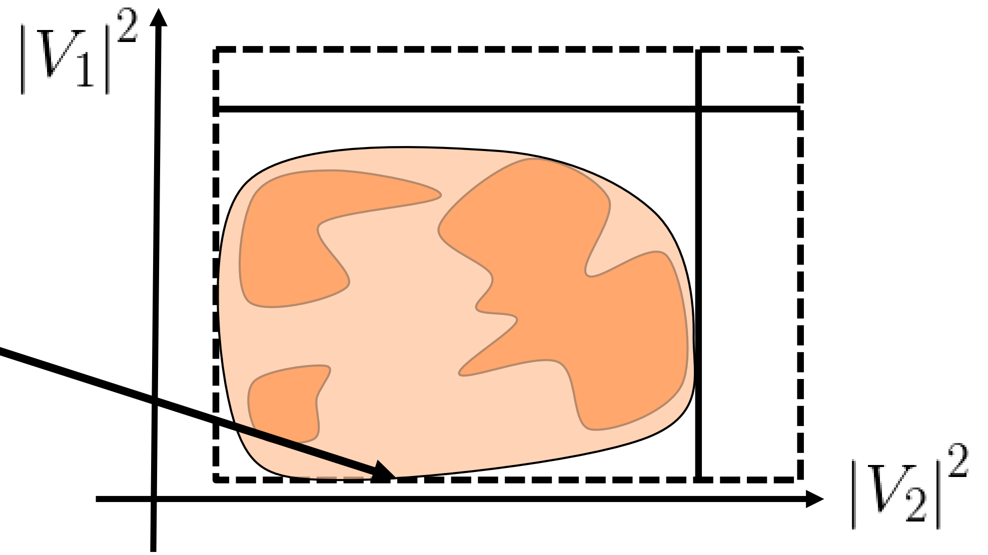
[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

$$\begin{aligned} &\min \quad |V_1|^2 \quad \text{subject to} \\ &1)\ \text{Operational constraints} \\ &2)\ \text{A power flow relaxation} \end{aligned}$$

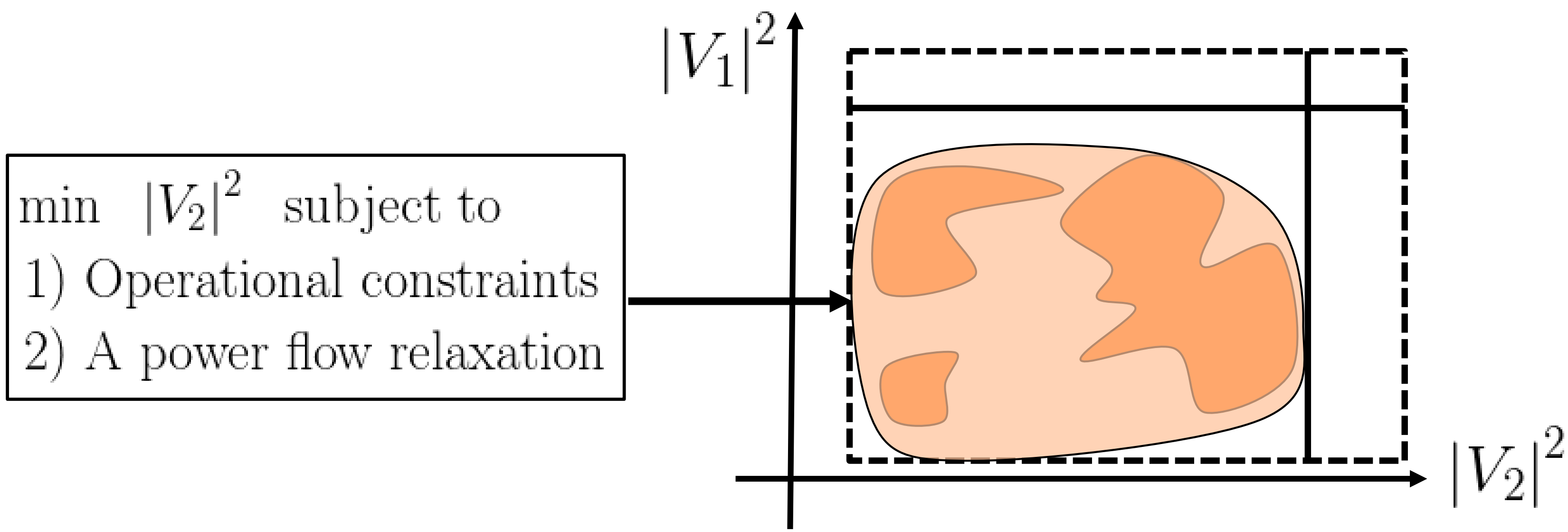


[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:



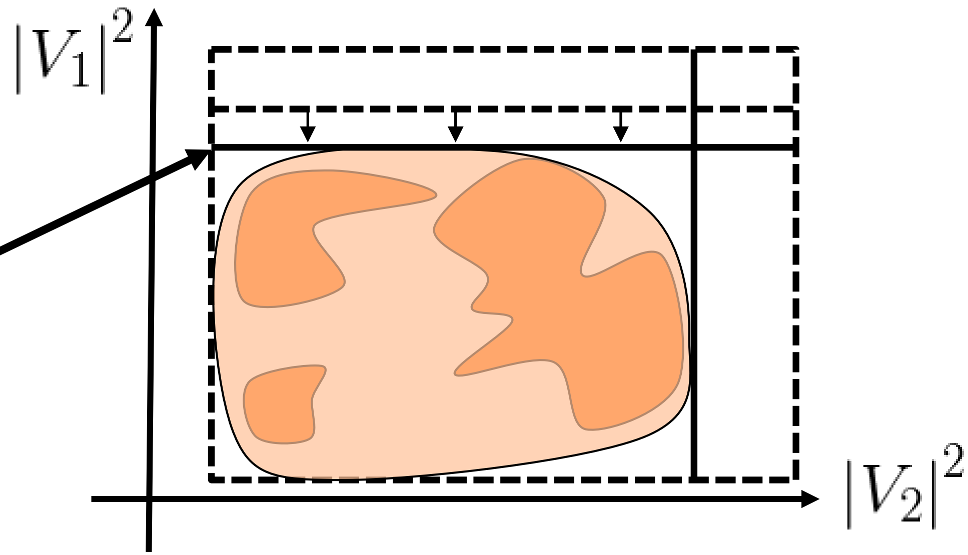
[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

max $|V_1|^2$ subject to
1) Operational constraints
2) A power flow relaxation

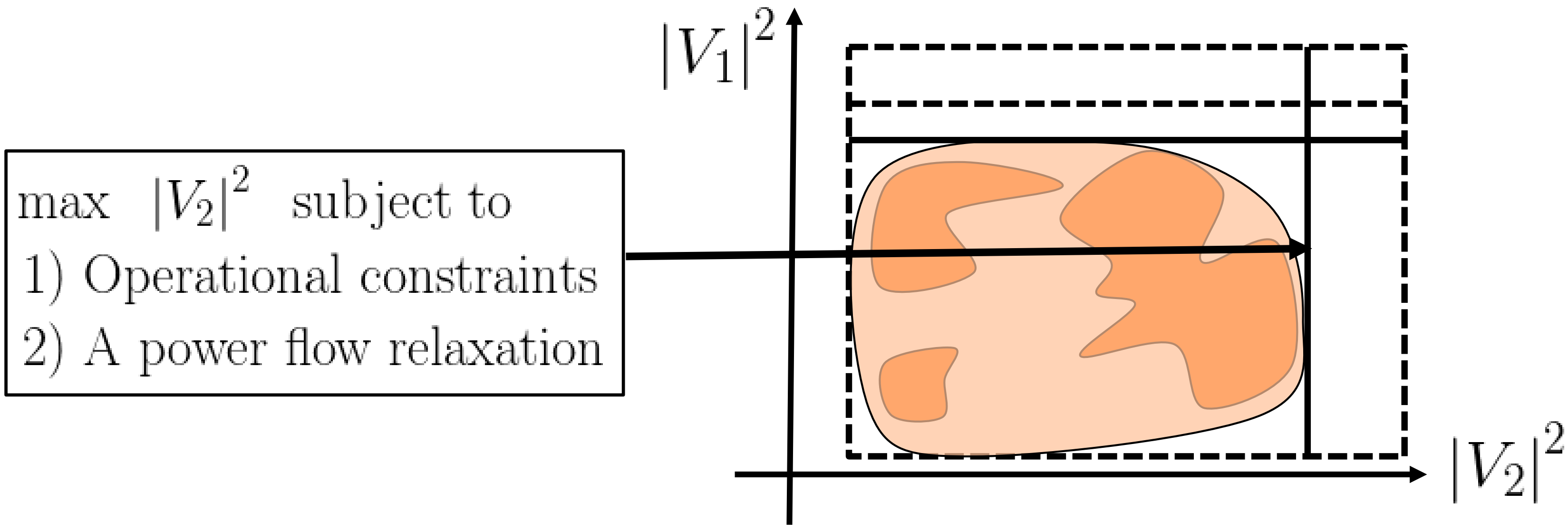


[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

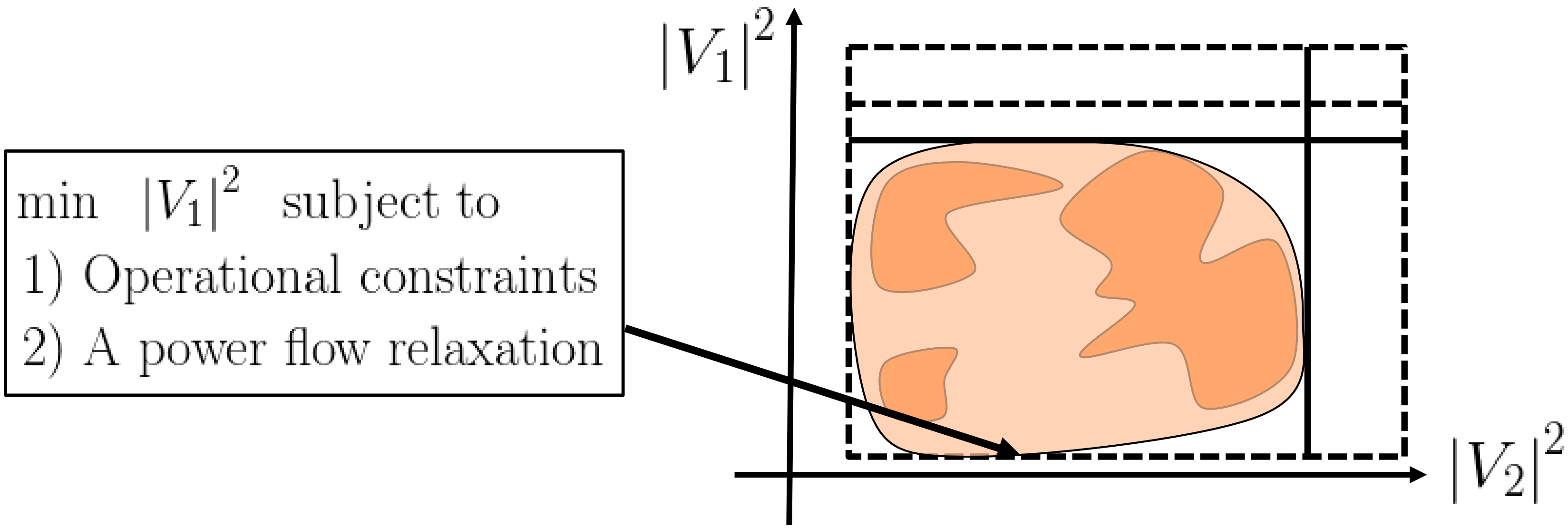


[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

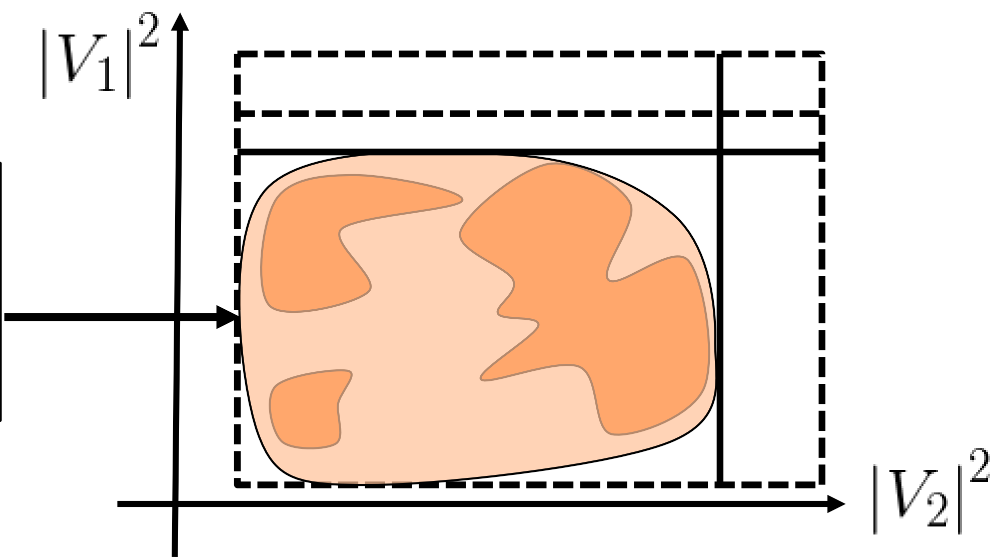


[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.
- Relaxations can be used to tighten the bounds:

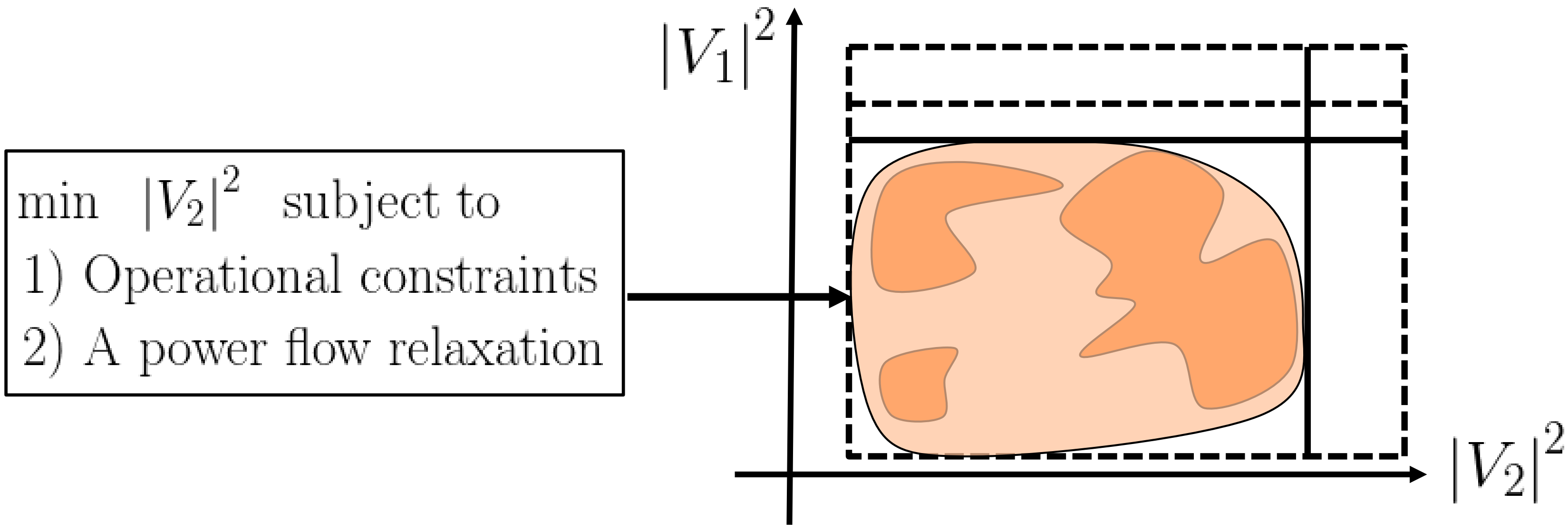
min $|V_2|^2$ subject to
1) Operational constraints
2) A power flow relaxation



[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.
- Relaxations can be used to tighten the bounds:



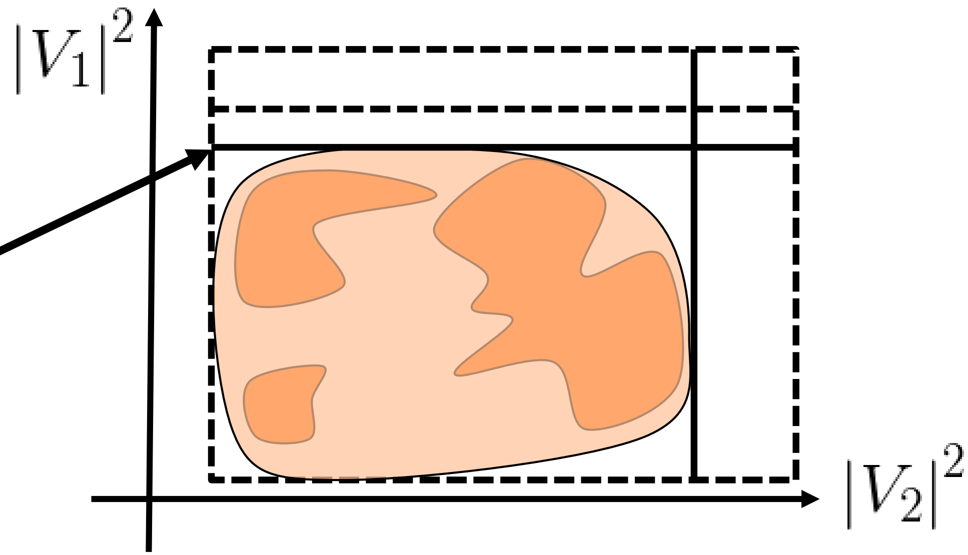
[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:

$$\begin{aligned}&\max \quad |V_1|^2 \quad \text{subject to}\\&\text{1) Operational constraints}\\&\text{2) A power flow relaxation}\end{aligned}$$

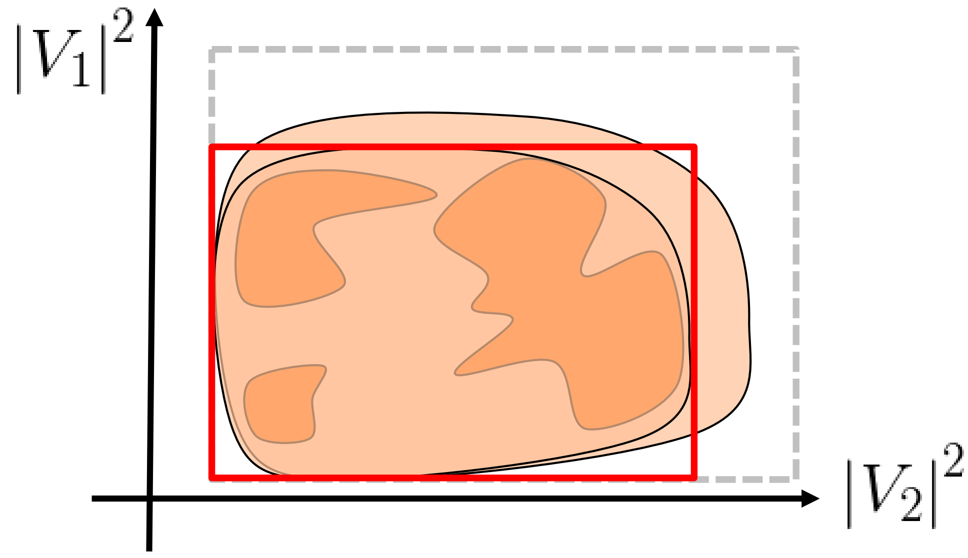


[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]

Bound Tightening

- Many of the relaxations require narrow bounds on voltage angles, voltage magnitudes, etc.

- Relaxations can be used to tighten the bounds:



[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16]