Power Systems Engineering Research Center

Measurement-Based Estimation of Linear Sensitivity Distribution Factors and Applications

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Outline

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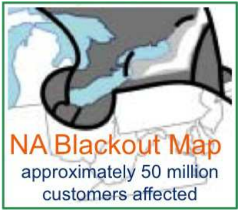
Motivation

- Power system operators rely on online studies conducted on a model of the system obtained offline based on:
  - Historical electricity demand patterns
  - Equipment maintenance schedules
  - Up-to-date network topology
- These model-based studies are not ideal since the results depend heavily on an accurate model with up-to-date network topology



- In the 2003 North American and 2011 San Diego blackouts
  1. Lack of accurate model — may not have accurate and timely information about key pieces of system
  2. Lack of situational awareness — limited visibility outside of system

Introduction

- Linear sensitivity distribution factors (DFs) are used in many online analysis tools:
  - Contingency analysis
  - Generation re-dispatch
  - Congestion relief
  - ...
- Existing approaches to computing DFs employ the system power flow model (typically the DC model); this is not ideal because
  1. Accurate model containing up-to-date network topology is required
  2. Results may not be applicable if actual system evolution does not match predicted operating points
- Phasor measurement units (PMUs) provide high-speed voltage and current measurements that are time-synchronized
- Objective: Estimate linear sensitivity DFs by exploiting measurements obtained from PMUs without the use of a power flow model

Outline

Injection shift factor (ISF)

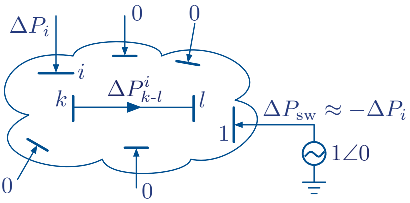
Definition (ISF of line $L_{k\text{-}l}$ w.r.t. bus i)

$\Psi^i_{k\text{-}l}$ is the partial derivative of $P_{k\text{-}l}$ — the real power flow through line $L_{k\text{-}l}$, with respect to P_i — the real power injection at bus i:

$$\Psi^i_{k\text{-}l} := \frac{\partial P_{k\text{-}l}}{\partial P_i}$$

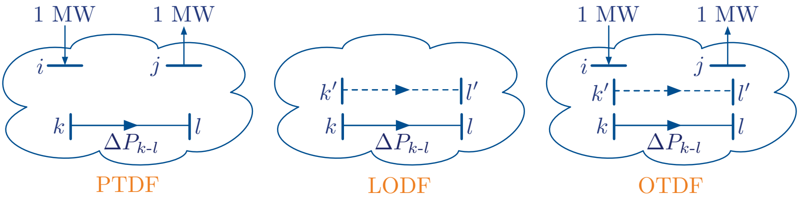
- Let $\Delta P_i(t) = P_i(t + \Delta t) - P_i(t)$
- Denote the change in line $L_{k\text{-}l}$ flow resulting from $\Delta P_i(t)$ by $\Delta P^i_{k\text{-}l}(t)$
- Based on the definition of ISF, it follows that

$$\Psi^i_{k\text{-}l} := \frac{\partial P_{k\text{-}l}}{\partial P_i} \approx \frac{\Delta P^i_{k\text{-}l}(t)}{\Delta P_i(t)}$$



Other Distribution Factors

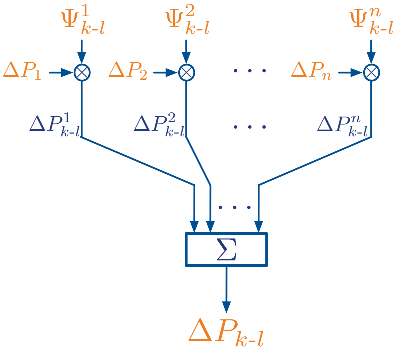
- Power transfer distribution factor (PTDF) — the MW change in a branch flow for a 1 MW exchange between two buses
- Line outage distribution factor (LODF) — the MW change in a branch flow due to the outage of a branch with 1 MW pre-outage flow
- Outage transfer distribution factor (OTDF) — the MW change in a branch flow for a 1 MW exchange between two buses with a line outage



These can all be computed once ISFs are known!

ISF Computation Approach

$$\Psi^i_{k\text{-}l} \approx \frac{\Delta P^i_{k\text{-}l}}{\Delta P_i}$$

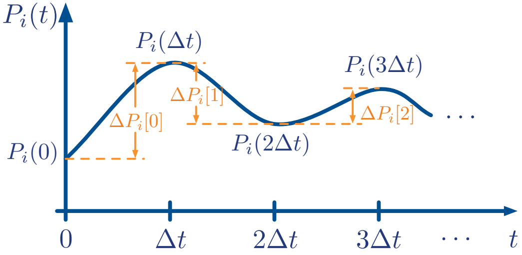


The total change in active power flow in line $L_{k\text{-}l}$ can be approximated as the sum of the change due to the real power injection at each bus by superposition:

$$\begin{aligned}\Delta P_{k\text{-}l} &\approx \Delta P^1_{k\text{-}l} + \cdots + \Delta P^i_{k\text{-}l} + \cdots + \Delta P^n_{k\text{-}l} \\ &\approx \Delta P_1 \Psi^1_{k\text{-}l} + \cdots + \Delta P_i \Psi^i_{k\text{-}l} + \cdots + \Delta P_n \Psi^n_{k\text{-}l}\end{aligned}$$

ISF Computation Approach

- Measurements are acquired every Δt units of time



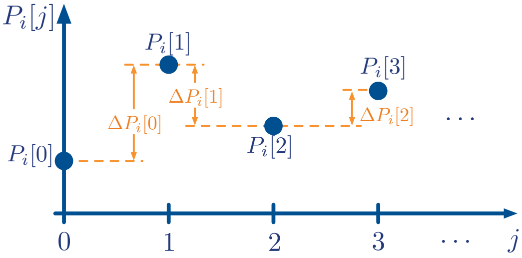
  - $\Delta P_i(t) = P_i(t + \Delta t) - P_i(t)$
  - $\Delta P_{k\text{-}l}(t) = P_{k\text{-}l}(t + \Delta t) - P_{k\text{-}l}(t)$

- The total change in active power flow in line $L_{k\text{-}l}$ can be approximated as:

$$\begin{aligned}\Delta P_{k\text{-}l}(t) &\approx \Delta P^1_{k\text{-}l}(t) + \cdots + \Delta P^i_{k\text{-}l}(t) + \cdots + \Delta P^n_{k\text{-}l}(t) \\ &\approx \Delta P_1(t)\Psi^1_{k\text{-}l} + \cdots + \Delta P_i(t)\Psi^i_{k\text{-}l} + \cdots + \Delta P_n(t)\Psi^n_{k\text{-}l}\end{aligned}$$

ISF Computation Approach

- Measurements are acquired every Δt units of time



  - Discretize with $t = j\Delta t$
  - $\Delta P_i[j] = P_i[j+1] - P_i[j]$
  - $\Delta P_{k\text{-}l}[j] = P_{k\text{-}l}[j+1] - P_{k\text{-}l}[j]$

- The total change in active power flow in line $L_{k\text{-}l}$ can be approximated as:

$$\begin{aligned}\Delta P_{k\text{-}l}[j] &\approx \Delta P^1_{k\text{-}l}[j] + \cdots + \Delta P^i_{k\text{-}l}[j] + \cdots + \Delta P^n_{k\text{-}l}[j] \\ &\approx \Delta P_1[j]\Psi^1_{k\text{-}l} + \cdots + \Delta P_i[j]\Psi^i_{k\text{-}l} + \cdots + \Delta P_n[j]\Psi^n_{k\text{-}l}\end{aligned}$$

ISF Computation Approach

- Stacking m of these measurement instances up:

$$\underbrace{\begin{bmatrix} \Delta P_{k\text{-}l}[1] \\ \vdots \\ \Delta P_{k\text{-}l}[j] \\ \vdots \\ \Delta P_{k\text{-}l}[m] \end{bmatrix}}_{\Delta P_{k\text{-}l} \in \mathbb{R}^m} \approx \underbrace{\begin{bmatrix} \Delta P_1[1] & \cdots & \Delta P_i[1] & \cdots & \Delta P_n[1] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta P_1[j] & \cdots & \Delta P_i[j] & \cdots & \Delta P_n[j] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta P_1[m] & \cdots & \Delta P_i[m] & \cdots & \Delta P_n[m] \end{bmatrix}}_{\Delta P \in \mathbb{R}^{m \times n}} \underbrace{\begin{bmatrix} \Psi_{k\text{-}l}^1 \\ \vdots \\ \Psi_{k\text{-}l}^i \\ \vdots \\ \Psi_{k\text{-}l}^n \end{bmatrix}}_{\Psi_{k\text{-}l} \in \mathbb{R}^n}$$

- Proposed measurement-based approach relies on inherent fluctuations in load and generation
- Other assumptions:
  - The ISFs are approximately constant across the $m+1$ measurements
  - The regressor matrix ΔP has full column rank

Outline

LSE-Based DF Estimation [Chen, D-G, Sauer, '13]

- Let $e = e_l + e_m$
  1. e_l: inherent error arising from the linearization assumption
  2. e_m: PMU measurement noise

$$\Delta P_{k\text{-}l} = \Delta P \Psi_{k\text{-}l} + e$$

- Assume that $\Delta P_{k\text{-}l} = \Delta P \Psi_{k\text{-}l} + e$ is overdetermined, i.e., $\Delta P \in \mathbb{R}^{m \times n}$, with $m > n$
- Least-squares errors estimation (LSE):

$$\min_{\Psi_{k\text{-}l}} \; e^T e$$

$$\implies \hat{\Psi}_{k\text{-}l} = (\Delta P^T \Delta P)^{-1} \Delta P^T \Delta P_{k\text{-}l}$$

Least-Squares Errors Estimation Variants

- Weighted least-squares (WLS) estimation:

$$\min_{\Psi_{k\text{-}l}} \; e^T W e$$

$$\Longrightarrow \; \hat{\Psi}_{k\text{-}l} = \left(\Delta P^T W \Delta P\right)^{-1} \Delta P^T W \Delta P_{k\text{-}l}$$

with

$$W = [w_{ij}] = \begin{cases} f^{m-i}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

where $f \in (0, 1]$ is the so-called forgetting factor, i.e., recent measurements are given more weight than older ones

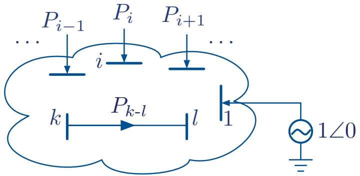
- Recursive least-squares (RLS) estimation:
  - Measurements obtained sequentially, hence update estimate as more data acquired

Case Study Methodology

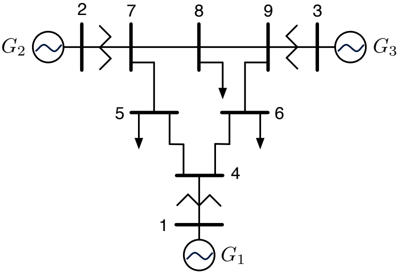
- Simulate PMU measurements of random fluctuations in active power injection at each bus

$$P_i = P_i^0 + \sigma_1 P_i^0 v_1 + \sigma_2 v_2$$

  - P_i^0 — nominal power injection at node i
  - $\sigma_1 P_i^0 v_1$ — inherent variability in power injection with time
  - $\sigma_2 v_2$ — measurement noise
  - v_1 and v_2 — pseudorandom values drawn from standard normal distribution
- For each set of random power injection data, compute the power flow, with the slack bus absorbing all power imbalances



Example: Changing Operating Point



- Simulate random real power fluctuations at each bus

Line	Actual [p.u.]		Model-based [p.u.]	WLS Estimation [p.u.]	
	Before	After	After	After ($f=1$)	After ($f=0.7$)
$\Delta P_{4\text{-}5}$	-0.2970	-0.2046	-0.3196	-0.2145	-0.2203
$\Delta P_{4\text{-}6}$	-0.1734	-0.1426	-0.1804	-0.0529	-0.1416
$\Delta P_{7\text{-}8}$	+0.1838	+0.2121	+0.1804	+0.1116	+0.2066

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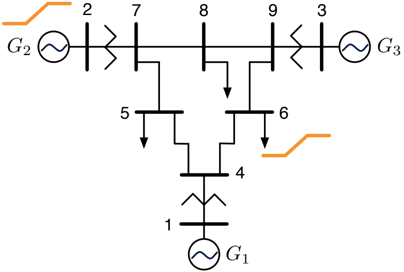

- Simulate random real power fluctuations at each bus
- Change operating point — increase load at bus 6 and generation at bus 2

Line	Actual [p.u.]		Model-based [p.u.]	WLS Estimation [p.u.]	
	Before	After	After	After ($f = 1$)	After ($f = 0.7$)
$\Delta P_{4\text{-}5}$	-0.2970	-0.2046	-0.3196	-0.2145	-0.2203
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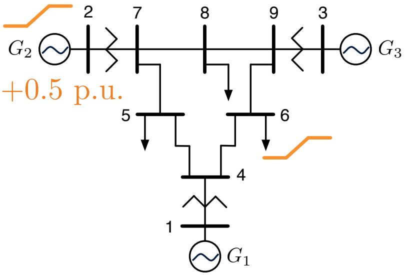
Example: Changing Operating Point



- Simulate random real power fluctuations at each bus
- Change operating point — increase load at bus 6 and generation at bus 2
- Predict effect of increasing bus 2 generation by 0.5 p.u.

Line	Actual [p.u.]		Model-based [p.u.]	WLS Estimation [p.u.]	
	Before	After	After	After ($f = 1$)	After ($f = 0.7$)
$\Delta P_{4\text{-}5}$	-0.2970	-0.2046	-0.3196	-0.2145	-0.2203
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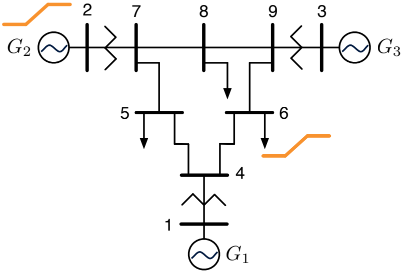
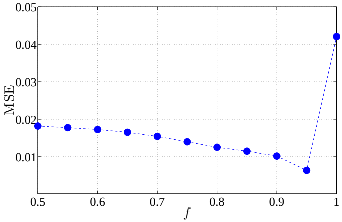
On the Choice of the Forgetting Factor



- Larger f: low misadjustment and good stability
- Smaller f: improved tracking capability



Outline

Contingency Analysis: Generator Outage

- Consider outage of generator G_{12} as contingency
- Lost generation divided among G_{10}, G_{25}, and G_{26}
- Compare post-contingency line flows obtained via full nonlinear power flow solution and model- and LSE-based ISF computations
  - Scenario 1: no undetected topology changes
  - Scenario 2: two undetected transmission line outage

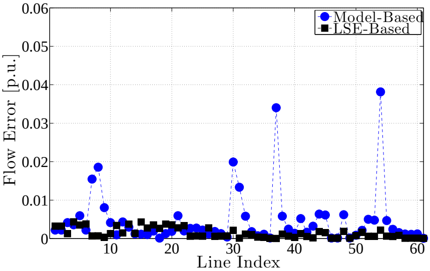


Figure: No undetected line outages: Error in line flows estimates with respect to full power flow solution

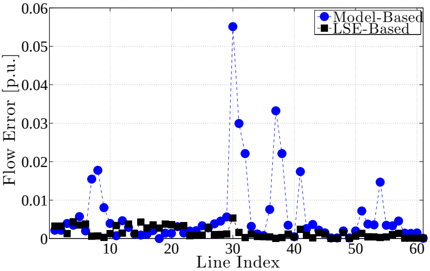


Figure: Two undetected line outages: Error in line flows estimates with respect to full power flow solution

Contingency Analysis: Line Outage

- Consider outage of line $L_{37\text{-}40}$ as contingency
- Undetected outages of lines $L_{41\text{-}42}$ and $L_{42\text{-}49}$ at $j = 200$
- ISFs computed using previous $m = 500$ sets of measurements at $j = 500$ and $j = 800$

Line $L_{k\text{-}l}$	Actual [p.u.]	Model-based [p.u.]	Measurement-based [p.u.] $j = 500$, $f = 1$	Measurement-based [p.u.] $j = 500$, $f = 0.98$	Measurement-based [p.u.] $j = 800$, $f = 1$
$L_{23\text{-}24}$	0.0344	0.0497	0.0523	0.0296	0.0360
$L_{26\text{-}30}$	2.2564	2.2509	2.2499	2.2589	2.2564
$L_{23\text{-}32}$	0.9465	0.9410	0.9402	0.9481	0.9459
$L_{15\text{-}33}$	0.0930	0.0860	0.0842	0.0933	0.0908
$L_{33\text{-}37}$	-0.1374	-0.1445	-0.1462	-0.1372	-0.1396
$L_{34\text{-}36}$	0.3088	0.3066	0.3064	0.3093	0.3085
$L_{34\text{-}37}$	-0.8849	-0.9049	-0.9125	-0.8855	-0.8928
$L_{38\text{-}37}$	2.6145	2.5585	2.5446	2.6274	2.6052
$L_{37\text{-}39}$	1.2548	1.1697	1.1451	1.2673	1.2346
$L_{37\text{-}40}$	—	—	—	—	—

Table: Post-outage actual and estimated line flows

Generation Re-Dispatch

- Consider outage of transformer $T_{37\text{-}38}$ as contingency
- Consider undetected outages of lines $L_{41\text{-}42}$ and $L_{42\text{-}49}$ at $j = 200$
  - Pre-outage flow through line $L_{k\text{-}l}$: $P^0_{k\text{-}l}$
  - Post-outage flow through line $L_{k\text{-}l}$: $\tilde{P}^0_{k\text{-}l}$
- ISFs computed using previous $m = 500$ sets of measurements at $j = 800$

Line	Pre-contingency [p.u.]		Post-contingency $P_{k\text{-}l}$ [p.u.]		
$L_{k\text{-}l}$	$P^0_{k\text{-}l}$	$\tilde{P}^0_{k\text{-}l}$	Actual	Model-based	Measurement-based
$L_{15\text{-}33}$	0.0470	0.0752	1.0378	0.9001	1.0742

- Suppose thermal limit of line $L_{15\text{-}33}$ is 1 p.u.
- Measurement-based method flags violation, while model-based approach does not

Generation Re-Dispatch (PJM Approach) to relieve $L_{15\text{-}33}$

- For each generating unit i, define

$$\rho_i := \frac{\overline{\gamma} - \gamma_i}{\Psi^i_{15\text{-}33}}$$

where $\overline{\gamma}$ is the so-called *dispatch* rate, and is determined by the pre-contingency economic dispatch solution

- In order to relieve $L_{15\text{-}33}$, choose generating unit with the lowest ρ_i

G_i	γ_i [$/MWh]	ISF $\Psi^i_{15\text{-}33}$ Model-based	ISF $\Psi^i_{15\text{-}33}$ Measurement-based	ρ_i [$/MW Effect] Model-based	ρ_i [$/MW Effect] Measurement-based
G_{34}	40.05	-0.0627	-0.0620	**10.6688**	10.7909
G_{36}	40.10	-0.0650	-0.0666	11.0480	10.7933
G_{40}	40.00	-0.0566	-0.0707	10.9217	**8.7546**

- Model-based: dispatch of G_{34} is optimal
- Measurement-based: dispatch of G_{40} is optimal

Approach	G_{i^*}	ΔP_{i^*} [MW]	$P_{15\text{-}33}$ [p.u.]	Cost [$/hr]
Model-based	G_{34}	32.55	0.9365	1314
Measurement-based	G_{40}	25.55	0.9604	**1028**

Outline

Motivation

So far: ISF estimation via LSE solution of an overdetermined system

- Accurate ISF estimation with undetected system topology changes
- Problem formulation necessitates at least as many sets of synchronized measurements as unknown ISFs
- For a large power system, such a restriction may be ill-advised

Next: a method to recover the ISF solution using fewer sets of measurements than unknown ISFs:

- Exploit a sparse representation (i.e., one in which many elements are zero) of the ISFs
- Solve for the transformed sparse representation

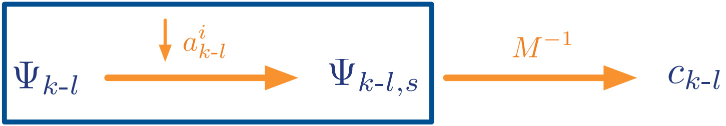
Sparsity-Exploiting DF Estimation [Chen, D-G, Sauer, '13]

ISF estimation via solution of an undetermined system:

$$\Delta P_{k\text{-}l} \approx \Delta P \Psi_{k\text{-}l}, \text{ where } \Delta P \in \mathbb{R}^{m \times n}, m < n$$

- Sparsify injection shift factors
  1. Rearrange elements of ISF vector $\Psi_{k\text{-}l}$ by electrical distance
  2. Transform ISF vector into sparse representation
- Computation of post-transformation ISF vector
  1. Greedy pursuit algorithms
  2. Convex relaxation algorithms
- Case studies
  1. IEEE 300-bus system
  2. Polish 2383-bus system

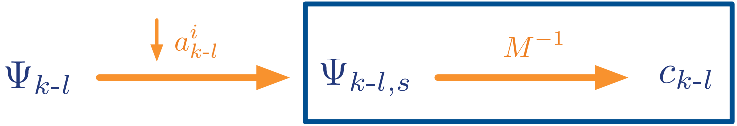
Sparsify Injection Shift Factors



Rearrange elements of ISF vector $\Psi_{k\text{-}l}$ by electrical distance:

- $a^i_{k\text{-}l}$: measure of electrical distance between bus i and line $L_{k\text{-}l}$
- Sort $\Psi_{k\text{-}l}$ by decreasing $a^i_{k\text{-}l}$
- Sort columns of ΔP accordingly $\rightarrow \Delta P_s$
- Intuition: injections at buses that are electrically far away from line $L_{k\text{-}l}$ have little effect on the active power flow through line $L_{k\text{-}l}$, while nearer ones have increased effect

Sparsify Injection Shift Factors



Apply difference transformation:

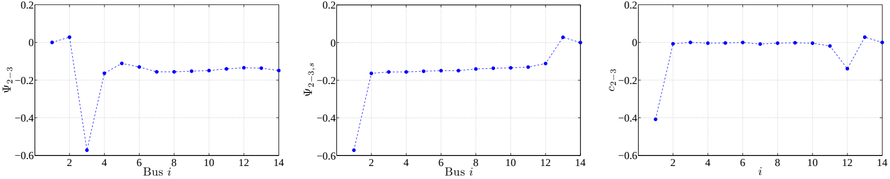
$$c_{k\text{-}l} = M^{-1}\Psi_{k\text{-}l,s}$$

where

$$M^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Sparsify Injection Shift Factors

$$\Psi_{k\text{-}l} \xrightarrow{\ \downarrow a^{i}_{k\text{-}l}\ } \Psi_{k\text{-}l,s} \xrightarrow{\ M^{-1}\ } c_{k\text{-}l}$$



Sparse ISF Vector Recovery Problem

- Since $c_{k\text{-}l}$ is assumed to be sparse, the ISF vector estimation problem can be cast as an optimization program:

$$\min_{c_{k\text{-}l}} ||c_{k\text{-}l}||_0$$

$$\text{subject to } \Delta P_{k\text{-}l} = \underbrace{\Delta P_s M}_{\Phi} c_{k\text{-}l}$$

  where $||c_{k\text{-}l}||_0$ denotes the number of nonzero elements in $c_{k\text{-}l}$
- This optimization problem is NP-hard due to the unavoidable combinatorial search
- There are numerous classes of computational techniques for obtaining approximate solutions; two major ones:
  - Greedy pursuit methods
  - Convex optimization methods

Algorithms for ISF Vector Recovery

In our work, we utilize two algorithms:

- Orthogonal matching pursuit (within the greedy pursuit class)
- Convex relaxation (within the convex optimization class)

Orthogonal Matching Pursuit

- Idea: successively build a set of the most likely locations of the nonzero terms in $c_{k\text{-}l}$
- ☺ Simple implementation
- ☺ Fast
- ☹ Sensitive to sparsity

Convex Relaxation

- Idea: replace l_0-norm in objective function ($||c_{k\text{-}l}||_0$) with convex l_1-norm ($||c_{k\text{-}l}||_1$)
- ☺ Less sensitive to sparsity
- ☺ More robust
- ☹ Generally slower

Case Study: IEEE 300-Bus System

- Compute benchmark values for $\Psi_{272\text{-}268}$, $\Psi_{30\text{-}73}$, and $\Psi_{112\text{-}148}$ using the nonlinear power flow model
- Compare $\hat{\Psi}_{272\text{-}268}$, $\hat{\Psi}_{30\text{-}73}$, and $\hat{\Psi}_{112\text{-}148}$ obtained via greedy pursuit, convex relaxation, and LSE algorithms to respective benchmarks

Line	$\|\|\hat{\Psi}_{k\text{-}l} - \Psi_{k\text{-}l}\|\|_2$			
$L_{272\text{-}268}$	$m = 110$	$m = 120$	$m = 130$	$m = 140$
via Orthogonal Matching Pursuit	0.0676	0.0693	0.0551	0.0777
via Convex Relaxation	0.0320	0.0341	0.0271	0.0279
	$m = 305$	$m = 350$	$m = 400$	$m = 600$
via LSE	0.5891	0.0962	0.0678	0.0339
$L_{30\text{-}73}$	$m = 190$	$m = 210$	$m = 230$	$m = 250$
via Orthogonal Matching Pursuit	0.0640	0.0649	0.0517	0.0494
via Convex Relaxation	0.0320	0.0323	0.0268	0.0288
	$m = 305$	$m = 350$	$m = 400$	$m = 600$
via LSE	0.2719	0.0372	0.0236	0.0162
$L_{112\text{-}148}$	$m = 230$	$m = 250$	$m = 270$	$m = 290$
via Orthogonal Matching Pursuit	0.9216	0.8255	0.7809	0.0964
via Convex Relaxation	0.0849	0.0716	0.0377	0.0474
	$m = 305$	$m = 350$	$m = 400$	$m = 600$
via LSE	0.1414	0.0289	0.0168	0.0116

Case Study: IEEE 300-Bus System

	Accuracy achieved using proposed approach		LSE-based approach	
$L_{272\text{-}268}$	$m = 130$	$>$	$m = 600$	$c_{272\text{-}268}$ most sparse
$L_{30\text{-}73}$	$m = 230$	$\approx$	$m = 400$	$\downarrow$
$L_{112\text{-}148}$	$m = 290$	$<$	$m = 350$	$c_{112\text{-}148}$ least sparse

- Proposed sparsity-exploiting approach achieves better accuracy with more sparse $c_{k\text{-}l}$
- In general, Convex Relaxation achieves better accuracy than Orthogonal Matching Pursuit
  - Attributed to interior point convex optimization's insensitivity to solution sparsity

Case Study: Polish 2383-Bus System

- Exorbitant simulation time for LSE-based approach
- $c_{53\text{-}52}$ is highly sparse
  - For the same m measurements, Orthogonal Matching Pursuit is more accurate (except in $m = 800$)
- Orthogonal Matching Pursuit incurs much lower computation time

Line	$\|\|\hat{\Psi}_{k\text{-}l} - \Psi_{k\text{-}l}\|\|_2$			
$L_{53\text{-}52}$	$m = 500$	$m = 600$	$m = 700$	$m = 800$
via Orthogonal Matching Pursuit	0.0490	0.0480	0.0413	0.0402
via Convex Relaxation	0.0562	0.0516	0.0459	0.0391

Line	Execution Time [s]			
$L_{53\text{-}52}$	$m = 500$	$m = 600$	$m = 700$	$m = 800$
via Orthogonal Matching Pursuit	0.2798	0.4502	0.5263	0.7828
via Convex Relaxation	15.5302	13.7511	15.8118	16.1992

Outline

Concluding Remarks and Future Work

- We discussed measurement-based methods for ISF estimation
  - They rely on PMU measurements
  - They do not use of a power flow model
  - Estimation can be achieved with fewer measuremens
- We demonstrated the effectiveness of the proposed methods
  - IEEE 118-bus, IEEE 300-bus, and Polish 2383-bus systems
  - Applications in contingency analysis and transmission loading relief
- Key advantages of the proposed measurement-based methods:
  - Eliminate reliance on system models and corresponding accuracy
  - Resilient to unexpected system topology and operating point changes
  - Independent of slack bus location designation
- Further Work:
  - Rigorous analysis of computational burden as system size increases
  - Explore distributed computation approaches
  - Explore ther power system health monitoring applications using DFs

Backup Slides

Least-Squares Errors Estimation Variants

- Measurements obtained sequentially, hence update estimate as more data acquired
- Recursive least-squares (RLS) estimation:

$$\hat{\Psi}_{k\text{-}l}[j] = \hat{\Psi}_{k\text{-}l}[j-1] + Q^{-1}[j]\Delta P^T[j]\left(\Delta P_{k\text{-}l}[j] - \Delta P[j]\hat{\Psi}_{k\text{-}l}[j-1]\right),$$
$$Q[j] = fQ[j-1] + \Delta P^T[j]\Delta P[j]$$

- Can avoid matrix inversion; set $R[j] = Q^{-1}[j]$:

$$\hat{\Psi}_{k\text{-}l}[j] = \hat{\Psi}_{k\text{-}l}[j-1] + R[j]\Delta P^T[j]\left(\Delta P_{k\text{-}l}[j] - \Delta P[j]\hat{\Psi}_{k\text{-}l}[j-1]\right),$$
$$R[j] = f^{-1}\left(R[j-1] - g[j]\Delta P[j]R[j-1]\right),$$
$$g[j] = \frac{R[j-1]\Delta P^T[j]}{f + \Delta P[j]R[j-1]\Delta P[j]}$$

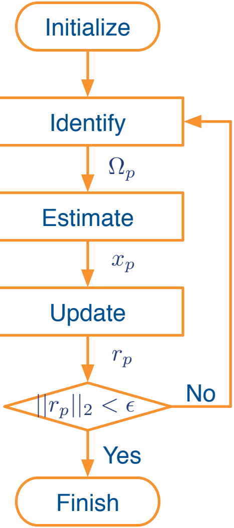
On Managing Bad Data

- Manage large errors by
  1. setting $f < 1$ so that earlier, possibly erroneous, data have less influence on the ISF estimation
  2. conducting estimation over a sliding window in time so that any erroneous data eventually become ineffectual

- Inject large random errors for measurements within time steps $j \in [201, 300]$

Line	Actual [p.u.]	WLS Estimation [p.u.] $j = 300$	$j = 400$	$j = 500$	$j = 600$
$\Delta P_{4\text{-}5}$	-0.2970	-0.0685	-0.2683	-0.2974	-0.3017
$\Delta P_{4\text{-}6}$	-0.1734	-0.0567	-0.1758	-0.1730	-0.1747
$\Delta P_{7\text{-}8}$	+0.1838	0.0395	0.1204	0.1781	0.1831
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
MSE	—	0.6742	0.1265	0.0132	0.0086

Computation of Sparse ISFs via Greedy Pursuit



Idea: successively build a set of the most likely locations of the nonzero terms in $c_{k\text{-}l}$

1. Initialize. Set residual $r_0 = \Delta P_{k\text{-}l}$, index set $\Omega_0 = \emptyset$, and counter $p = 1$
2. Identify. Find column n_p of Φ that is most strongly correlated with the residual r_{p-1}, and set $\Omega_p = \Omega_{p-1} \cup n_p$
3. Estimate. Find the best fit x_p with the columns chosen so far via LSE
4. Update. $r_p = \Delta P_{k\text{-}l} - \Phi x_p$, $p = p + 1$
5. Check. If $||r_p||_2 < \epsilon$, finish, else go to step 2

Computation of Sparse ISFs via Greedy Pursuit

Possible stopping criteria:

1. Residual magnitude $||r_p||_2 < \epsilon$ or $p = m$, whichever occurs sooner
2. $p = \kappa$, where κ is an upper bound to the sparsity of $c_{k\text{-}l}$

Computation of Sparse ISFs via Convex Relaxation

Idea: replace l_0-norm in objective function with convex l_1-norm and solve

$$\min_{c_{k\text{-}l}} ||c_{k\text{-}l}||_1$$
$$\text{subject to } \Delta P_{k\text{-}l} = \Phi c_{k\text{-}l}$$

- Relax linear constraints to accommodate measurement error

$$\min_{c_{k\text{-}l}} ||c_{k\text{-}l}||_1$$
$$\text{subject to } ||\Delta P_{k\text{-}l} - \Phi c_{k\text{-}l}||_2 \leq \epsilon$$

- Convert to standard second-order cone program and solve via log-barrier algorithm

Estimation of ISFs with a Subset of Measurements

- Entire set of PMU measurements may not be wholly available at every sample time
- $\mathcal{B}$: set of all buses
- Consider $L_{k\text{-}l}$, where $k, l \in \mathcal{B}_1 \subseteq \mathcal{B}$

$$\Delta P_{k\text{-}l} = \begin{bmatrix} \Delta P_{\mathcal{B}_1} & \Delta P_{\mathcal{B}_1^c} \end{bmatrix} \begin{bmatrix} \Psi_{k\text{-}l}^{\mathcal{B}_1} \\ \Psi_{k\text{-}l}^{\mathcal{B}_1^c} \end{bmatrix}, \ \mathcal{B}_1^c := \mathcal{B} \backslash \mathcal{B}_1$$

- $\Delta P_{\mathcal{B}_1}$ $\left(\Delta P_{\mathcal{B}_1^c}\right)$: real power injection fluctuation at buses in $\mathcal{B}_1$ $(\mathcal{B}_1^c)$
- $\Psi_{k\text{-}l}^{\mathcal{B}_1}$ $\left(\Psi_{k\text{-}l}^{\mathcal{B}_1^c}\right)$: ISFs of line $L_{k\text{-}l}$ with respect to buses in $\mathcal{B}_1$ $(\mathcal{B}_1^c)$
- Assume $\Psi_{k\text{-}l}^{\mathcal{B}_1^c}$ is negligibly small compared to $\Psi_{k\text{-}l}^{\mathcal{B}_1}$

$$\Delta P_{k\text{-}l} = \Delta P_{\mathcal{B}_1} \Psi_{k\text{-}l}^{\mathcal{B}_1} + \theta,$$

- $\theta = \Delta P_{\mathcal{B}_1^c} \Psi_{k\text{-}l}^{\mathcal{B}_1^c}$ can be viewed as a measurement error

Case Study: Polish 2383-Bus System

- $\mathcal{B}_1$ contains 1000 buses electrically nearest to $L_{53\text{-}52}$
- Reduced case performance is similar to full case, both in accuracy as well as computation time

Line	$\|\|\hat{\Psi}_{k\text{-}l} - \Psi_{k\text{-}l}\|\|_2$			
$L_{53\text{-}52}$	$m = 500$	$m = 600$	$m = 700$	$m = 800$
via Orthogonal Matching Pursuit	0.0509	0.0482	0.0440	0.0420
via Convex Relaxation	0.0579	0.0525	0.0565	0.0632

Line	Execution Time [s]			
$L_{53\text{-}52}$	$m = 500$	$m = 600$	$m = 700$	$m = 800$
via Orthogonal Matching Pursuit	0.2547	0.4706	0.6899	0.9705
via Convex Relaxation	18.6132	15.6137	17.2752	19.7156