A New Method for Estimating Maximum Power Transfer and Voltage Stability Margins to Mitigate the Risk of Voltage Collapse

Bernie Lesieutre

Dan Molzahn

University of Wisconsin-Madison

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Voltage Stability

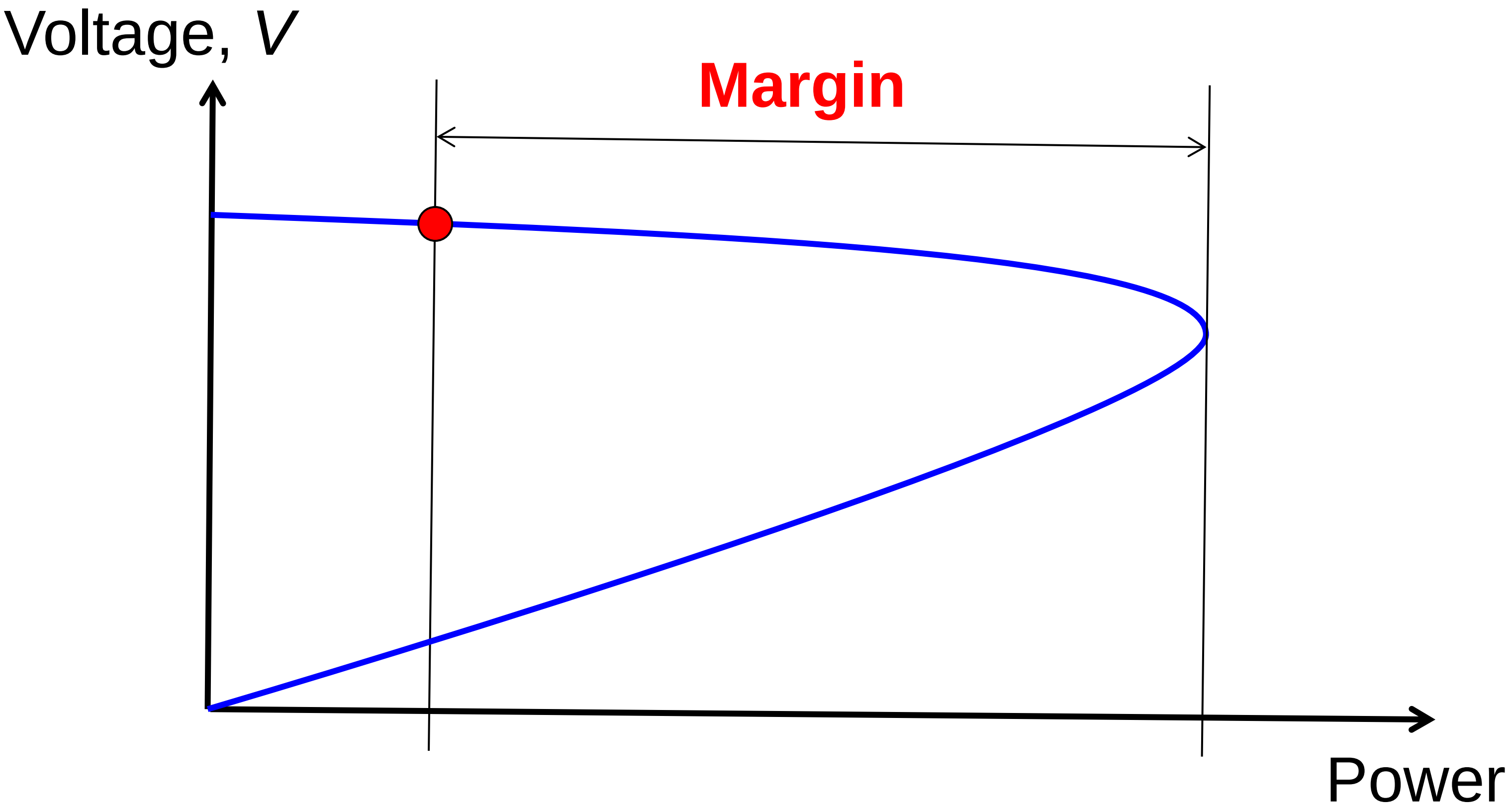
Static:

- **Loading, Power Flows**
- Local Reactive Power Support

Dynamic:

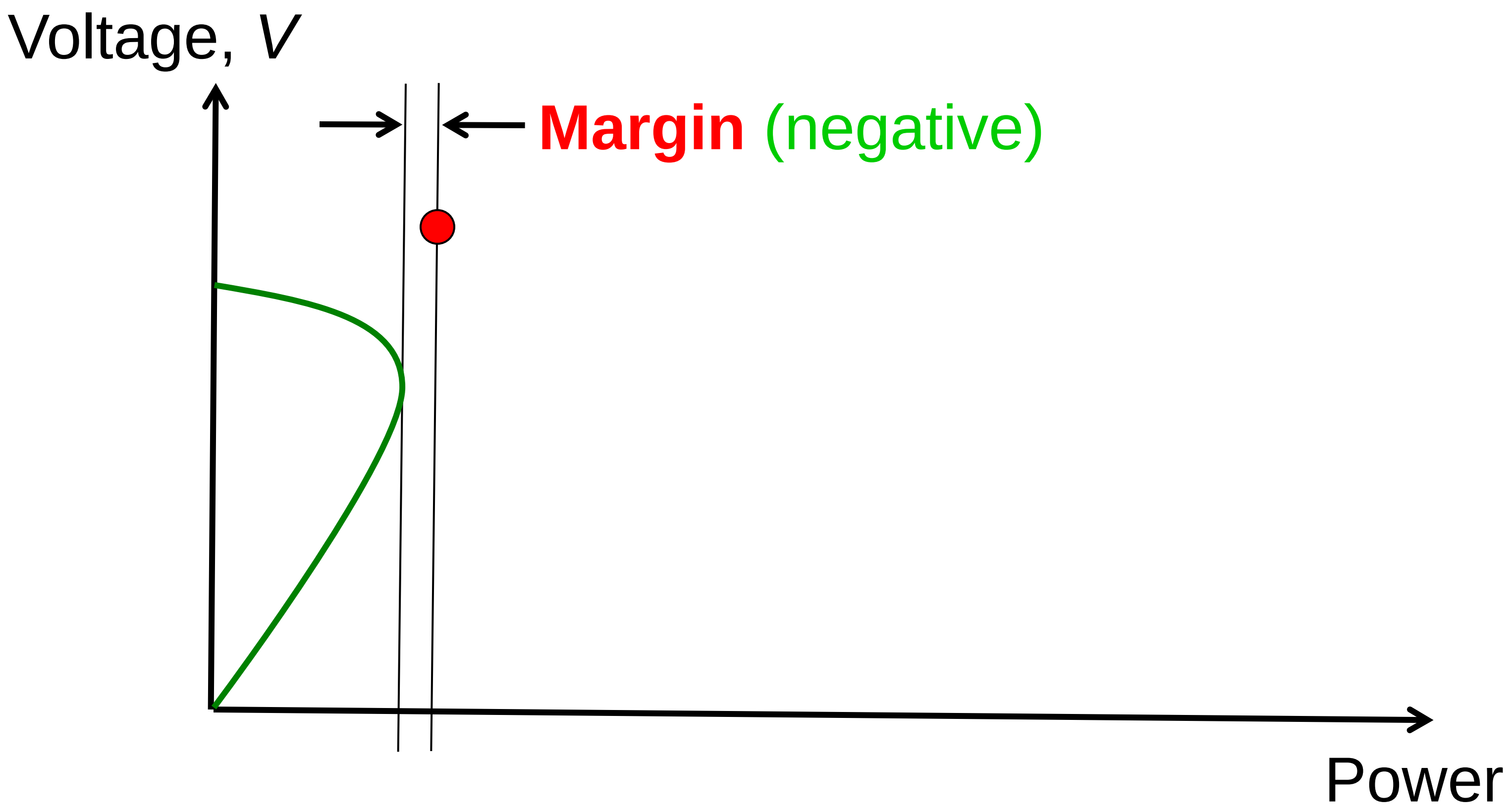
- Component Controls
- Network Controls

OVERVIEW: Static Voltage Stability Margin



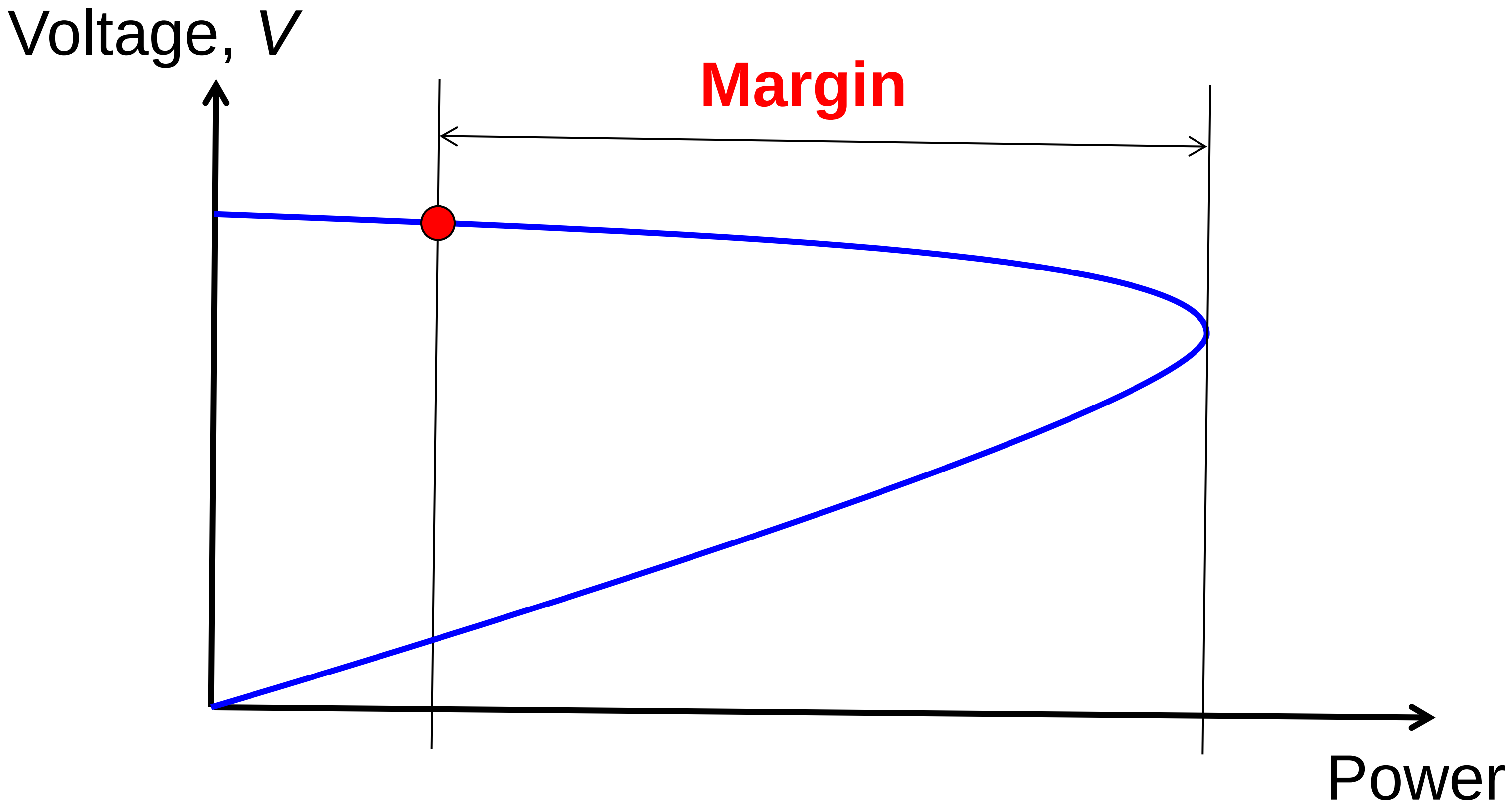
A common voltage stability margin measures the distance from a **post-contingency** operating point to the **"nose point"** on a power-voltage curve.

OVERVIEW: Static Voltage Stability Margin



It's even possible that a normal power flow solution won't exist, post contingency.

ISSUES



1. We don’t know the values on the curve.
2. We don’t know whether a post-contingency operating point exists!

Typical Approach

- Run post-contingency power flow. This may or may not converge.

- If successful, determine nose point: use a sequence of power flows with increasing load, or a continuation power flow.

Our Approach

Cast as an optimization problem:

• Minimize the controlled voltages while a solution exists. (Claim: a solution exists.)

• Exploit the quadratic nature of the power flow equations to directly obtain

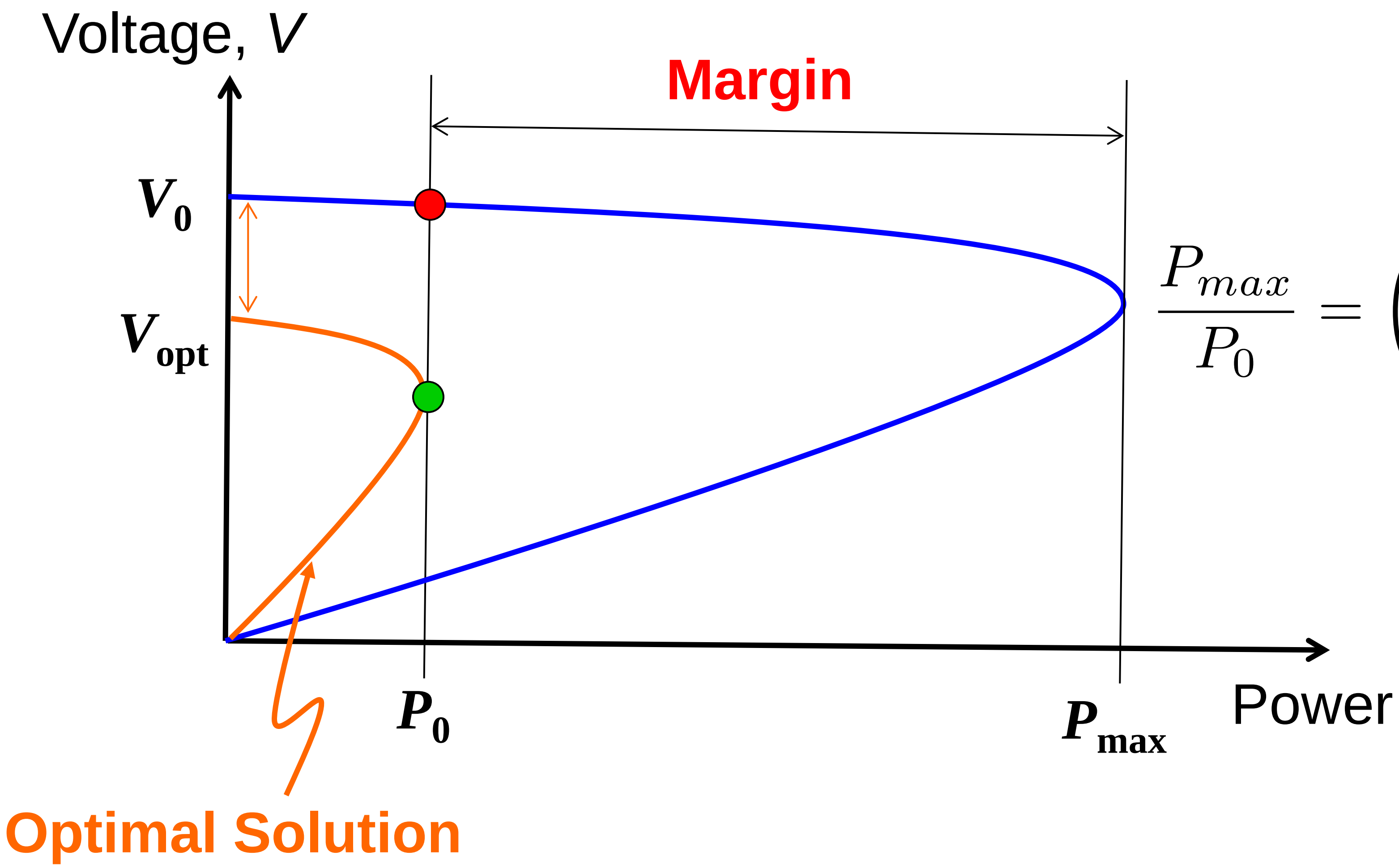
Traditional Voltage Stability Margin

even when the margin is negative.

Advantages

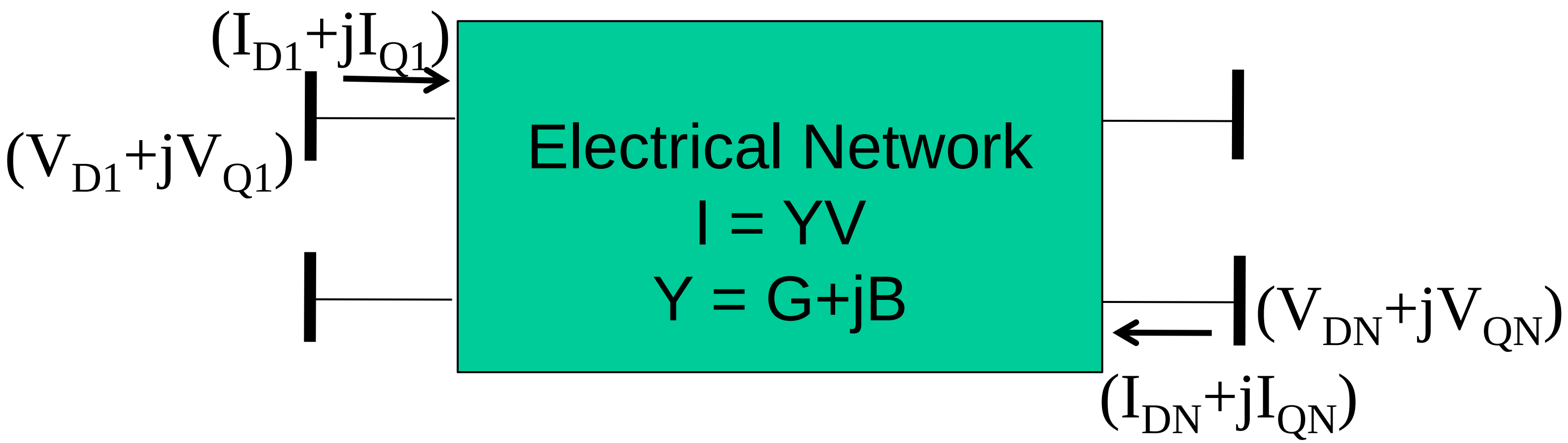
- Eliminates need for repeated solutions (multiple power flows, continuation power flows)
- Often offers provably **globally** optimal results
- Works when the margin is negative, i.e. when there isn't a solution.

OVERVIEW: Static Voltage Stability Margin



$$\frac{P_{max}}{P_0} = \left(\frac{V_0}{V_{opt}}\right)^2$$

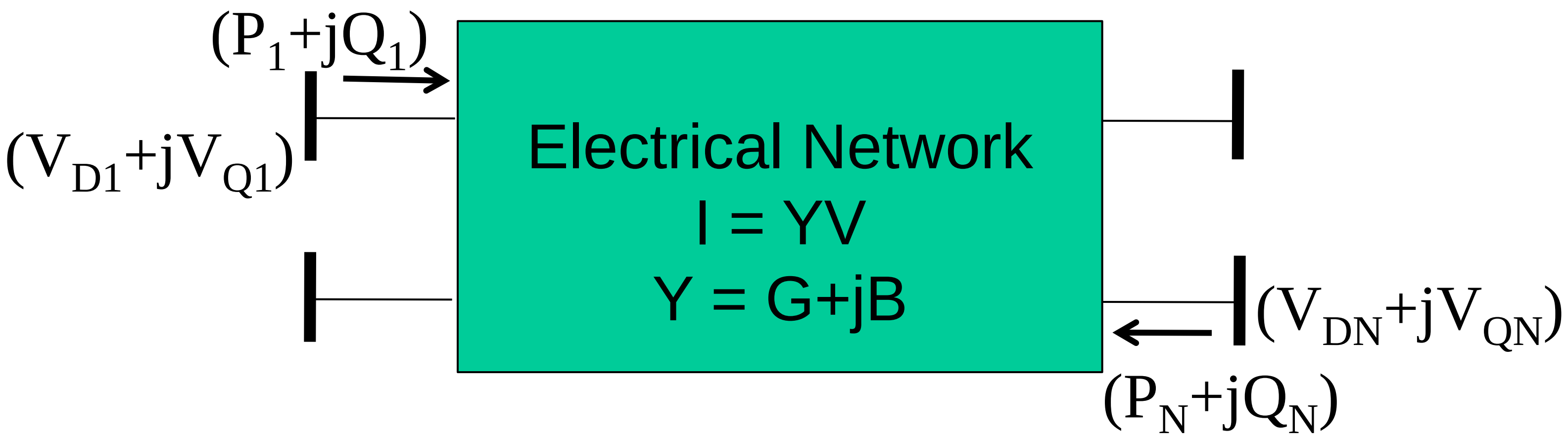
Power Flow Equations: Review



Current "Injection" equations in Rectangular Coordinates:

$$\begin{bmatrix} (I_{D1} + jI_{Q1}) \\ \vdots \\ (I_{DN} + jI_{QN}) \end{bmatrix} = \begin{bmatrix} G + jB \end{bmatrix} \begin{bmatrix} (V_{D1} + jV_{Q1}) \\ \vdots \\ (V_{DN} + jV_{QN}) \end{bmatrix}$$

Power Flow Equations: Review



Power "Injection" equations in Rectangular Coordinates:

$$\begin{bmatrix} (P_1 + jQ_1) \\ \vdots \\ (P_N + jQ_N) \end{bmatrix} = \begin{bmatrix} (V_{D1} + jV_{Q1}) & & \\ & \ddots & \\ & & (V_{DN} + jV_{QN}) \end{bmatrix} \begin{bmatrix} G - jB \end{bmatrix} \begin{bmatrix} (V_{D1} - jV_{Q1}) \\ \vdots \\ (V_{DN} - jV_{QN}) \end{bmatrix}$$

Power Flow Equations

The power flow equations are quadratic in voltage variables:

$$P_i = V_{Di}\sum_{k=1}^{N}(G_{ik}V_{Dk} - B_{ik}V_{Qk}) \quad + V_{Qi}\sum_{k=1}^{N}(B_{ik}V_{Dk} + G_{ik}V_{Qk})$$

$$Q_i = -V_{Di}\sum_{k=1}^{N}(B_{ik}V_{Dk} + G_{ik}V_{Qk}) \quad + V_{Qi}\sum_{k=1}^{N}(G_{ik}V_{Dk} - B_{ik}V_{Qk})$$

For reference, power engineers almost always express these equations in voltage polar coordinates:

$$P_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik}\cos\left(\delta_k - \delta_i\right) + B_{ik}\sin\left(\delta_k - \delta_i\right)\right)$$

$$Q_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik}\sin\left(\delta_k - \delta_i\right) - B_{ik}\cos\left(\delta_k - \delta_i\right)\right)$$

Classical Power Flow Constraints

Bus Type	Specified	Calculated
PQ (load)	P, Q	V, δ
PV (generator)	P, V	Q, δ
Slack	V, $\delta = 0^o$	P, Q

The "controlled voltages" are the problem-specified slack and generator voltage magnitudes.

Locking the controlled voltages in constant proportion, and allowing them to scale by α, we claim a power solution exists for any power flow injection profile. (subject to the unimportant small print.)

Optimization Problem

- Modify the power flow formulation to $(\dagger)$
- Slack bus voltage magnitude unconstrained
- PV bus voltage magnitudes scale with slack bus voltage
- Minimize slack bus voltage

$$(\dagger)\quad \begin{aligned} V_{\text{slack}}^{\text{opt}} = \min \quad & V_{\text{slack}} \\ \text{subject to} \quad & \\ & P_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik} \cos\left(\delta_k - \delta_i\right) + B_{ik} \sin\left(\delta_k - \delta_i\right)\right) && \forall\, k \in \{\mathcal{PQ}, \mathcal{PV}\} \\ & Q_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik} \sin\left(\delta_k - \delta_i\right) - B_{ik} \cos\left(\delta_k - \delta_i\right)\right) && \forall\, k \in \mathcal{PQ} \\ & V_k = \alpha_k V_{\text{slack}} && \forall\, k \in \mathcal{PV} \end{aligned}$$

Optimization Problem

This optimization problem can be solved many different ways…

We've been using the convex relaxation formulation for the power flow equations (Lavaei, Low) because we really want (provably) the minimum solution.

- The problem has a feasible solution
- The optimization using the convex relaxation can be solved for a global minimum (and hopefully a feasible power flow solution).

Relaxed Problem Formulation

In Rectangular coordinates, define

$$x = [V_{d1} \dots V_{dN} V_{q1} \dots V_{qN}]^T$$

Then, power flow equations can be written in the form

$$P_k = c_k^T (x\ x^T) c_k \quad = tr(C_k W)$$

$$Q_k = \bar{c}_k^T (x\ x^T) \bar{c}_k \quad = tr(\bar{C}_k W)$$

$$V_k^2 = tr(M_k W)$$

where

$$W = x\ x^T$$

Which is a rank one matrix by construction.

Relaxed Problem Formulation

The convex relaxation is introduced by relaxing the rank of *W.* With that, we pose the following convex optimization problem:

$$V_{slack}^{opt} = \min \ V_{slack}$$

$$\text{subject to}$$

$$P_k = tr(Y_k W)$$

$$Q_k = tr(\bar{Y}_k W)$$

$$\alpha_k^2 V_{slack}^2 = tr(M_k W)$$

$$W \succeq 0$$

W is positive semi-definite

Semidefinite Relaxation Example

$$x = \begin{bmatrix} V_{d1} & V_{d2} & \dots & V_{dn} & V_{q1} & V_{q2} & \dots & V_{qn} \end{bmatrix}^T$$

$$\mathbf{M}_1 \qquad\qquad \mathbf{W} = xx^T$$

$$\text{trace}\left(\begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} V_{d1}^2 & V_{d1}V_{d2} & \cdots & V_{d1}V_{q1} & \cdots & V_{d1}V_{qn} \\ V_{d1}V_{d2} & V_{d2}^2 & \cdots & V_{d2}V_{q1} & \cdots & V_{d2}V_{qn} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ V_{d1}V_{q1} & V_{d2}V_{q1} & \cdots & V_{q1}^2 & \cdots & V_{q1}V_{qn} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ V_{d1}V_{qn} & V_{d2}V_{qn} & \cdots & V_{q1}V_{qn} & \cdots & V_{qn}^2 \end{bmatrix} \right)$$

$$= V_{d1}^2 + V_{q1}^2 = V_1^2$$

Semidefinite Relaxation Example

$$\mathbf{M}_1 \qquad\qquad \mathbf{W} \succeq 0$$

$$\operatorname{trace}\left(\begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1,n+1} & \cdots & W_{1,2n} \\ W_{12} & W_{22} & \cdots & W_{2,n+1} & \cdots & W_{2,2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{1,n+1} & W_{2,n+1} & \cdots & W_{n+1,n+1} & \cdots & W_{n+1,2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{1,2n} & W_{2,2n} & \cdots & W_{n+1,2n} & \cdots & W_{2n,2n} \end{bmatrix}\right)$$

$$= W_{11} + W_{n+1,n+1}$$

Bonus result concerning solution to the power flow equations

- The existence of a power flow solution requires

$$V_{\text{slack}}^{\text{opt}} \leq V_0 \quad \text{(Specified slack bus voltage)}$$

  - Necessary, but not sufficient, condition for existence

- Conversely, no solution exists if $V_{\text{slack}}^{\text{opt}} > V_0$

  - Sufficient, but not necessary, condition for non-existence

Controlled Voltage Margin

- A controlled voltage margin to the solvability boundary

$$\sigma = \frac{V_0}{V_{slack}^{min}}$$

  - Upper bound (non-conservative)

- No power flow solution exists for $\sigma < 1$

  - Increasing the slack bus voltage (with proportional increases in PV bus voltages) by at least $\frac{1}{\sigma}$ is required for solution.

Power Injection Margin

- Uniformly scaling all power injections scales $\left(V_{\text{slack}}^{\text{opt}}\right)^2$

$$\text{if} \quad \left(V_{\text{slack}}^{\text{opt}}\right)^2 = f\left(P_{\text{inj}} + jQ_{\text{inj}}\right)$$

$$\text{then} \quad \eta\left(V_{\text{slack}}^{\text{opt}}\right)^2 = f\left(\eta\left(P_{\text{inj}} + jQ_{\text{inj}}\right)\right)$$

- Uniformly scale power injections until

$$\eta\left(V_{\text{slack}}^{\text{opt}}\right)^2 = V_0^2$$

- Corresponding η gives a power flow voltage stability margin in the direction of uniformly increasing power injections at constant power factor.

- $\eta < 1$ indicates that no solution exists for the original power flow problem

Power Injection Margin

The power injection margin answers the question

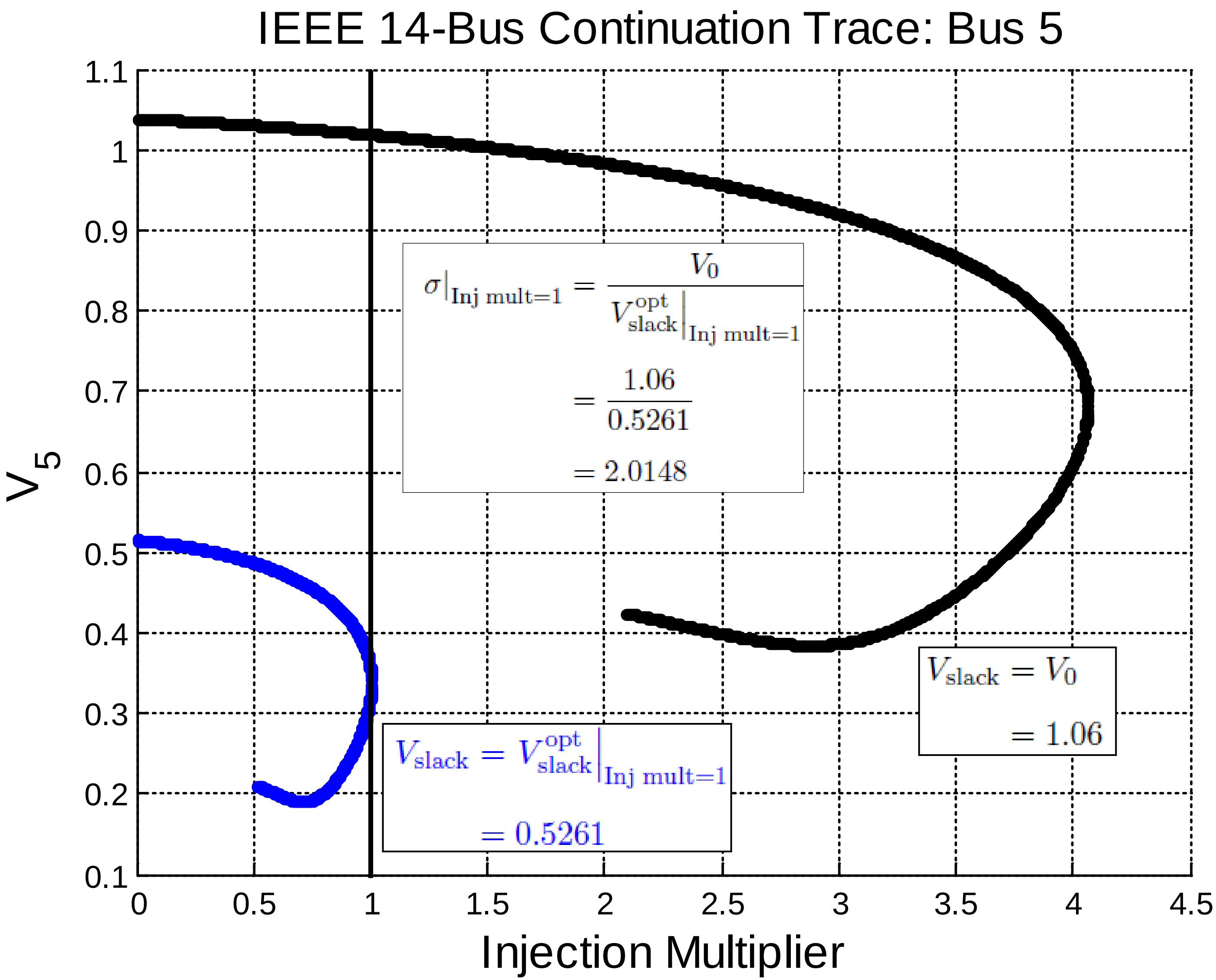
For a given voltage profile, by what factor can we change our power injections (uniformly at all buses) while still potentially having a solution?

Answer: $$\eta = \left(\frac{V_0}{V_{\text{slack}}^{\text{opt}}} \right)^2$$

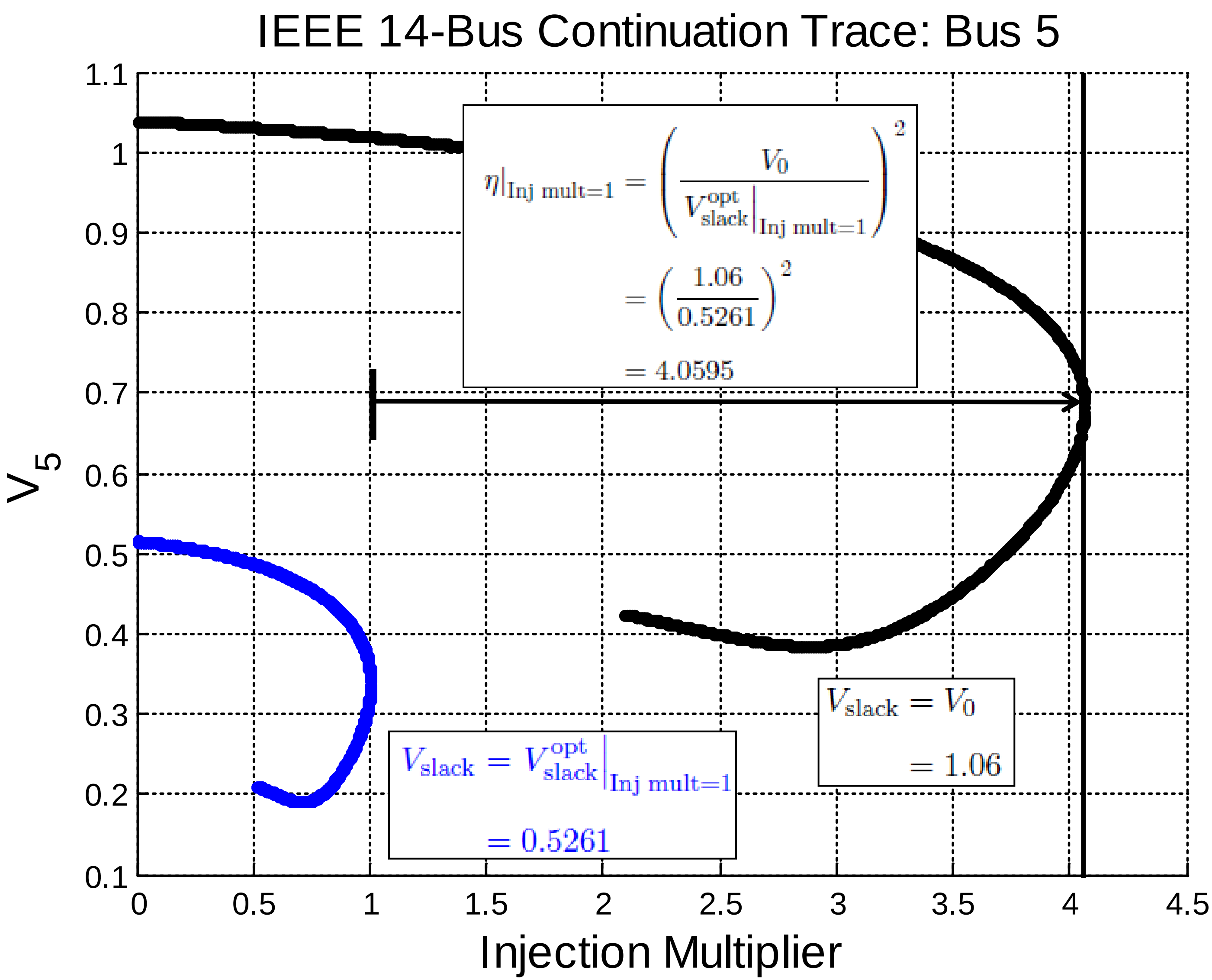
Examples

- IEEE 14-Bus System
- IEEE 118-Bus System
- Tested many other systems and loadings

Controlled Voltage Margin



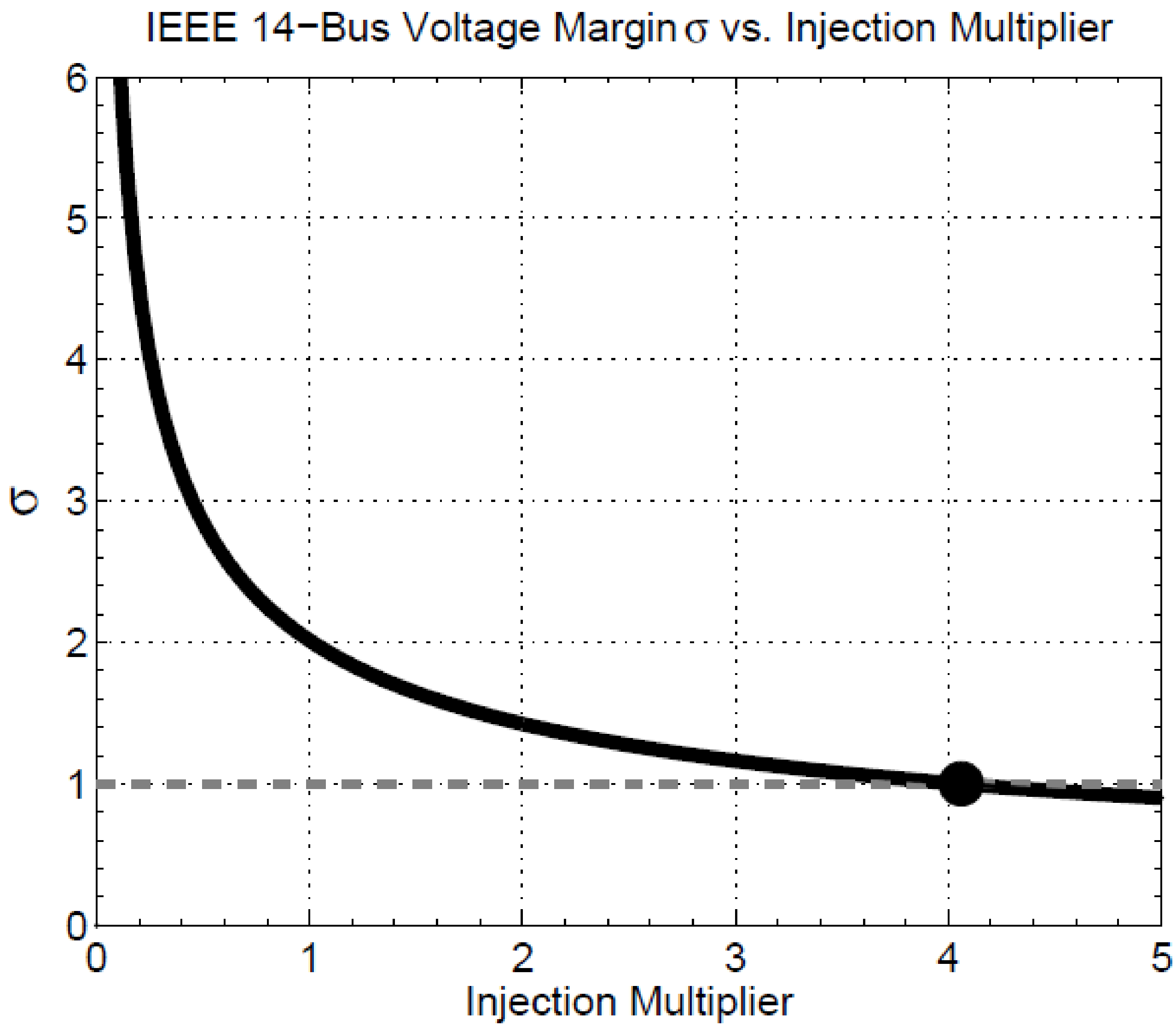
Power Injection Margin



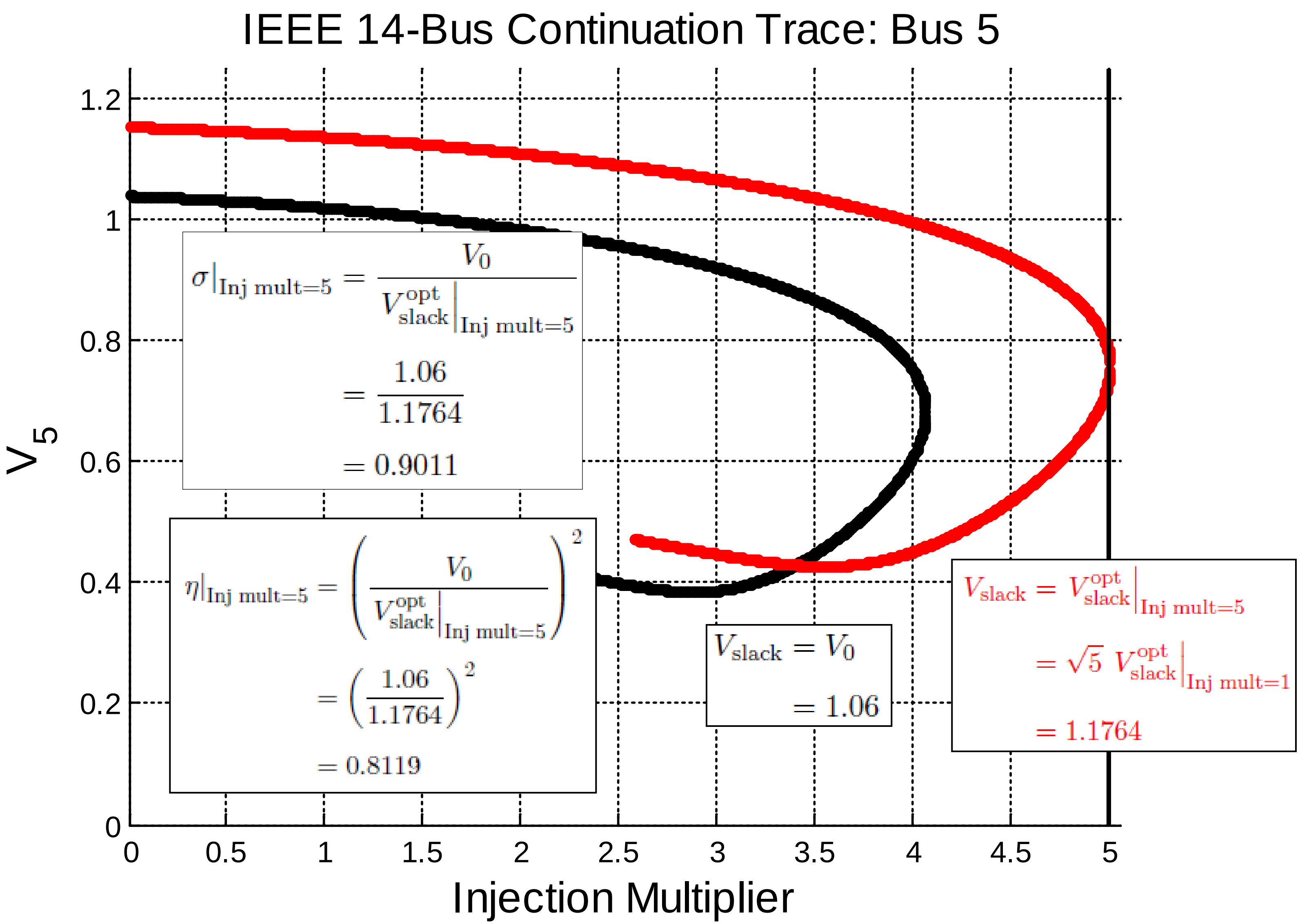
IEEE 14 Bus System

Injection Multiplier	Newton-Raphson Converged?	V_0	$V_{\text{slack}}^{\text{opt}}$	dim(null(A))
1.000	Yes	1.06	0.5261	2
2.000	Yes	1.06	0.7440	2
3.000	Yes	1.06	0.9112	2
4.000	Yes	1.06	1.0522	2
4.010	Yes	1.06	1.0535	2
4.020	Yes	1.06	1.0548	2
4.030	Yes	1.06	1.0561	2
4.040	Yes	1.06	1.0575	2
4.050	Yes	1.06	1.0588	2
4.055	Yes	1.06	1.0594	2
4.056	Yes	1.06	1.0595	2
4.057	Yes	1.06	1.0597	2
4.058	Yes	1.06	1.0598	2
4.059	Yes	1.06	**1.0599**	2
4.060	No	1.06	**1.0601**	2
4.061	No	1.06	1.0602	2
4.062	No	1.06	1.0603	2
4.063	No	1.06	1.0605	2
4.064	No	1.06	1.0606	2
4.065	No	1.06	1.0607	2
5.000	No	1.06	1.1764	2

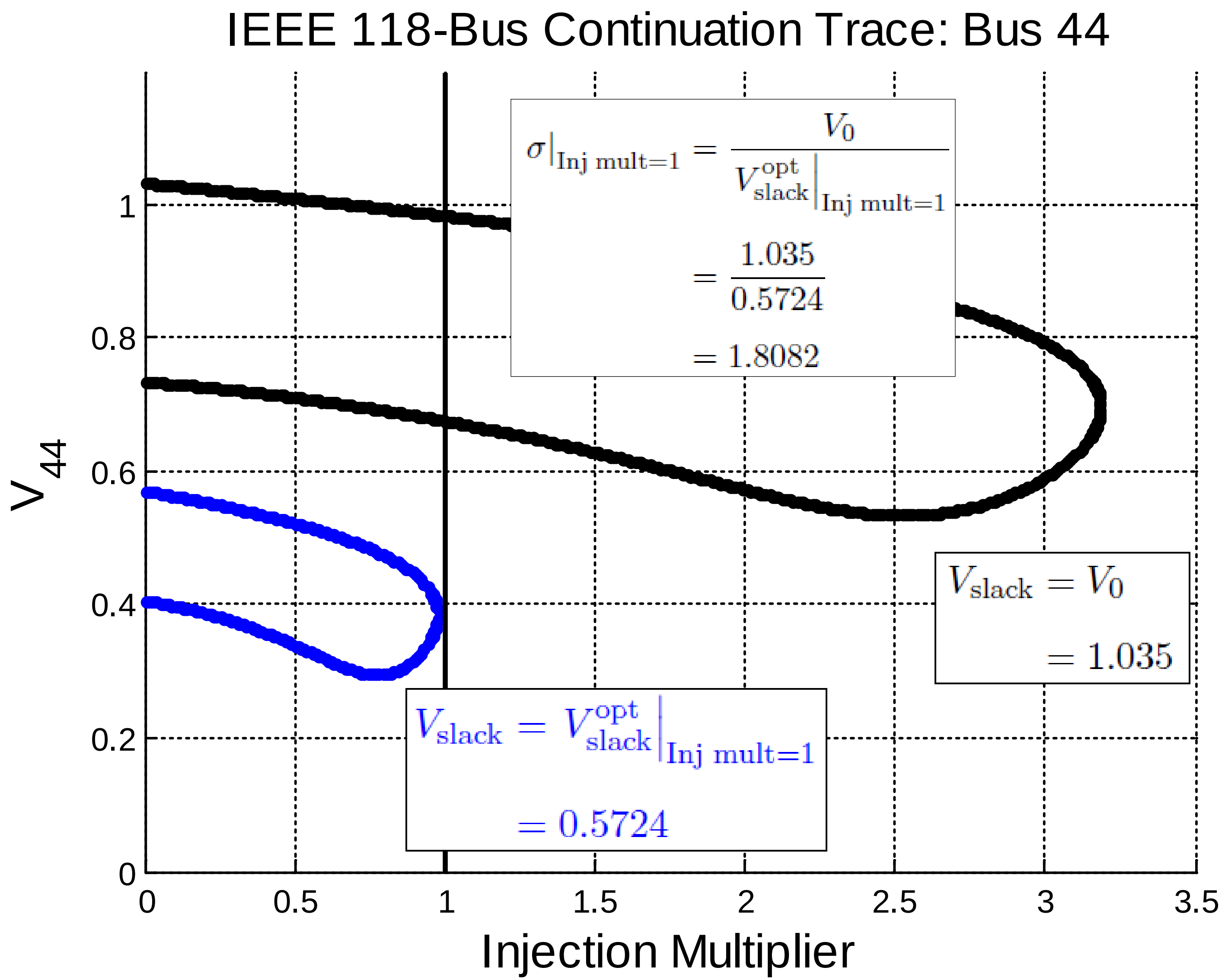
Voltage Margin



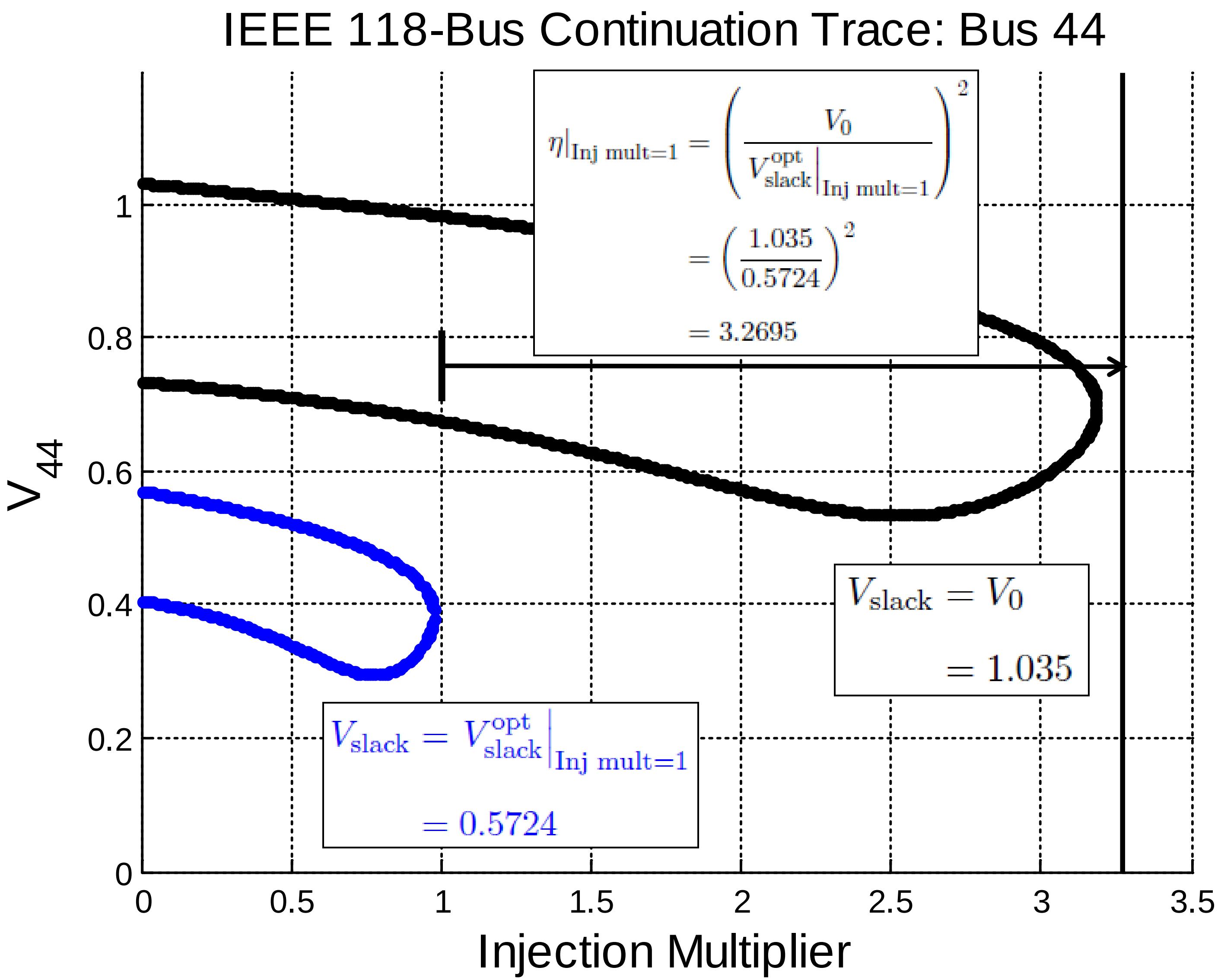
Voltage and Power Injection Margins



Controlled Voltage Margin



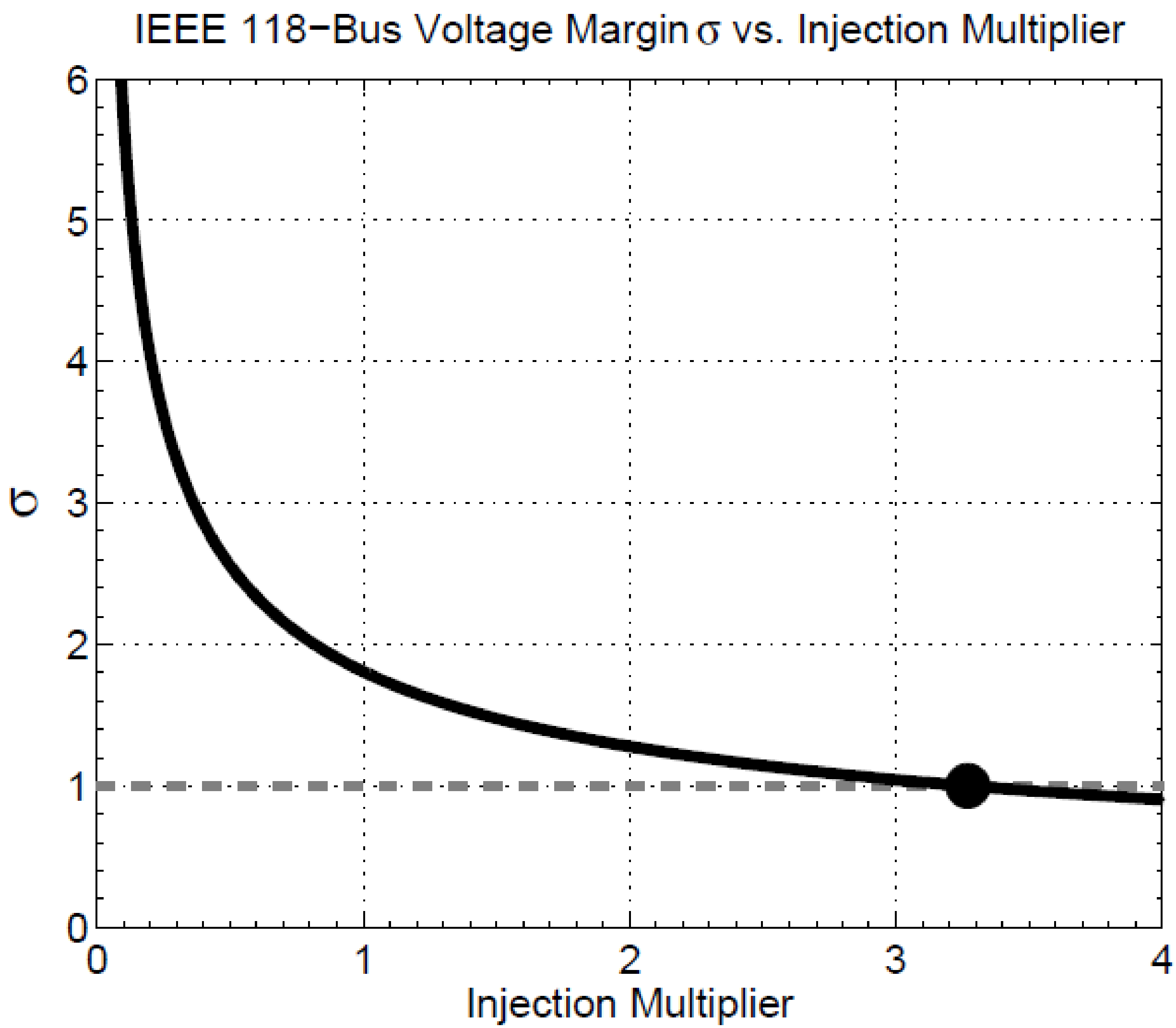
Power Injection Margin



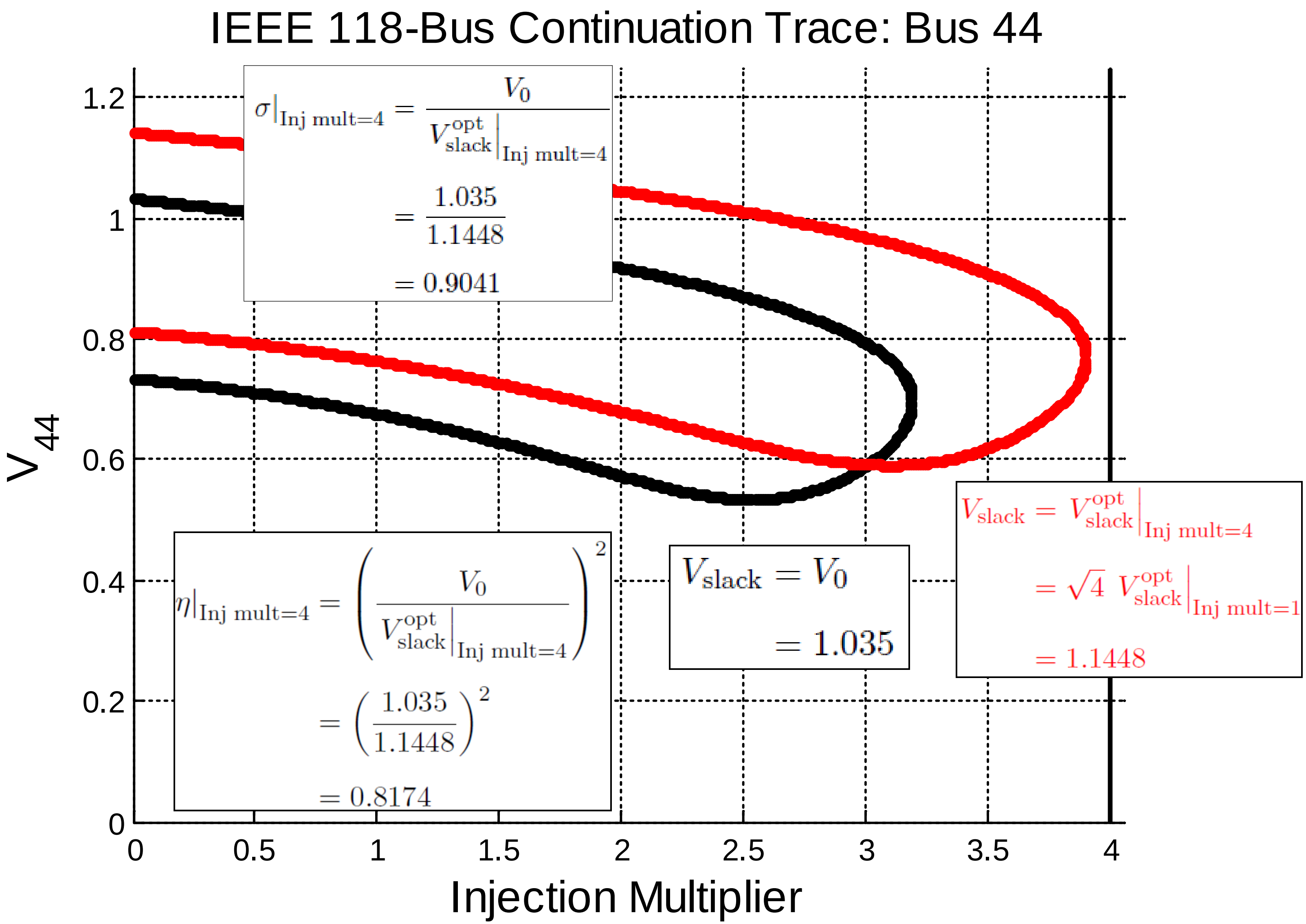
IEEE 118 Bus System

Injection Multiplier	Newton-Raphson Converged?	V_0	$V_{\text{slack}}^{\text{opt}}$ (lower bound)	dim(null(A))
1.00	Yes	1.035	0.5724	4
1.50	Yes	1.035	0.7010	4
2.00	Yes	1.035	0.8095	4
2.50	Yes	1.035	0.9050	4
3.00	Yes	1.035	0.9914	4
3.15	Yes	1.035	1.0159	4
3.16	Yes	1.035	1.0175	4
3.17	Yes	1.035	1.0191	4
3.18	Yes	1.035	1.0207	4
3.19	No	1.035	1.0223	4
3.20	No	1.035	1.0239	4
3.21	No	1.035	1.0255	4
3.22	No	1.035	1.0271	4
3.23	No	1.035	1.0287	4
3.24	No	1.035	1.0303	4
3.25	No	1.035	1.0319	4
3.26	No	1.035	1.0335	4
3.27	No	1.035	1.0351	4
3.28	No	1.035	1.0366	4
3.29	No	1.035	1.0382	4
4.00	No	1.035	1.1448	4

Voltage Margin
IEEE 118 Bus System



Voltage and Power Injection Margins



Alternate Power Injection Profiles

Power injection margin is in the direction of a uniform, constant-power-factor injection profile

We can alternatively specify any profile that is a linear function of powers and squared voltages

- However, insolvability condition $\eta < 1$ is not necessarily valid

$$\max \eta \qquad \text{subject to}$$

$$\operatorname{trace}\left(\mathbf{Y}_k \mathbf{W}\right) = f_k\left(P, Q, V^2, \eta\right) \qquad \forall k \in \{\mathcal{PQ}, \mathcal{PV}\}$$

$$\operatorname{trace}\left(\bar{\mathbf{Y}}_k \mathbf{W}\right) = g_k\left(P, Q, V^2, \eta\right) \qquad \forall k \in \{\mathcal{PQ}\}$$

$$\operatorname{trace}\left(\mathbf{M}_k \mathbf{W}\right) = \alpha V_0 \qquad \forall k \in \{\mathcal{PV}\}$$

$$\operatorname{trace}\left(\mathbf{M}_{slack} \mathbf{W}\right) = V_0$$

$$\mathbf{W} \succeq 0$$

Reactive Power Limits

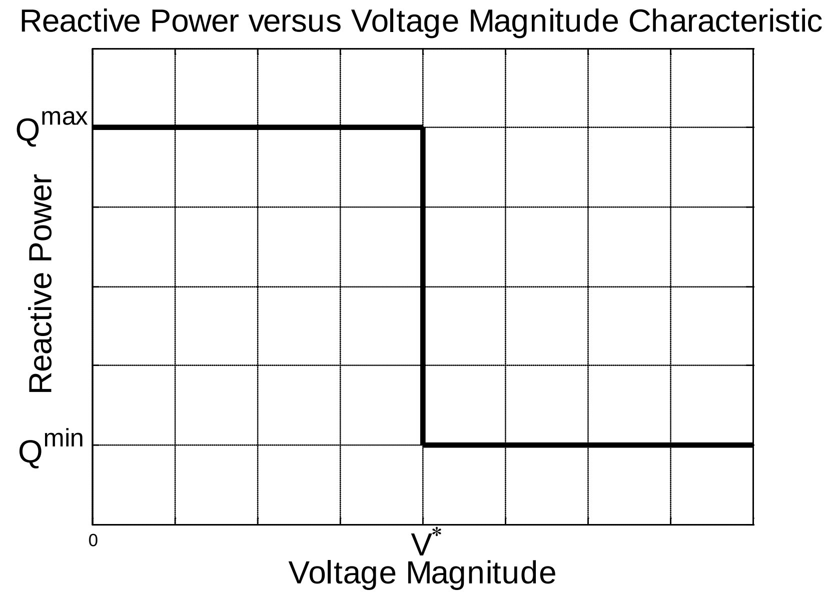
Previous work models generators as ideal voltage sources

Detailed models limit reactive outputs

- Limit-induced bifurcations

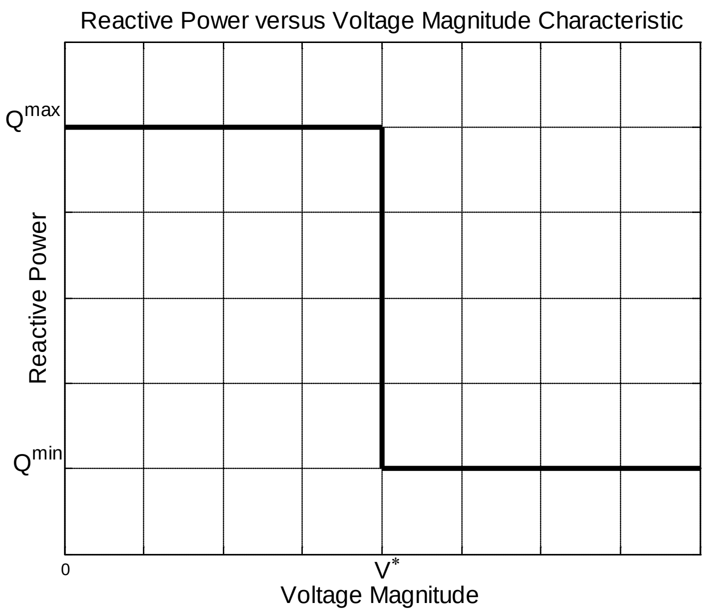
Two approaches to modeling these limits:

- Mixed-integer semidefinite programming
- Infeasibility certificates using sum of squares programming



MISDP Formulation

Model reactive power limits using **binary variables**



$$\max_{\mathbf{W},\psi_U,\psi_L,\eta} \quad \eta \qquad \text{subject to}$$

$$\mathrm{tr}\left(\mathbf{Y}_k\mathbf{W}\right) = P_k\,\eta \qquad \forall k \in \{\mathcal{PQ},\,\mathcal{PV}\}$$

$$\mathrm{tr}\left(\bar{\mathbf{Y}}_k\mathbf{W}\right) = Q_{Dk}\,\eta \qquad \forall k \in \mathcal{PQ}$$

$$\begin{cases} \mathrm{tr}\left(\bar{\mathbf{Y}}_k\mathbf{W}\right) \geq Q_k^{max}\psi_{Uk} + Q_k^{min}\left(1-\psi_{Uk}\right) \\ \mathrm{tr}\left(\bar{\mathbf{Y}}_k\mathbf{W}\right) \leq Q_k^{min}\psi_{Lk} + Q_k^{max}\left(1-\psi_{Lk}\right) \end{cases} \qquad \forall k \in \{\mathcal{PV},\,\mathcal{S}\}$$

$$\begin{cases} \mathrm{tr}\left(\mathbf{M}_k\mathbf{W}\right) \geq \left(V_k^*\right)^2\left(1-\psi_{Uk}\right) \\ \mathrm{tr}\left(\mathbf{M}_k\mathbf{W}\right) \leq \left(V_k^*\right)^2\left(1-\psi_{Lk}\right) + d\psi_{Lk} \end{cases} \qquad \forall k \in \{\mathcal{PV},\,\mathcal{S}\}$$

$$\psi_{Lk} + \psi_{Uk} \leq 1 \qquad \forall k \in \{\mathcal{PV},\,\mathcal{S}\}$$

$$\sum_{k\in\{\mathcal{PV},\mathcal{S}\}} \left(\psi_{Lk} + \psi_{Uk}\right) \leq n_g - 1$$

$$\mathbf{W} \succeq 0$$

$$\psi_{Uk} \in \{0,\,1\} \qquad \psi_{Lk} \in \{0,\,1\} \qquad \forall k \in \{\mathcal{PV},\,\mathcal{S}\}$$

for some large d

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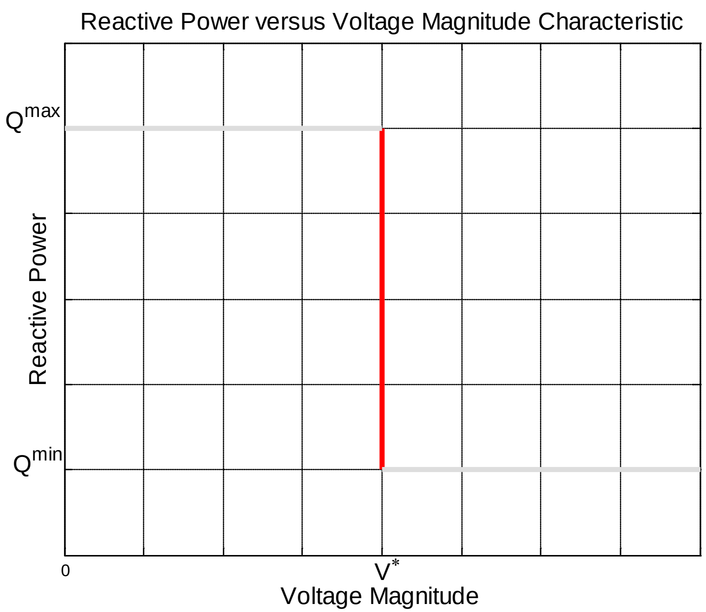
Model reactive power limits using **binary variables**



$$\psi_U = 0$$
$$\psi_L = 0$$

$$\max_{\mathbf{W}, \psi_U, \psi_L, \eta} \quad \eta \quad \text{subject to}$$

$$\text{tr}\left(\mathbf{Y}_k \mathbf{W}\right) = P_k \, \eta \qquad \forall k \in \{\mathcal{PQ}, \mathcal{PV}\}$$

$$\text{tr}\left(\bar{\mathbf{Y}}_k \mathbf{W}\right) = Q_{Dk} \, \eta \qquad \forall k \in \mathcal{PQ}$$

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$$\begin{cases} \text{tr}\left(\mathbf{M}_k \mathbf{W}\right) \geq \left(V_k^*\right)^2 \left(1 - \psi_{Uk}\right) \\ \text{tr}\left(\mathbf{M}_k \mathbf{W}\right) \leq \left(V_k^*\right)^2 \left(1 - \psi_{Lk}\right) + d \psi_{Lk} \end{cases} \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

$$\psi_{Lk} + \psi_{Uk} \leq 1 \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

$$\sum_{k \in \{\mathcal{PV}, \mathcal{S}\}} \left(\psi_{Lk} + \psi_{Uk}\right) \leq n_g - 1$$

$$\mathbf{W} \succeq 0$$

$$\psi_{Uk} \in \{0, 1\} \qquad \psi_{Lk} \in \{0, 1\} \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

for some large d

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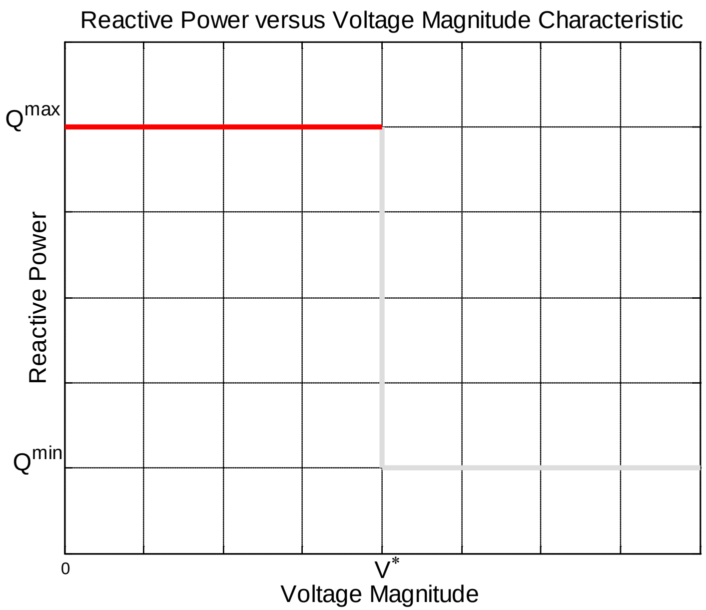

$$\psi_U = 1$$
$$\psi_L = 0$$

$$\max_{\mathbf{W},\psi_U,\psi_L,\eta} \quad \eta \quad \text{subject to}$$

$$\mathrm{tr}\left(\mathbf{Y}_k\mathbf{W}\right) = P_k\,\eta \qquad \forall k \in \{\mathcal{PQ},\,\mathcal{PV}\}$$

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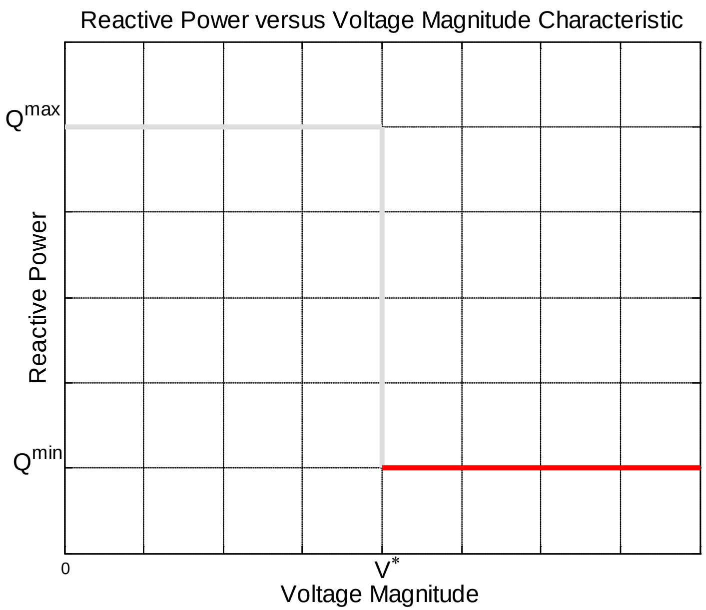


$$\psi_U = 1$$
$$\psi_L = 0$$

$$\max_{\mathbf{W}, \psi_U, \psi_L, \eta} \quad \eta \quad \text{subject to}$$

$$\mathrm{tr}\left(\mathbf{Y}_k \mathbf{W}\right) = P_k \, \eta \qquad \forall k \in \{\mathcal{PQ}, \mathcal{PV}\}$$

$$\mathrm{tr}\left(\bar{\mathbf{Y}}_k \mathbf{W}\right) = Q_{Dk} \, \eta \qquad \forall k \in \mathcal{PQ}$$

$$\begin{cases} \mathrm{tr}\left(\bar{\mathbf{Y}}_k \mathbf{W}\right) \geq Q_k^{max} \psi_{Uk} + Q_k^{min}\left(1 - \psi_{Uk}\right) \\ \mathrm{tr}\left(\bar{\mathbf{Y}}_k \mathbf{W}\right) \leq Q_k^{min} \psi_{Lk} + Q_k^{max}\left(1 - \psi_{Lk}\right) \end{cases} \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

$$\begin{cases} \mathrm{tr}\left(\mathbf{M}_k \mathbf{W}\right) \geq \left(V_k^*\right)^2 \left(1 - \psi_{Uk}\right) \\ \mathrm{tr}\left(\mathbf{M}_k \mathbf{W}\right) \leq \left(V_k^*\right)^2 \left(1 - \psi_{Lk}\right) + d \psi_{Lk} \end{cases} \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

$$\psi_{Lk} + \psi_{Uk} \leq 1 \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

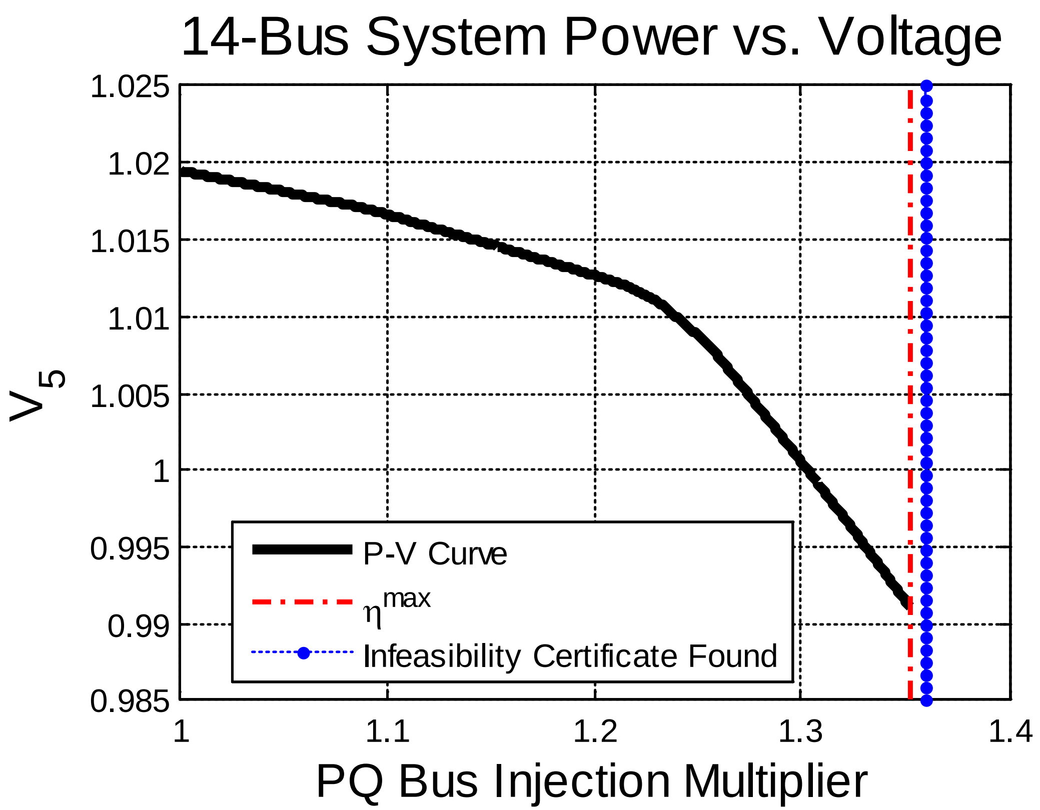
$$\sum_{k \in \{\mathcal{PV}, \mathcal{S}\}} \left(\psi_{Lk} + \psi_{Uk}\right) \leq n_g - 1$$

$$\mathbf{W} \succeq 0$$

$$\psi_{Uk} \in \{0, 1\} \qquad \psi_{Lk} \in \{0, 1\} \qquad \forall k \in \{\mathcal{PV}, \mathcal{S}\}$$

for some large d

Reactive Power Limits Results



System	Trace Nose Point	η^{max}	Infeasibility Certificate
14-bus	1.3522	1.3522	1.36
30-bus	2.8609	2.8609	2.86
57-bus	1.6486	1.6486	1.65

Conclusions

- Cast the problem of computing voltage stability margins as an optimization problem – to minimize the slack bus voltage.
- Calculated voltage stability margins – power injection/flows, and controlled voltages.
- Tested with numerical examples

Advantages:

- Eliminates repeated solution (multiple power flows, continuation power flows)
- Often offers provably globally optimal results
- Works when the margin is negative, i.e. when there isn't a solution.

Related Publications

[1] B.C. Lesieutre, D.K. Molzahn, A.R. Borden, and C.L. DeMarco, "Examining the Limits of the Application of Semidefinite Programming to Power Flow Problems," *49th Annual Allerton Conference on Communication, Control, and Computing (Allerton), 2011*, pp.1492-1499, 28-30 Sept. 2011.

[2] D.K. Molzahn, J.T. Holzer, and B.C. Lesieutre, and C.L. DeMarco, "Implementation of a Large-Scale Optimal Power Flow Solver Based on Semidefinite Programming," To appear in *IEEE Transactions on Power Systems*.

[3] D.K. Molzahn, B.C. Lesieutre, and C.L. DeMarco, "An Approximate Method for Modeling ZIP Loads in a Semidefinite Relaxation of the OPF Problem," submitted to *IEEE Transactions on Power Systems, Letters.*

[4] D.K. Molzahn, B.C. Lesieutre, and C.L. DeMarco, "A Sufficient Condition for Global Optimality of Solutions to the Optimal Power Flow Problem," to appear in *IEEE Transactions on Power Systems, Letters.*

[5] D.K. Molzahn, B.C. Lesieutre, and C.L. DeMarco, "A Sufficient Condition for Power Flow Insolvability with Applications to Voltage Stability Margins," *IEEE Transactions on Power Systems*, Vol 28, No. 3, pp. 2592-2601.

[6] D.K. Molzahn, V. Dawar, B.C. Lesieutre, and C.L. DeMarco, "Sufficient Conditions for Power Flow Insolvability Considering Reactive Power Limited Generators with Applications to Voltage Stability Margins," presented at *Bulk Power System Dynamics and Control - IX. Optimization, Security and Control of the Emerging Power Grid, 2013 IREP Symposium*, 25-30 Aug. 2013.

[7] D.K. Molzahn, B.C. Lesieutre, and C.L. DeMarco, "Investigation of Non-Zero Duality Gap Solutions to a Semidefinite Relaxation of the Power Flow Equations," To be presented at the Hawaii International Conference on System Sciences, January 2014.

Questions?

Extra Slides: Feasibility

Feasibility

For lossless systems:

1. Show a solution exists for zero power injections at PQ buses and zero active power injection at PV buses, for $\alpha = 1$.
2. Use implicit function theorem to argue that perturbations to zero power injections solutions also exist. Specifically choose one in the direction of desired power injection profile.
3. Exploit the quadratic nature of power flow equations to scale voltages and power to match injection profile.

Feasibility: Zero Power Injection Solution

Easy: Construct a solution.

- **Open Circuit PQ buses for zero power, zero current injection.**
- **Use Ward-type reduction to eliminate PQ buses. (not really necessary, but clean)** small print
- **Choose uniform angle solution for all buses.**
- **Directly use reactive power flow equation to calculated the reactive power injections at generator buses.**

0 0

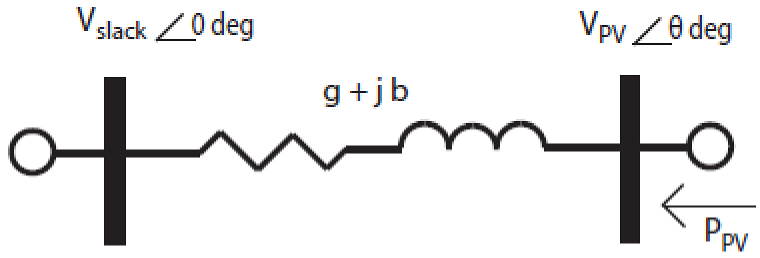
$$P_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik} \cos\left(\delta_k - \delta_i\right) + B_{ik} \sin\left(\delta_k - \delta_i\right)\right)$$

0

$$Q_k = V_k \sum_{i=1}^{n} V_i \left(G_{ik} \sin\left(\delta_k - \delta_i\right) - B_{ik} \cos\left(\delta_k - \delta_i\right)\right)$$

Note: Zero Power Injection Solutions for Lossy Systems

- Not all systems have a zero-power injection solution



$$P_{\mathrm{PV}} = gV_{\mathrm{PV}}^2 - V_{\mathrm{PV}}V_{\mathrm{slack}}\left(g\cos(\theta) + b\sin(\theta)\right)$$

- Ability to choose θ such that $P_{PV} = 0$ depends on
  - Ratio of V_{PV} to V_{slack}
  - Ratio of g to b

$$\left(\frac{V_{PV}}{V_{slack}}\right)^2 \leq 1 + \left(\frac{b}{g}\right)^2$$

  - Systems with small resistances and small voltage magnitude differences are expected to have a zero power injection solution

Nearby Solutions

A nearby non-zero solution exists

$$f((V_{Di} + jV_{Qi}) + (\Delta V_{Di} + j\Delta V_{Qi})) = \Delta P_i + j\Delta Q_i$$

provided the Jacobian is nonsingular. For a connected lossless system at the zero-power injection solution, the appropriate Jacobian is nonsingular, generically.

small print

Feasible Solution

Exploit the quadratic nature of power flow equations to scale voltages to match desired power profile:

$$f(\beta(V_{Di} + jV_{Qi}) + \beta(\Delta V_{Di} + j\Delta V_{Qi})) = \beta^2(\Delta P_i + j\Delta Q_i)$$
$$= P_i + jQ_i$$

Extra Slides: Infeasibility Certificates

Infeasibility Certificates

Guarantee that a system of polynomial is infeasible

$$f_i(x) = 0 \qquad i = 1, \ldots, m$$
$$g_i(x) \geq 0 \qquad i = 1, \ldots, p$$

Positivstellensatz Theorem

$$\text{ideal}(f_1, \ldots, f_m) = \left\{ f \mid f = \sum_{i=1}^{m} t_i f_i, \qquad t_i \in \mathbb{R}[x] \right\}$$

$$\text{cone}(g_1, \ldots, g_p) = \left\{ g \mid g = s_0 \sum_i g_i + \sum_{\{i,j\}} s_{ij} g_i g_j + \sum_{\{i,j,k\}} s_{ijk} g_i g_j g_k + \cdots \right\}$$

If $F(x) \in \text{ideal}(f_1, \ldots, f_m)$, $G(x) \in \text{cone}(g_i, \ldots, g_p)$ such that $F(x) + G(x) = -1$

then the system of polynomials has no solution

Power Flow in Polynomial Form

Power Injection and Voltage Magnitude Polynomials

$$P_i = f_{Pi}\left(V_d, V_q\right) = V_{di}\sum_{k=1}^{n}\left(\mathrm{G}_{ik}V_{dk} - \mathrm{B}_{ik}V_{qk}\right) + V_{qi}\sum_{k=1}^{n}\left(\mathrm{B}_{ik}V_{dk} + \mathrm{G}_{ik}V_{qk}\right)$$

$$Q_i = f_{Qi}\left(V_d, V_q\right) = V_{di}\sum_{k=1}^{n}\left(-\mathrm{B}_{ik}V_{dk} - \mathrm{G}_{ik}V_{qk}\right) + V_{qi}\sum_{k=1}^{n}\left(\mathrm{G}_{ik}V_{dk} - \mathrm{B}_{ik}V_{qk}\right)$$

$$V_i^2 = f_{Vi}\left(V_d, V_q\right) = V_{di}^2 + V_{qi}^2$$

Reactive Power Limit Polynomials

$$f_{Vi} = \left(V_i^*\right)^2 - V_i^- + V_i^+$$

$$Q_i^{max} - f_{Qi} = x_i$$

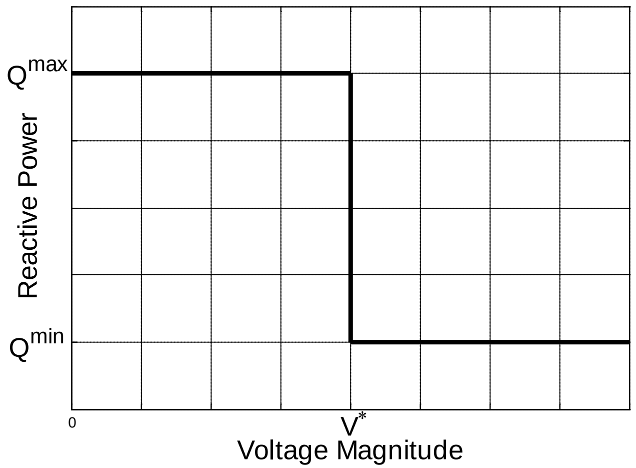
$$V_i^- x = 0$$

$$V_i^+\left(Q_i^{max} - Q_i^{min} - x\right) = 0$$

$$Q_i^{max} - Q_i^{min} - x \geq 0$$

$$V_i^+ \geq 0, \quad V_i^- \geq 0, \quad x_i \geq 0$$

Reactive Power versus Voltage Magnitude Characteristic



Power Flow Infeasibility Certificates

Find a sum of squares polynomial of the form

$$H\left(V_d, V_q, x, V^+, V^-\right) = \tau V_{q,slack} + \sum_{i \in \{\mathcal{PV},\mathcal{PQ}\}} \lambda_i \left(f_{Pi} - P_i\right) + \sum_{i \in \mathcal{PQ}} \gamma_i \left(f_{Qi} - Q_i\right)$$
$$+ \sum_{i \in \{\mathcal{S},\mathcal{PV}\}} \Big\{ \psi_{1i} \left(\left(V_i^*\right)^2 - V_i^- + V_i^+ - f_{Vi}\right) + \psi_{2i} \left(Q_i^{max} - f_{Qi} - x_i\right) + \psi_{3i} V_i^- x$$
$$+ \psi_{4i} \left(Q_i^{max} - Q_i^{min} - x\right) V_i^+ + s_{1i} \left(Q_i^{max} - Q_i^{min} - x\right) + s_{2i} V_i^+ + s_{3i} V_i^- + s_{4i} x_i \Big\}$$

such that

$$\left(-H\left(V_d, V_q, x, V^+, V^-\right) - 1\right) \quad \text{is sum of squares}$$

by finding polynomials $\tau,\ \lambda,\ \gamma,\ \phi_1,\ \phi_2,\ \phi_3,\ \phi_4$

and sum of squares polynomials $s_1,\ s_2,\ s_3,\ s_4$

Then the power flow equations have no solution.