Power Systems Engineering Research Center

Architectures and Algorithms for Distributed Generation Control of Inertia-Less AC Microgrids

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Outline

Microgrid Notion

A group of loads and distributed energy resources (DERs) interconnected via an electrical network with a small physical footprint with the possibility of operating:

M1. as part of a large power system [Grid-connected mode]

M2. as an autonomous power system [Islanded mode]

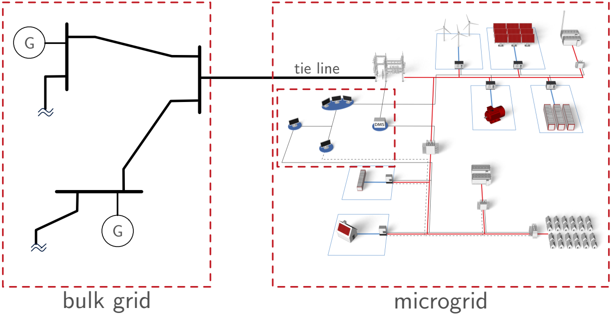
Examples of Distributed Energy Resources (DERs)

PV systems

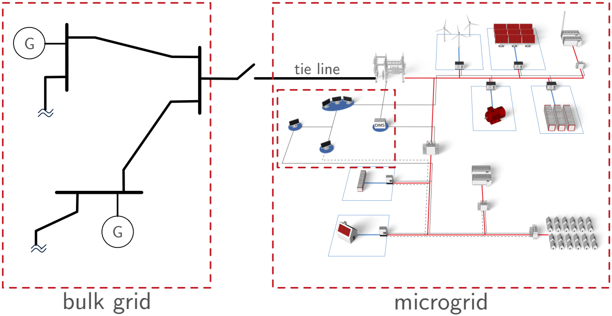
Electric Vehicles

Fuel Cells

Residential Storage

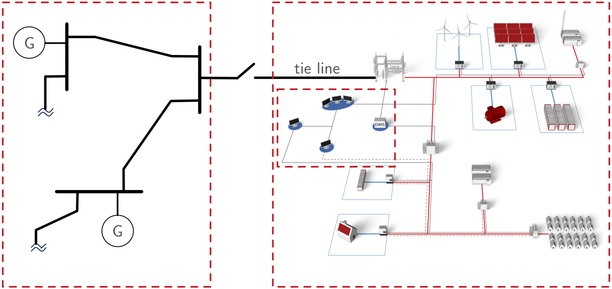


- Grid-Connected: May be viewed as a single entity with the idea of controlling the DERs within its boundaries to provide services to the bulk grid

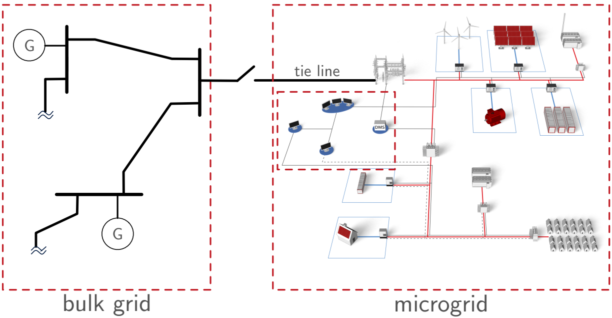


- Grid-Connected: May be viewed as a single entity with the idea of controlling the DERs within its boundaries to provide services to the bulk grid

- Islanded: Enables consumers to maintain electricity supply by appropriately controlling the DERs within its boundaries



- Grid-Connected: May be viewed as a single entity with the idea of controlling the DERs within its boundaries to provide services to the bulk grid
- Islanded: Enables consumers to maintain electricity supply by appropriately controlling the DERs within its boundaries
- Different control objectives for each operational mode and each type of DER



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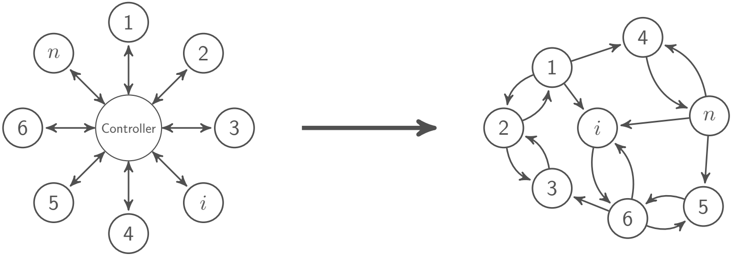
Focus of the Talk

Control of inverter-interfaced DERs in islanded inertia-less AC microgrids

Architectural Solutions

- Centralized:
  - Requires communication between a central processor and the various generation resources (and possibly loads)
  - Requires up-to-date knowledge of generation resource availability
  - Subject to failures at the decision maker (single-point-of-failure)

- Distributed:
  - Inherent ability to handle incomplete global knowledge
  - Potential resiliency to faults and/or unpredictable behavior



Centralized

Distributed

Outline

Modeling Assumptions

A1. Short lossless lines

A2. Balanced three-phase sinusoidal regime

A3. Generators and loads interfaced through voltage source inverters

A4. Inverter frequency and voltage magnitude droop-based controls

A5. All quantities in p.u.

A6. Base voltages $=$ voltage control reference values

A7. Network, inverter filter, and voltage controller dynamics are the fastest

A8. Voltage droop control much faster than frequency droop control

A9. Voltage and current inner controller much faster than droop controls

A10. Inverter reactive power capability sufficient to support voltage control

Assumptions A1, A5 – A10 $\Longrightarrow$ bus voltages approximately equal to 1 p.u.

Generator Buses

- For each generator bus $i \in \mathcal{V}_p^{(g)} := \{1, 2, \ldots, m\}$:

$$D_i \frac{d\theta_i(t)}{dt} = u_i(t) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)),$$
$$\underline{u}_i \leq u_i(t) \leq \overline{u}_i,$$

where $\mathcal{N}_p(i)$ is the set of buses to which bus i is electrically connected

$\theta_i(t)$	Phase angle in reference frequency coordinate frame
$u_i(t)$	Active power set point
$\underline{u}_i$, $\overline{u}_i$	Set point lower and upper limits
D_i	Frequency control droop coefficient
B_{ij}	Absolute value of susceptance of line connecting nodes i and j

Load Buses

- For each load bus $i \in \mathcal{V}_p^{(\ell)} := \{m+1, m+2, \ldots, n\}$:

$$D_i \frac{d\theta_i(t)}{dt} = -(\ell_i^0 + \Delta\ell_i(t)) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)),$$

where $\mathcal{N}_p(i)$ is the set of buses to which bus i is electrically connected

$\theta_i(t)$	Phase angle in reference frequency coordinate frame
ℓ_i^0	Nominal active power demand
$\Delta\ell_i(t)$	Active power demand perturbation
D_i	Frequency control droop coefficient
B_{ij}	Absolute value of susceptance of line connecting nodes i and j

Generation Control Objectives

O1. For $\ell_i^0,\ i \in \mathcal{V}_p^{(\ell)}$, find constant generator set points, $u_i^*,\ i \in \mathcal{V}_p^{(g)}$, so that

P1. $\sum_{i \in \mathcal{V}_p^{(g)}} u_i^* = \sum_{i \in \mathcal{V}_p^{(\ell)}} \ell_i^0$, $\underline{u}_i \le u_i^* \le \overline{u}_i,\ i \in \mathcal{V}_p^{(g)}$,

and there is a corresponding equilibrium point, $\theta_i^*, i \in \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$, satisfying

P2. $|\theta_i^* - \theta_j^*| \le \phi$ for all i, j electrically connected, and $\phi \in [0, \pi/2)$

O2. For sufficiently small changes in $\Delta\ell_i(t),\ i \in \mathcal{V}_p^{(\ell)}$, regulate the value of $u_i(t),\ i \in \mathcal{V}_p^{(g)}$ around u_i^*, $i \in \mathcal{V}_p^{(g)}$ so that

P3. $\frac{d\theta_i(t)}{dt} \to 0$ as $t \to \infty$

Outline

Overview of Frequency Regulation Architecture

- For given ℓ_i^0's, u_i^*'s are computed to meet the properties in Objective O1
- After a load perturbation occurs, set points are iteratively adjusted around the u_i^*'s via closed-loop control
- These adjustments serve to eliminate the frequency error that results from the load perturbation
- In general, load perturbations and subsequent set point adjustments move the system away from the stable equilibrium point corresponding to the u_i^*'s
- The u_i^*'s are recomputed based upon an estimate for the amount by which the system has deviated from the aforementioned stable equilibrium point

Choosing Set Points

- For $\ell_i^0,\ i \in \mathcal{V}_p^{(\ell)}$, find constant generator set points, u_i^*, $i \in \mathcal{V}_p^{(g)}$, so that

  P1. $\sum_{i \in \mathcal{V}_p^{(g)}} u_i^* = \sum_{i \in \mathcal{V}_p^{(\ell)}} \ell_i^0$, $\underline{u}_i \leq u_i^* \leq \overline{u}_i,\ i \in \mathcal{V}_p^{(g)}$

  and there is a corresponding equilibrium point, $\theta_i^*, i \in \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$, satisfying

  P2. $|\theta_i^* - \theta_j^*| \leq \phi$, for all i, j, electrically connected, and $\phi \in [0, \pi/2)$

The u_i^*'s can be obtained by solving a feasibility problem involving

U1. Node power balance equations

U2. Set point lower and upper limits

Feasibility Problem Formulation

- Let $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p)$ be an undirected graph, where
  - $\mathcal{V}_p = \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$,
  - $\mathcal{E}_p \subseteq \mathcal{V}_p \times \mathcal{V}_p$, with $\{i, j\} \in \mathcal{E}_p$ if nodes i and j are electrically connected

- Then, the u_i^*'s can be obtained as a solution of

$$\begin{aligned} \text{find} \quad & u^*,\ \theta^* \\ \text{subject to} \quad & \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB\mathbf{sin}(M^{\mathrm{T}}\theta^*) \\ & \|M^{\mathrm{T}}\theta^*\|_\infty \leq \phi \\ & \underline{u} \leq u^* \leq \overline{u}, \end{aligned}$$

$u^* \in \mathbb{R}^m$	Vector of active power set points
$\theta^* \in \mathbb{R}^n$	Vector of phase angles
$\ell^0 \in \mathbb{R}^{n-m}$	Vector of nominal active power demands
$\underline{u}$, $\overline{u} \in \mathbb{R}^m$	Vector of set point lower and upper limits
$M \in \mathbb{R}^{n \times \|\mathcal{E}_p\|}$	Graph Incidence matrix for arbitrary edge orientation
$B \in \mathbb{R}^{\|\mathcal{E}_p\| \times \|\mathcal{E}_p\|}$	Diagonal matrix of line susceptance absolute values
$\mathbf{sin}(M^{\mathrm{T}}\theta^*) \in \mathbb{R}^{\|\mathcal{E}_p\|}$	Vector of angle across line sine's

Flow Space Formulation

- The feasibility problem

$$\textbf{F0:} \quad \text{find} \quad u^*,\ \theta^*$$
$$\text{subject to} \quad \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB\,\mathbf{sin}(M^{\mathrm{T}}\theta^*)$$
$$\|M^{\mathrm{T}}\theta^*\|_\infty \leq \phi$$
$$\underline{u} \leq u^* \leq \overline{u},$$

is equivalent to

$$\textbf{F1:} \quad \text{find} \quad u^*,\ f^*$$
$$\text{subject to} \quad \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = Mf^*$$
$$N\,\mathbf{arcsin}(B^{-1}f^*) = \mathbf{0}$$
$$-\sin(\phi)b \leq f^* \leq \sin(\phi)b$$
$$\underline{u} \leq u^* \leq \overline{u}$$

$f^* \in \mathbb{R}^{\lvert\mathcal{E}_p\rvert}$	Vector of line flows
$b \in \mathbb{R}^{\lvert\mathcal{E}_p\rvert}$	Vector of line susceptance absolute values
$N \in \mathbb{R}^{(\lvert\mathcal{E}_p\rvert-n+1)\times\lvert\mathcal{E}_p\rvert}$	Graph fundamental loop matrix
$\mathbf{arcsin}(B^{-1}f^*) \in \mathbb{R}^{\lvert\mathcal{E}_p\rvert}$	Vector of normalized line flows arcsin's

Convexifying the Problem [Zholbaryssov, D-G, '16]

The constraint $N\mathbf{arcsin}(B^{-1}f^*) = \mathbf{0}$ makes feasibility problem F1 non convex

- Trees: Nothing to do as $N = 0$
- Networks with non-overlapping loops:
  - Remove $N\mathbf{arcsin}(B^{-1}f^*) = 0$, and
  - Replace $-\sin(\phi)b \leq f^* \leq \sin(\phi)b$ by

$$\underline{f} \leq f^* \leq \overline{f},$$

with $\overline{f} = -\underline{f} = \sin(\phi)\left(b - |N^{\mathrm{T}}|\beta\right)$, where $\beta = [\beta_1, \cdots, \beta_{|\mathcal{E}_p|-n+1}]$, and

$$\beta_i = \frac{1}{2}\left(\overline{b}_i - \sin\left(\frac{\phi}{|\mathcal{C}_i| - 1}\right)\underline{b}_i\right)$$

$\overline{b}_i$	Maximum of line susceptance absolute values along loop i
$\underline{b}_i$	Minimum of of line susceptance absolute values along loop i
$\lvert\mathcal{C}_i\rvert$	Number of lines in loop i

Set Point Computation

- Feasibility Problem F0:

$$\begin{aligned} \text{find} \quad & u^*,\ \theta^* \\ \text{subject to} \quad & \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB\mathbf{sin}(M^{\mathrm{T}}\theta^*) \\ & \|M^{\mathrm{T}}\theta^*\|_\infty \leq \phi \\ & \underline{u} \leq u^* \leq \overline{u} \end{aligned}$$

- Feasibility Problem F2:

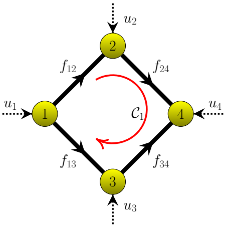
$$\begin{aligned} \text{find} \quad & u^*,\ f^* \\ \text{subject to} \quad & \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = Mf^* \\ & \underline{f} \leq f^* \leq \overline{f} \\ & \underline{u} \leq u^* \leq \overline{u} \end{aligned}$$

Lemma

For every solution (u, f) of F2, there exists a θ so that (u, θ) is a solution of F0

- F2 is a special case of the class of problems studied in commodity networks
- F2 can be used to find set points that satisfy desired properties P1 and P2

Example



$$M = \begin{array}{c} \begin{array}{cccc} f_{12} & f_{13} & f_{24} & f_{34} \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$N = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$$

$$|b_{ij}| = 1, \ \forall \{i, j\} \in \mathcal{E}_p$$

- In this case, $\beta_1 = 0.25$ for $\phi = \frac{\pi}{2}$; thus, $\overline{f}_{ij} = 0.75$, $\underline{f}_{ij} = -0.75$
- Assume $u_4 = -1$, and u_1, u_2, and u_3 are constrained to lie within $[0, 1]$
- $u = [1, 0, 0, -1]^{\mathrm{T}}$ and $f = [0.6, 0.4, 0.6, 0.4]^{\mathrm{T}}$ form a solution for F2
- From (u, f), we can form (u, f') that solves F1 by adding to f a term laying in the null space of M: $f' = f - 0.1N^{\mathrm{T}} = [0.5, 0.5, 0.5, 0.5]^{\mathrm{T}}$; clearly

$$\sin^{-1}(f'_{12}) - \sin^{-1}(f'_{13}) + \sin^{-1}(f'_{24}) - \sin^{-1}(f'_{34}) = 0$$

- Not every solution to F1 is a solution to F2

Average Frequency Error

- Recall microgrid dynamic model:

$$D_i \frac{d\theta_i(t)}{dt} = u_i(t) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)), \quad i \in \mathcal{V}_p^{(g)}$$

$$D_i \frac{d\theta_i(t)}{dt} = -(\ell_i^0 + \Delta\ell_i(t)) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)), \quad \in \mathcal{V}_p^{(\ell)}$$

- Define the average frequency error at $t \geq 0$ as follows

$$\Delta\overline{\omega}(t) := \frac{\sum_{i=1}^n D_i \frac{d\theta_i(t)}{dt}}{\sum_{i=1}^n D_i}$$

- Then, by adding the microgrid dynamics equations

$$\Delta\overline{\omega}(t) = \frac{1}{\sum_{i=1}^n D_i} \left(\sum_{i \in \mathcal{V}_p^{(g)}} u_i(t) - \sum_{i \in \mathcal{V}_p^{(\ell)}} (\ell_i^0 + \Delta\ell_i(t)) \right)$$

Control Scheme [Cady, Zholbaryssov, D-G, Hadjicostis, '16]

- The intervals of control scheme execution are referred to as *rounds*
  - Rounds are indexed by $r = 0, 1, 2, \ldots$
  - The duration of each round is T_0 units of time
  - $t_r := rT_0$ denotes the time at the beginning of round r

- Assume a large number of rounds elapsed between load perturbations, i.e.,
  - If the power demanded by load $i \in \mathcal{V}_p^{(\ell)}$ is perturbed by $\Delta\ell_i$ at $t = t_0$, then $\ell_i(t) = \ell_i^0 + \Delta\ell_i$ for $t_0 < t < r_0 T_0$ and r_0 sufficiently large

- Let $u_i[r] := u_i(t),\ t_r \leq t < t_{r+1}$, then

$$\Delta\overline{\omega}[r] = \overline{D}\left(\sum_{i\in\mathcal{V}_p^{(g)}} u_i[r] - \sum_{i\in\mathcal{V}_p^{(\ell)}} (\ell_i^0 + \Delta\ell_i)\right),$$

where $\overline{D} := \frac{1}{\sum_{i=1}^{n} D_i}$

Control Objective

Iteratively adjust set-points so as to drive the average frequency error to zero

Control Scheme [Cady, Zholbaryssov, D-G, Hadjicostis, '16

- For a given u_i^*, generator i adjusts its set point as follows:

$$e_i[r+1] = e_i[r] + \kappa_i \Delta\overline{\omega}[r],$$
$$u_i[r] = u_i^* + \alpha_i e_i[r],$$

where $e_i[0] = 0$, and α_i and κ_i are appropriately chosen gains

- Let $e[r] = [e_1[r], e_2[r], \cdots, e_m[r]]^{\mathrm{T}}$, then:

$$\begin{bmatrix} \Delta\overline{\omega}[r+1] \\ e[r+1] \end{bmatrix} = \underbrace{\begin{bmatrix} \beta & \overline{D}\alpha^{\mathrm{T}} \\ \kappa & I_m \end{bmatrix}}_{:=\Phi} \begin{bmatrix} \Delta\overline{\omega}[r] \\ e[r] \end{bmatrix} + \begin{bmatrix} -\overline{D} \\ 0_m \end{bmatrix} \sum_{i \in \mathcal{V}_p^{(\ell)}} \Delta\ell_i,$$

where $\beta := \overline{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i$, $\alpha := [\alpha_1, \ldots, \alpha_m]^{\mathrm{T}}$, and $\kappa := [\kappa_1, \ldots, \kappa_m]^{\mathrm{T}}$

Gain Choice

Proposition

If α_i and κ_i for $i \in \mathcal{V}_p^{(g)}$ are chosen such that

$$-2 < \overline{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i < 0,$$

then

- *The system is marginally stable and the average frequency error asymptotically approaches zero, i.e., $\rho(\Phi) \leq 1$ and $\Delta\overline{\omega}[r] \to 0$ as $r \to \infty$*
- *The value of the average frequency error at any round r is given by*

$$\Delta\overline{\omega}[r] = (1+\beta)^{r-1}(\beta - 1)\overline{D}\left(\sum_{i \in \mathcal{V}_p^{(\ell)}} \Delta\ell_i\right)$$

Outline

Control Dependence on Global Information

- Recall the set-point adjustment scheme::

$$e_i[r+1] = e_i[r] + \kappa_i \Delta\overline{\omega}[r],$$
$$u_i[r] = u_i^* + \alpha_i e_i[r],$$

where $e_i[0] = 0$, and α_i and κ_i chosen so that $-2 < \overline{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i < 0$

- In order to implement and execute this control, each generator requires some global information:
  - Q1. Set point u_i^*
  - Q2. Gains α_i and κ_i
  - Q3. Average frequency error $\Delta\overline{\omega}[r]$

Objective

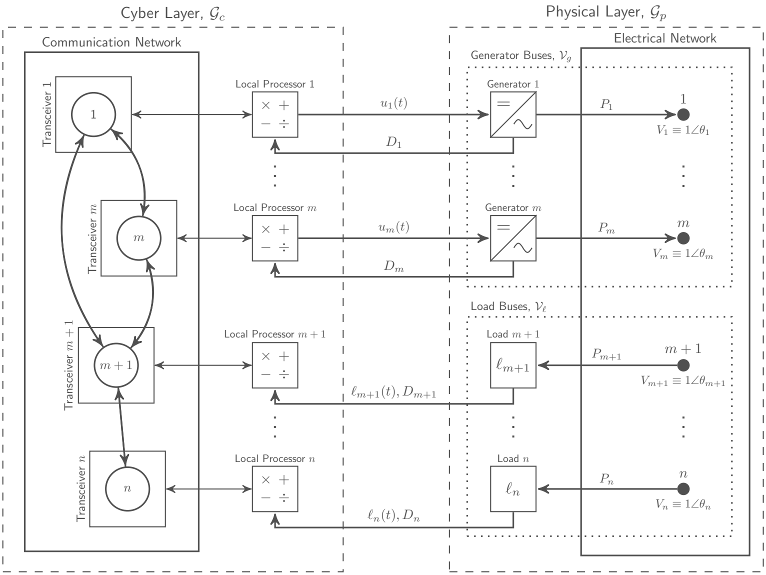
Design distributed algorithms that enables the computation of Q1-Q3

Cyber Layer for Distributed Computation and Control

- Bus local processors can perform simple computations $(\times, \div, +, -)$
- Exchange of information between local processors is described by a connected undirected graph $\mathcal{G}_c = \{\mathcal{V}_c, \mathcal{E}_c\}$:
  - $\mathcal{V}_c := \{1, 2, \dots, n\}$ is the set of nodes
  - $\mathcal{E}_c \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, where $\{i, j\} \in \mathcal{E}_c$ if nodes i and j can exchange information
  - Each element in $\mathcal{V}_c$ corresponds to a bus of the physical network, i.e., there is a one-to-one mapping between $\mathcal{V}_c$ and $\mathcal{V}_p$
  - Each element in $\mathcal{E}_c$ corresponds to a line of the physical network, i.e., there is a one-to-one mapping between $\mathcal{E}_c$ and $\mathcal{E}_p$
- Nodes that exchange information which node i are said to be its neighbors and are represented by the set

$$\mathcal{N}_c(i) = \{i \in \mathcal{V} : \{i, j\} \in \mathcal{E}_c\}$$

Interaction Between Physical and Cyber Layers



Set Point Computation [Cady, Hadjicostis, D-G, '15]

- Recall feasibility problem F2:

$$\begin{aligned} \text{find} \quad & u^*,\ f^* \\ \text{subject to} \quad & \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = Mf^* \\ & \underline{f} \le f^* \le \overline{f} \\ & \underline{u} \le u^* \le \overline{u} \end{aligned}$$

- Distributed algorithm over graph $\mathcal{G}_c$ for solving F2:
  - Each node $i \in \mathcal{V}_p$ maintains and updates estimates of flows with its neighbors: $f_{ji}^{(i)}[k],\ j \in \mathcal{N}_p(i),\ k = 0, 1, 2, \cdots$
  - Each generator node $i \in \mathcal{V}_p^{(g)}$ maintains an estimate for its set point: $u_i[k],\ k = 0, 1, 2, \cdots$
  - At each iteration k, each node $i \in \mathcal{V}_p$ computes its power flow balance:

$$d_i[k] := \begin{cases} -\sum_{j \in \mathcal{N}_p(i)} f_{ij}^{(i)}[k] + u_i[k], & i \in \mathcal{V}_p^{(g)} \\ -\sum_{j \in \mathcal{N}_p(i)} f_{ij}^{(i)}[k] - \ell_i, & i \in \mathcal{V}_p^{(\ell)} \end{cases}$$

Algorithm Progress

Each node iteratively updates its estimates so as to drive its power flow balance to zero

Iteration Three-Step Process

- **[Step 1]** Estimates are adjusted to interim values using the flow balance:

$$\tilde{f}_{ij}^{(i)}[k+1] = f_{ij}^{(i)}[k] + \frac{d_i[k]}{|\mathcal{N}_p(i)|+1}$$

$$\hat{u}_i[k+1] = u_i[k] - \frac{d_i[k]}{2\left(|\mathcal{N}_p(i)|+1\right)}$$

- **[Step 2]** Pairs of neighbors average interim estimates of flow between them:

$$\hat{f}_{ij}^{(i)}[k+1] = \frac{1}{2}\left(\tilde{f}_{ij}^{(i)}[k+1] - \tilde{f}_{ji}^{(j)}[k+1]\right)$$

- **[Step 3]** New estimate values are obtained by enforcing limits:

$$f_{ij}^{(i)}[k+1] = \left[\hat{f}_{ij}^{(i)}[k+1]\right]_{\underline{f}_{ij}}^{\overline{f}_{ij}}$$

$$u_i[k+1] = \left[\hat{u}_i[k+1]\right]_{\underline{u}_i}^{\overline{u}_i}$$

Feasible Flow Algorithm Convergence

- Since $f_{ij}^{(i)}[k] = -f_{ji}^{(j)}[k] =: f_{ij}[k]$, Steps 1-3 can be collapsed:

$$f_{ij}[k+1] = \left[f_{ij}[k] + \frac{1}{2}\left(\frac{d_i[k]}{|\mathcal{N}_p(i)|+1} - \frac{d_j[k]}{|\mathcal{N}_p(j)|+1} \right) \right]_{\underline{f}_{ij}}^{\overline{f}_{ij}} \quad (1)$$

$$u_i[k+1] = \left[u_i[k] - \frac{d_i[k]}{2(|\mathcal{N}_p(i)|+1)} \right]_{\underline{u}_i}^{\overline{u}_i} \quad (2)$$

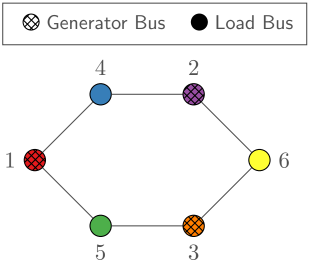
Proposition

Suppose there exists a solution, (u^*, f^*), to feasibility problem F2. Then, as $k \to \infty$, $u_i \to u_i^*$, $f_{ij}[k] \to f_{ij}^*$, $d_i[k] \to 0$

- Convergence proof relies on the observation that the iterations in (1) - (2) corresponds to those of a gradient descent algorithm for a quadratic program

Outline

Six-Node Network Case Study



Generator bus parameters

i	$\underline{u}_i$	$\overline{u}_i$	D_i
1	0.1	1.15	0.225
2	0.15	2.65	0.679
3	0.05	1.68	0.95

Load bus parameter

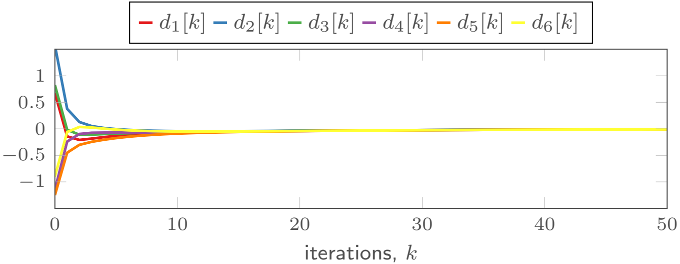
i	ℓ_i^0	D_i
4	1.15	0.0125
5	1.25	0.0679
6	0.9	0.0479

Line parameters

Line Index	$\{1,4\}$	$\{1,5\}$	$\{2,4\}$	$\{2,6\}$	$\{3,5\}$	$\{3,6\}$
Susceptance	-2.919	-6.685	-4.474	-4.375	-7.435	-6.274

Initial Set-Point Computation

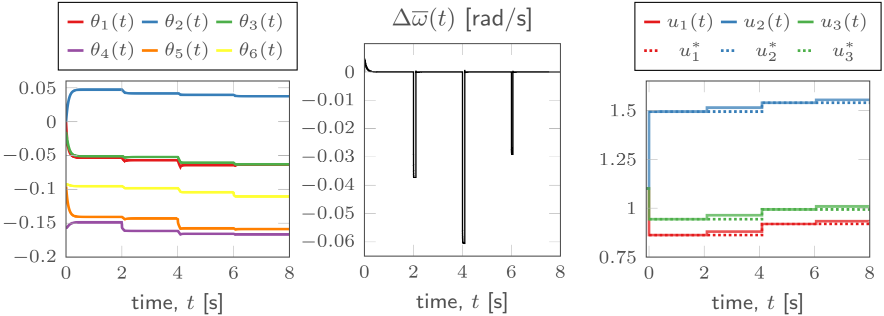
- Flow balances converge to within ± 0.001 in the first 50 iterations, yielding:
  - $u_1^* = 0.85$
  - $u_2^* = 1.5$ pu
  - $u_3^* = 0.95$ pu



System Response to Load Perturbations

- ℓ_4, ℓ_6, and ℓ_5 perturbed at $t = 2$ s, $t = 4$ s, and $t = 6$ s, respectively:
  - $\Delta \ell_4 = 0.0575$ pu
  - $\Delta \ell_5 = 0.09375$ pu
  - $\Delta \ell_6 = 0.045$ pu

- u_1^*, u_2^*, u_3^* recomputed at $t = 2$ s



(a) Bus voltage angles (b) Average frequency error (c) Set-points

Outline

Summary

- Things discussed today:
  - Architecture for frequency regulation in islanded inertia-less AC microgrids
  - Computation of set points guaranteed to yield a stable equilibrium point
  - Distributed algorithm for set-point computation
- Things not discussed today [Cady, Zholbaryssov, D-G, Hadjicostis, '16]:
  - Distributed computation of controller gains
  - Distributed computingutation of average frequency error
  - Criteria andd distributed protocol for triggering for set-point recomputation
- Future work:
  - Voltage control architecture
  - Extensions to lossy networks

Questions?

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