Input-Output Characteristics of the Power Transmission Network's Swing Dynamics: A Graph-Theory Perspective

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Swing Dynamics: Input-Output Properties

- Oscillations and transients in the power transmission network are governed by the swing equations.
  - The model is a nonlinear DAE, but admits linear approximations.
  - Lots of work on the internal properties (modes)

- Engineers care about input-output characteristics of models (e.g., the swing-dynamics):

$$\left.\begin{aligned} \frac{d(\Delta x)}{dt} &= J\,\Delta x + Q\,\mathrm{u} \\ z &= R\Delta x \end{aligned}\right\}$$

May represent:
control channel,
disturbance response, etc.

- The linearized input-output map can be expressed, in the Laplace domain, as a transfer function:

$$\left.\frac{Z(s)}{U(s)} = H(s) = \frac{(s+q_1)\times\cdots\times(s+q_m)}{p(s)}\right\}$$

Numerator roots are **zeros.**
System is **non-minimum-phase** if a zero has positive real part.

Why Should Power Engineers Care About Input-Output Properties?

- Changing paradigm for wide-area analysis and control of transients.
  - New technologies: power electronics, wide deployment of PMUs, pervasive communications.
  - New needs: increased stress and variability (e.g. due to renewable integration), new sources of disruptions, decentralization, etc.

- Input-output dynamics (transfer functions) matter!
  - *Analysis:* Will a disruption at one location cause swings at other locations?
  - *Control:* can multiple, remotely-located sensors and actuators be used to damp oscillations and transients?
  - *Model reduction:* do standard reduced-order models maintain input-output properties?
  - *The zeros are the essential invariants of these input-output dynamics.*
    - *In particular, it matters if the system is minimum-phase or not.*

- Engineers need simple, graph-theoretic insights…

Modeling and Analysis Goals

- We consider the simplest evocative model for the swing-dynamics, but impose control input(s) and measurements:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -H^{-1}L(\Gamma) & -H^{-1}D \end{bmatrix} \begin{bmatrix} \delta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ e_i \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & e_j \end{bmatrix} \begin{bmatrix} \delta \\ \omega \end{bmatrix}$$

where Γ is a graph of line susceptances, *L* is the graph's Laplacian matrix, the diagonal matrix *H* captures generator inertias, the diagonal matrix *D* captures dampings, and *i and j are input/output locations.*

  - The poles have been extensively analyzed in the power literature.

Main Goal: characterize the zeros (and specifically minimum-phase characteristics) in terms of the network's structure (Γ, H, D), and input/output locations.

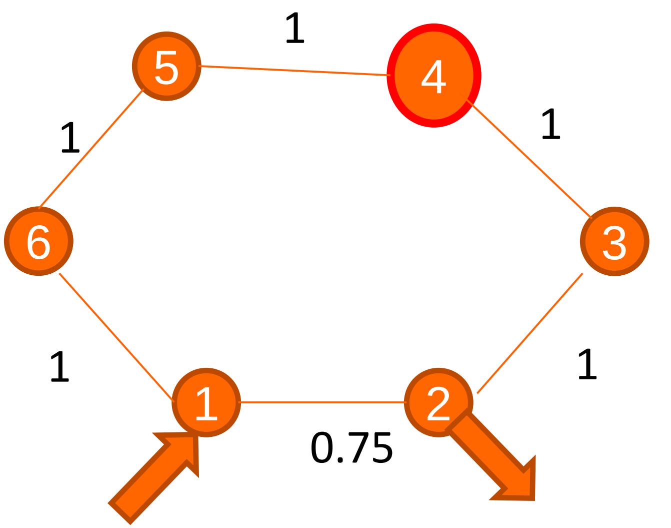
  - Use these results to gain insight into control, disturbance analysis, and model reduction

Related Literature

- N. Martins and co-workers explored numerical computation of zeros for the swing-dynamics model.

- Recent work in the controls-engineering community, on relating the zeros to the network's graph.
  - Mostly for simpler models.
  - Initial work by Allgower's group, Selec's group, and my group.

Exploratory Example



--Graph Γ is shown.
--Dampings are all equal to 0.1,
--Inertia at bus 4 is 2 while other inertias are 1 (there's a large conventional generator at bus 4).
--Input at bus 1, output at bus 2

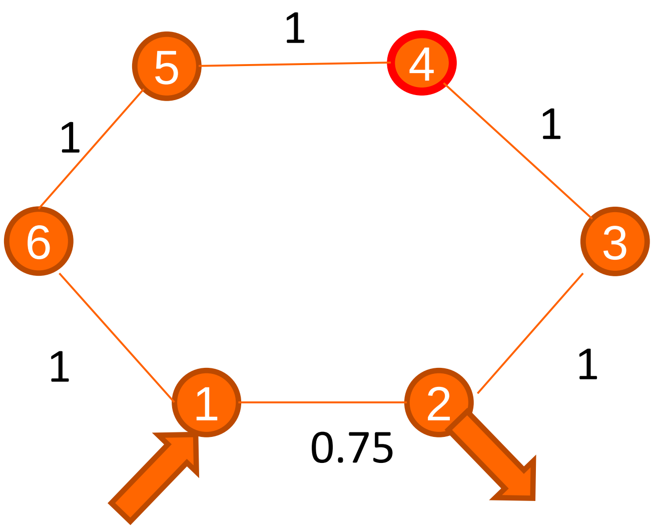
- This system is minimum phase! (Transmission zeros are shown)

Note: if the output is the frequency instead of angle, there is one additional zero at s=0.

ans =

-0.0631 + 1.6934i
-0.0631 - 1.6934i
-0.0298 + 1.6678i
-0.0298 - 1.6678i
-0.0292 + 0.6585i
-0.0292 - 0.6585i
-0.0529 + 0.9536i
-0.0529 - 0.9536i

Example (continued)



--Graph is shown.
--Dampings are all equal to 0.1,
--**Wind farm replaces conventional generation at bus 4, inertia decreases to 1.**
--Input at bus 1, output at bus 2

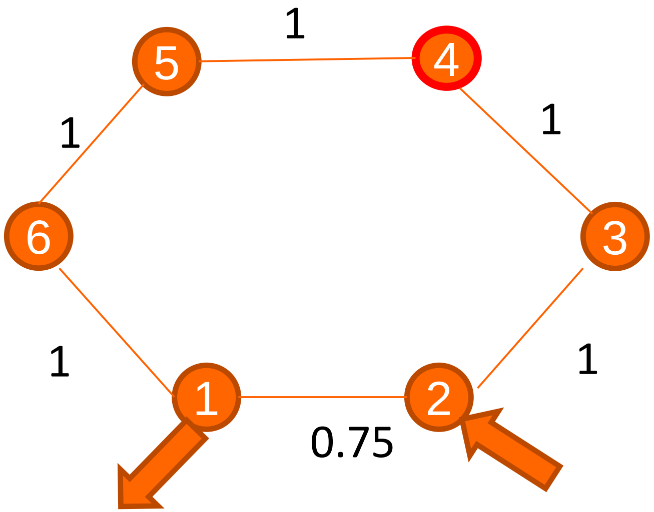
ans =

-0.1168 + 0.8784i
-0.1168 - 0.8784i
0.0168 + 0.8784i
0.0168 - 0.8784i
-0.0826 + 1.7969i
-0.0826 - 1.7969i
-0.0174 + 1.7969i
-0.0174 - 1.7969i

- Becomes nonminimum phase! (Transmission zeros are shown)

- More power flow on line 1—2 exacerbates the problem

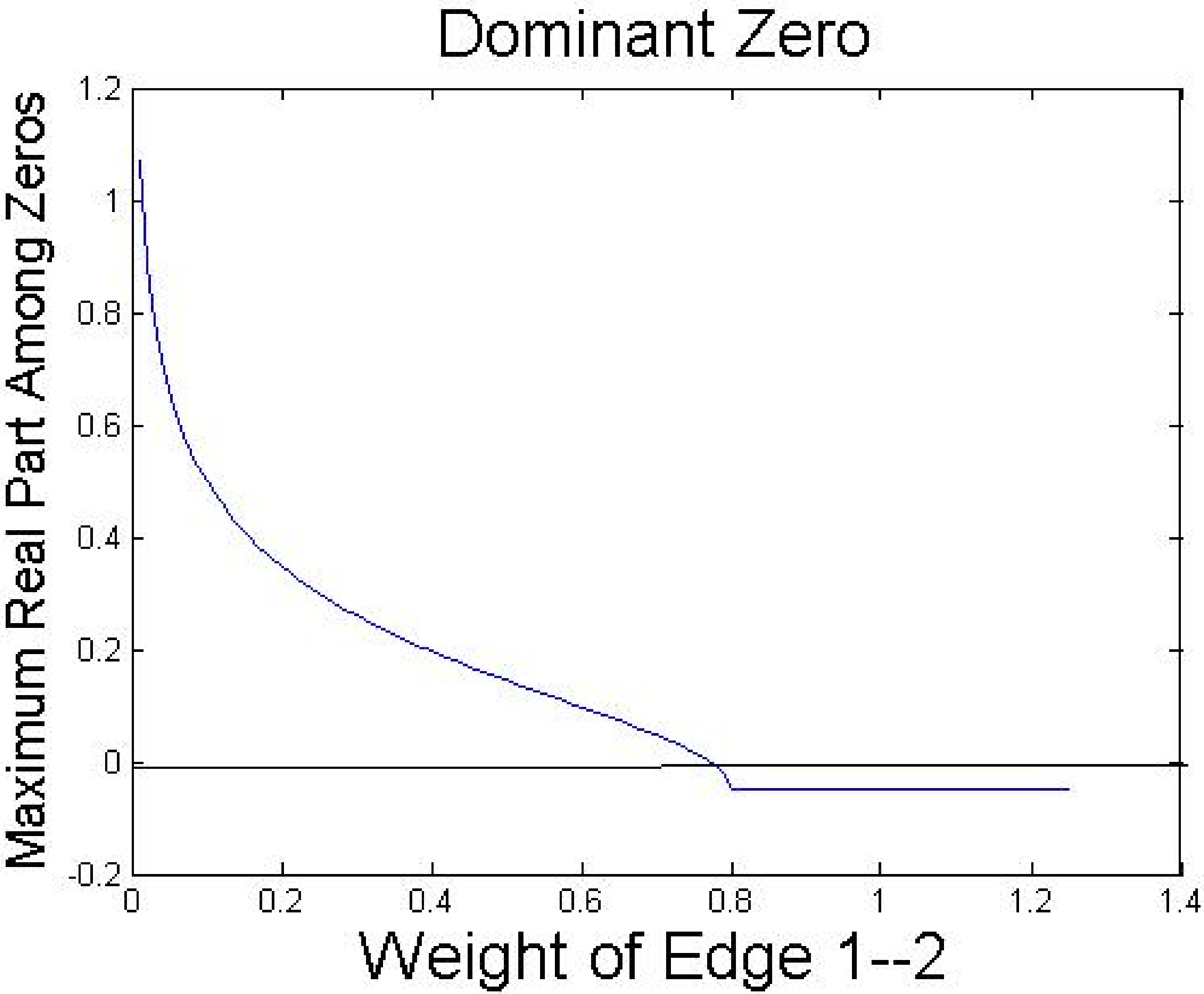
Example (continued)



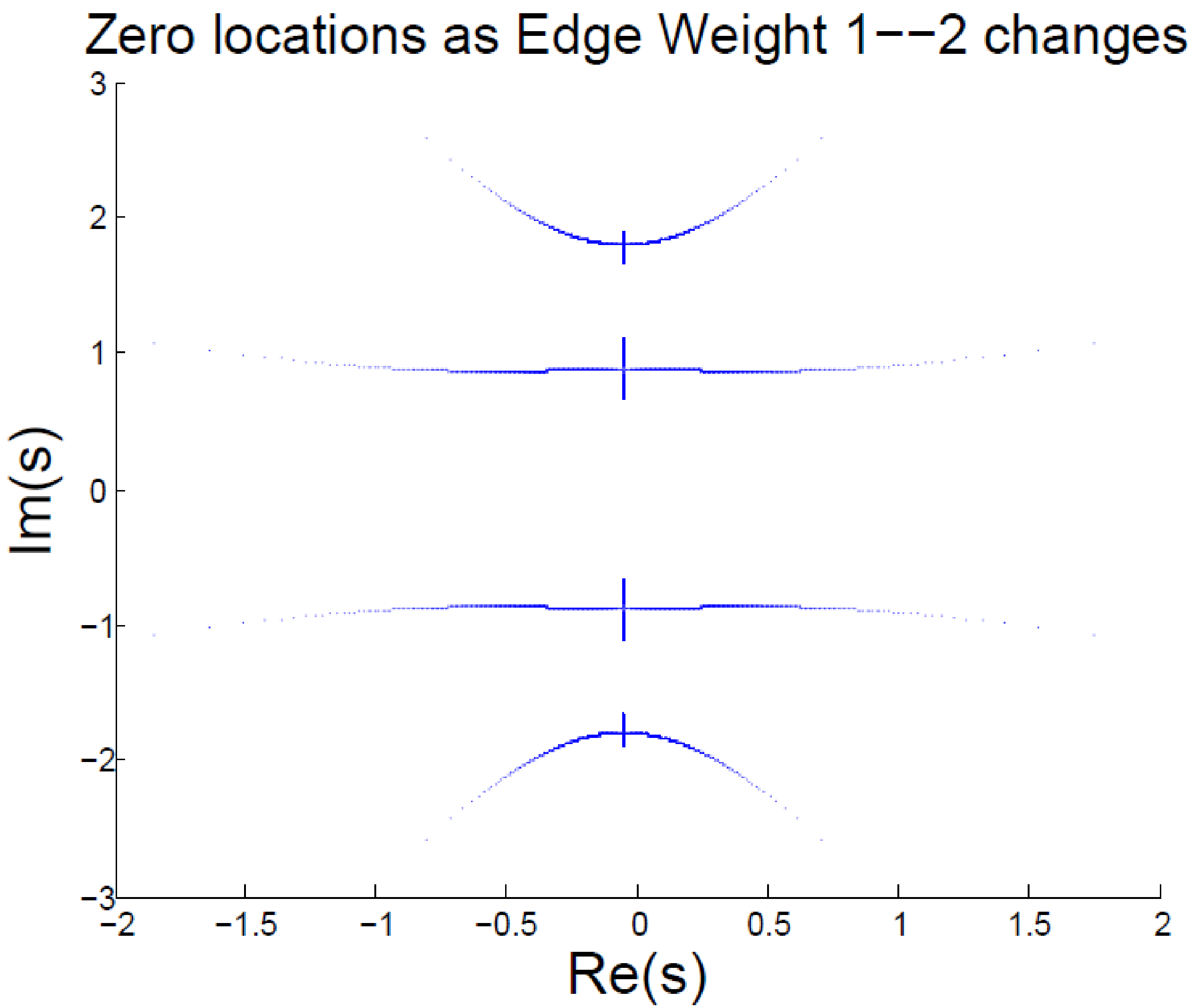
- Still nonminimum phase with reversed input and output.

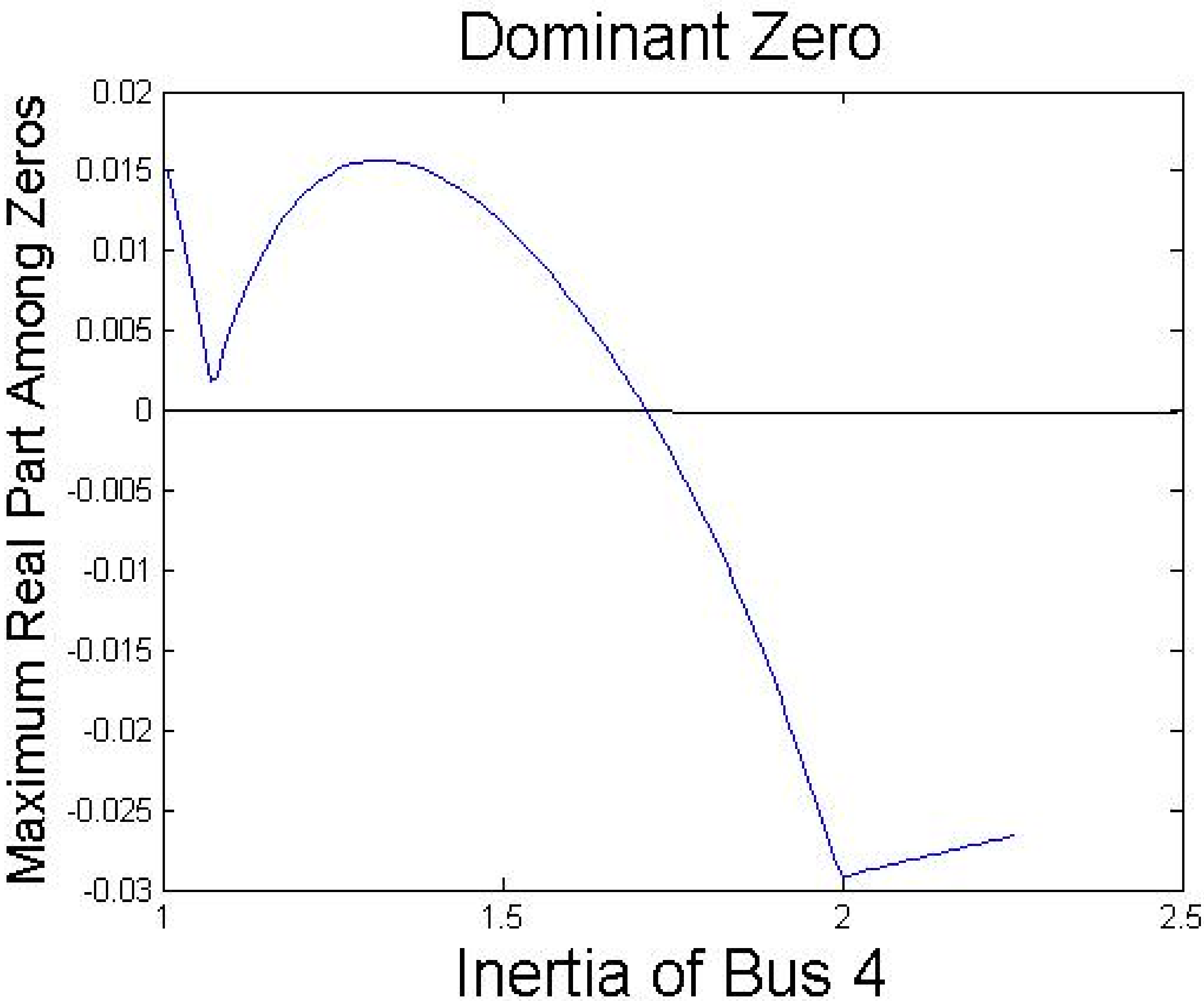
(Transmission zeros are same as previous slide.)

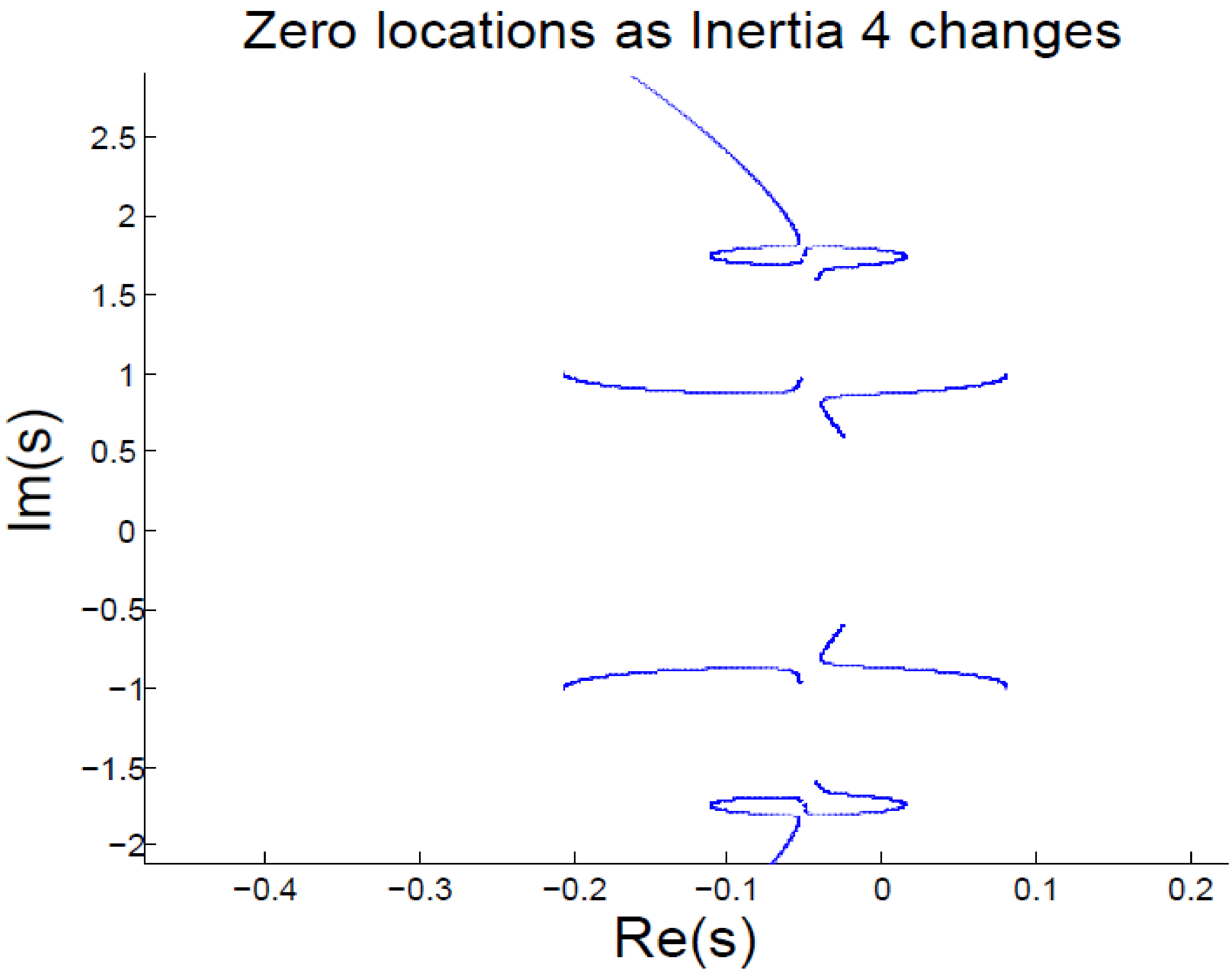
Dependence of Dominant Zero on Network Parameters



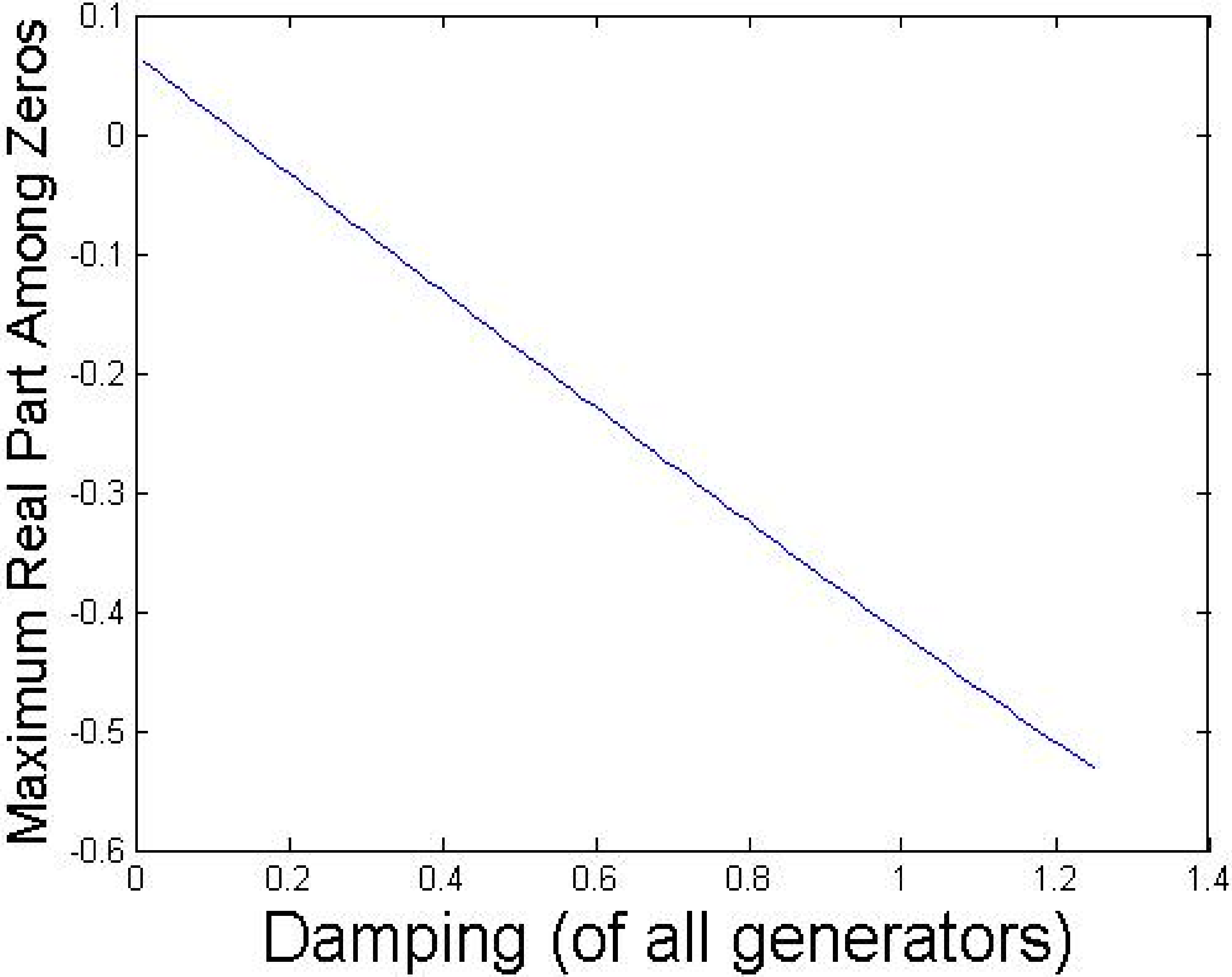
Root locus viewpoint



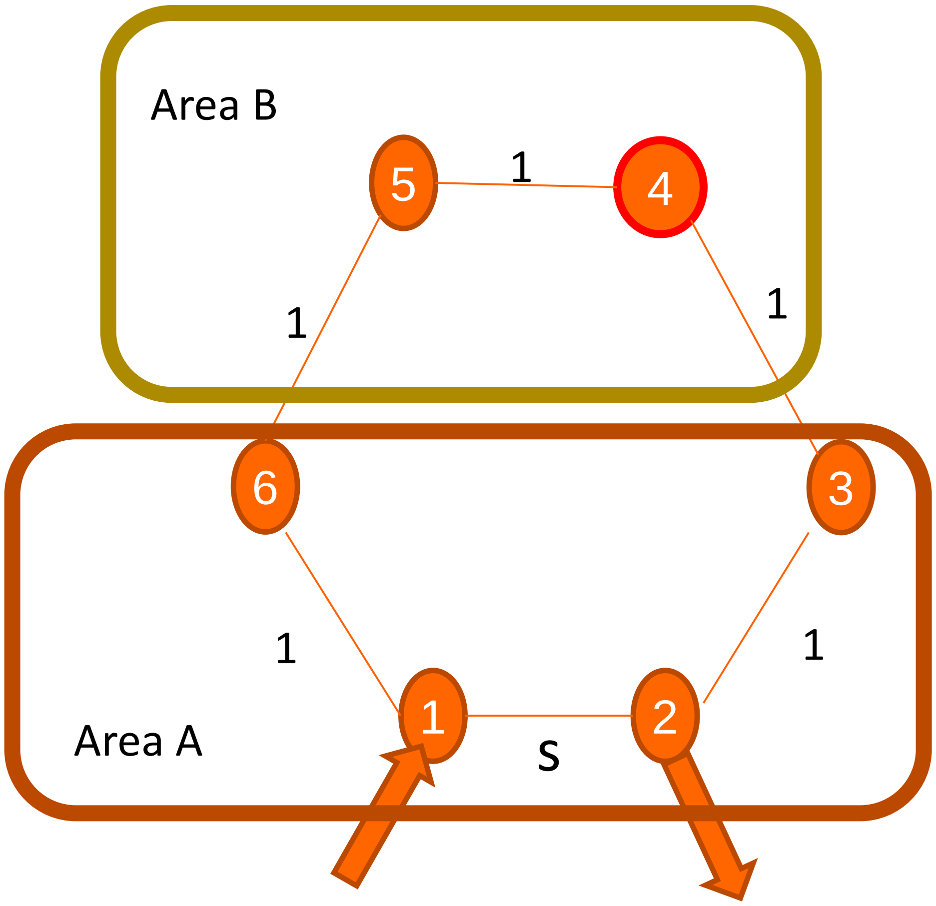

Dominant Zero
Maximum Real Part Among Zeros
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
1
1.5
2
2.5
Inertia of Bus 4


Zero locations as Inertia 4 changes
Im(s)
Re(s)
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2

Dominant Zero



Model Reduction and Zeros



Imagine the buses 4 and 5 are geographically distant, so we develop a reduced model for them (or only have a reduced model).

-- In particular, replace them with a single generator with twice the inertia (as obtained from a coherency-based reduction).

- Modes are preserved fairly well…

Full:

```
ans =

  -0.0500 + 1.9674i
  -0.0500 - 1.9674i
  -0.0500 + 1.7313i
  -0.0500 - 1.7313i
  -0.0500 + 1.6507i
  -0.0500 - 1.6507i
  -0.0500 + 0.9987i
  -0.0500 - 0.9987i
  -0.0500 + 0.9470i
  -0.0500 - 0.9470i
   0.0000
  -0.1000
```

Reduced:

```
ans =

  -0.0500 + 1.8106i
  -0.0500 - 1.8106i
  -0.0458 + 1.7313i
  -0.0458 - 1.7313i
  -0.0500 + 1.1031i
  -0.0500 - 1.1031i
  -0.0375 + 0.9991i
  -0.0375 - 0.9991i
   0.0000
  -0.0834
```

- But zeros change…
  - Reduced system is minimum-phase!

Full:

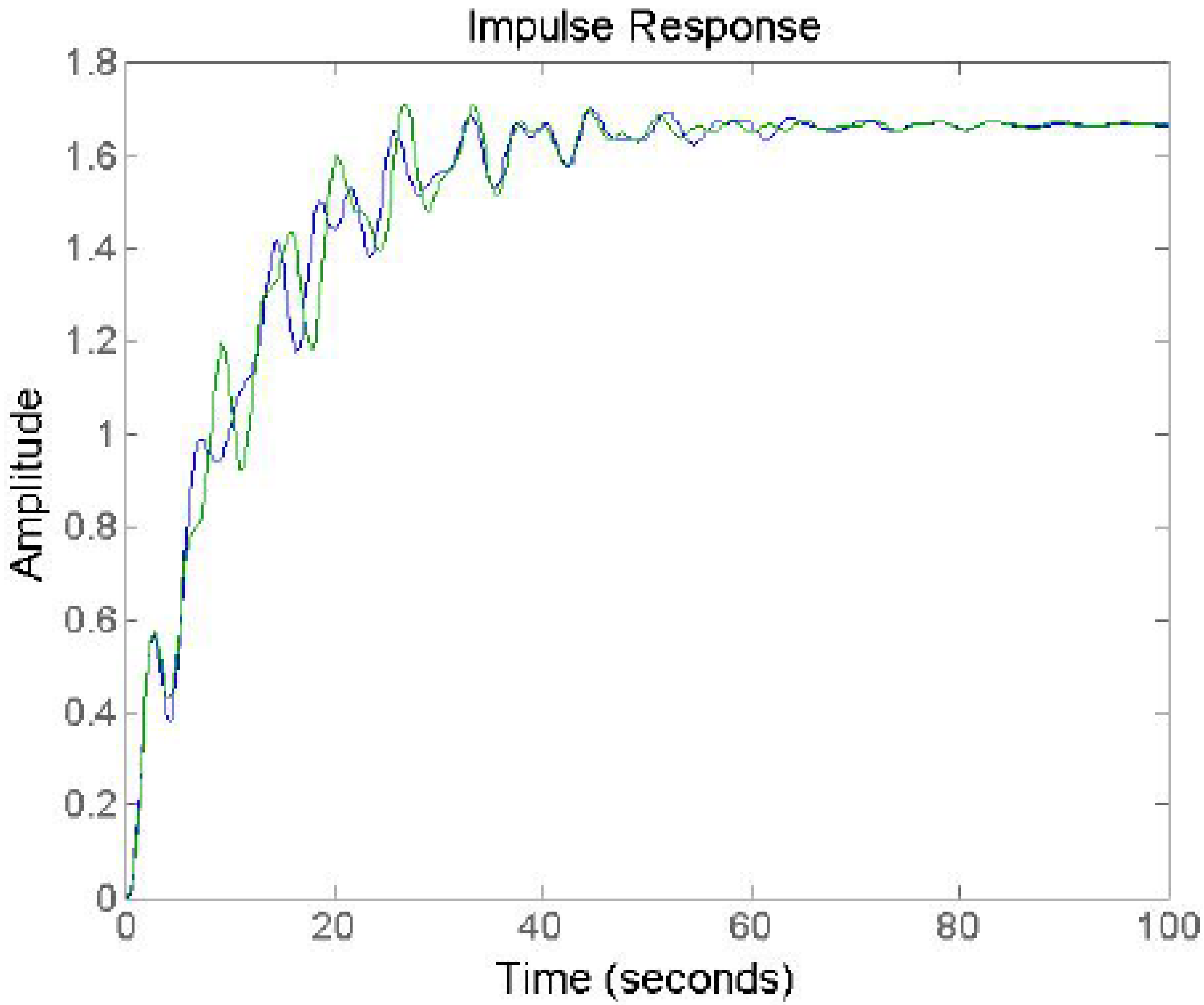
```
ans =

 -0.0826 + 1.7969i
 -0.0826 - 1.7969i
 -0.0174 + 1.7969i
 -0.0174 - 1.7969i
 -0.1168 + 0.8784i
 -0.1168 - 0.8784i
  0.0168 + 0.8784i
  0.0168 - 0.8784i
```

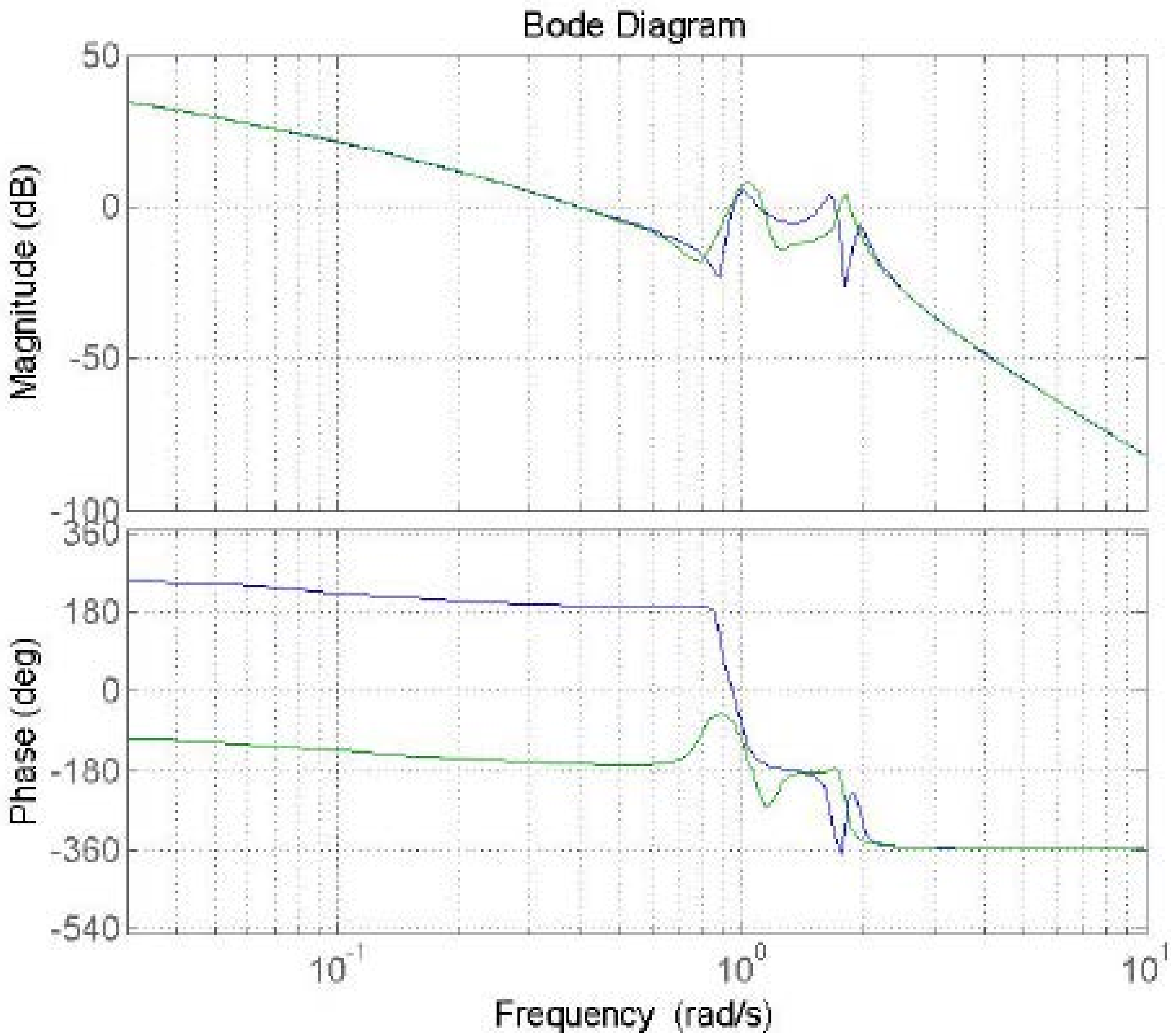
Reduced:

```
ans =

 -0.0437 + 1.7047i
 -0.0437 - 1.7047i
 -0.0254 + 0.7902i
 -0.0254 - 0.7902i
 -0.0559 + 1.2099i
 -0.0559 - 1.2099i
```



Impulse responses are very similar!



…but the frequency response (specifically, phase response) changes drastically.

How Does Congestion (High Loading) Impact Zeros?

- Increasing penetration of intermittent renewables is causing increasing variability in operating points.
  - Need to understand how changing operating profiles impact zeros
- Analysis thus far has assumed lightly-loaded lines.
  - Linearization around zero operating point
- Need to understand impact of changed loading profile.

Changing Operating Points and Zeros

- Changing operating points can be viewed as modifying susceptances in the linearized model.
  - Congested lines become “weak”
  - Increased loading exacerbates fragility
- We have identified nonminimum-phase pairs for the France-Spain power network, for varying loadings.
  - Aggregated model with 8 generators, 56 load buses

Generator Type	Bus name	Node number
Offshore	OFFS3P61	1
Offshore	OFFS4P61	2
Nuclear	PALUEP71	3
Nuclear	PALUEP72	4
Nuclear	PALUEP73	5
Nuclear	PALUEP74	6
Nuclear	PENLYP71	7
Nuclear	PENLYP72	8

Congestion and Zeros

Generally, heavily-loaded networks have more nonminimum-phase channels.

load	0		10%		20%		30%		40%	
	in-out		in-out		in-out		in-out		in-out	
pair of non-minimum phase nodes	1	7	1	2	1	2	1	2	1	2
	7	1	1	8	2	1	1	7	1	7
			2	1	2	4	2	1	2	1
			2	4	2	6	2	4	2	4
			2	6	3	4	2	6	2	6
			3	4	4	2	3	4	4	2
			4	2	4	7	4	2	4	3
			4	8	6	2	4	5	4	5
			6	2	7	1	4	7	4	7
			6	8	7	4	6	2	4	8
			7	1			7	1	5	1
			7	4			7	4	5	4
									6	2
									7	1
									7	4
									8	4

Enhanced Model: DC Line Control

- Oscillations caused by controllers for DC lines is a concern.
  - Experiment in WECC many years ago, recent interest for the France-Spain interchange.

- The classical model has been enhanced to capture DC line controls.
  - Proportional, PD, and lead-lag controls; also measurement delays (Pade approximation).
  - Real-world experience suggests that measurement delays cause problems, strong filters or PD control may resolve.

- Main Result: PD control across DC lines does not nominally introduce nonminimum-phase behavior, however measurement delays can lead to nonminimum-phase behaviors.

Formal Analysis Approach

- Main goal: develop graphical conditions for nonminimum-phase dynamics.

Step 1: Characterize the relative degree in terms of the distance *d* between the input and output in the network graph.

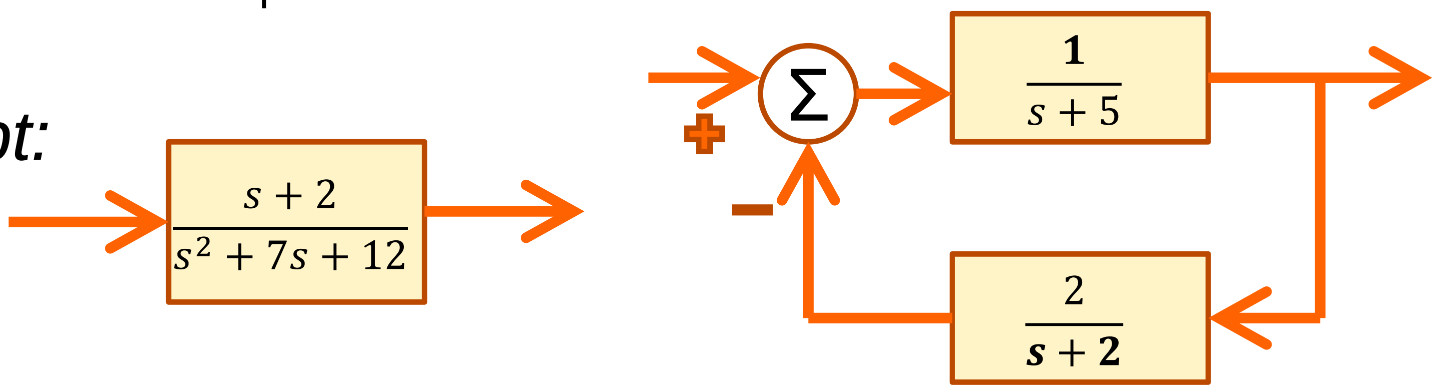
Theorem 1: The relative degree of the input-output swing-dynamics model (1), and hence the number of infinite zeros, is $n_d = 2d+1$. The number of finite zeros is $n_a = 2n - 2d - 1$.

Formal Analysis

Step 2. Algebraic result: express the zeros as the eigenvalues of a matrix, say A_{aa}.

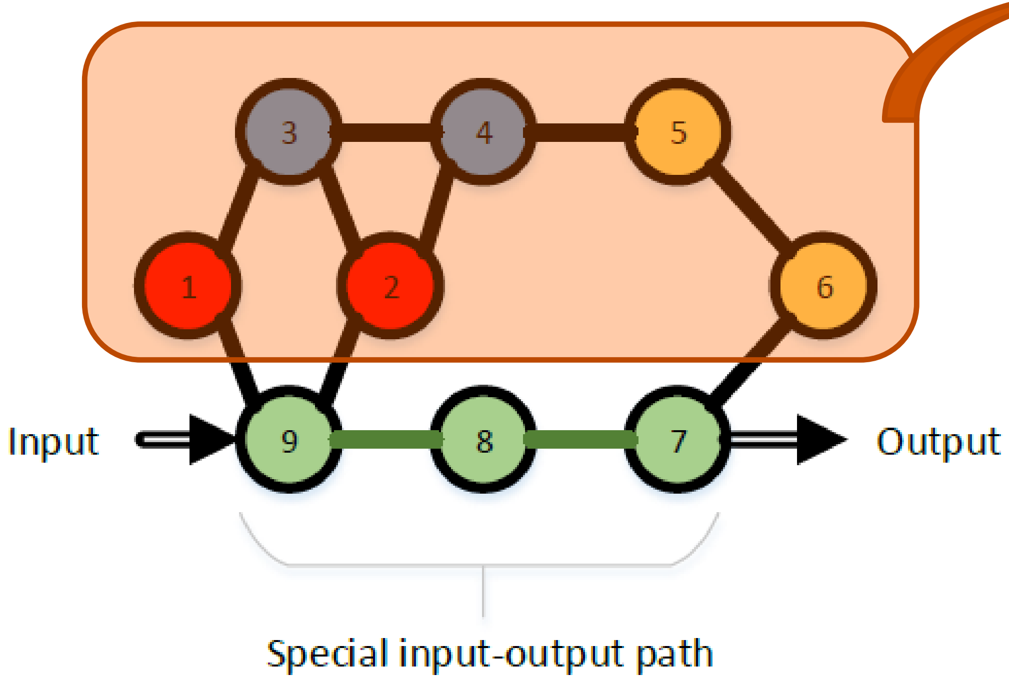
- Normal way to find zeros is via a generalized eigenvalue problem.
- The *special coordinate basis* instead allows analysis of zeros via a true eigenvalue problem.
  - This is a starting point for graph-theoretic results; also appealing for computation.

Concept:



Key insight for graphical results: the matrix A_{aa} is a sparse perturbation of a submatrix of the state matrix A.

Formal Analysis



$$A = \begin{bmatrix} A_{n_a} & A_{n_{ad}} \\ A_{n_{da}} & A_{n_d} \end{bmatrix}$$

Theorem 2: The finite zeros of the system (1) are the eigenvalues of matrix $A_{aa} = A_{n_a} - A_{n_{ad}} Z_{n_d}^{-1} Z_{n_{ad}}$, where A_{n_a} and $A_{n_{ad}}$ are submatrices of A as defined above, where

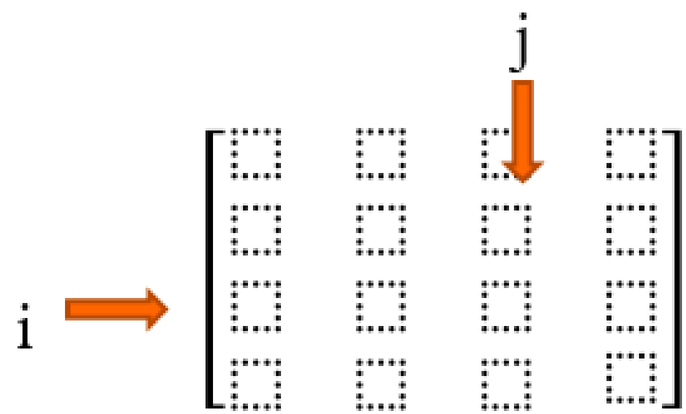
$$
Z_{n_{ad}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \{A\}_{n_a+1,1} & \{A\}_{n_a+1,2} & \cdots & \{A\}_{n_a+1,n_a} \\ \{A^2\}_{n_a+1,1} & \{A^2\}_{n_a+1,2} & \cdots & \{A^2\}_{n_a+1,n_a} \\ \vdots & \vdots & \cdots & \vdots \\ \{A^{n_d-1}\}_{n_a+1,1} & \{A^{n_d-1}\}_{n_a+1,2} & \cdots & \{A^{n_d-1}\}_{n_a+1,n_a} \end{bmatrix} \tag{5}
$$

and where Z_{n_d} is the following lower triangular matrix:

$$
Z_{n_d} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \{A\}_{n_a+1,n_a+1} & \{A\}_{n_a+1,n_a+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \{A^{n_d-1}\}_{n_a+1,n_a+1} & \{A^{n_d-1}\}_{n_a+1,n_a+2} & \cdots & \{A^{n_d-1}\}_{n_a+1,2n} \end{bmatrix} \tag{6}
$$

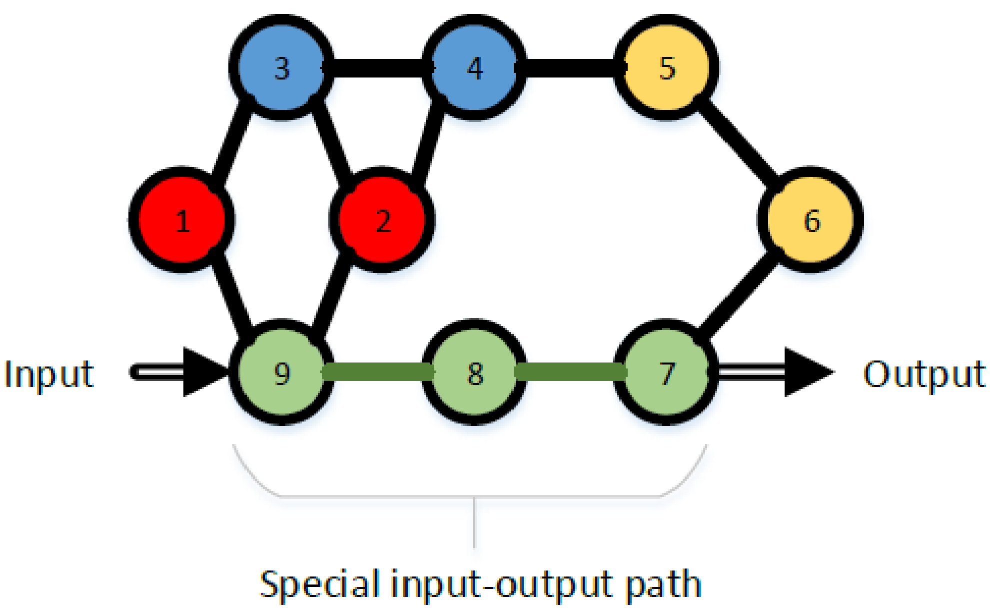
Formal Analysis

- Step 3. Understand how the zeros state matrix A_{aa} relates to a submatrix of A (A_{n_a}).
  - Only a few entries are different, depends on the graph…



$d(input, i) + d(j, output) \leq d +$

i adjacent to special



Formal Analysis: Graphical Results

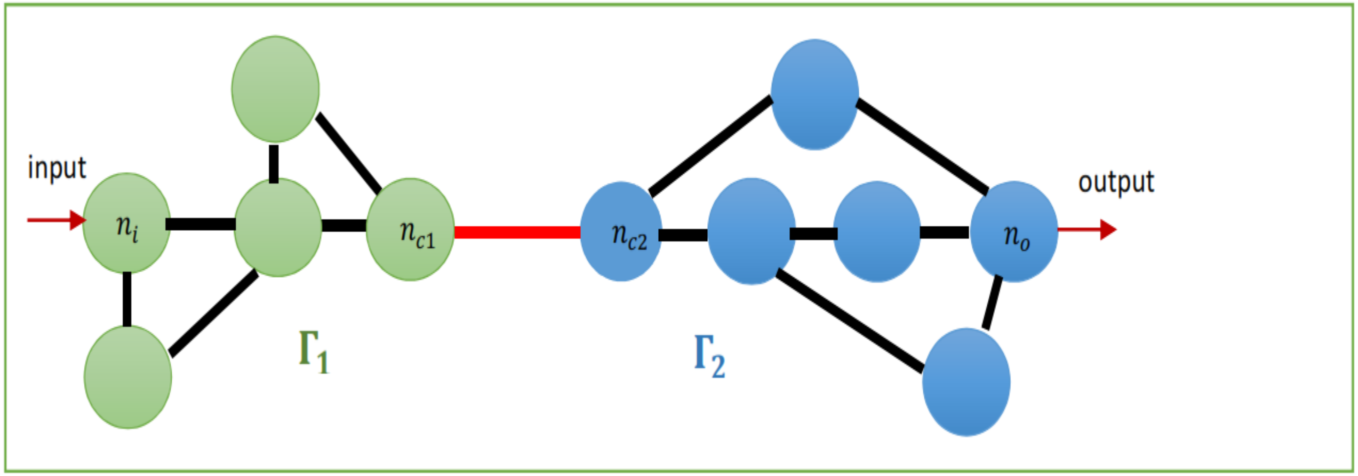
- Tree networks are always minimum-phase.
  - Single path between input and output is sufficient.

- If the shortest input-output path is sufficiently weak compared to other paths, then the dynamics is non-minimum phase.
  - If it is sufficiently strong, then minimum phase.

- Well-damped networks are minimum phase.

More Graphical Results

- The damping of generators at input node and output node do not have any effect on the zeros.
- If two separate networks are minimum phase and are connected by only one line, the new network will be also minimum phase.



Formal Results: HVDC Line

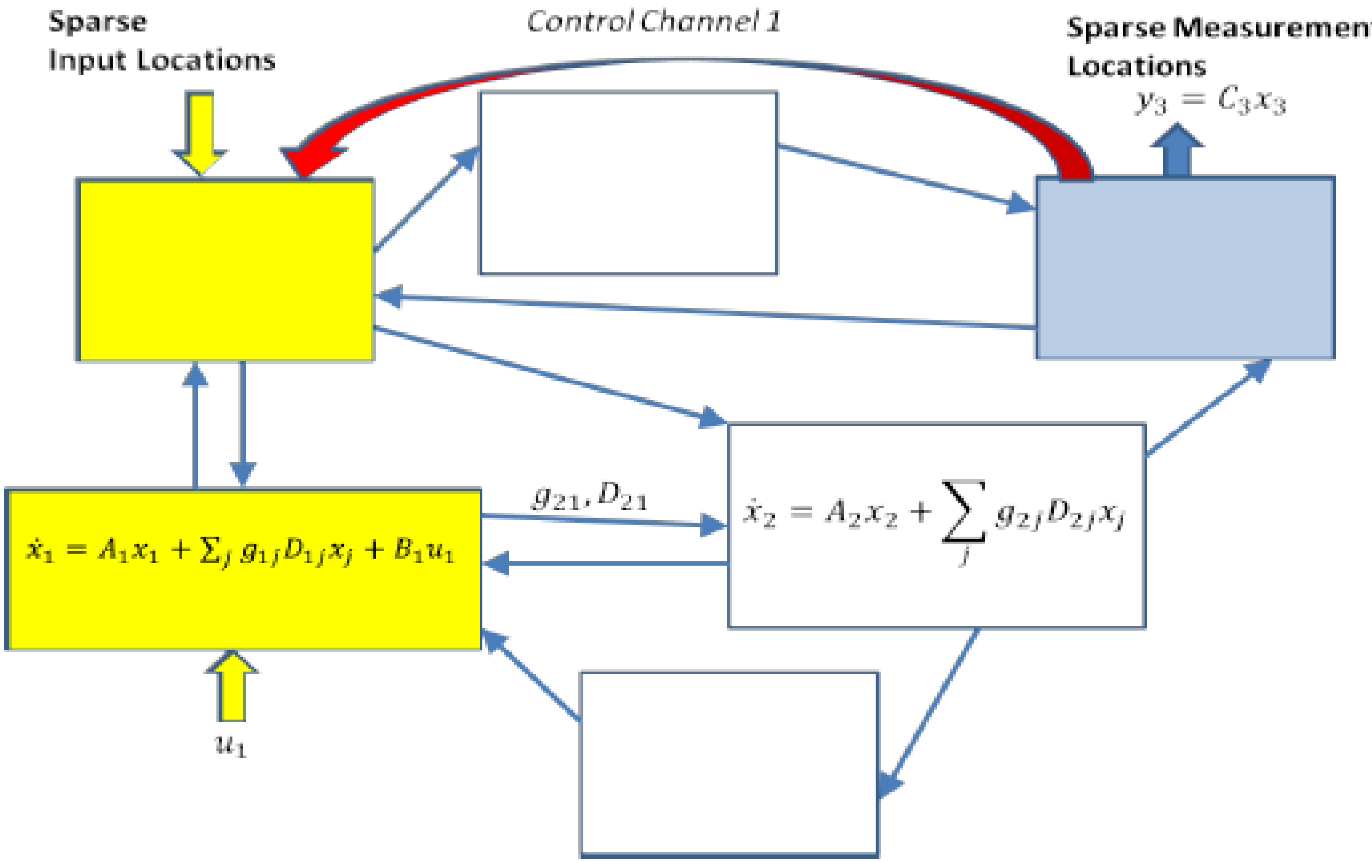
- Consider a network with P-controlled HVDC line between the input and the output nodes. For large enough proportional gains, the network is minimum phase.

- If a PD or lead-lag network is used, either the proportional gain or the derivative term can be increased to make the dynamics minimum phase.

- However, measurement delays cause non-minimum-phase dynamics, when these high gain controllers are used.

Just for Fun

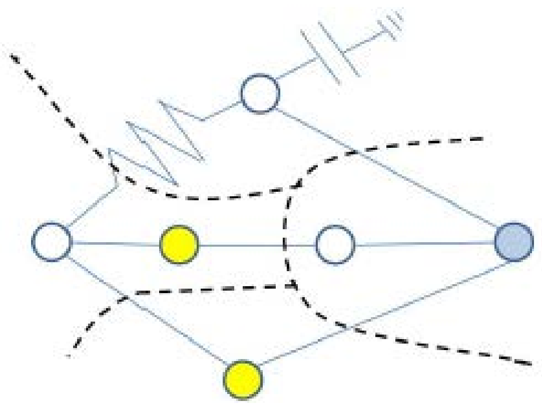
- This research is part of a broader effort to understand input-output dynamics in networks.
  - *Applies to diverse wide-area control problems for infrastructures*.

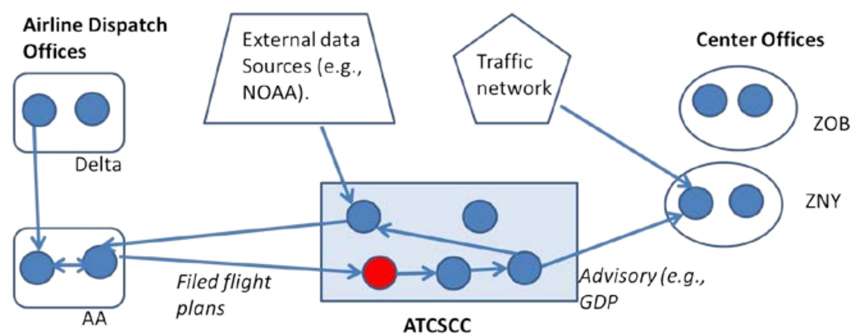


Application 1: Cyber- Risk Assessment for the Air Traffic Management System

- From the linearized model:

$$\begin{bmatrix} x_t[k+1] \\ x_c[k+1] \end{bmatrix} = \begin{bmatrix} G_{tt}(\Gamma_t) & G_{ct}(\Gamma_{ct}) \\ ?? & G_{cc}(\Gamma_c) \end{bmatrix} \begin{bmatrix} x_t[k] \\ x_c[k] \end{bmatrix} + \begin{bmatrix} 0 \\ B_c \end{bmatrix} u[k]$$

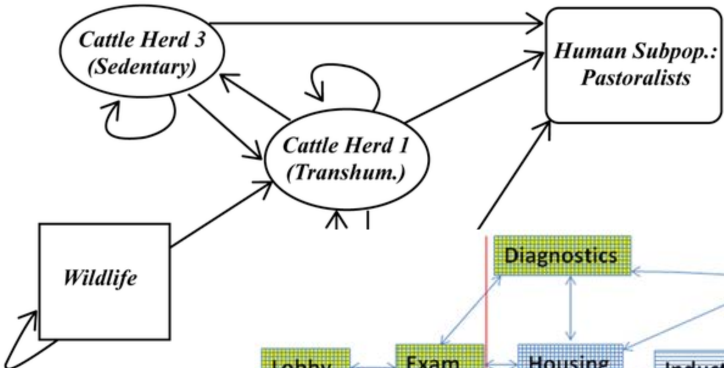


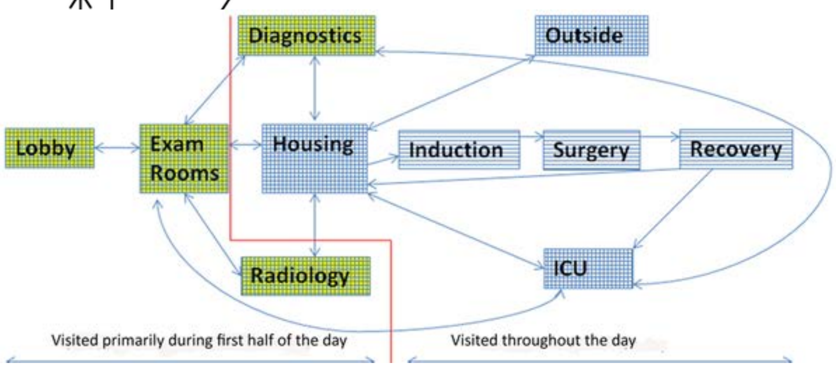


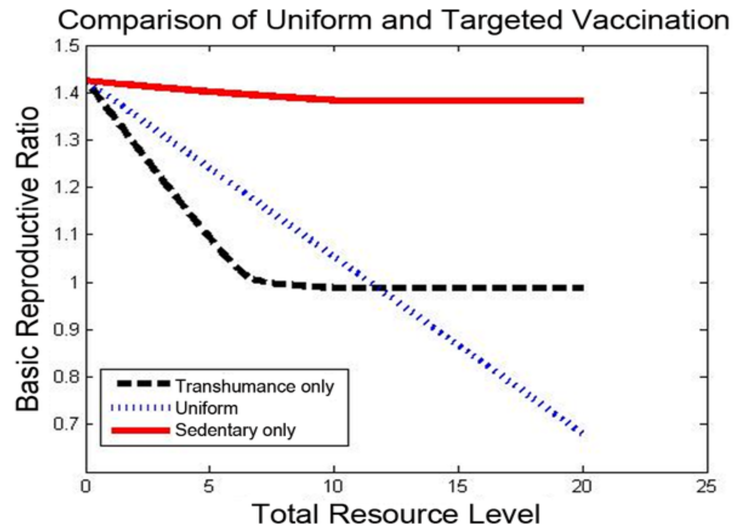
  - Can a blunt disruption u[k] cause big deviations, or induce oscillations? This is a *stability and robustness* analysis.
  - If a sophisticated attack designs u[k], how much effort is needed to achieve a specific disruption? This is a *controllability or state-hijacking problem*.
  - ***Most important:*** how does a disruption at one location (cyber or physical) affect traffic at key bottleneck locations – this is an input-output analysis!

Application 2: Epidemiology/Biology

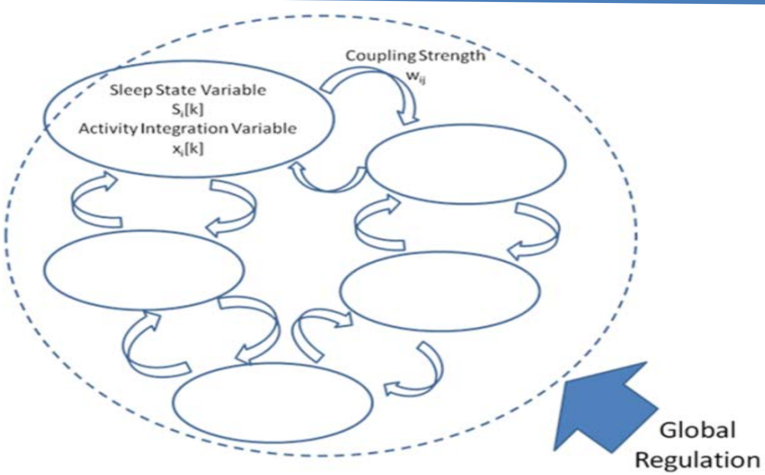
Zoonotic Disease Control

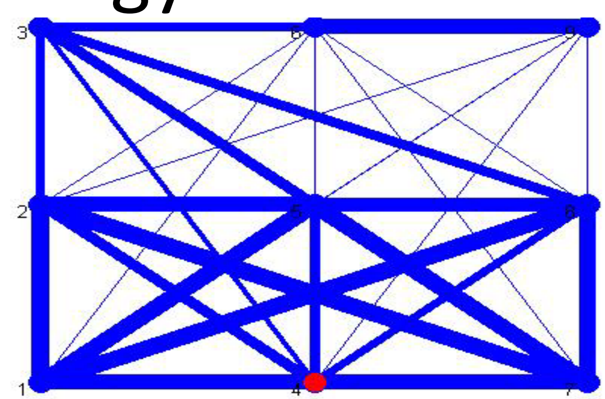






Sleep Biology





Thanks!

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- Collaborators:
  - Kasra Koorehdavoudi (WSU)
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  - Mani Venkatasubramanian (WSU)
  - Patrick Panciatici (RTE)

Questions?

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