Variational optimal power flow and dispatch problems and their approximations

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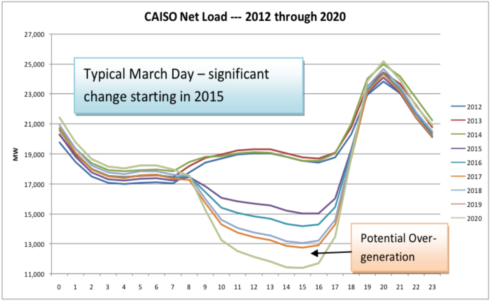
Motivation



- Problem: Shortage of ramping resources in the real-time operation of power systems
  $\rightarrow$ ramping is not appropriately represented and incentivized

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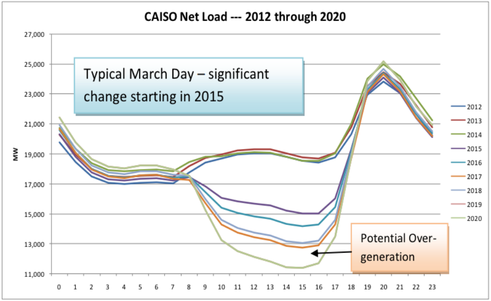

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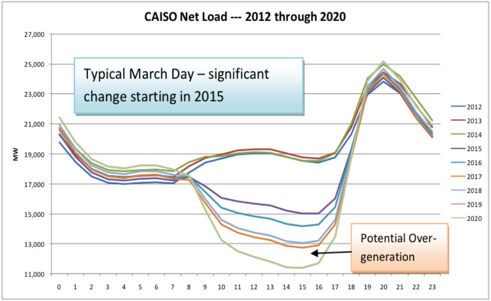
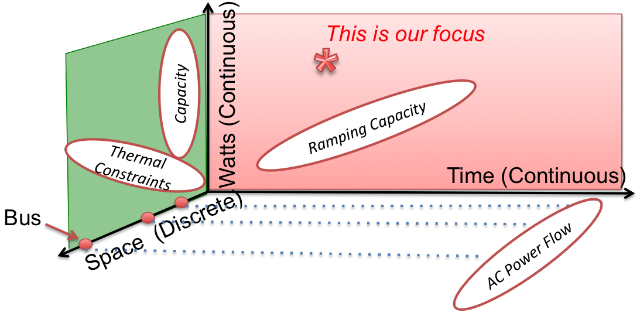


- Problem: Shortage of ramping resources in the real-time operation of power systems
  $\rightarrow$ ramping is not appropriately represented and incentivized
- Flexible ramping products (e.g. CAISO and MISO)
- Tenet: Better handling of both *variability* and *uncertainty*

Modeling errors in time

- Load demand is a continuous time random process
- Generators have continuous time inter-temporal constraints (ramping, on-off time)

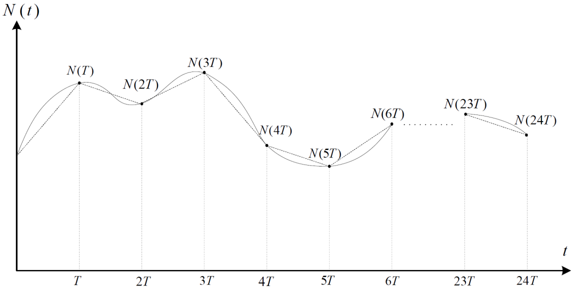


Objective

Mapping the variational stochastic problems into tractable approximations.

Where is ramping first accounted for?

1. In the Unit Commitment (UC) we schedule a piecewise constant generation trajectory based on a single forecast
2. Trajectory Interpretation: Hourly ramping constraints $\rightarrow$ piecewise linear generation trajectory



Agenda

Information loss $\rightarrow$ In the conventional practice, continuity and higher order stochastic features are being relaxed

- continuous trajectories & derivatives are replaced by samples & finite differences
- deterministic approximation: single forecast for the net-load
- stochastic approximation: only marginal distributions, Markov chains, discrete time quantized scenario trees/fan

In this talk we introduce:

- Continuous Time Economic Dispatch (CT-ED), marginal pricing and approximation via Splines of CT DC OPF
- Continuous Time Unit Commitment: Deterministic (CT-DUC) and Stochastic Multi-Stage formulations (CT-SMUC)

Nomenclature for Continuous Time Optimization

OPF and UC variables, Deterministic case

- Generator index $g \in \mathcal{G}$: Set of generation units,
- Bus index $b \in \mathcal{B}$: Set of buses,
- $(l, l') \in \mathcal{B} \times \mathcal{B}$: Set of transmission lines,
- $\xi^b(t) \in \mathbb{R}_+$: Net-Load Demand
- Schedule for $g \in \mathcal{G}^b$
  - $\mathrm{x}^g(t) \in \mathbb{R}_+$: Scheduled power
  - $\dot{\mathrm{x}}^g(t) \in \mathbb{R}_+$: Ramping decision
  - $\mathrm{y}^g(t) \in \{0, 1\}$: Commitment decision
  - $\overline{\mathrm{s}}^g(t)$ switching action from *off* to on,
  - $\underline{\mathrm{s}}^g(t)$ switching action from *on* to off.
- Costs: C^g and startup $\overline{S}^g$, shut-down $\underline{S}^g$

Continuous Time Economic Dispatch and Marginal Pricing

Economic Dispatch in continuous Time

Continuous Time Economic Dispatch:

$\min \sum_{b\in\mathcal{B}} \sum_{g\in\mathcal{G}^b} \int_{t_0}^{t_0+T} C^g(\mathrm{x}^g, t)dt \quad \text{w.r.t} \quad \mathbf{x}(t)$	Objective and decision var.
$\sum_{b\in\mathcal{B}} \left(\sum_{g\in\mathcal{G}^b} \mathrm{x}^g(t) - \xi^b(t) \right) = 0$	Balance constraint
$\underline{G}^g \leq \mathrm{x}^g(t) \leq \overline{G}^g$	Production capacity
$-\underline{G}^{g\prime} \leq \dot{\mathrm{x}}^g(t) \leq \overline{G}^{g\prime}$	Ramping constraint

- Note: $C^g(\mathrm{x}^g, t)$ is a cost per unit of time (may depend on the ramp $\dot{x}^g$ too, optional)

The DC OPF version simply adds:

$-L_{ll'} \leq \sum_{b\in\mathcal{B}} D^b_{ll'} \left(\sum_{g\in\mathcal{G}^b} \mathrm{x}^g(t) - \xi^b(t) \right) \leq L_{ll'}$	Thermal constraints

Variational formulation of the CT-ED

Lagrangian of the CT-ED:

$$\mathcal{L} = \sum_{b\in\mathcal{B}} \sum_{g\in\mathcal{G}^b} \int_{t_0}^{t_0+T} f^{(g,b)}(\mathrm{x}^g, \dot{\mathrm{x}}^g, t)dt$$

$$\begin{aligned} f^{(g,b)}(\mathrm{x}^g, \dot{\mathrm{x}}^g, t) =& C^g(\mathrm{x}^g, t) + \lambda(t)\left(\frac{\xi^b(t)}{|\mathcal{G}^b|} - \mathrm{x}^g(t)\right) \\ &+ \overline{\mu}^g(t)(\mathrm{x}^g(t) - \overline{G}^g) + \underline{\mu}^g(t)(\underline{G}^g - \dot{\mathrm{x}}^g(t)) \end{aligned}$$

The variational problem:

$$\min_{\mathbf{x}(t)} \mathcal{L} = \min_{\mathbf{x}(t)} \sum_{b\in\mathcal{B}} \sum_{g\in\mathcal{G}^b} \int_{t_0}^{t_0+T} f^{(g,b)}(\mathrm{x}^g, \dot{\mathrm{x}}^g, t)dt$$

is a special case of the isoperimetric problem in Physics.

Optimum solutions and Euler-Lagrange equations

- The optimum trajectories $x_o^g(t)$ are solutions of the Euler-Lagrange partial differential equations:

$$\frac{\partial f^{(g,b)}(\mathrm{x}_o^g, \dot{\mathrm{x}}_o^g, t)}{\partial x^g} - \frac{d}{dt}\frac{\partial f^{(g,b)}(\mathrm{x}_o^g, \dot{\mathrm{x}}_o^g, t)}{\partial \dot{\mathrm{x}}^g} = 0, \ \forall b \in \mathcal{B}, g \in \mathcal{G}^b$$

  plus the remaining KKT conditions...

- Hence, the Lagrange multiplier function, the marginal cost and the other Lagrange multipliers functions:

$$\lambda_o(t) = \frac{\partial C^g(\mathrm{x}_o^g, t)}{\partial \mathrm{x}^g} - \underbrace{\frac{d}{dt}\frac{\partial C^g(\mathrm{x}_o^g, t)}{\partial \dot{\mathrm{x}}^g}}_{=0}$$

$$+ \overline{\mu}_o^g(t) - \underline{\mu}_o^g(t) - \frac{d\overline{\gamma}_o^g(t)}{dt} + \frac{d\underline{\gamma}_o^g(t)}{dt}$$

$$\forall t_0 \leq t \leq t_0 + T, \quad g \in \mathcal{G}$$

Observations

- Due to complementarity slackness if constraints are not tight $\overline{\mu}_o^g(t) = \underline{\mu}_o^g(t) = 0$ and/or $\overline{\gamma}_o^g(t) = \underline{\gamma}_o^g(t) = 0$.
- For feasibility each time instant $t_0 \leq t \leq t_0 + T$ the always exist an extra unit to meet demand
- The *marginal unit* is the unit g^* for which at time t and so $\underline{\mu}_o^{g*}(t) = 0$ and/or $\overline{\gamma}_o^{g*}(t) = 0$

$$\lambda_o(t) = \frac{\partial C^{g^*}(\mathrm{x}_o^{g^*}, t)}{\partial \mathrm{x}^g}$$

- Note that since the marginal unit in general will be different at different times, $\lambda_o(t)$ is naturally a discontinuous function (piece-wise constant if costs are linear in $\mathrm{x}^g(t)$)

Marginal Price

- Suppose we increase the entire load trajectory at an arbitrary bus by a constant $\xi^b(t) \to \tilde{\xi}^b(t) = \xi^b(t) + \epsilon$ without any change in ramp
- It is not difficult to see that the rate of change of the objective w.r.t. ϵ is:

$$\lim_{\epsilon \to 0} \frac{\mathcal{L}^*(\epsilon) - \mathcal{L}(\epsilon)}{\epsilon} = \int_{t_0}^{t_0+T} \lambda_o(t) dt$$

which in turn implies that $\lambda_o(t)$ could be interpreted as a shadow price per unit of time.

Approximation of the CT DC-OPF

- Without loss of generality let $t_0 = 0$ and $T = 1$
- Suppose also that $C^g(\mathrm{x}^g, t) = C^g(\mathrm{x}^g) = \Lambda^g \mathrm{x}^g + const.$
- If the net-load lies approximately in an $n+1$ dimensional signal space, spanned by the linearly independent functions $\{b_i^{(n)}(t)\}_{i=0}^n$ can we approximate the variational solution?

$$\xi^b(t) \approx \sum_{i=0}^{n} \xi_i^b b_{i,n}(t) \quad \rightarrow \quad \mathrm{x}^g(t) \approx \sum_{i=0}^{n} x_i^g b_{i,n}(t)$$

There are uncountable constraints

- Balance: OK if $\forall b \in \mathcal{B}, i = 0, .., n, \quad \xi_i^b - \sum_{g \in \mathcal{G}^b} x_i^g = 0$
- Inequalities: Capacity and ramping constraints, flows need attention $\rightarrow$ this goal guides the choice of $\{b_i^{(n)}(t)\}_{i=0}^n$

Bernstein Polynomials

Bernstein polynomials of degree n are defined as

$$b_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i} \Pi(t), \quad i \in [0, n]$$

$$\Pi(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{else} \end{cases}$$

And the vector of polynomials of degree n is denoted by $\mathbf{b}_n(t)$.

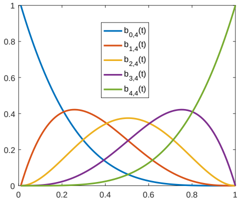


Figure: Bernstein polynomial basis for $n = 4$

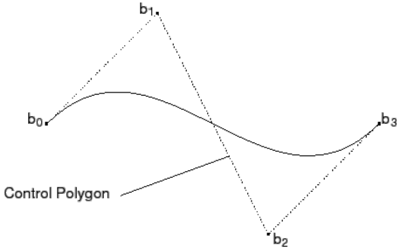
Convex Hull Property

- The coefficients of the Bernstein polynomial expansion define control points for the corresponding curves are called Bérzier curves
- A Bérzier curve is always contained in the convex hull of the control points
- For a $1D$ function:

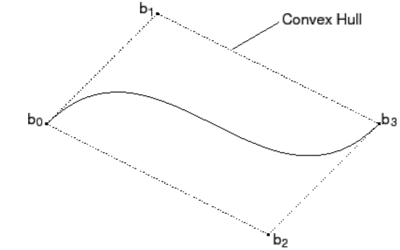
$$\min_i x_i \leq \mathrm{x}(t) \leq \max_i x_i$$

- The derivative is also a Bérzier curve of order $n-1$ such that

$$\dot{\mathrm{x}}(t) = \sum_{i=0}^{n-1} \underbrace{n(x_{i+1} - x_i)}_{\dot{x}_i} b_{i(n-1)}(t)$$



(a)



(b)

Approximation of DC -OPF

Indicating by $\mathbf{x}$ the $(n+1) \times |\mathcal{G}|$ matrix of all coefficients:

$$\min_{\mathbf{x}(t)} \sum_{g \in \mathcal{G}} \int_0^1 C^g(\mathrm{x}^g) dt = \min_{\mathbf{X}} \sum_{g \in \mathcal{G}} \Lambda^g \sum_{i=0}^{n} x_i^g \underbrace{\int_0^1 b_{in}(t) dt}_{\frac{1}{n+1}}$$

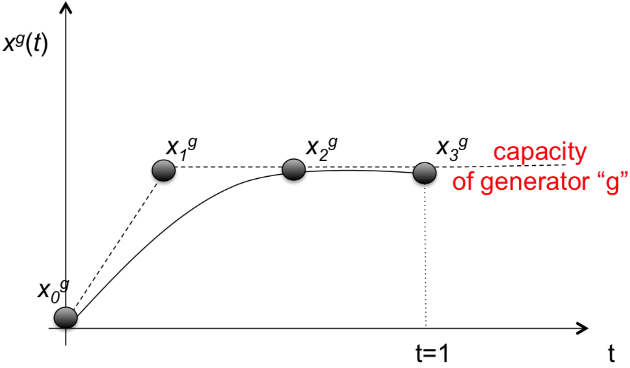
$$\text{s.t. Balance } \forall b \in \mathcal{B}, i = 0, .., n, \quad \xi_i^b - \sum_{g \in \mathcal{G}^b} x_i^g = 0$$

- Capacity: $\max_i x_i^g \leq \overline{G}^g$ & $\min_i x_i^g \geq \underline{G}^g$ imply $\underline{G}^g \leq \mathrm{x}^g(t) \leq \overline{G}^g$
- Ramping: Similarly $\max_i (x_{i+1}^g - x_i^g) \geq \overline{G'}^g / n$ & $\min_i (x_{i+1}^g - x_i^g) \geq -\underline{G'}^g / n$ imply $-\underline{G'}^g \leq \dot{\mathrm{x}}^g(t) \leq \overline{G'}^g$
- Flow constraints : Analogously, sufficient conditions are:

$$\min_i \left(\sum_{b \in \mathcal{B}} D_{ll'}^b \left(\sum_{g \in \mathcal{G}^b} x_i^g - \xi_i^b \right) \right) \geq -L_{ll'} \quad \max_i \left(\sum_{b \in \mathcal{B}} D_{ll'}^b \left(\sum_{g \in \mathcal{G}^b} x_i^g - \xi_i^b \right) \right) \leq L_{ll'}$$

Cost and price

- In the approximate solution constraints become tight $1/(n+1)$ earlier than in reality
- It forces C^1 continuity of generation trajectory $\rightarrow$ it imposes the generators to go smoothly towards their limits



Continuous Time Deterministic and Stochastic Multi-Stage Unit Commitment

Deterministic CT UC

- In principle the commitment function $\mathrm{y}^g(t) \in \{0, 1\}$ could switch units at any time $\rightarrow$ UC then non-linear problem
- State of the art MILP approximation: switching only at the beginning of hour h:

$$\mathrm{y}^g(t) = \sum_{h=1}^{H} y_h^g \Pi \left(\frac{t - t_{h-1}}{t_h - t_{v-1}} \right)$$

i.e. one hourly variable $y_h^g \in 0, 1$ describes the degrees of freedom for $\mathrm{y}^g(t)$ in $(t_{h-1}, t_h]$

The idea of the CT UC:

- Allow the scheduled trajectories $\mathrm{x}^g(t)$ within each $(t_{h-1}, t_h]$ to be a Bérzier curve of the order needed to represent accurately the Bérzier curve of net-load $\xi^b(t)$
- Keep things continuous from one hour to the next

Polynomial Interpolation of Net-Load

Let $(v_-, v) = (h-1, h)$, $(v, v_+) = (h, h+1)$ and $\mathcal{V} = \{1, \ldots, H\}$

- In $t_{v_-} < t \leq t_v$ the vector of *control points*:

$$\boldsymbol{\xi}_{v_-,v} = [\xi^{(0)}_{v_-,v}, \ldots, \xi^{(n-1)}_{v_-,v}, \xi^{(n)}_{v_-,v}]^{\mathrm{T}}$$

- The continuous time approximation $\forall h$ in $t_{h-1} < t \leq t_h$:

$$\xi_{v_-,v}(t) = \sum_{i=0}^{n} \xi^{(i)}_{v_-,v} b_{in}\left(\frac{t - t_{v_-}}{t_v - t_{v_-}}\right) = \mathbf{b}_n^{(v_-,v)}(t)\boldsymbol{\xi}_{v_-,v}$$

  with $\mathbf{b}_n^{(v_-,v)}(t) := \mathbf{b}_n\left(\frac{t-t_{v_-}}{t_v-t_{v_-}}\right)$.

- Continuity:
  - C^0 is equivalent to $\xi^{(n)}_{v_-,v} = \xi^{(0)}_{v,v_+}$
  - C^1 is equivalent to $\xi^{(n)}_{v_-,v} - \xi^{(n-1)}_{v_-,v} = \xi^{(1)}_{v,v_+} - \xi^{(0)}_{v,v_+}$

New Convention for Minimum-up/down Constraints

Two *state* variables o_v^g and d_v^g are introduced to handle minimum-up: O_n^g and minimum-down: O_f^g time for each unit g.

Definition: o_v^g (d_v^g) is the residual time unit g needs to stay on (off) after time t_v, which depends on the state $o_{v_-}^g$ ($d_{v_-}^g$) and only when $o_v^g = 0$ ($d_v^g = 0$) the unit can be turned off (on).

- the state persists for the next generations as long as the unit continues to stay on (off), or
- if is switched off (on), for as long as it is off (on) and not switched on (off) again.

Observations

(1) With these new definitions the on and off constraints can be expressed on a purely nodal basis in the Stochastic MUC. (2) Need to add $\frac{o_v^g}{O_n^g} + \frac{d_v^g}{O_f^g}$ to the cost to relax integrality.

Decision Variables

In continuous time, decision variables:

$$(\mathrm{x}^g(t), \dot{\mathrm{x}}^g(t), \mathrm{y}^g(t), \overline{\mathrm{s}}^g(t), \underline{\mathrm{s}}^g(t), \mathrm{o}^g(t), \mathrm{d}^g(t))$$

may vary continually at all time instances t, providing ultimate flexibility to optimal balancing the load.

Assumption: Commitment and therefore start-up, shut-down, minimum-up/down variables are constant $\forall t, t_{v_-} < t \leq t_v$ and the control point at the end of the interval $(t_{v_-}, t_v]$ carry all the information on the edge (v_-, v).

CT-UC Coefficients Corresponding to Decision Variables

The the polynomial coefficients for continuous-time generation and ramping[1], commitment, start-up, shut-down, minimum-up/down trajectories, for the interval (v_-, v):

$$\mathbf{x}^g_{v_-,v} = [x^{g(0)}_{v_-,v}, x^{g(1)}_{v_-,v}, x^{g(2)}_{v_-,v} \ldots, x^{g(n-1)}_{v_-,v}, x^{g(n)}_{v_-,v}]^{\mathrm{T}}$$

$$\dot{\mathbf{x}}^g_{v_-,v} = [\dot{x}^{g(0)}_{v_-,v}, \dot{x}^{g(1)}_{v_-,v}, \dot{x}^{g(2)}_{v_-,v} \ldots, \dot{x}^{g(n-1)}_{v_-,v}, \dot{x}^{g(n)}_{v_-,v}]^{\mathrm{T}}$$

$$\mathbf{y}^g_{v_-,v} = y^{g(n)}_{v_-,v} = y^g_v$$

$$\overline{\mathbf{s}}^g_{v_-,v} = \overline{s}^{g(n)}_{v_-,v} = \overline{s}^g_v$$

$$\underline{\mathbf{s}}^g_{v_-,v} = \underline{s}^{g(n)}_{v_-,v} = \underline{s}^g_v$$

$$\mathbf{o}^g_{v_-,v} = o^{g(n)}_{v_-,v} = o^g_v$$

$$\mathbf{d}^g_{v_-,v} = d^{g(n)}_{v_-,v} = d^g_v$$

[1] Elements of vector $\dot{\mathbf{x}}^g_{v_-,v}$ can be expressed as linear combination of elements of $\mathbf{x}^g_{v_-,v}$.

Decision Variables ctd.

- Continuous-time generation:

$$\mathrm{x}^{g}_{v_-,v}(t) = \mathbf{b}_n^{(v_-,v)}(t)\mathbf{x}^{g}_{v_-,v} \quad t_{v_-} \le t \le t_v$$

- Continuous-time ramping:

$$\dot{\mathrm{x}}^{g}_{v_-,v}(t) = \mathbf{b}_{n-1}^{(v_-,v)}(t)\overbrace{\mathbf{M}\mathbf{x}^{g}_{v_-,v}}^{\dot{\mathbf{x}}^{g}_{v_-,v}} \quad t_{v_-} < t \le t_v$$

  where the matrix $\mathbf{M}$ changes basis from $d\mathbf{b}_n(t)/dt$ to $\mathbf{b}_{n-1}(t)$
- Continuous-time commitment (similar for switch & on off):

$$\mathrm{y}^{g}_{v_-,v}(t) = y^{g}_{v}\Pi\left(\frac{t - t_{v_-}}{t_v - t_{v_-}}\right) \quad t_{v_-} < t \le t_v$$

- Continuity conditions:
  - C^0 is equivalent to $x^{g(n)}_{v_-,v} = x^{g(0)}_{v,v_+}$
  - C^1 is equivalent to $x^{g(n)}_{v_-,v} - x^{g(n-1)}_{v_-,v} = x^{g(1)}_{v,v_+} - x^{g(0)}_{v,v_+}$
  - (Smooth switch): For generation schedule the last two variables of the expansion $(x^{g(n-1)}_{v_-,v}, x^{g(n)}_{v_-,v})$ are zero or not depending on the next hour commitment $y^{g}_{v^+}$

Constraints: Generation and Ramping Limits

Convexhull property: The entire generation and ramping trajectories for edge (v_-, v) is contained in the convexhull of their control point $\mathbf{x}^g_{v_-,v}$ and $\dot{\mathbf{x}}^g_{v_-,v}$ respectively.

Therefore, bounds on continuous-time generation and ramping trajectories for interval $t_{v_-} \leq t \leq t_v$ can be expressed:

$$\min\{\mathbf{x}^g_{v_-,v}\} \leq \min_{t_{v\_} < t \leq t_v} \mathrm{x}^g_{v_-,v}(t)$$

$$\max_{t_{v\_} < t \leq t_v} \mathrm{x}^g_{v_-,v}(t) \leq \max\{\mathbf{x}^g_{v_-,v}\}$$

$$\min\{\dot{\mathbf{x}}^g_{v_-,v}\} \leq \min_{t_{v\_} < t \leq t_v} \dot{\mathrm{x}}^g_{v_-,v}(t)$$

$$\max_{t_{v\_} < t \leq t_v} \dot{\mathrm{x}}^g_{v_-,v}(t) \leq \max\{\dot{\mathbf{x}}^g_{v_-,v}\}$$

Balance and Transmission Capacity

- The continuous-time balance between generation works like in the CT DC-OPF and load is guarantied and expressed by balancing the polynomial coefficients of load and generation:

$$\sum_{b \in \mathcal{B}} \left(\sum_{g \in \mathcal{G}^b} \mathbf{x}^g_{v_-,v} - \xi^b_{v_-,v} \right) = 0$$

- For the flow constraints we need to use the convex hull property again as we did for CT DC-OPF . . .
- Start-up, Shut-down, and Minimum-up/down Constraints are analogous to conventional UC

Objective Function

- Note that the generation costs terms are linear:

$$C^g(\mathrm{x}^g(t)) = c^g_{1v}\mathrm{x}^g(t) + c^g_{0v}\mathrm{y}^g_v(t)$$

$$S^g(\bar{\mathrm{s}}^g_v(t), \underline{\mathrm{s}}^g_v(t))\overline{S}^g\bar{\mathrm{s}}^g_v(t) + \underline{S}^g\underline{\mathrm{s}}^g_v(t)$$

- Also the following holds:

$$\forall i = 0, \ldots, n \quad \int_{t_{v\_}}^{t_v} b_{in}(\frac{t - t_{v-}}{t_v - t_{v-}})dt = \frac{t_v - t_{v-}}{n+1}$$

- Thus, substituting the variables and nodal notation:

$$\sum_{v\in\mathcal{V}}\sum_{g\in\mathcal{G}}\int_{t_{v\_}}^{t_v}\left(c^g_{1v}\mathbf{b}_n^{(v_-,v)}(t)\mathbf{x}^g_{v_-,v} + c^g_0 y^g_v(t) + \overline{S}^g\bar{\mathrm{s}}^g_v(t) + \underline{S}^g\underline{\mathrm{s}}^g_v(t) + \frac{\mathrm{o}^g_v(t)}{O^g_n} + \frac{\mathrm{d}^g_v(t)}{O^g_f}\right)dt$$

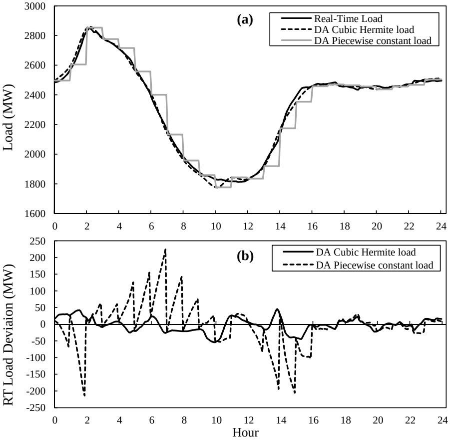
$$= \sum_{v\in\mathcal{V}}(t_v - t_{v\_})\sum_{g\in\mathcal{G}}\frac{c^g_{1v}}{n+1}\left(\sum_{i=0}^{n} x^{g(i)}_{v_-,v}\right) + c^g_0 y^g_v + \overline{S}^g\overline{s}^g_v + \underline{S}^g\underline{s}^g_v + \frac{o^g_v}{O^g_n} + \frac{d^g_v}{O^g_f}$$

CT Deterministic Unit Commitment

$\min \; \sum_{v\in\mathcal{V}}\sum_{g\in\mathcal{G}} \frac{c^g_{1v}}{n+1}\left(\sum_{i=0}^{n} x^{g(i)}_{v_-,v}\right) + c^g_0 y^g_v + \overline{S}^g\overline{s}^g_v + \underline{S}^g\underline{s}^g_v + \frac{o^g_v}{O^g_n} + \frac{d^g_v}{O^g_f}$	Cost $(t_v - t_{v\_}) = const.$
w.r.t $(\mathbf{y}, \mathbf{o}, \mathbf{d}, \mathbf{x}, \overline{\mathbf{s}}, \underline{\mathbf{s}})$	Decision variables
$\mathbf{y} \in \mathbb{B}^{\lvert\mathcal{G}\rvert\times\lvert\mathcal{V}\rvert},\; \mathbf{o}, \mathbf{d}, \mathbf{x} \in \mathbb{R}_+^{\lvert\mathcal{G}\rvert\times\lvert\mathcal{V}\rvert},\; \overline{\mathbf{s}}, \underline{\mathbf{s}} \in [0,1]^{\lvert\mathcal{G}\rvert\times\lvert\mathcal{V}\rvert}$	Bounds
$y^g_v - y^g_{v_-} \le \overline{s}^g_v$	Start up constraints
$\underline{s}^g_v = y^g_{v_-} - y^g_v + \overline{s}^g_v$	Shut down constraint
$o^g_v \ge \overline{s}^g_v(O^g_n - 1)$ $\max\{0, o^g_{v_-} - y^g_{v_-}\} \le o^g_v \le o^g_{v_-} + \overline{s}^g_v(O^g_n - 1)$ $o^g_{v_-} - o^g_v \le y^g_v \le 1$	Minimum-up time
$d^g_v \ge \underline{s}^g_v(O^g_f - 1)$ $\max\{0, d^g_{v_-} - 1 + y^g_v\} \le d^g_v \le d^g_{v_-} + \underline{s}^g_v(O^g_f - 1)$ $0 \le y^g_v \le 1 - d^g_v + d^g_{v_-}$	Minimum-down time
like in CT DC-OPF ...	Balance constraint
like in CT DC-OPF ...	Flow constraints
$\underbrace{\max\left(\max_{0\le i\le n-2} x^{g(i)}_{v,v_+}, x^{g(n-1)}_{v_-,v}, x^{g(n)}_{v_-,v}\right) \le \overline{G}^g y^g_{v_+} \ldots}_{\text{Smooth switch}}$	Production limits
Similar ...	Ramping constraint
$x^{g(n)}_{v_-,v} = x^{g(0)}_{v,v_+}$	C^0 Continuity
$x^{g(n)}_{v_-,v} - x^{g(n-1)}_{v_-,v} = x^{g(1)}_{v,v_+} - x^{g(0)}_{v,v_+}$	C^1 Continuity

Simulation Results: IEEE-RTS + CAISO Load

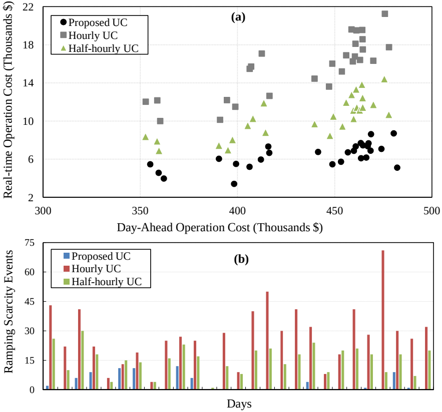
- 32 units of the IEEE-RTS and load data from the CAISO used here.
- The five-minute net-load forecast data of CAISO for Feb. 2, 2015 (scaled down to peak load of 2850MW)
- Both the day-ahead (DA) and real-time (RT) operations are simulated.
- Hourly day-ahead load forecast error standard deviation %1 of the load at the time.

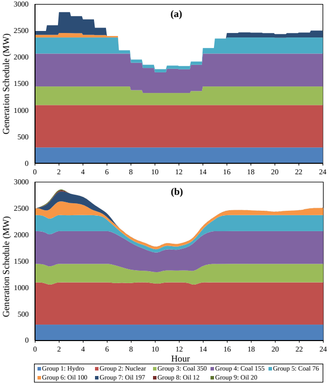


Reduced Operation Cost and Ramping Scarcity

- Case 1: Current UC Model
- Case 2: The Proposed UC Model

Case	DA Operation Cost ($)	RT Operation Cost ($)	Total DA and RT Operation Cost ($)	RT Ramping Scarcity Events
Case 1	471,130.7	16,882.9	488,013.6	27
Case 2	476,226.4	6,231.3	482,457.7	0





Stochastic Multi-Stage CT UC - Sampling

- The net load is a continuous random process $\Xi^b(t)$
- The process is continuous in time and sample space, it is intractable
- We are seeking to find a discrete time replacement, such that:

$$\lim_{n \to +\infty} \int_0^1 \mathbb{E}\left[\left(\Xi^b(t) - \underbrace{\sum_{i=0}^{n} \Xi^{b(i)} b_{in}(t)}_{\hat{\Xi}^b(t)}\right)^2\right] dt = 0$$

- Using a finite finite n each time segment of the process is mapped onto n dimensional random vector of coefficients

Polynomial Interpolation of Stochastic Load

- We can assume that in $t_{v_-} < t \leq t_v$ each realization $\xi^b(t)$ of $\Xi^b(t)$ can be mapped onto a polynomial approximation
- Given the corresponding sample path (scenario) vector of *control points*:

$$\boldsymbol{\xi}_{v_-,v} = [\xi^{(0)}_{v_-,v}, \ldots, \xi^{(n-1)}_{v_-,v}, \xi^{(n)}_{v_-,v}]^{\mathrm{T}}$$

The continuous time approximation of load scenarios is obtained:

$$\hat{\xi}_{v_-,v}(t) = \mathbf{b}_n^{(v_-,v)}(t)\boldsymbol{\xi}_{v_-,v}, \quad t_{v_-} \leq t < t_v$$

the approximate process of all such scenarios $\hat{\Xi}^b(t)$ is actually amenable to the Multi-Stage formulation since we can describe a filtration

Edge variables $\xi_{v_-,v}$ and Filtration structure

Non-anticipativity is obtained describing the stochastic process causally. This is called *Filtration*

Definition: Filtration $\mathfrak{F}$, is an increasing sequence of σ-algebras $\mathcal{F}_t, t \geq 0$ of subsets of Ω.

In continuous time, *filtrations* have additional structure:

- **Right-continuity**: if for each $t \geq 0$,

$$\mathcal{F}_t = \mathcal{F}_{t_+} = \bigcap_{\epsilon > 0} \mathcal{F}_{t+\epsilon}$$

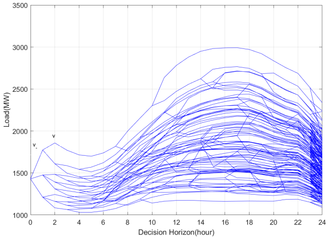
- specifically, for $\hat{\Xi}^b(t)$ the filtration

$$\mathcal{F}_{t_{v_-}} = \bigcap_{t_{v\_} < t \leq t_v} \mathcal{F}_t$$

Scenario Tree

The scenario tree $\mathcal{T} = \{\mathcal{V}, \mathcal{E}, \mathbb{P}; \boldsymbol{\xi}\}$ is the basic structure for multi-stage stochastic optimization.

- is a directed graph
- $\mathcal{V}$ set of all nodes v,
  - each node $v \in \mathcal{V}$ has a corresponding value $\xi_v \in \boldsymbol{\xi}$,
  - the present: ξ_0 is deterministic and represent the root of the tree



- $\mathcal{E}$ set of all edges (v_-, v),
- $\mathbb{P}$ is the probability law
  - associates to edge (v_-, v) the conditional probability $p_{v_-,v}$ of outcome ξ_v given unique path $\boldsymbol{\xi}_{0:v_-}$
  - recursive rule: $\pi_v = p_{v_-,v}\pi_{v_-}, \quad \pi_0 = 1.$

While normally the stochastic variables ξ_v are nodal we have each edge associated with $\boldsymbol{\xi}_{v_-,v}$,

CT Stochastic Multi-Stage UC formulation

The CT-SMUC problem is, of course, tractable only if the $\mathcal{T} = \{\mathcal{V}, \mathcal{E}, \mathbb{P}; \boldsymbol{\xi}\}$ is a finite (quantized) approximation of the true filtration

- Constraints: $(v_-, v) \in \mathcal{E}$ are edges of our scenario tree instead of indexes of consecutive hours $(v_-, v) = (h-1, h)$. With this difference the CT SMUC constraints are written exactly in the same way as in the CT- DUC (..no extra work)
- Objective: The objective of the CT-SMUC is different since it is the expected cost over all scenarios:

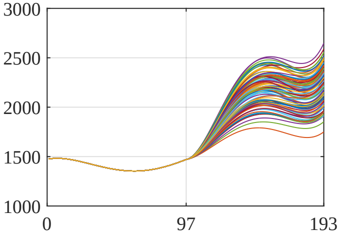
$$\mathbb{E}[Cost] = \sum_{v \in \mathcal{V}} \pi_v \sum_{g \in \mathcal{G}} \frac{c^g_{1v}}{n+1} \left(\sum_{i=0}^{n} x^{g(i)}_{v_-,v} \right) + c^g_0 y^g_v + \overline{S}^g \overline{s}^g_v + \underline{S}^g \underline{s}^g_v + \frac{o^g_v}{O^g_n} + \frac{d^g_v}{O^g_f}$$

C^1 Continuity of Load Scenarios on the Tree

Sufficient Condition: In order to maintain the C^1 continuity of load scenarios on the tree, it is sufficient to enforce the condition that at each segment of the scenario tree, the continuous load curve is tangent to the coefficients' polygon at the endpoints:

$$\left.\frac{d\xi_{v_-,v}(t)}{dt}\right|_{t=t_{v_-}} = n\left(\xi^{(1)}_{v_-,v} - \xi^{(0)}_{v_-,v}\right)$$

$$\left.\frac{d\xi_{v_-,v}(t)}{dt}\right|_{t=t_v} = n\left(\xi^{(n)}_{v_-,v} - \xi^{(n-1)}_{v_-,v}\right)$$



Discrete-time : Inaccuracies & Problem Size

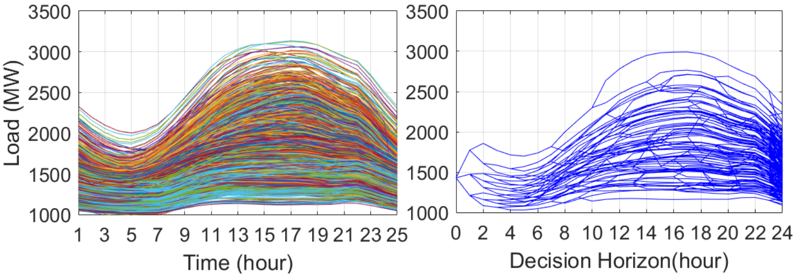


Figure: (left) Discrete-time hourly summer Load Trajectories from PJM. (right) Discrete-time hourly Load Scenario Tree

Continuous Time: Smoothness & Tractability

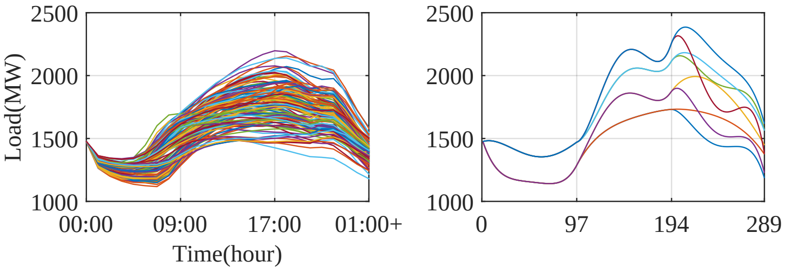


Figure: (left) Hourly summer load trajectories from PJM,(right) Corresponding scenario tree (with binary structure [2 2 2]) in continuous time with C^1-Continuity imposed at the nodes. The entire horizon is split in 3 stages of 8 hours each.

Conclusion

What we did

- We started casting the classic ED problem in continuous time to understand the meaning of the variational problem
- The rest of the talk is essentially building on the generalized notion of *sampling* from sampling trajectories to sampling random processes to provide tractable numerical solutions
- With this first step we show that it is possible to adopt the machinery of stochastic optimization to variational problems

What we left out

- We did not touch upon non-linearities (e.g. AC power flow)
- We are exploring the possibility of including dynamic constraints (ODEs), e.g. generator inertia
- We did not quantify the error due to finite n and quantization in the SMUC

Questions ?

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