# Recent Advances in Solving AC OPF & Robust UC

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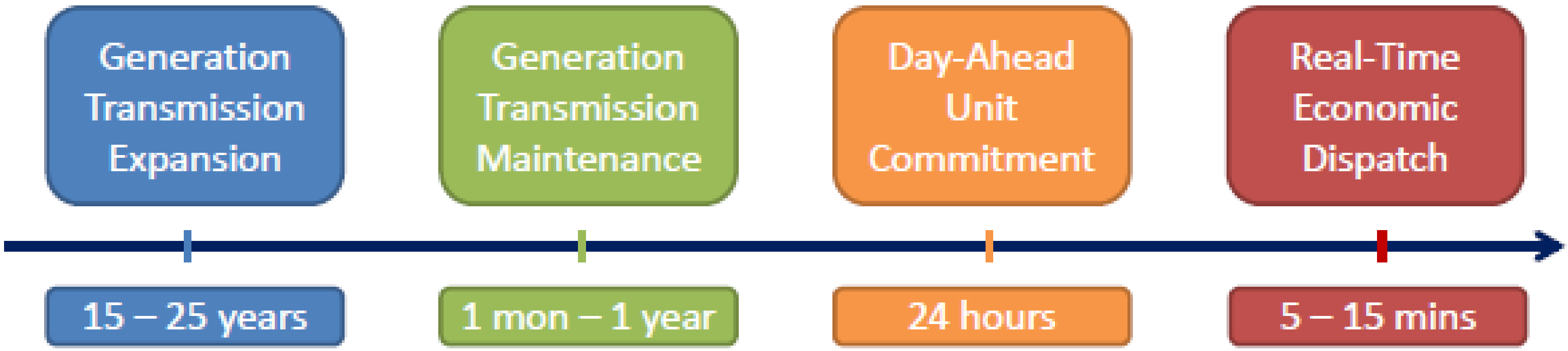
# Major Challenges

- Non-convexity:
  - **Discrete decisions:**
    - On/off operational/maintenance decisions
    - Network expansion/topology change decisions
  - **Continuous non-convex constraints**:
    - Non-convex quadratic constraints
    - Differential equation constraints
- Uncertainty:
  - Data-driven uncertainty modeling
  - Solving huge multistage robust/stochastic programming

# Electric Power Systems Problems

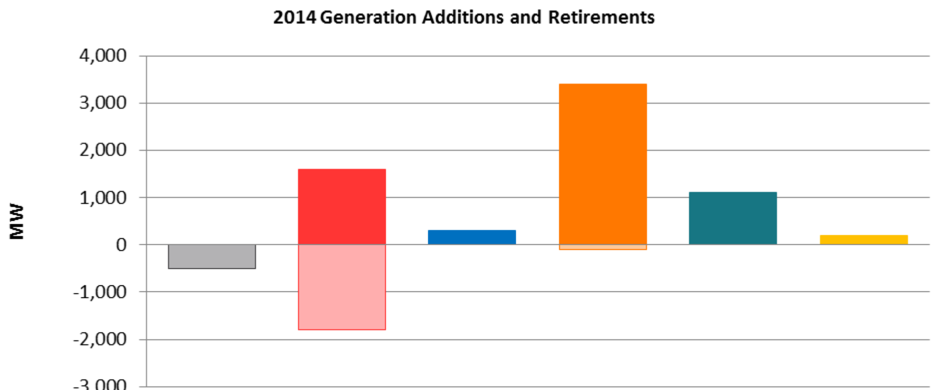
- Key Optimization Problems in power system operations from System perspective:



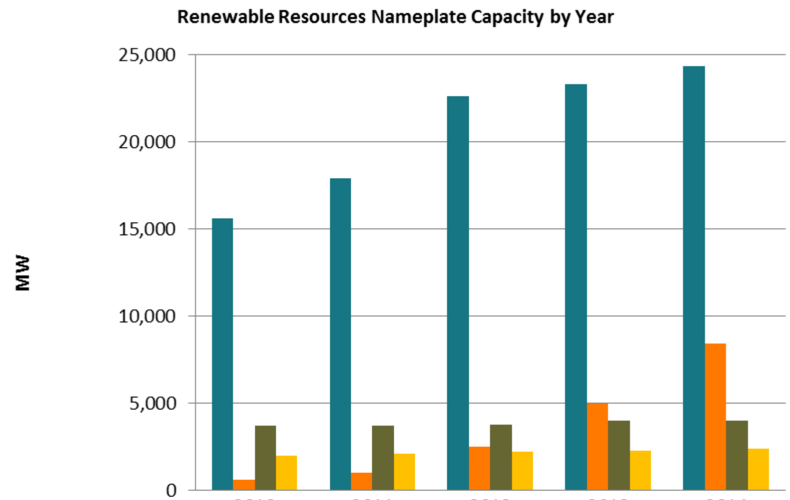

  - Real-Time Economic Dispatch:
    - Hourly bidding and ISO 5, 15 min dispatch
  - Day-Ahead Unit Commitment:
    - A day prior to operation to determine unit commitment
  - Yearly generation/transmission maintenance:
  - Long-term generation/transmission expansion:

# Challenge: Renewable Integration

- **Renewable Energy** Integration in Western Interconnection




| | Coal | Gas | Hydro | Solar | Wind | Other Renewable |
|---|---|---|---|---|---|---|
| Commissioned | 0 | 1,600 | 300 | 3,400 | 1,100 | 200 |
| Retired | -500 | -1,800 | 0 | -100 | 0 | 0 |
| Net | -500 | -200 | 300 | 3,300 | 1,100 | 200 |

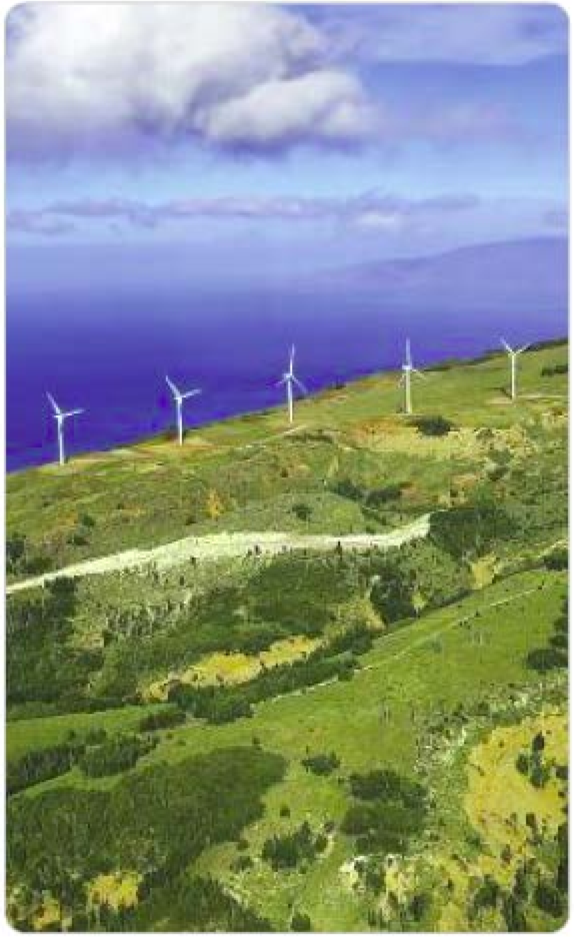


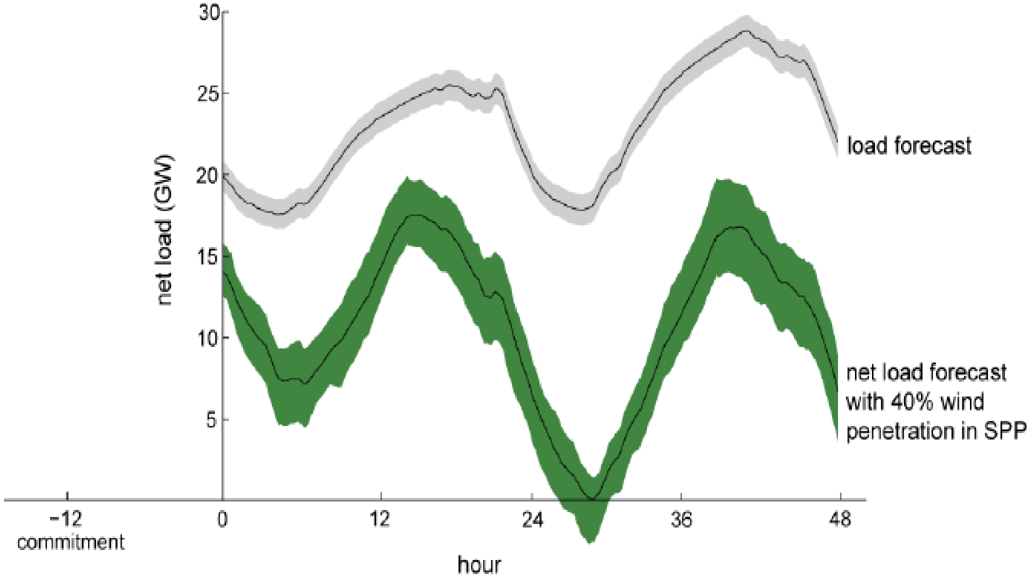

| | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|
| Wind | 15,600 | 17,900 | 22,600 | 23,300 | 24,300 |
| Solar | 600 | 1,000 | 2,500 | 5,000 | 8,400 |
| Geothermal | 3,700 | 3,700 | 3,800 | 4,000 | 4,000 |
| Other Renewable | 2,000 | 2,100 | 2,200 | 2,300 | 2,400 |

- WECC's Largest generation addition in 2014: **3,400 MW utility-scale solar**
- **Behind-the-meter solar** at least 3,200 MW
- Since 2010, nearly **10,000 MW wind** and **8,000 MW solar** added

# Challenge: Supply/Demand Uncertainty

- Renewable Integration





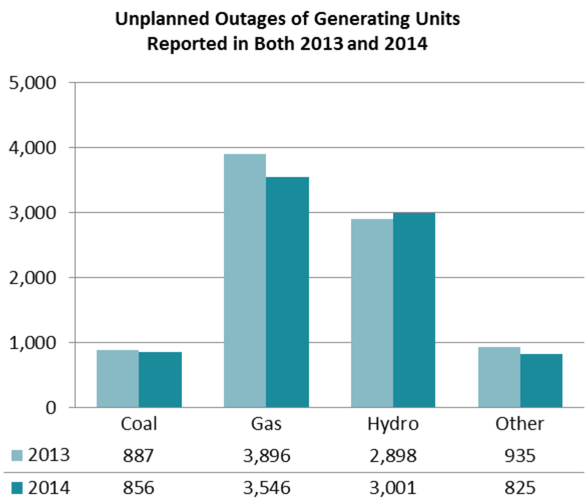

[Ruiz, Philbrick 10]

**Supply Variation:**
**Wind/Solar Power Penetration**
**Behind-the-Meter installation**

Net Load Uncertainty
Can be Huge!

# Challenge: Unplanned Outages

- Unplanned Generator Outages:

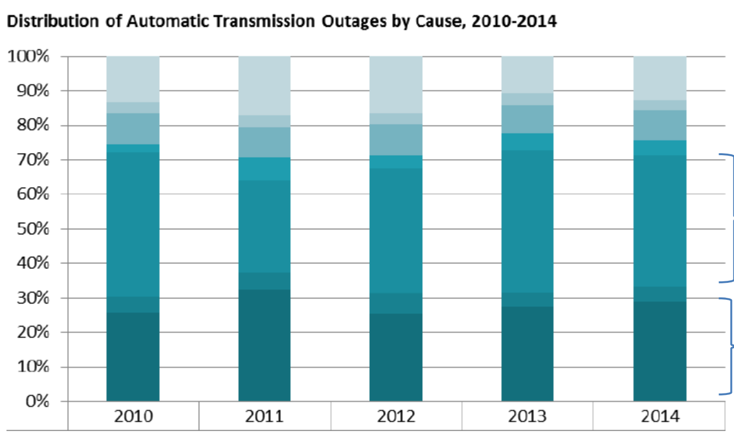



**Median Unplanned Outages Per Unit Per Year, 2013-2014**

| | 2013 | 2014 |
|---|---|---|
| Coal | 9 | 9 |
| Gas | 5 | 5 |
| Hydro | 3 | 3 |

**Lack of monitoring**
Entails high economic
Cost and threaten
system security

- Unplanned Transmission Outages:

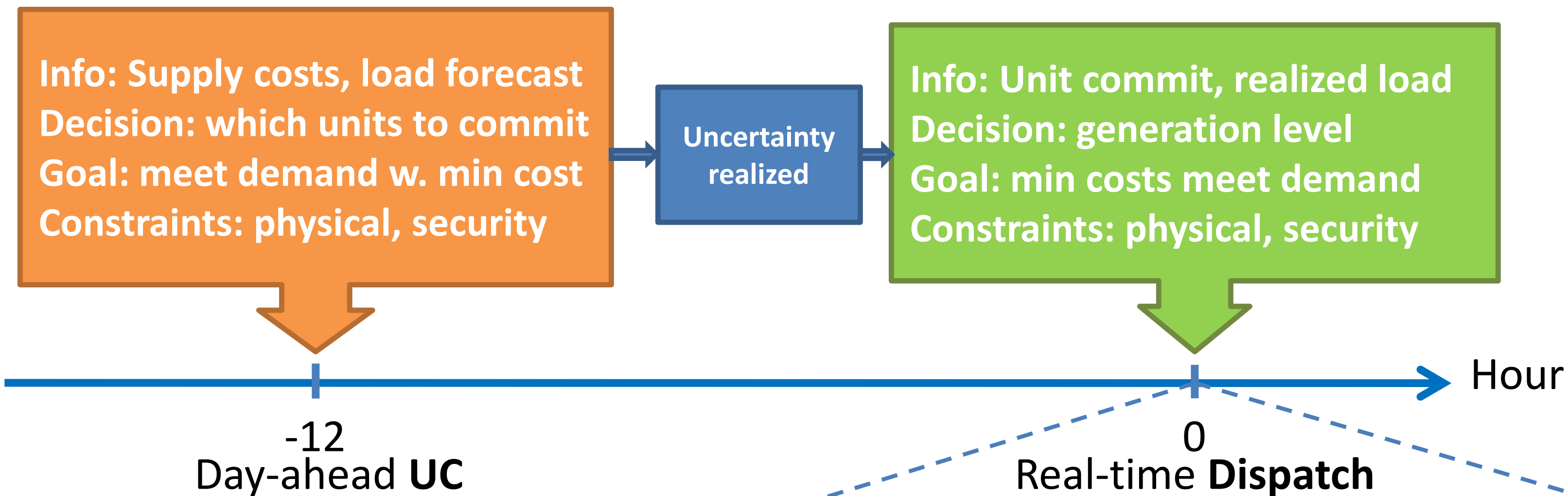



Environmental & Weather: 558/1471=38%

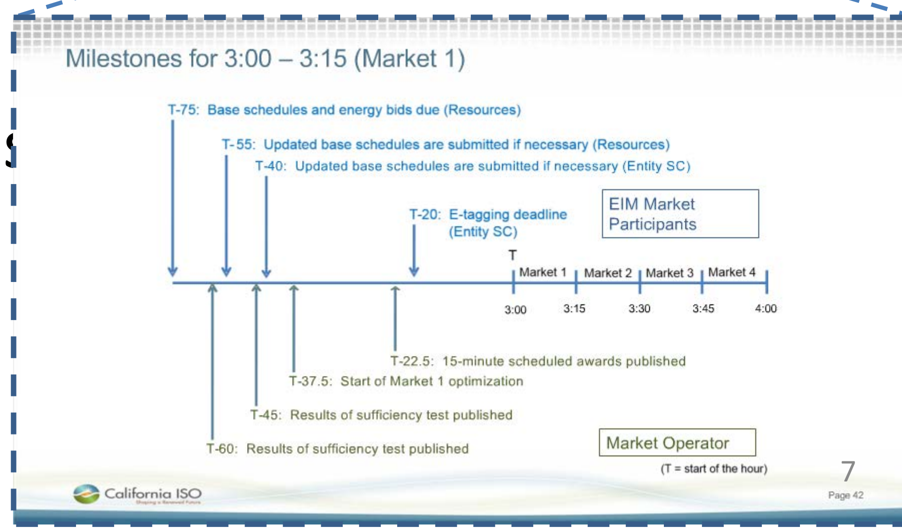
**Unknown causes**: 425/1471 = 29%!

# Challenge: Dynamic Decision Making

- Uncertainty in Dynamic Decision Making




  - Wind/solar rapid changes
  - Limited ramping



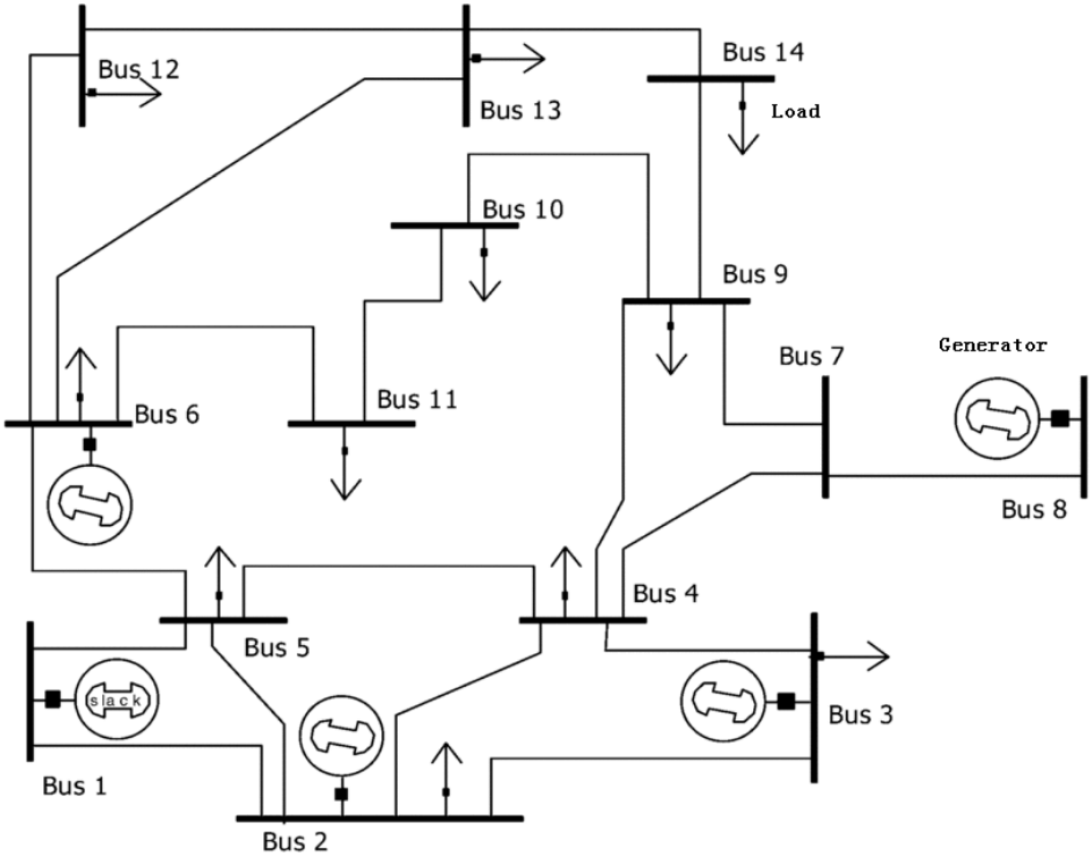
# Outline

- Part1: Dealing with **Non-convexity:**
  Optimal Power Flow (OPF)
- Part 2: Dealing with **Uncertainty:**
  Data-driven Robust Unit Commitment (UC)

# Part 1

Optimal Power Flow (OPF): Fast Convexification and Cutting Plane Method to deal with **non-convexity**

# AC Optimal Power Flow




<u>Data</u>:

- Network:

$$\mathcal{N} = (\mathcal{B}, \mathcal{L})$$

- Load at bus $i$:

$$p_i^d,\ q_i^d$$

- Generator at bus $i$:

$$[p_i^{\min}, p_i^{\max}]$$

$$[q_i^{\min}, q_i^{\max}]$$

- Voltage bounds at bus $i$:

$$[V_i^{\min}, V_i^{\max}]$$

- Network line admittance:

$$(G_{ij}, B_{ij})_{(i,j)\in\mathcal{L}}$$

- Line flow limit:

$$\bar{S}_{ij}$$

# AC OPF Formulation: Variables and Objective

Variables:

1. Active and reactive **power** at generator $i$: $(p_i^g, q_i^g)$
2. Active and reactive **power flow** on line $(i, j)$: $(p_{ij}, q_{ij})$
3. Complex **voltage** at bus $i$: $V_i = |V_i|(\cos\theta_i + \mathrm{i}\sin\theta_i) = e_i + \mathrm{i} f_i$

Objective:

$$\min \sum_{i \in \mathcal{G}} C_i(p_i^g)$$

Usually a separable increasing function.

# AC OPF Formulation: Constraints

$$p_i^g - p_i^d = \sum_{j \in \delta(i)} p_{ij} \qquad \text{(active flow balance)}$$

$$q_i^g - q_i^d = \sum_{j \in \delta(i)} q_{ij} \qquad \text{(reactive flow balance)}$$

$$p_{ij}^2 + q_{ij}^2 \leq \overline{S}_{ij}^2 \qquad \text{(apparent flow limit)}$$

$$p_i^{\min} \leq p_i^g \leq p_i^{\max} \qquad \text{(active power limits)}$$

$$q_i^{\min} \leq q_i^g \leq q_i^{\max} \qquad \text{(reactive power limits)}$$

Power flow equations and voltage bounds in **polar coordinates**

$$\begin{cases} p_{ij} = -G_{ij}|V_i|^2 + G_{ij}|V_i||V_j|\cos(\theta_i - \theta_j) + B_{ij}|V_i||V_j|\sin(\theta_i - \theta_j) \\ q_{ij} = \phantom{-}B_{ij}|V_i|^2 - B_{ij}|V_i||V_j|\cos(\theta_i - \theta_j) + G_{ij}|V_i||V_j|\sin(\theta_i - \theta_j) \\ \underline{V}_i \leq |V_i| \leq \overline{V}_i \end{cases}$$

Power flow equations and voltage bounds in **rectangular coordinates**

$$\begin{cases} p_{ij} = -G_{ij}(e_i^2 + f_i^2) + G_{ij}(e_i e_j + f_i f_j) - B_{ij}(e_i f_j - e_j f_i) \\ q_{ij} = \phantom{-}B_{ij}(e_i^2 + f_i^2) - B_{ij}(e_i e_j + f_i f_j) - G_{ij}(e_i f_j - e_j f_i) \\ \underline{V}_i^2 \leq e_i^2 + f_i^2 \leq \overline{V}_i^2 \end{cases}$$

# Recent Literature on OPF

- **Local solvers** by Newton-Raphson and Interior-Point methods
- **Convex relaxations** using **semidefinite programming** (SDP) and Lasserre hierarchy: (Lavaei and Low, 2012; Madani et. al., 2013; Zhang and Tse, 2012; Lavaei et al., 2014, Molzahn et al. 2013, Molzahn and Hiskens, 2014, Chen et al. 2015, Madani et al. 2017)
- **Second order cone program** (SOCP) relaxation: (Jabr 2006, Hijazi et al., 2014)
- **Approximate LPs** with guaranteed bounds for the AC-OPF problem on graphs with bounded tree-width (Bienstock and Munoz, 2015)
- **Global optimal solutions** based on branch-and-bound (Phan, 2012)

# AC OPF Reformulation

- Introduce Hermitian matrix $X = (e + if)(e + if)^H$:

$$
\begin{aligned}
p_{ij} &= -G_{ij} X_{ii} + G_{ij} \mathcal{R}(X_{ij}) + B_{ij} \mathcal{I}(X_{ij}) \\
q_{ij} &= B_{ij} X_{ii} - B_{ij} \mathcal{R}(X_{ij}) + G_{ij} \mathcal{I}(X_{ij}) \\
&\underline{V}_i^2 \leq X_{ii} \leq \overline{V}_i^2 \\
&X \text{ is hermitian} \\
&X \succeq 0 \\
&\boxed{\operatorname{rank}(X) = 1}
\end{aligned}
$$

- Standard SDP relaxation: Ignore rank constraint
- **Our proposal**: A new minor reformulation of rank constraints and use simpler relaxation than SDP

# Minor Representation

- **Proposition**: For a nonzero Hermitian matrix $X$, $X \succcurlyeq 0$ and $rank(X) = 1$ if and only if all the $2 \times 2$ minors of $X$ are 0 and $X_{ii} \geq 0$ for all $i$.
- AC OPF constraints can be equivalently reformulated as:

$$p_i^g - p_i^d = \sum_{j \in \delta(i)} p_{ij}$$

$$q_i^g - q_i^d = \sum_{j \in \delta(i)} q_{ij}$$

$$p_{ij}^2 + q_{ij}^2 \leq \overline{S}_{ij}^2$$

$$p_i^{\min} \leq p_i^g \leq p_i^{\max}$$

$$q_i^{\min} \leq q_i^g \leq q_i^{\max}$$

$$p_{ij} = -G_{ij} X_{ii} + G_{ij} \mathcal{R}(X_{ij}) + B_{ij} \mathcal{I}(X_{ij})$$

$$q_{ij} = B_{ij} X_{ii} - B_{ij} \mathcal{R}(X_{ij}) + G_{ij} \mathcal{I}(X_{ij})$$

$$\underline{V}_i^2 \leq X_{ii} \leq \overline{V}_i^2$$

$X$ is hermitian

all $2 \times 2$ minors of $X$ are zero

# First Type of $2 \times 2$ Submatrices

- Type 1: $\begin{bmatrix} X_{ii} & X_{ij} \\ X_{ji} & X_{jj} \end{bmatrix}$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} \end{bmatrix}$$

# Second Type of $2 \times 2$ Submatrices

- Type 2: $\begin{bmatrix} X_{ii} & X_{ij} \\ X_{ki} & X_{kj} \end{bmatrix}$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} \end{bmatrix}$$

# Third Type of $2 \times 2$ Submatrices

- Type 3: $\begin{bmatrix} X_{ij} & X_{ik} \\ X_{lj} & X_{lk} \end{bmatrix}$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} \end{bmatrix}$$

# First Type $2 \times 2$ Minors: Edge minor

- Let $c_{ij} = \mathcal{R}(X_{ij}), s_{ij} = -\mathcal{I}(X_{ij})$
- Type 1: Edge minor.
  - $\begin{vmatrix} X_{ii} & X_{ij} \\ X_{ji} & X_{jj} \end{vmatrix} = 0$

  Implies $X_{ii}X_{jj} - X_{ij}X_{ij}^* = 0$ ➔ $c_{ii}c_{jj} - \left(c_{ij}^2 + s_{ij}^2\right) = 0$
  - This is the boundary of a rotated Lorentz cone in $R^4$
    - One direction is convex: $c_{ij}^2 + s_{ij}^2 \leq c_{ii}^j c_{jj}^i$
    - Other direction is reverse convex:
      - $f(c_{ij}, s_{ij}) = \sqrt{c_{ij}^2 + s_{ij}^2} \geq \sqrt{c_{ii}^j c_{jj}^i} = g(c_{ii}^j, c_{jj}^i)$
      - $f$ is convex, $g$ is concave
      - Overestimate of $f$ and underestimate of $g$ by hyperplanes

# Second Type $2 \times 2$ Minors: 3-Cycle minor

- Type 2: 3-Cycle minor.
  - $\begin{vmatrix} X_{ii} & X_{ij} \\ X_{ki} & X_{kj} \end{vmatrix} = 0$
  - $0 = X_{ii}X_{kj} - X_{ij}X_{ki}$

$= c_{ii}\left(c_{kj} - is_{kj}\right) - \left(c_{ij} - is_{ij}\right)\left(c_{ki} - is_{ki}\right)$

$= \left(c_{ii}c_{kj} - c_{ij}c_{ki} + s_{ij}s_{ki}\right) - i\left(c_{ii}s_{kj} - c_{ij}s_{ki} - s_{ij}c_{ki}\right)$

  - So we have two bilinear constraints:
    - $\left(c_{ii}c_{kj} - c_{ij}c_{ki} + s_{ij}s_{ki}\right) = 0$
    - $\left(c_{ii}s_{kj} - c_{ij}s_{ki} - s_{ij}c_{ki}\right) = 0$

# Third Type $2 \times 2$ Minors: 4-Cycle minor

- Type 3: 4-Cycle minor.
  - $\begin{vmatrix} X_{ij} & X_{ik} \\ X_{lj} & X_{lk} \end{vmatrix} = 0$
  - $0 = X_{ij}X_{lk} - X_{ik}X_{lj}$

    $= (c_{ij}c_{lk} - s_{ij}s_{lk} - c_{lj}c_{ik} + s_{lj}s_{ik})$

    $\quad -i(s_{ij}c_{lk} - c_{ij}s_{lk} - s_{lj}c_{ik} - c_{lj}s_{ik})$
  - So we have two bilinear constraints:
    - $(c_{ij}c_{lk} - s_{ij}s_{lk} - c_{lj}c_{ik} + s_{lj}s_{ik}) = 0$
    - $(s_{ij}c_{lk} - c_{ij}s_{lk} - s_{lj}c_{ik} - c_{lj}s_{ik}) = 0$

# 3-, 4-Cycle Minors = Cycle Constraints

For a cycle $C$, instead of satisfying:

$$\sum_{(i,j)\in C} \arctan\left(\frac{\mathbf{s_{ij}}}{\mathbf{c_{ij}}}\right) = 0,$$

We write "angles sum to zero over the cycle" by the following relaxation:

$$\sum_{(i,j)\in C} \theta_{ij} = 2\pi k, \quad \text{for some } k \in \mathbb{Z}. \tag{1}$$

Condition (1) is equivalent to:

$$\textit{Cycle constraint:} \quad \cos\Big(\sum_{(i,j)\in C} \theta_{ij}\Big) = 1. \tag{2}$$

Cycle constraint (2) can be reformulated as a degree $|C|$ homogeneous polynomial $p_C = 0$ in $\mathbf{s_{ij}}$ and $\mathbf{c_{ij}}$ for $(i,j) \in C$.

# 3-Cyle, 4-Cycle, and Larger Cycles

- 3-cycle:

For a 3-cycle: $\cos(\theta_{12} + \theta_{23} + \theta_{31}) = 1$ can be written as

$$\left.\begin{aligned} s_{12}c_{33} + c_{23}s_{31} + s_{23}c_{31} &= 0 \\ c_{12}c_{33} - c_{23}c_{31} + s_{23}s_{31} &= 0. \end{aligned}\right\} \text{Exactly 3-cycle minor = 0}$$

- 4-cycle:

For a 4-cycle: $\cos(\theta_{12} + \theta_{23} + \theta_{34} + \theta_{41}) = 1$ can be written as

$$\left.\begin{aligned} s_{12}c_{34} + c_{12}s_{34} + s_{23}c_{41} + c_{23}s_{41} &= 0 \\ c_{12}c_{34} - s_{12}s_{34} + c_{23}c_{41} - s_{23}s_{41} &= 0. \end{aligned}\right\} \text{Exactly 4-cycle minor = 0}$$

# Our Strategy for Solving AC-OPF

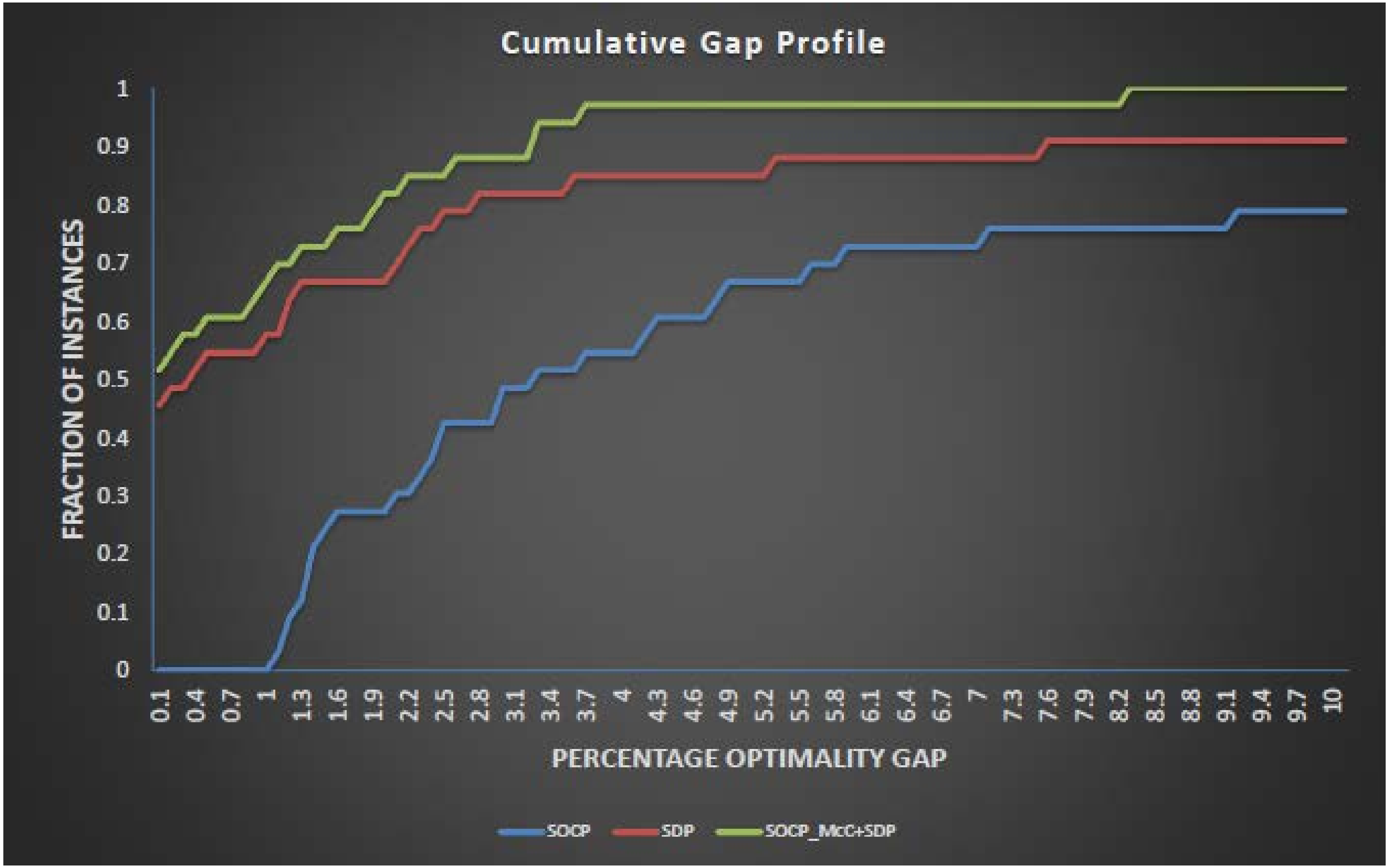
- Workhorse: SOCP relaxation for fast computation
- Strengthen SOCP relaxation for key non-convexities:
  - **Minor constraints**: Characterize convex hull and linear outer envelopes
  - **Arctangent envelopes + SDP separation**:
    - **Arctangent envelope:** Linear upper/lower approximation
    - **SDP separation:** Lift-and-project
- **Results:**
  - IEEE instances (Easy): up to 3375-bus

| | %gap | Time (s) |
|---|---|---|
| SOCP | 0.43 | 2.62 |
| SOCP_cuts | 0.08 | 207.81 |
| SDP | 0.04 | 380.37 |

  - NESTA (Hard): up to 3375-bus

| | Plain SOCP | | SDP | | SOCP w. Cuts | |
|---|---|---|---|---|---|---|
| case | %gap | time (s) | %gap | time (s) | %gap | time (s) |
| Typical | 5.14 | 1.97 | 0.35 | 291.20 | 1.09 | 54.03 |
| Congested | 6.29 | 0.72 | 2.91 | 306.69 | 1.59 | 101.27 |
| Small Angle | 5.22 | 1.62 | 2.30 | 305.25 | 1.79 | 62.34 |

# Cumulative Gap Profile



# Part 2

Data-Driven Robust Unit Commitment

# Part 2: Data-Driven Robust UC

- **Dynamic Uncertainty Models for Temporal-Spatial Correlations of Wind/Solar/Demand**
- **Respecting Physical Causality Improves Ramping Capabilities of Power System**
- **Computational Results:**
  - **Practical computation time 2718-bus**
  - **Near-optimal performance**
  - **Reduced reserve requirement and increased reliability level**

# Recent Works on Robust UC and ED

- Robust Optimization for unit commitment
  - Adaptive two-stage robust SCUC models
    - [Jiang et. al. 2012], [Zhao, Zeng 2012],
    - [Bertsimas, Litvinov, Sun, Zhao, Zheng 2013] (joint w. ISO-NE)
  - RO for security optimization
    - [Street et. al. 2011], [Wang et. al. 2013]
  - Unifying RO with Stochastic UC
    - [Wang et. al. 2013]
  - New types uncertainty set
    - [Guan Wang 2014] [Lorca Sun 2014] [Chen et. al. 2015]
- Robust Optimization for economic dispatch
  - AGC control (two-stage: dispatch + AGC)
    - [Zheng et. al. 2012]
  - Affine policy (dispatch as linear function of total load)
    - [Jabr 2013][Warrington2012,2013]

# Dynamic Uncertainty Sets

- In a multi-period problem:

  Let $\xi_t$ be the uncertainty vector at time *t*

  Uncertainty set of $\xi_t$ *depends on* the realization of uncertainties before *t*

$$\Xi_t(\xi_{[1:t-1]}) = \left\{ \xi_t : \exists u_{[t]} \text{ s.t. } f(\xi_{[t]}, u_{[t]}) \leq 0 \right\}$$

- For example: a dynamic interval uncertainty set:

$$\xi_t \in \left[ \underline{\xi}_t(\xi_{[t-1]}), \overline{\xi}_t(\xi_{[t-1]}) \right]$$

- Polyhedral dynamic uncertainty sets:

$$\sum_{\tau=1}^{t} (A_\tau \xi_\tau + B_\tau u_\tau) \leq 0$$

# Dynamic Uncertainty Sets for Wind Speed

- A dynamic uncertainty set for wind speed:

$$\mathcal{R}_t(\boldsymbol{r}_{[t-L:t-1]}) = \Big\{ \boldsymbol{r}_t : \exists \widetilde{\boldsymbol{r}}_{[t-L:t]},\ \boldsymbol{u}_t \quad \text{s.t.}$$

$$\boldsymbol{r}_\tau = \boxed{\boldsymbol{g}_\tau} + \boxed{\widetilde{\boldsymbol{r}}_\tau} \quad \forall \tau = t-L, \ldots, t$$

$$\boxed{\widetilde{\boldsymbol{r}}_t = \sum_{s=1}^{L} \boldsymbol{A}_s \widetilde{\boldsymbol{r}}_{t-s} + \boldsymbol{B}\boldsymbol{u}_t}$$

$$\sum_{i \in \mathcal{N}^w} |u_{it}| \le \Gamma^w \sqrt{N^w}$$

$$|u_{it}| \le \Gamma^w \qquad \forall i \in \mathcal{N}^w$$

$$\boldsymbol{r}_t \ge \boldsymbol{0} \Big\},$$

Residual

Seasonal pattern

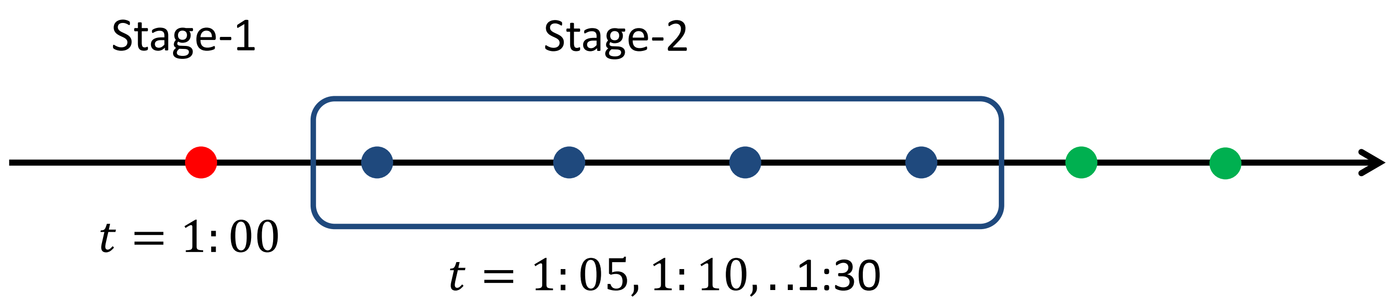
Linear dynamics: Temporal & Spatial correlation

Uncertainty in Estimation with Budget Constraints

# Two-Stage Robust ED and Rolling Horizon

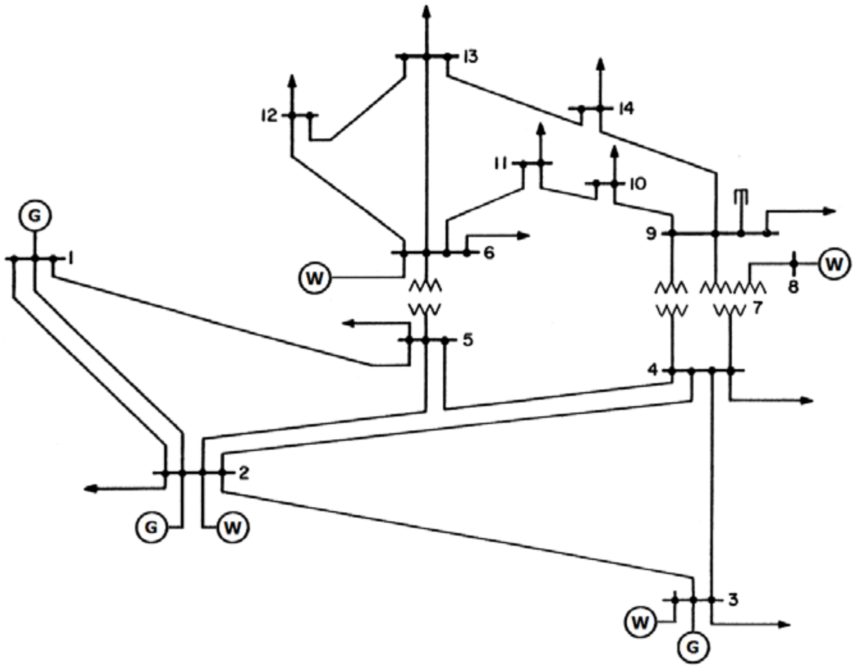
- Adaptive robust ED:
  - **Time period 1** is decision to be implemented
  - **Future periods** with dynamic uncertainty sets

$$\min_{\boldsymbol{x}\in\Omega_1^{det}} \left\{ \boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{d}\in\mathcal{D},\overline{\boldsymbol{p}}^w\in\overline{\mathcal{P}}^w} \; \min_{\boldsymbol{y}\in\Omega(\boldsymbol{x},\boldsymbol{d},\overline{\boldsymbol{p}}^w)} \boldsymbol{b}^\top \boldsymbol{y} \right\}$$



# Experiment Setup

- IEEE Test Systems with 14-bus and 118-bus
- 14-bus system: 3 thermal gen, 4 wind farms, 11 loads, 20 transmission lines




THERMAL GENERATORS IN 14-BUS SYSTEM

| Gen | Pmax (MW) | Pmin (MW) | Ramp (MW/10min) | Cost ($/MWh) |
|---|---|---|---|---|
| 1 | 300 | 50 | 5 | 20 |
| 2 | 100 | 10 | 10 | 40 |
| 3 | 100 | 10 | 15 | 60 |

Daily system demand:

132.6MW-319.1MW

Avg: 252.5MW

# Experiment Setup

- Wind farms:
  - 4 wind farms, each of 75MW (50 GE 1.5MW)
  - Real wind data: 5 min wind speed for a year
  - Exhibit significant temporal/spatial correlations
  - Avg wind speeds: 4.8m/s, 5.6m/s, 5.1m/s, 5.5m/s
  - Avg total available wind power: 104.2MW
    - Equivalent to 34.7% capacity factor
    - Or 32.7% of peak load, 20% of thermal generation
    - Represent significant level of wind penetration

# Robust ED Improves Cost and Reliability

- Adaptive robust ED v.s. Determ Look-Ahead ED:

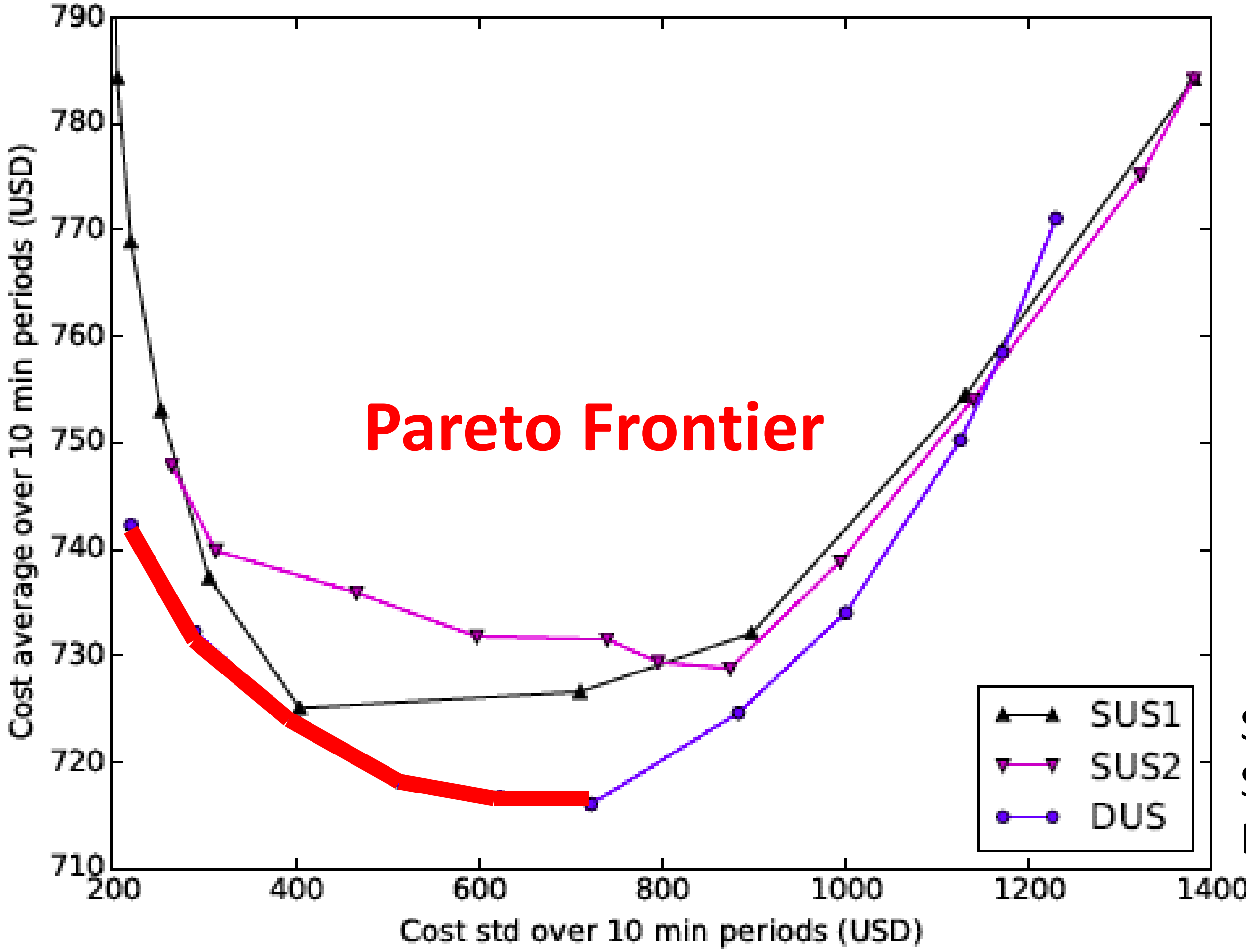
PERFORMANCE OF ROBUST AND DETERMINISTIC ED

| | LA-ED | Rob-ED | | | | |
|---|---|---|---|---|---|---|
| $\Gamma^w$ | **0.0** | 0.1 | 0.3 | **0.5** | 0.7 | **1.0** |
| Total Cost Avg ($) | **771.1** | 758.5 | 734.0 | **716.0** | 718.2 | **742.2** |
| Total Cost Std ($) | **1231** | 1172 | 1000 | **723** | 513 | **221** |
| Penalty Avg ($) | **88.2** | 77.1 | 54.2 | **30.6** | 15.8 | **2.4** |
| Penalty Freq (%) | **1.41** | 1.21 | 0.95 | **0.67** | 0.46 | **0.28** |

- Cost Avg: Rob-ED 7.1% ($\Gamma = 0.5$) lower than LA-ED
- Cost StD: Rob-ED 41.2% lower than LA-ED;
  Rob-ED up to 82.0% lower than LA-ED
- Penalty freq: Rob-ED 52.4% lower than LA-ED;
  Rob-ED up to 80.1% lower than LA-ED

# Dynamic U Sets Pareto Dominate

- Dynamic uncertainty sets v.s. Static uncert sets

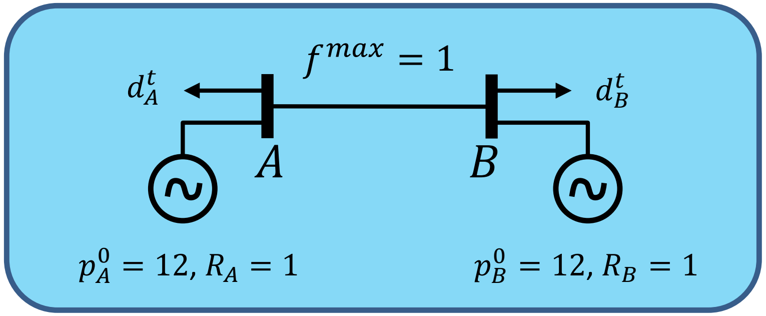



SUS1: No temp
SUS2: No temp/spatial
DUS: w. temp/spatial

A. Lorca, A. Sun Adaptive Robust Optimization with Dynamic Uncertainty Sets for Multi-Period Economic Dispatch under Significant Wind, to appear *IEEE Trans Power Syst* 2015

# Issues with Two-Stage Robust UC

- A simple two-bus two-period example:




Demand uncertainty sets:
$D^1 = \{(12,12)\}$,
$D^2 = \{(d_A^2, d_B^2): d_A^2 + d_B^2 = 25, d_i^2 \in [10,15]\}$

- Claim: **Two-stage robust UC is feasible**
  - UC solution: $(x_A^t, x_B^t) = (1,1)$ for $t = 1,2$
  - Feasible dispatch solution:
    - $p_A^1(\boldsymbol{d}) = 12 + \frac{2}{5}(d_A^2 - 12.5), p_B^1(\boldsymbol{d}) = 12 - \frac{2}{5}(d_A^2 - 12.5)$
    - $p_A^2(\boldsymbol{d}) = 12.5 + \frac{3}{5}(d_A^2 - 12.5), p_B^2(\boldsymbol{d}) = 12.5 - \frac{3}{5}(d_A^2 - 12.5)$
  - Satisfy $p_A^t(\boldsymbol{d}) + p_B^t(\boldsymbol{d}) = d_A^t + d_B^t, f_{AB}(\boldsymbol{d}) \leq f^{max}, \forall \boldsymbol{d} \in D$

# Respecting Physical Causality is Important

- Can we find a policy $p(\cdot)$ that does not look into the future? i.e. $\boldsymbol{p}^1(\boldsymbol{d}^1), \boldsymbol{p}^2(\boldsymbol{d}^1, \boldsymbol{d}^2)$?
  - Because real-time dispatch cannot depend on future

- No feasible **non-anticipative** policy exists!
  - No feasible $\boldsymbol{p}^1$ s.t. for any $\boldsymbol{d}^2 \in D^2$ there exists $\boldsymbol{p}^2$
  - If $p_A^1 \in [11,12]$: $p_A^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (15,10)$
  - If $p_A^1 \in [12,13]$: $p_B^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (10,15)$

- Bottleneck: Ramping constraint

# Multistage Robust UC

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{p}(\cdot)} \left\{ \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} (G_i x_i^t + S_i u_i^t) \; + \; \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i \, p_i^t(\boldsymbol{d}^{[t]}) \right\}$$

s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \le p_i^t(\boldsymbol{d}^{[t]}) \le p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\, i \in \mathcal{N}_g,\, t \in \mathcal{T}$$

$$-RD_i x_i^t - SD_i v_i^t \le \boxed{p_i^t(\boldsymbol{d}^{[t]})} - p_i^{t-1}(\boldsymbol{d}^{[t-1]}) \le RU_i x_i^{t-1} + SU_i u_i^t$$

$$\forall\, \boldsymbol{d} \in \mathcal{D},\, i \in \mathcal{N}_g,\, t \in \mathcal{T}$$

$$-f_l^{max} \le \alpha_l^\top \left( B^p\, \boldsymbol{p}^t(\boldsymbol{d}^{[t]}) - B^d\, \boldsymbol{d}^t \right) \le f_l^{max} \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\, t \in \mathcal{T},\, l \in \mathcal{N}_l$$

$$\sum_{i\in\mathcal{N}_g} p_i^t(\boldsymbol{d}^{[t]}) = \sum_{j\in\mathcal{N}_d} d_j^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\, t \in \mathcal{T}$$

Notation: $\boldsymbol{d}^{[t]} = (\boldsymbol{d}^1, \dots, \boldsymbol{d}^t)$

# Affine Multistage Robust UC

- Tractable alternative for $p(\cdot)$:

$$p_i^t(\boldsymbol{d}^1, \ldots, \boldsymbol{d}^t) = w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs}\, d_j^s$$

- Multistage robust UC with affine policy:

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{w},\boldsymbol{W}} \left\{ \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} (G_i x_i^t + S_i u_i^t) + \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i \left( w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs} d_j^s \right) \right\}$$

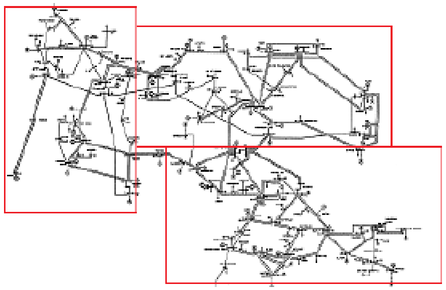
s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \le w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs} d_j^s \le p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d}\in\mathcal{D},\ i\in\mathcal{N}_g,\ t\in\mathcal{T}$$

$\ldots$

# Simplified Affine Policies




**Temporal Aggregation**

**Spatial Aggregation**

General affine policy: $p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{s \in \{1,\ldots,t\}} \sum_{j \in \mathcal{N}_d} W_{itjs}\, d_j^s$

Simpler information basis: $p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{j \in \mathcal{N}_d} W_{itj}\, d_j^t$

All loads aggregated: $p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t$

Loads and time periods aggregated: $p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_i \sum_{j \in \mathcal{N}_d} d_j^t$

# Solution Method

- **Dualization** approach does not work:
  - Traditionally, robust constraints are dualized
  - Resulting problem is too large for power systems

- **Constraint generation** makes sense:

$$p_i^{min} x_i^t \leq w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t \leq p_i^{max} x_i^t \qquad \forall \boldsymbol{d} \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T}$$

- However, naïve CG also does not work

# Solution Method

- **Valid inequalities** for $x$ and specific $d$'s for ramping, generating limits, and line flow
- **Fixing** binary decisions and finding cuts by CG with an LP master
- **Iteratively improving** policy structure (e.g. $W_i \rightarrow W_{it}$) with approximate warm-start (not solving $W_i$ fully)
- **Exploiting structure** of special policy form: e.g. pre-computing all needed constraints for ramping and generation limit constraints for $W_{it}$-policy.

# Computational Study

- How good is the proposed algorithm?
  - Effectiveness of various algorithmic improvements
- How good is the simplified affine policy?
  - Compared to the “true” multi-stage robust UC
- Why should we use multi-stage formulation?
  - Worst case infeasibility of two-stage robust UC
  - Managing Ramping capability
- How good is affine UC “on average”?
  - Rolling-horizon Monte-Carlo simulation
  - Average performance in cost, std, reliability

# How Good is the Algorithm?

**Solution time (s) for three test systems using $W_{it}$ policy:**

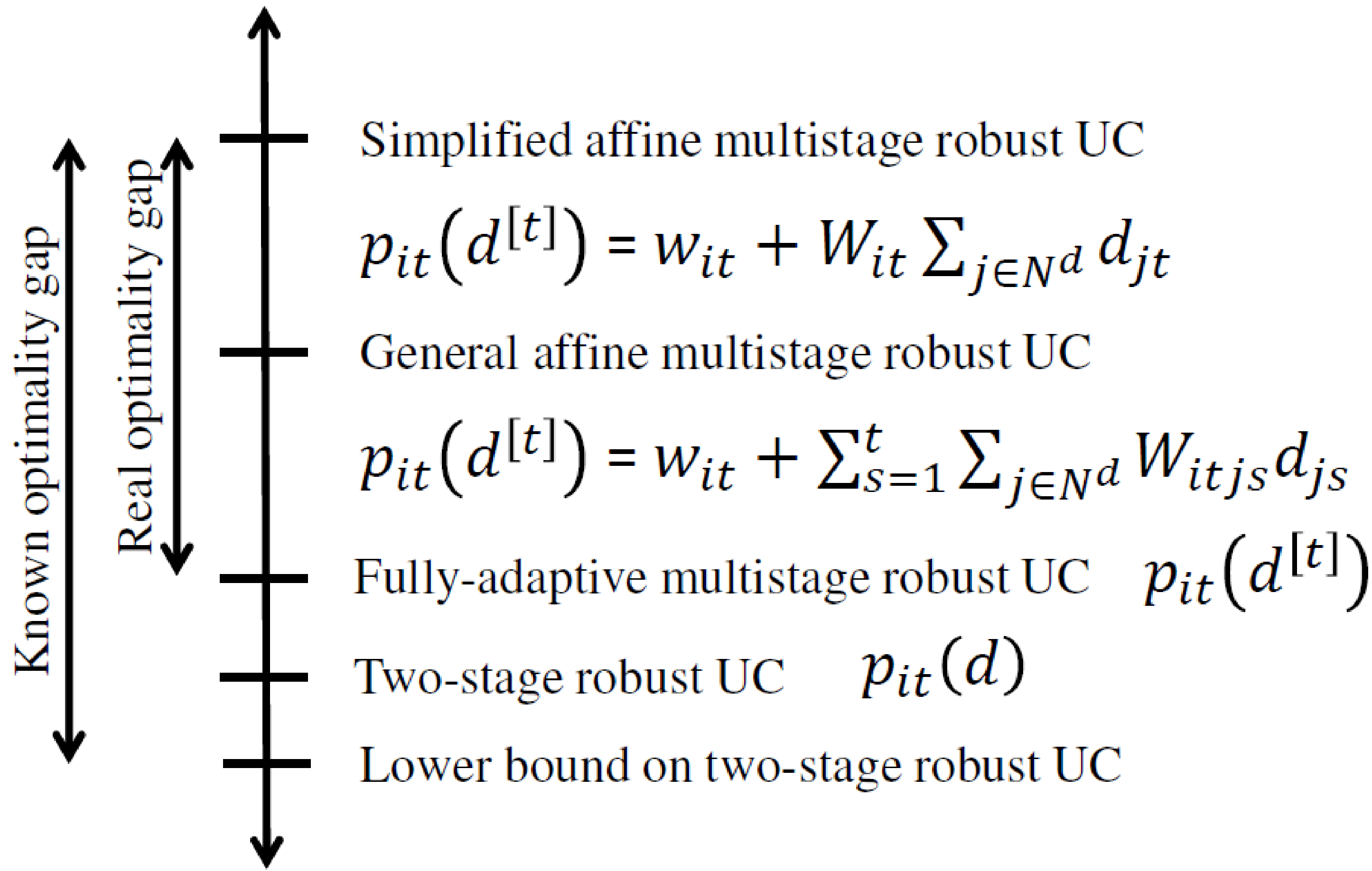
| System | $\Gamma = 0.25$ | $\Gamma = 0.5$ | $\Gamma = 1$ | $\Gamma = 2$ | $\Gamma = 4$ |
|---|---|---|---|---|---|
| 30 bus | 6s | 3s | 8s | 6s | 29s (inf) |
| 118 bus | 66s | 64s | 47s | 63s | 178s |
| 2718 bus | 3.6h | 3.2h | 2.3h | 2.0h | 0.4h (inf) |

Note: "inf" indicates that the problem is infeasible

MIP optimality gap used for 30, 118, 2718 bus systems: 0.1%, 0.1%, 1%

# How Good is the Simplified Affine Policy?

- How good is the simplified affine policy?



# How Good is the Simplified Affine Policy?

Table : Opt. gap under different policy structures, for the 118 bus system.

| $(n_g, n_{\mathcal{T}}, n_d, L)$ | $\Gamma = 0.5$ | $\Gamma = 1$ | $\Gamma = 2$ | $\Gamma = 4$ |
|---|---|---|---|---|
| (10,1,1,0) | 0.03% | 0.06% | 0.11% | 0.95% |
| (21,1,1,0) | 0.03% | 0.05% | 0.11% | 0.77% |
| (31,1,1,0) | 0.02% | 0.04% | 0.10% | 0.74% |
| (54,1,1,0) | 0.02% | 0.04% | 0.10% | 0.67% |
| (54,4,1,0) | 0.02% | 0.03% | 0.10% | 0.52% |
| (54,24,1,0) | 0.02% | 0.03% | 0.07% | 0.35% |

Table : Opt. gap for the 2718 bus system under the "$W_{it}$" policy.

| $(n_g, n_{\mathcal{T}}, n_d, L)$ | $\Gamma = 0.25$ | $\Gamma = 0.5$ | $\Gamma = 1$ | $\Gamma = 1.5$ | $\Gamma = 2$ |
|---|---|---|---|---|---|
| (289,1,1,0) | 0.09% | 0.22% | 0.42% | 0.55% | 1.05% |
| (289,24,1,0) | 0.07% | 0.11% | 0.25% | 0.35% | 0.53% |

# Why Multistage? Worst-Case

- Worst-case (US$) of multistage robust dispatch under two-stage and Multistage UC solutions for the 2718-bus system.

| | $\Gamma = 0.5$ | $\Gamma = 1$ | $\Gamma = 1.5$ | $\Gamma = 2$ | $\Gamma = 3$ |
|---|---|---|---|---|---|
| Affine multistage UC solutions | | | | | |
| Total Cost | 9,445,069 | 9,596,788 | 9,746,685 | 9,905,527 | 10,234,459 |
| Penalty | 0 | 0 | 0 | 0 | 0 |
| Two-stage UC solutions | | | | | |
| Total Cost | 9,505,651 | 9,745,889 | 10,183,433 | 10,975,403 | 12,864,719 |
| Penalty | 96,313 | 224,952 | 591,661 | 1,165,324 | 2,703,522 |
| Rel Diff | 0.64% | 1.55% | 4.49% | 10.80% | 25.70% |

# How Good is Affine UC on Average?

- Average performance over independent demand

Affine multistage robust UC with policy-enforcement robust ED

| Γ | 0.25 | 0.5 | 1 | 1.5 | 2 | 3 |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 9,397,528 | 9,319,396 | 9,342,754 | 9,360,359 | 9,379,464 | 9,442,858 |
| Cost Std ($) | 113,725 | 15,970 | 12,828 | 12,509 | 12,363 | 12,092 |
| Penalty Cost Avg ($) | 93,552 | 3497 | 727 | 61 | 5 | 0 |
| Penalty Freq Avg | 10.00% | 1.47% | 0.40% | 0.01% | 0.00% | 0.00% |

0.46%

Two-stage robust UC with look-ahead ED

| Γ | 0.25 | 0.5 | 1 | 1.5 | 2 | 3 |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 9,398,109 | 9,456,599 | 9,408,732 | 9,383,569 | 9,407,290 | 9,362,379 |
| Cost Std ($) | 93,470 | 195,774 | 173,884 | 144,698 | 162,469 | 45,584 |
| Penalty Cost Avg ($) | 80,127 | 152,637 | 98,113 | 66,801 | 82,864 | 6,103 |
| Penalty Freq Avg | 9.93% | 12.26% | 7.80% | 5.11% | 5.57% | 0.37% |

Deterministic UC with reserve and look-ahead ED 0.95%

| Reserve | 2.5% | 5% | 10% | 15% | 20% | 30% |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 9,556,549 | 9,575,446 | 9,424,678 | 9,561,024 | 9,408,173 | 9,411,741 |
| Cost Std ($) | 261,464 | 288,777 | 121,122 | 196,354 | 92,268 | 69,050 |
| Penalty Cost Avg ($) | 254,627 | 271,672 | 119,127 | 248,658 | 83,938 | 51,907 |
| Penalty Freq Avg | 15.93% | 13.37% | 14.31% | 18.16% | 10.03% | 7.22% |

# How Good is Affine UC on Average?

- Average performance over wind power and persistent demand

Affine multistage robust UC with policy-enforcement robust ED

| $\Gamma$ | 0.25 | 0.5 | 1 | 1.5 | 2 | 3 |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 10,996,931 | 9,459,785 | 8,502,923 | 8,581,532 | 8,646,665 | 9,415,693 |
| Cost Std ($) | 3,665,301 | 2,007,317 | 490,457 | 466,999 | 424,801 | 458,865 |
| Penalty Cost Avg ($) | 2,679,299 | 1,110,032 | 101,234 | 81,834 | 27,344 | 218 |
| Penalty Freq Avg | 18.84% | 14.44% | 1.67% | 0.47% | 0.18% | 0.01% |

**1.23%**

Two-stage robust UC with look-ahead ED

| $\Gamma$ | 0.25 | 0.5 | 1 | 1.5 | 2 | 3 |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 10,390,214 | 11,365,568 | 8,734,840 | 8,863,975 | 8,609,160 | 8,947,959 |
| Cost Std ($) | 1,831,279 | 1,059,427 | 620,301 | 802,441 | 522,881 | 793,447 |
| Penalty Cost Avg ($) | 2,064,045 | 1,032,109 | 380,451 | 490,562 | 195,681 | 443,401 |
| Penalty Freq Avg | 12.73% | 3.68% | 7.37% | 5.19% | 2.07% | 2.66% |

**24.52%**

Deterministic UC with reserve and look-ahead ED

| Reserve | 2.5% | 5% | 10% | 15% | 20% | 30% |
|---|---|---|---|---|---|---|
| Cost Avg ($) | 13,186,705 | 14,272,477 | 13,110,030 | 13,617,194 | 11,879,817 | 11,248,546 |
| Cost Std ($) | 5,557,309 | 7,023,964 | 5,596,039 | 6,082,173 | 4,095,780 | 3,113,902 |
| Penalty Cost Avg ($) | 4,905,635 | 6,003,861 | 4,827,766 | 5,334,746 | 3,578,986 | 2,912,186 |
| Penalty Freq Avg | 30.45% | 29.94% | 33.00% | 32.43% | 23.61% | 15.03% |

# References


- A. Lorca, X. A. Sun Adaptive Robust Optimization with Dynamic Uncertainty Sets for Multi-Period Economic Dispatch under Significant Wind, *IEEE Trans Power Syst 30(4): 1702-1713,* 2015
- A. Lorca, X. A. Sun, E. Litvinov, T. Zheng, Multistage Robust Optimization for Unit Commitment Problem, *Operations Research*, 64(1): 32-51, 2016
- A. Lorca, X. A. Sun, Multistage Robust Unit Commitment with Dynamic Uncertainty Sets and Energy Storage, accepted for publication at *IEEE Trans Power Syst*, 2017

# Some Concluding Remarks

- Significant challenges:
  - AC Optimal Flow Problem with Discrete Decisions
  - Voltage-stability constrained OPF
  - Robust UC with AC OPF
  - Sensor-driven real-time operation and maintenance scheduling
- Many more challenging computational problems!

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- Nov 9 – 10, 2017 at Georgia Tech
- Please come to GT and continue our discussion!

Thank you!

Questions?

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