

Real Time Market Power Monitoring

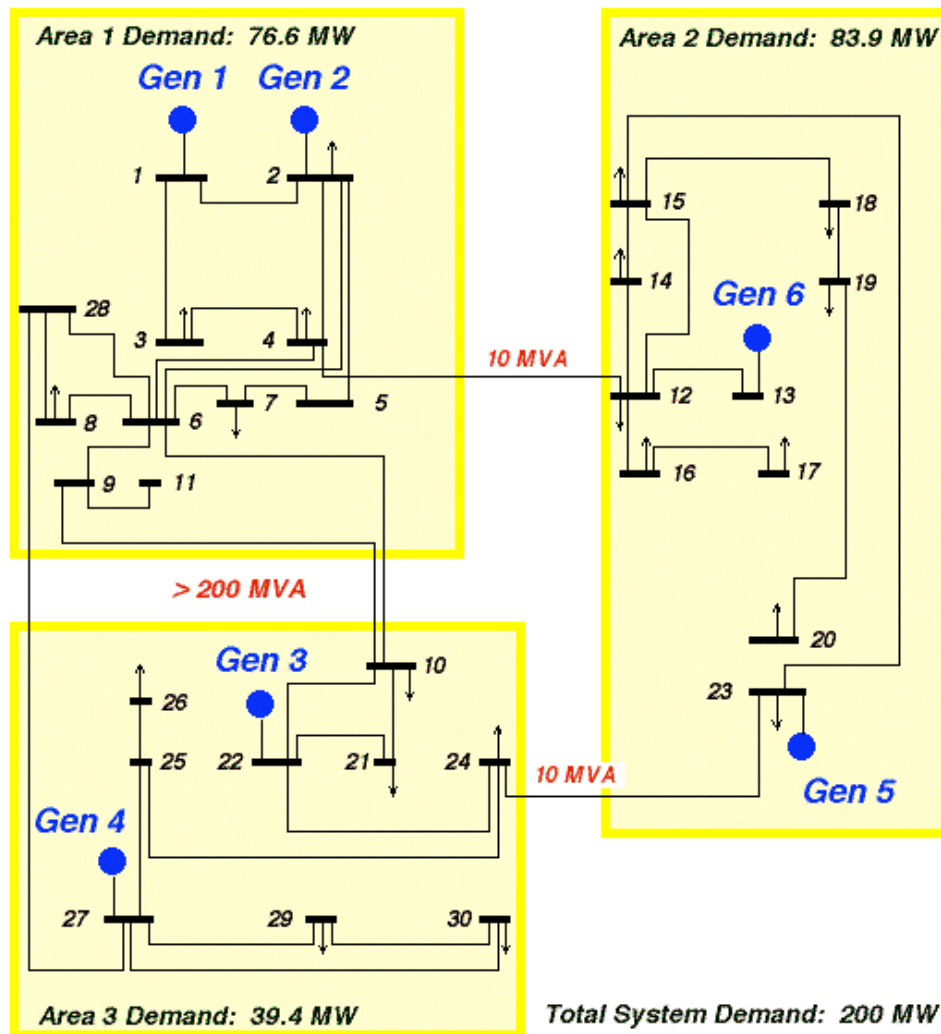
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December 9, 2008
PSERC

Overview

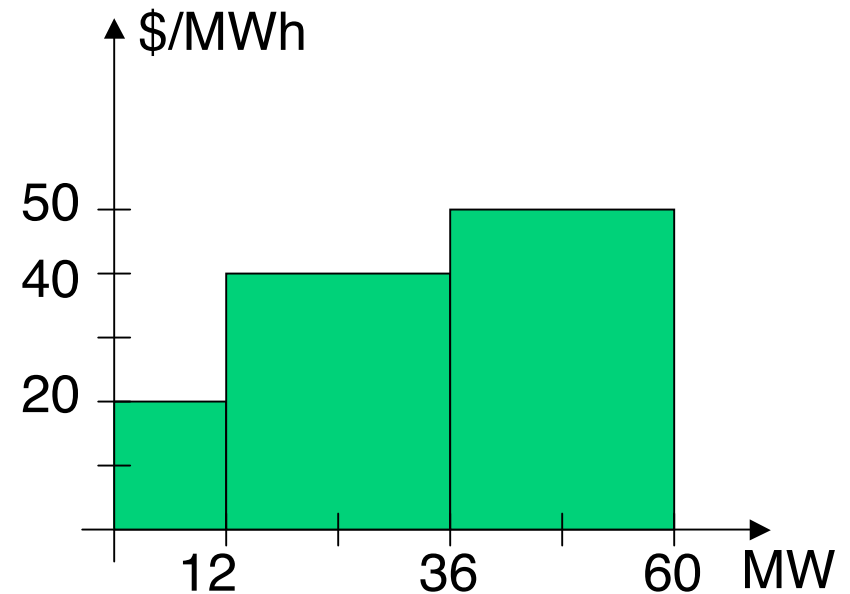
- Electricity Markets are complicated.
- Market power will be exploited (CAISO, PJM).
- Traditional concentration measures do not account for engineering constraints.
- Our approach: Examine revenue (or profit) and dispatch sensitivities to identify participants who have potential market power.

Markets in a minute: generators, network, and loads

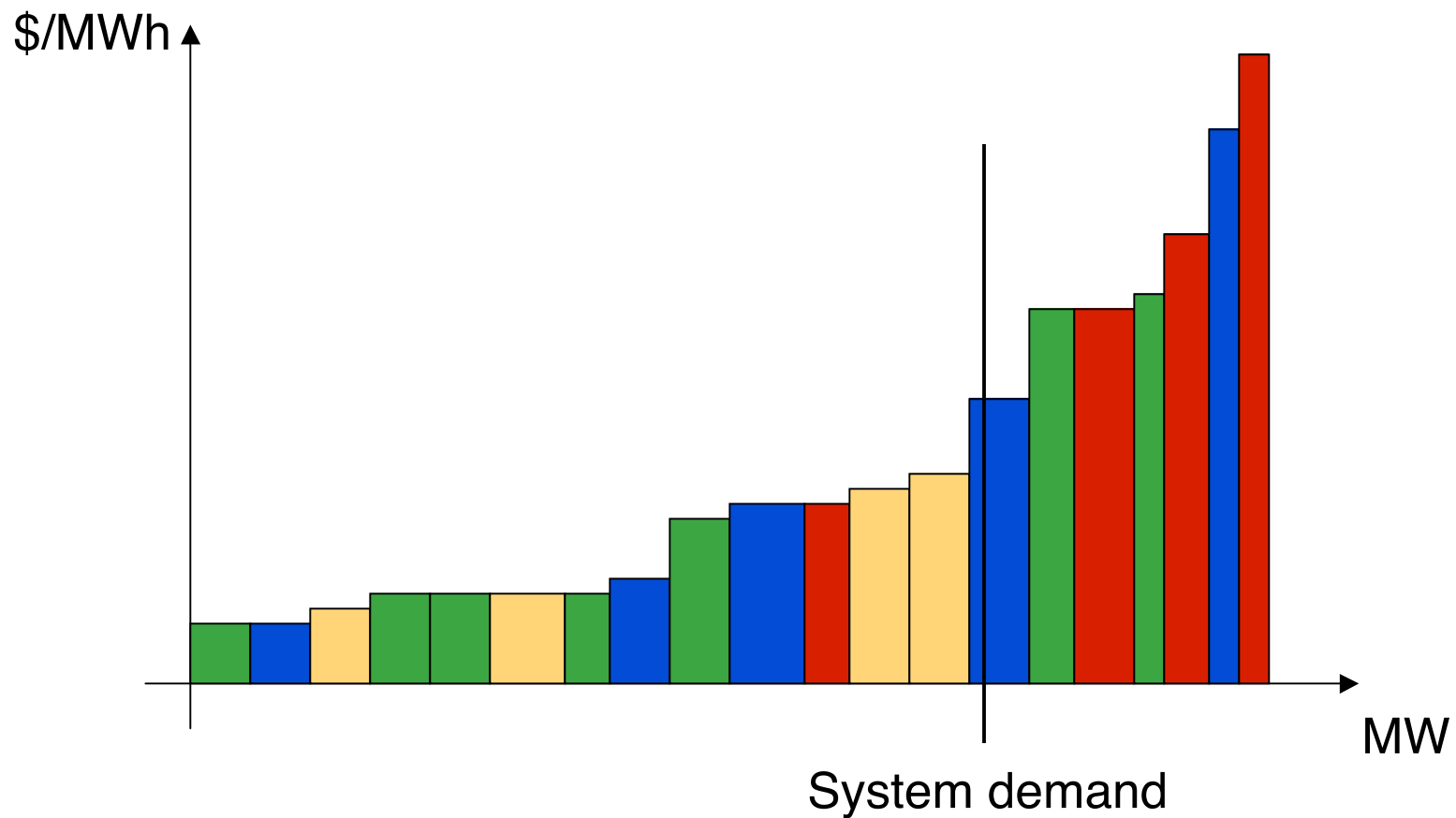


In a market setting, suppliers offer to sell energy:

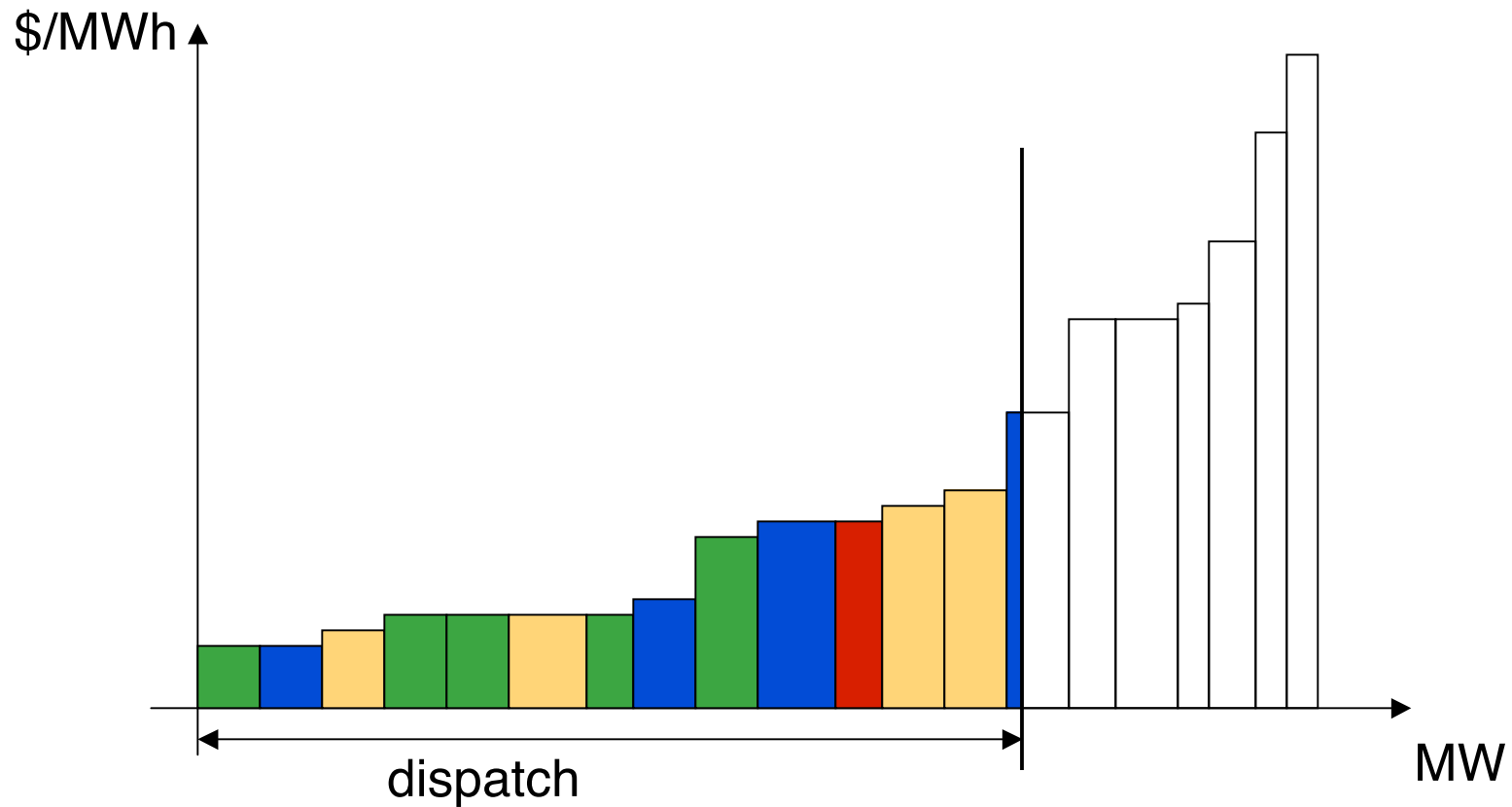
- quantity
- price



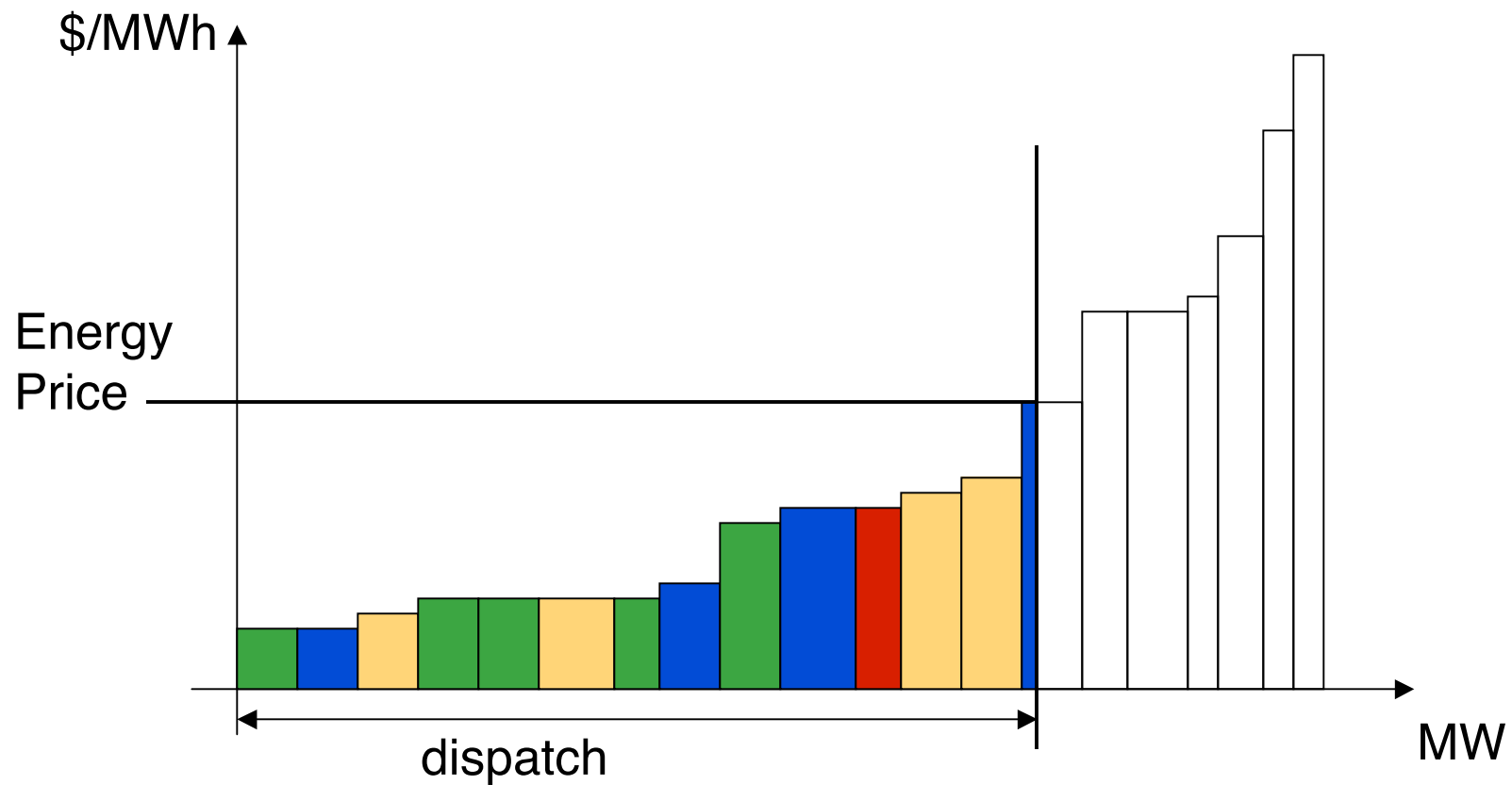
Markets in a minute: Ideal dispatch



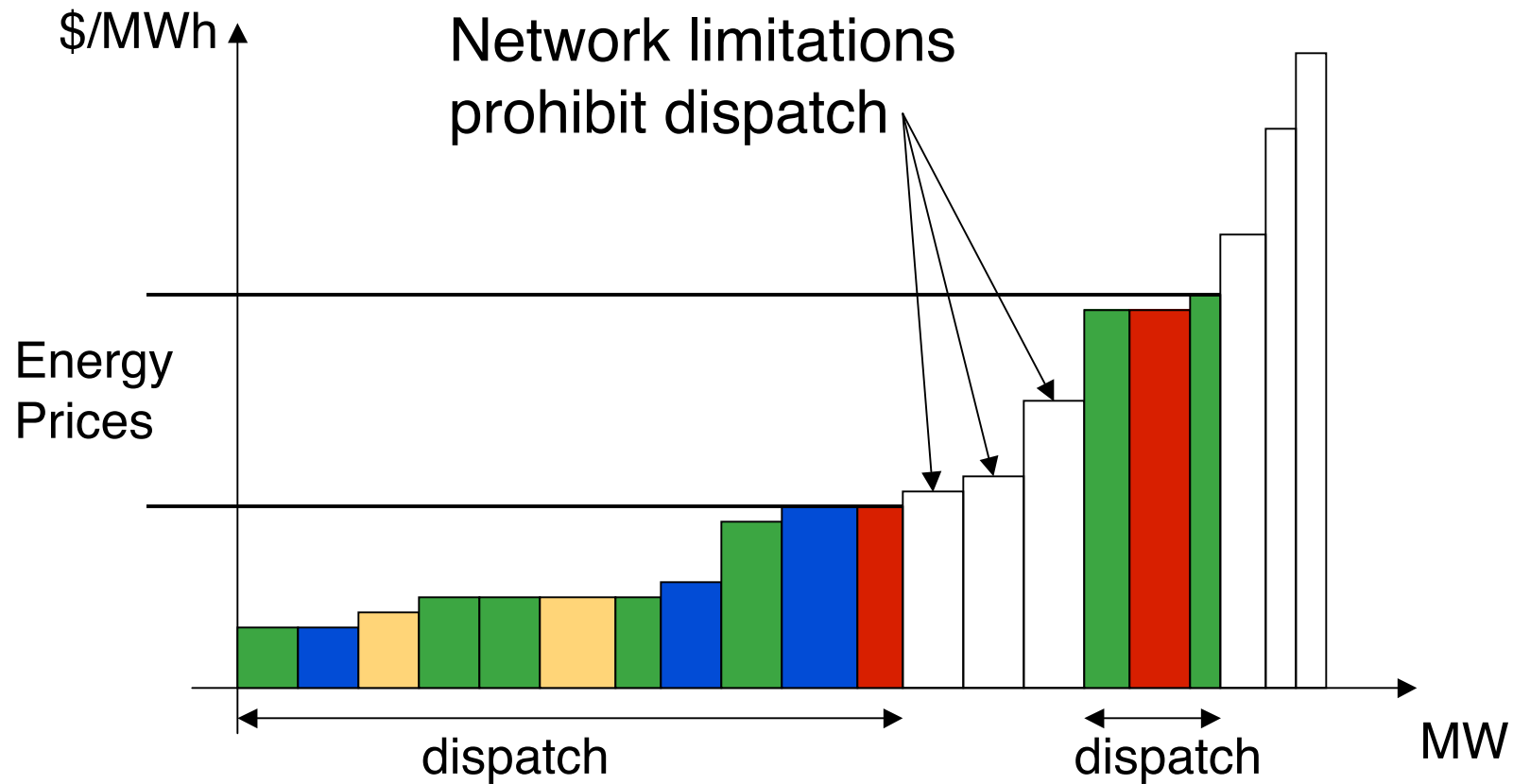
Markets in a minute: Ideal dispatch



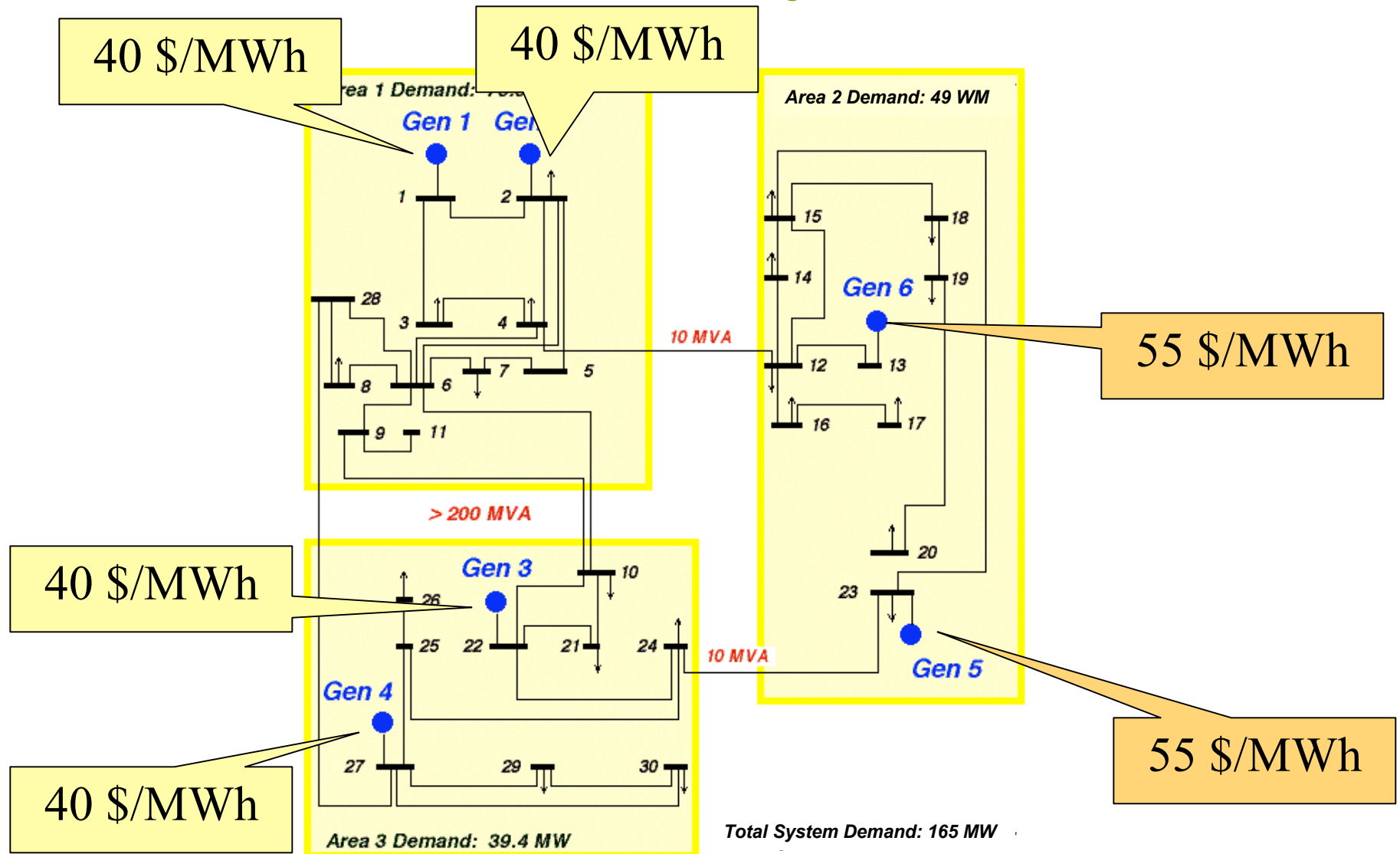
Markets in a minute: Ideal dispatch



Markets in a minute: Non-ideal dispatch



Example System



Locational Advantage and Market Power

Key Issue: Substitutability

- Can suppliers exploit locational advantages?

If so,

- Can we identify when this is possible?

Market Competitiveness: Concentration Measures

- Pivotal/Residual Supplier Indices
- Herfindahl-Hirshman Index (HHI):

$$\sum_{\text{participants}} (\% \text{ market share})^2$$

ISO Market monitors: examine concentrations in Capacity, Dispatch, and Capacity and Dispatch by Fuel Type...

FERC: evaluate concentrations in applications for “market-based rate authority.”

Market Competitiveness: Concentration Measures

- Pivotal/Residual Supplier Indices
- Herfindahl-Hirshman Index (HHI):

$$\sum_{\text{participants}} (\% \text{ market share})^2$$

Concentration measures are coarse, indirect, and often inconclusive.

Market Power: Our Approach

Identify those individuals or groups with the ability **to increase revenues and price.**

Competitive Environment: Increasing (offer) price will serve to decrease a participant's own revenues.

Non Competitive Environment: A participant may increase revenues and price.

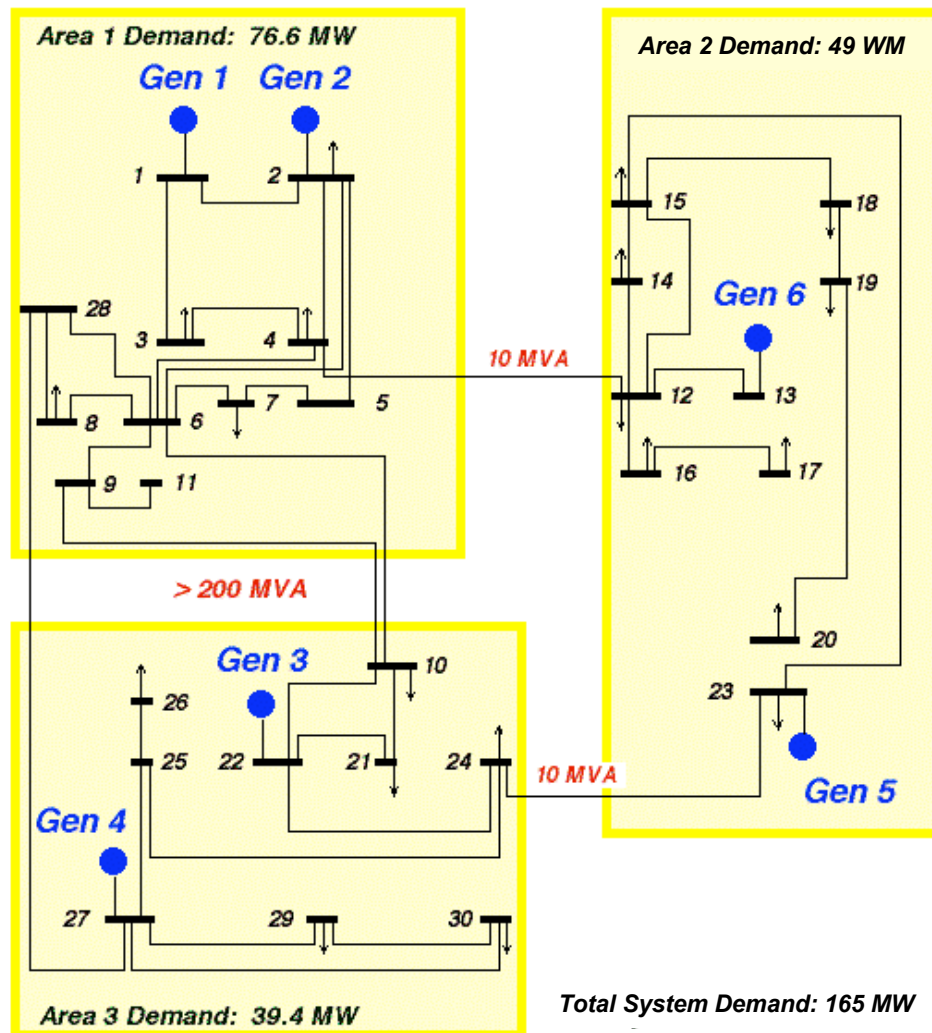
Identifying Market Power: Sensitivity Approach

Examine dispatch and revenue/offer sensitivities

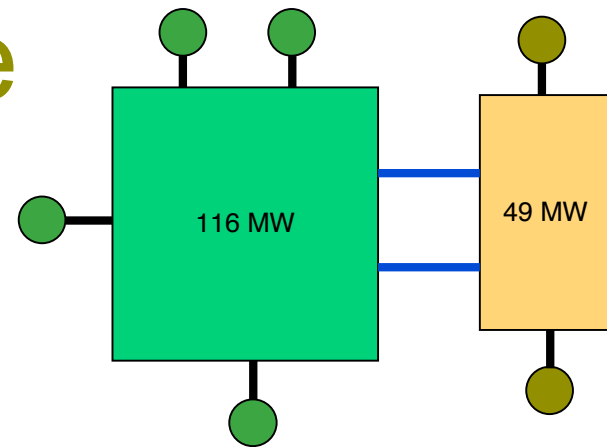
$$\frac{\Delta r}{\Delta \lambda}$$

If a small set of suppliers can simultaneously raise prices and increase revenues, they have some amount of market power.

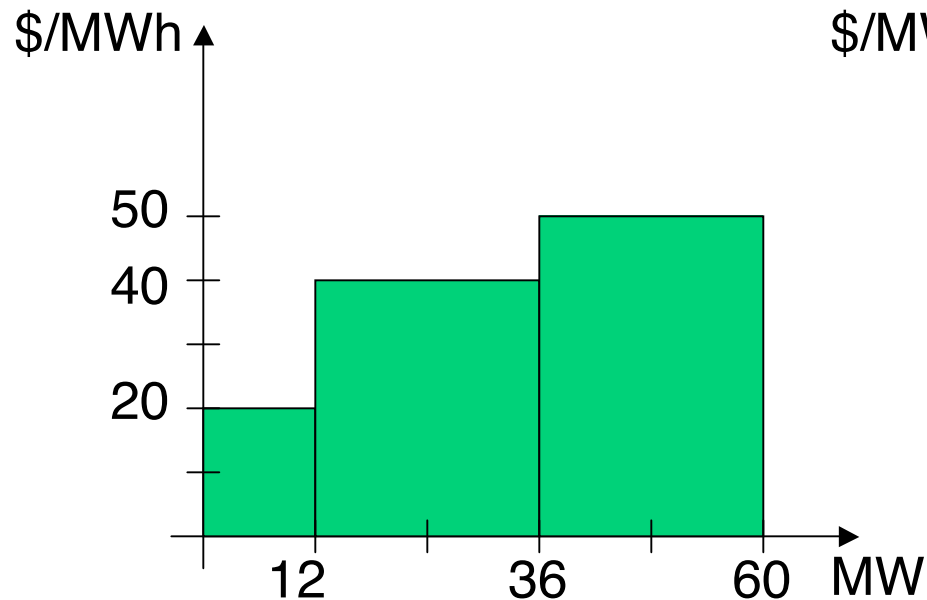
Example System



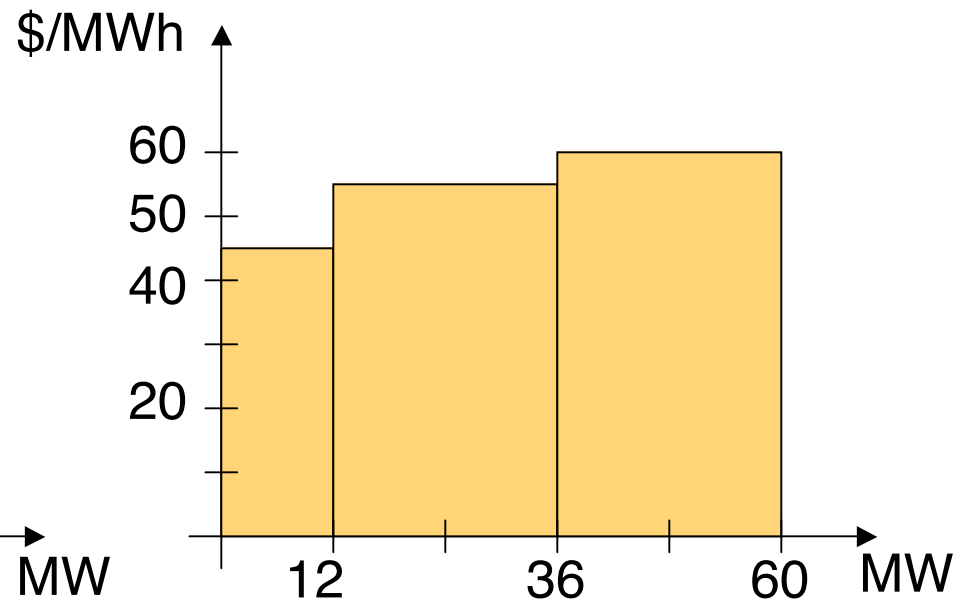
Six Supplier Example (Market Power)



Base Case Offers:



Generators 1-4



Generators 5-6

Higher to create
load pocket

Results (75 round experiment)

Base Case Solution

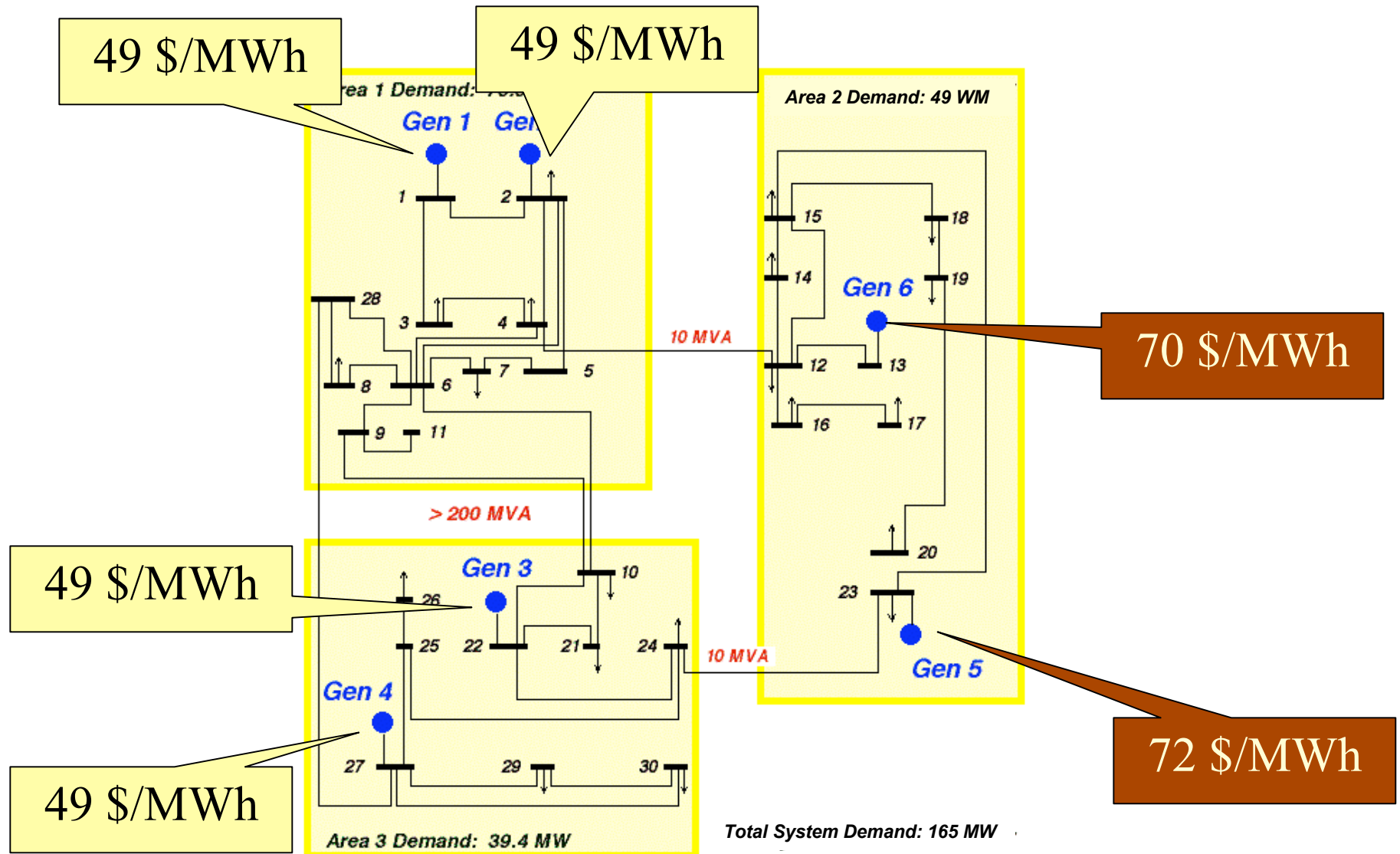
	G1	G2	G3	G4	G5	G6
Dispatch(MW)	31.7	36.0	34.0	36.0	17.6	12.0
Price (\$/MWh)	40.0	40.1	40.0	40.1	55.0	54.3

Actual Experiment*

* 80 \$/MWh offer cap

	G1	G2	G3	G4	G5	G6
Dispatch(MW)	37.9	34.9	30.1	34.9	14.9	14.6
Price (\$/MWh)	48.5	48.7	48.5	48.6	72.0	70.0

Example System - Experiments



Matrix of Revenue/Offer Sensitivities

Base Case Solution

$$\begin{bmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \\ \Delta r_4 \\ \Delta r_5 \\ \Delta r_6 \end{bmatrix} = \begin{bmatrix} -3298 & 3231 & 31 & 65 & 52 & -49 \\ 3219 & -3695 & 244 & 263 & 315 & -310 \\ 31 & 244 & -544 & 308 & -234 & 229 \\ 65 & 263 & 307 & -597 & -127 & 125 \\ 38 & 230 & -170 & -93 & -160 & 173 \\ -36 & -229 & 169 & 92 & 175 & -159 \end{bmatrix} \begin{bmatrix} \Delta \lambda_1 \\ \Delta \lambda_2 \\ \Delta \lambda_3 \\ \Delta \lambda_4 \\ \Delta \lambda_5 \\ \Delta \lambda_6 \end{bmatrix}$$

Observations:

- If any supplier, acting alone, raises its (offer) price, that supplier will lose revenue.
- If all suppliers, acting together, raise (offer) prices, everyone's revenue increases.

Matrix of Revenue/Offer Sensitivities

Base Case Solution

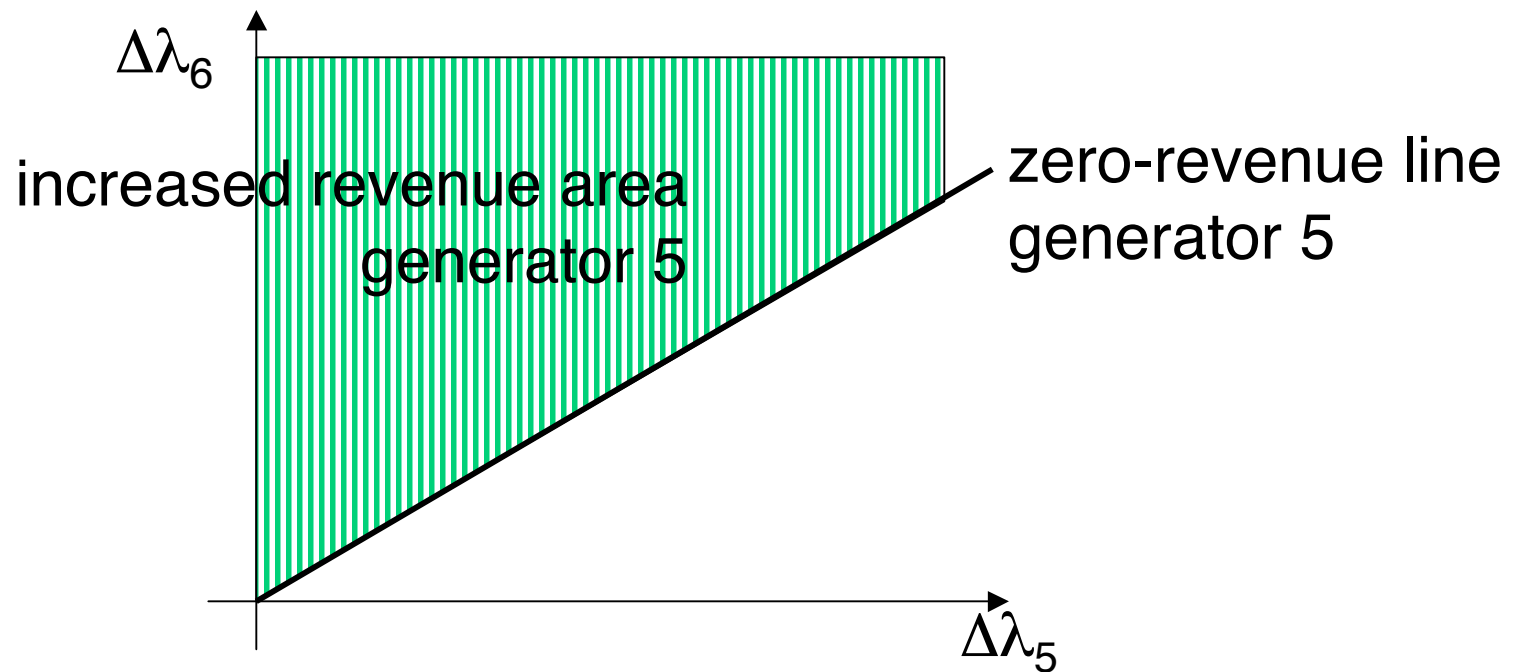
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Observations:

- If the load pocket generators (5 and 6) raise their (offer) prices together, *their own revenues increase* with almost no effect on the other revenues.

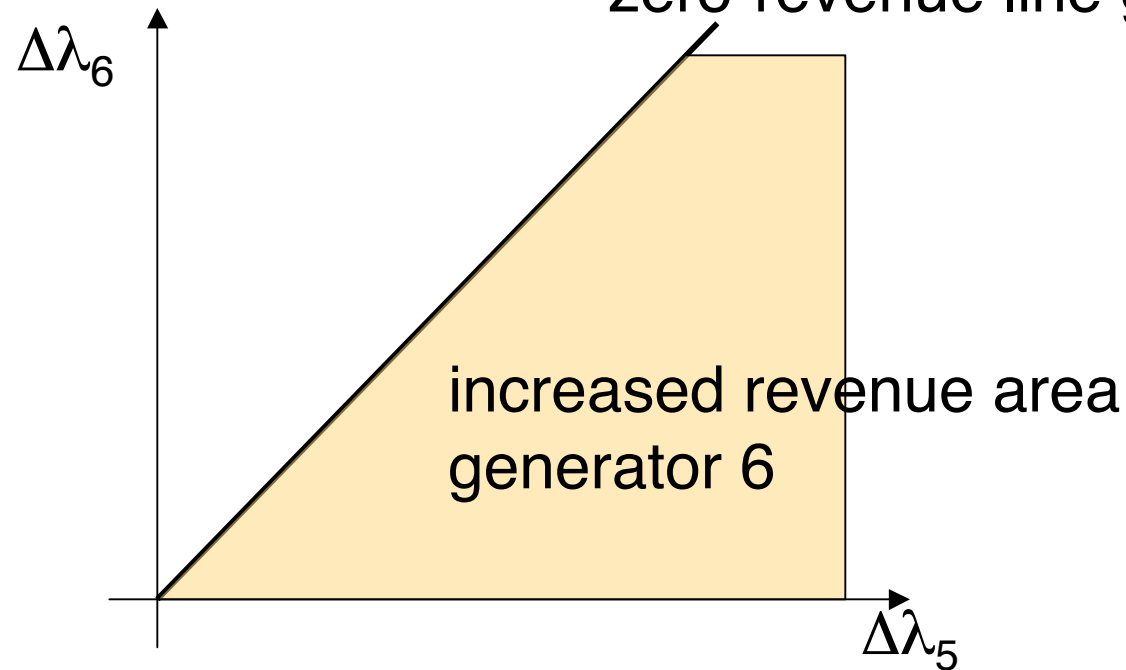
Load Pocket Market Power

Closer examination:
$$\begin{bmatrix} \Delta r_5 \\ \Delta r_6 \end{bmatrix} = \begin{bmatrix} -160 & 173 \\ 175 & -159 \end{bmatrix} \begin{bmatrix} \Delta \lambda_5 \\ \Delta \lambda_6 \end{bmatrix}$$



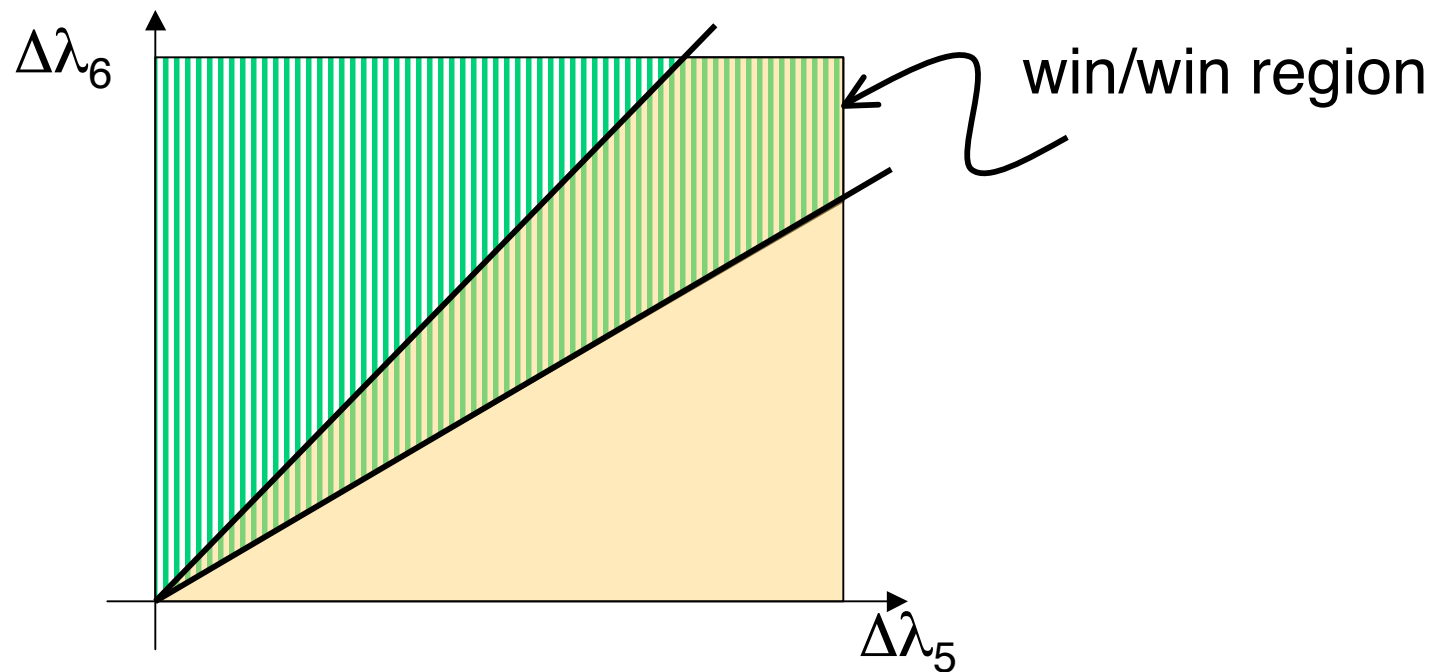
Load Pocket Market Power

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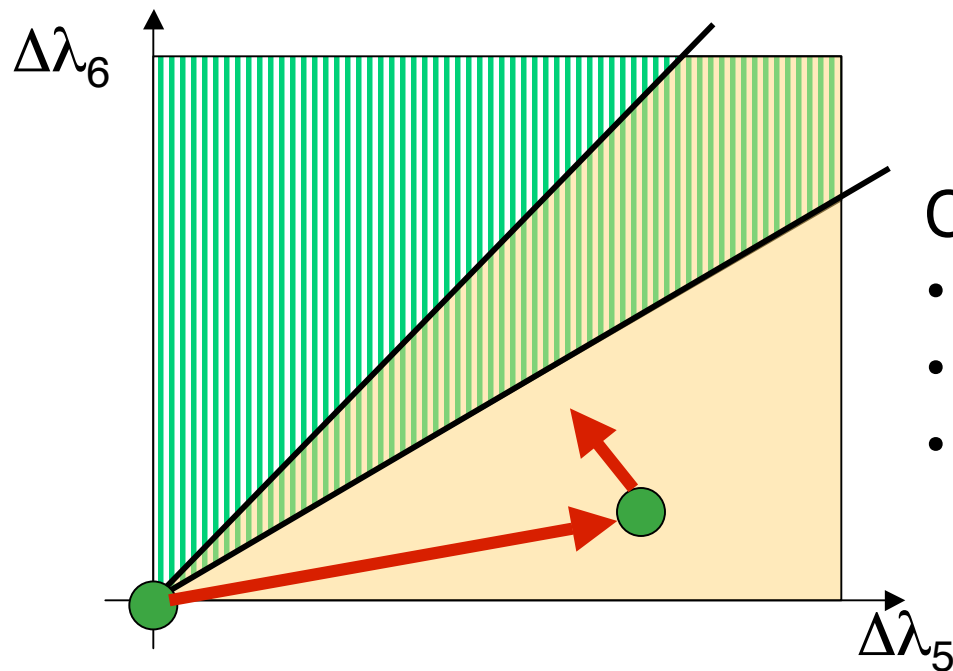


Load Pocket Market Power

Generators 5 and 6 will find their win/win region.



Load Pocket Market Power



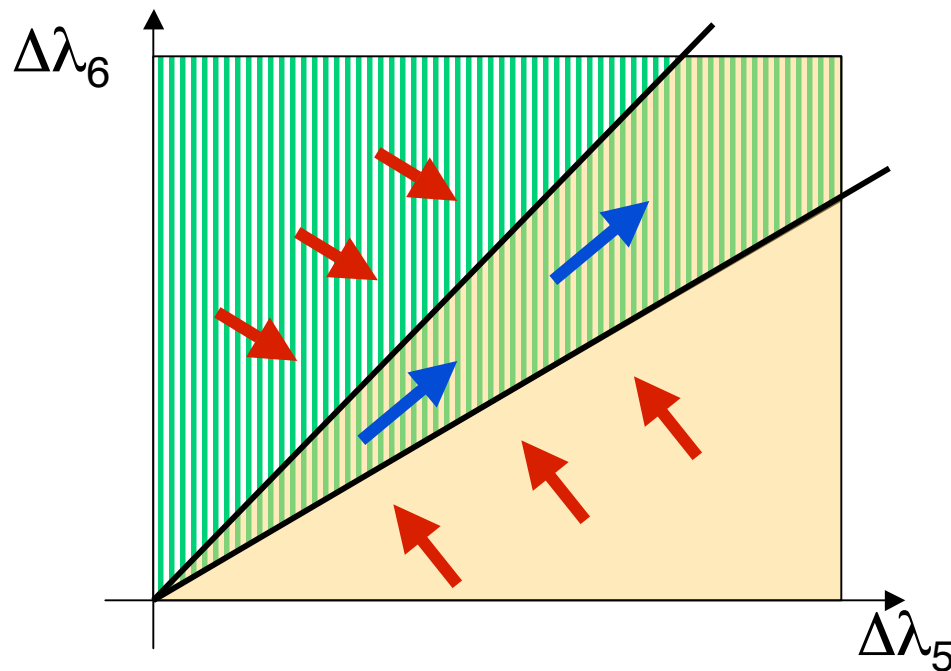
Consider:

- both increase offers and
- generator 5 loses revenue
- generator 6 gains revenue

Then

- generator 5 will decrease its offer and
- generator 6 will increase its offer
- and they will move towards the win/win region

Load Pocket Market Power



It will take time, but

- prices will increase and
- revenues will increase.

There isn't direct collusion – each generator acts on its own price and revenue signals.

Load Pocket Identification

How can we identify generators in a load pocket?

A property of a load pocket is that a subset of generators serve the load (in excess of limited imports).

- Examine the matrix of dispatch/price sensitivities
- Seek price increase combinations that don't affect dispatch. (Who can print \$\$?)

Matrix of Dispatch/Price Sensitivities

Construct a matrix M whose elements

$$m_{ij} = \frac{\partial g_i}{\partial \lambda_j} \quad \text{are dispatch/price sensitivities}$$

Then

$$\Delta g = M \Delta \lambda$$

Search for vectors

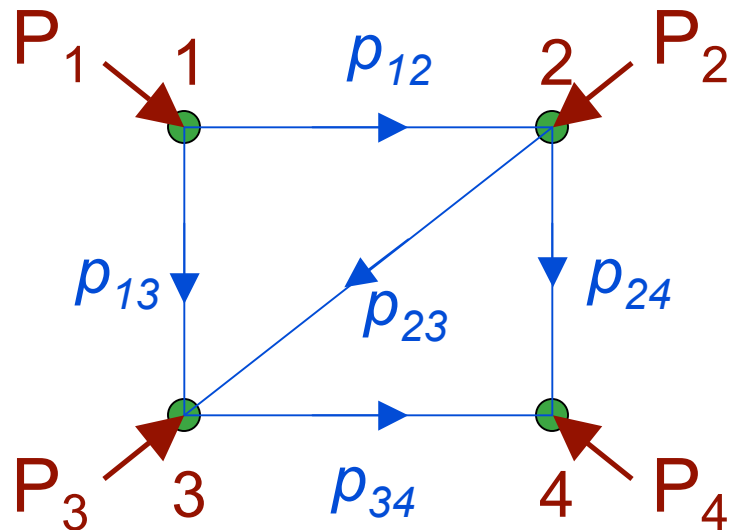
$$\Delta\lambda = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ * \\ \vdots \\ * \end{bmatrix}$$

zero-valued elements

(strictly) positive elements

That satisfy $0 = M\Delta\lambda$

A Power Flow Model



Assuming voltages are constant at 1 per unit (100% rated), line powers are related to angle differences.

$$p_{12} = b_{12} \sin(\theta_1 - \theta_2)$$

$$p_{13} = b_{13} \sin(\theta_1 - \theta_3)$$

$$p_{23} = b_{23} \sin(\theta_2 - \theta_3)$$

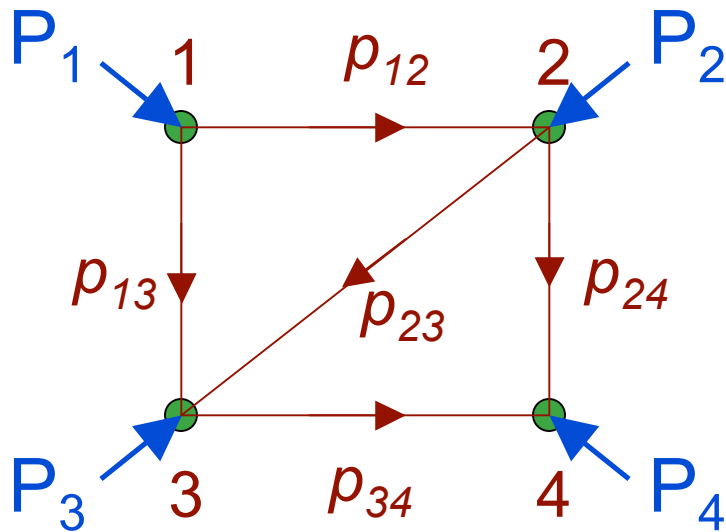
$$p_{24} = b_{24} \sin(\theta_2 - \theta_4)$$

$$p_{34} = b_{34} \sin(\theta_3 - \theta_4)$$

$$\underbrace{\begin{bmatrix} p_{12} \\ p_{13} \\ p_{23} \\ p_{24} \\ p_{34} \end{bmatrix}}_{p_{line}} = \underbrace{\begin{bmatrix} b_{12} & & & & \\ & b_{13} & & & \\ & & b_{23} & & \\ & & & b_{24} & \\ & & & & b_{34} \end{bmatrix}}_B \sin \left(\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{A\theta} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \right)$$

A Power Flow Model

$$p_{line} = B \sin(A\theta)$$



“Injected Powers”

$$\underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}}_P = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}}_{A^T p_{line}} \begin{bmatrix} p_{12} \\ p_{13} \\ p_{23} \\ p_{24} \\ p_{34} \end{bmatrix}$$

Simplified power
flow equation:

$$P = A^T B \sin(A\theta) \\ \approx A^T B A \theta$$

Injected powers

Simplified OPF Model

$$\min_g \sum_i C_i(g_i; w_i) \quad \text{Cost Function parameterized by } w.$$

Subject to

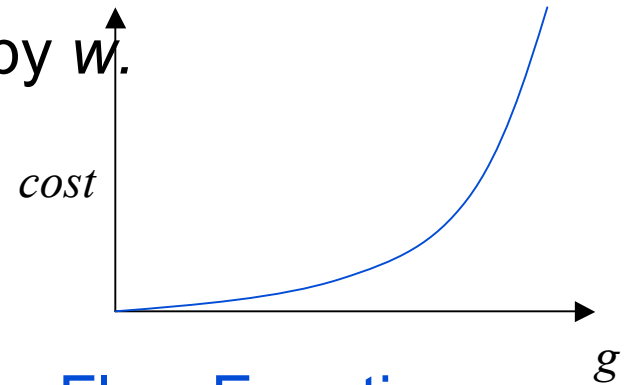
$$P_{branch} = BA\theta$$

$$P_{injected} = \begin{bmatrix} g \\ l \end{bmatrix} = A^T P_{branch}$$

$$P_{\max} = P_b$$

Subset of branches at maximum flow limit.

Power Flow Equations



GOAL: find sensitivities of g to *price (incremental cost)*.

(The same fundamental structure appears in the nonlinear equations.) 30

Simplified OPF Model

Set up the Lagrangian,

$$L = \sum_i (C_i(g_i; w_i)) + \mu^T (P_{branch} - BA\theta) + \sigma^T \left(\begin{bmatrix} g \\ l \end{bmatrix} - A^T P_{branch} \right) + \xi^T (P_{\max} - P_b)$$

The KKT conditions require

$$\begin{bmatrix} \frac{\partial C(g; w)}{\partial g} \left(\frac{1}{g} \right) & \cdot & \cdot & \cdot & [I \ 0] & \cdot \\ \cdot & \cdot & \cdot & -A^T B & \cdot & \cdot \\ \cdot & \cdot & \cdot & I & -A & -\begin{bmatrix} 0 \\ I \end{bmatrix} \\ \cdot & -BA & I & \cdot & \cdot & \cdot \\ \begin{bmatrix} I \\ 0 \end{bmatrix} & \cdot & -A^T & \cdot & \cdot & \cdot \\ \cdot & \cdot & -[0 \ I] & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} g \\ \theta \\ P_{branch} \\ \mu \\ \sigma \\ \xi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \begin{bmatrix} 0 \\ -l \end{bmatrix} \\ -P_{\max} \end{bmatrix}$$

Incremental Model

$$\begin{bmatrix} \frac{\partial^2 C(g;w)}{\partial g^2} & \cdot & \cdot & \cdot & [I \ 0] & \cdot \\ \cdot & \cdot & \cdot & -A^T B & \cdot & \cdot \\ \cdot & \cdot & \cdot & I & -A & -\begin{bmatrix} 0 \\ I \end{bmatrix} \\ \cdot & -BA & I & \cdot & \cdot & \cdot \\ \begin{bmatrix} I \\ 0 \end{bmatrix} & \cdot & -A^T & \cdot & \cdot & \cdot \\ \cdot & \cdot & -\begin{bmatrix} 0 & I \end{bmatrix} & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \Delta g \\ \Delta \theta \\ \Delta P_{branch} \\ \Delta \mu \\ \Delta \sigma \\ \Delta \xi \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 C(g;w)}{\partial g \partial w} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

At this point we could solve for $\Delta g = M \Delta \lambda$
but it is instructive to examine the larger model for solutions

$$\Delta g = 0 \quad \Delta \theta = 0 \quad \Delta P_{branch} = 0$$

and work backwards to characterize $\Delta \lambda = \frac{\partial^2 C(g;w)}{\partial g \partial w}$

Zero Dispatch Change

$$\begin{bmatrix} \cdot & [I \ 0] & \cdot \\ -A^T B & \cdot & \cdot \\ I & -A & -\begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Delta\mu \\ \Delta\sigma \\ \Delta\xi \end{bmatrix} = \begin{bmatrix} -\Delta\lambda \\ 0 \\ 0 \end{bmatrix}$$

Separate parts,

$$-\Delta\lambda = [I \ 0] \Delta\sigma$$

“Cost” perturbations that
Result in no change in
dispatch

$$\begin{bmatrix} -A^T B & \cdot & \cdot \\ I & -A & -\begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Delta\mu \\ \Delta\sigma \\ \Delta\xi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Examine Null Space of this matrix

Dimension: buses+branches x buses+branches+constraints

Zero Dispatch Change

Simplify

$$A^T B A \Delta \sigma + A^T B \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta \xi = 0$$

A basis for vectors $\Delta \sigma$, Σ , is the given by the solution of

$$A^T B A \Sigma = - \underbrace{A^T B \begin{bmatrix} 0 \\ I \end{bmatrix}}$$

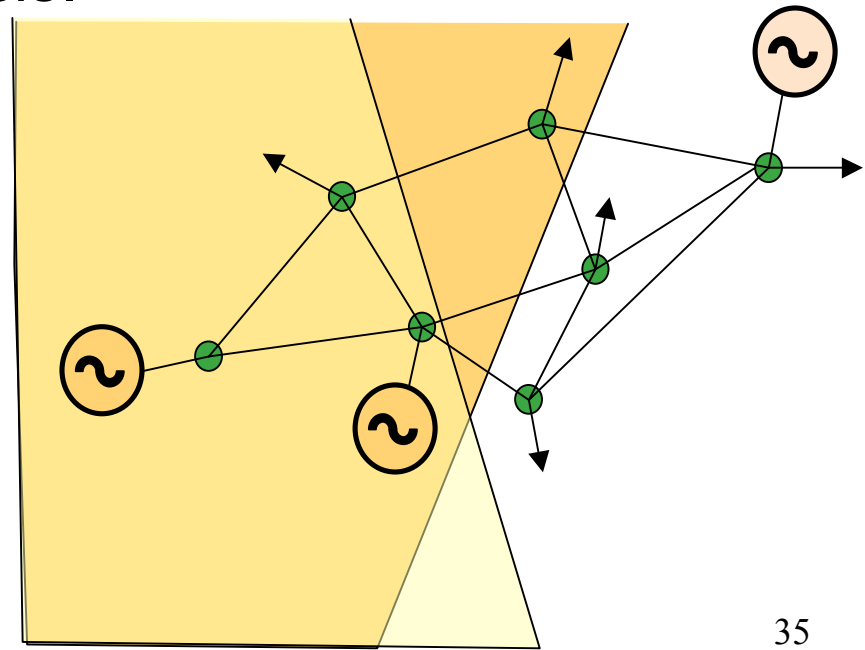
each vector takes the form $c e_{st}$

$$e_{st} = \begin{bmatrix} \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ 0 \\ \vdots \end{bmatrix}$$

We Claim...

Each vector in a basis of the null space of the dispatch/price sensitivity matrix corresponds to a **graph partitioning indicator** vector. The intersection of different partitions gives rise to correlated rows in the basis.

$$\text{Basis} = \begin{bmatrix} + & + & + \\ + & + & + \\ + & + & + \\ + & - & + \\ + & - & - \\ + & - & - \\ + & - & - \end{bmatrix}$$



Proof Outline– Graph Theory

Rewrite $A^T B A x = c e_{st}$
as

$$\begin{bmatrix} Q_+ & R \\ R^T & Q_- \end{bmatrix} \begin{bmatrix} x_+ \\ x_- \end{bmatrix} = \alpha \begin{bmatrix} e_s \\ e_t \end{bmatrix}$$

necessary because

$$e_s = \begin{bmatrix} \vdots \\ 0 \\ +1 \\ 0 \\ \vdots \end{bmatrix} \quad e_t = \begin{bmatrix} \vdots \\ 0 \\ -1 \\ 0 \\ \vdots \end{bmatrix}$$

NOTE: Elements of R are non-positive. At least one element is negative since the graph is connected. Q_+ and Q_- are both positive definite.

$$\begin{bmatrix} x_+^T & 0 \end{bmatrix} \begin{bmatrix} Q_+ & R \\ R^T & Q_- \end{bmatrix} \begin{bmatrix} x_+ \\ 0 \end{bmatrix} = x_+^T Q_+ x_+ = \alpha \begin{bmatrix} x_+^T & 0 \end{bmatrix} e_s > 0$$

Proof Outline– Graph Theory

$$\begin{bmatrix} Q_+ & R \\ R^T & Q_- \end{bmatrix} \begin{bmatrix} x_+ \\ x_- \end{bmatrix} = \alpha \begin{bmatrix} e_s \\ e_t \end{bmatrix} \quad \text{Vector } x_+ \text{ must correspond to a connected subgraph.}$$

Proof by contradiction. Assume that x_+ corresponds to two or more disconnected subgraphs. Then

$$\begin{bmatrix} x_{1+}^T & x_{2+}^T \end{bmatrix} \begin{bmatrix} Q_{1+} & 0 \\ 0 & Q_{2+} \end{bmatrix} \begin{bmatrix} x_{1+} \\ x_{2+} \end{bmatrix} = \alpha \begin{bmatrix} x_{1+}^T & x_{2+}^T \end{bmatrix} e_s > 0$$

Since e_s has a single nonzero element, either

$$x_{1+}^T Q_{1+} x_{1+} = 0 \quad \text{or} \quad x_{2+}^T Q_{2+} x_{2+} = 0$$

There is a contradiction because both Q_{1+} and Q_{2+} are positive definite!

Load Pocket Identification

The columns of Σ are partition indicator vectors.
A basis for the “cost” perturbation vectors

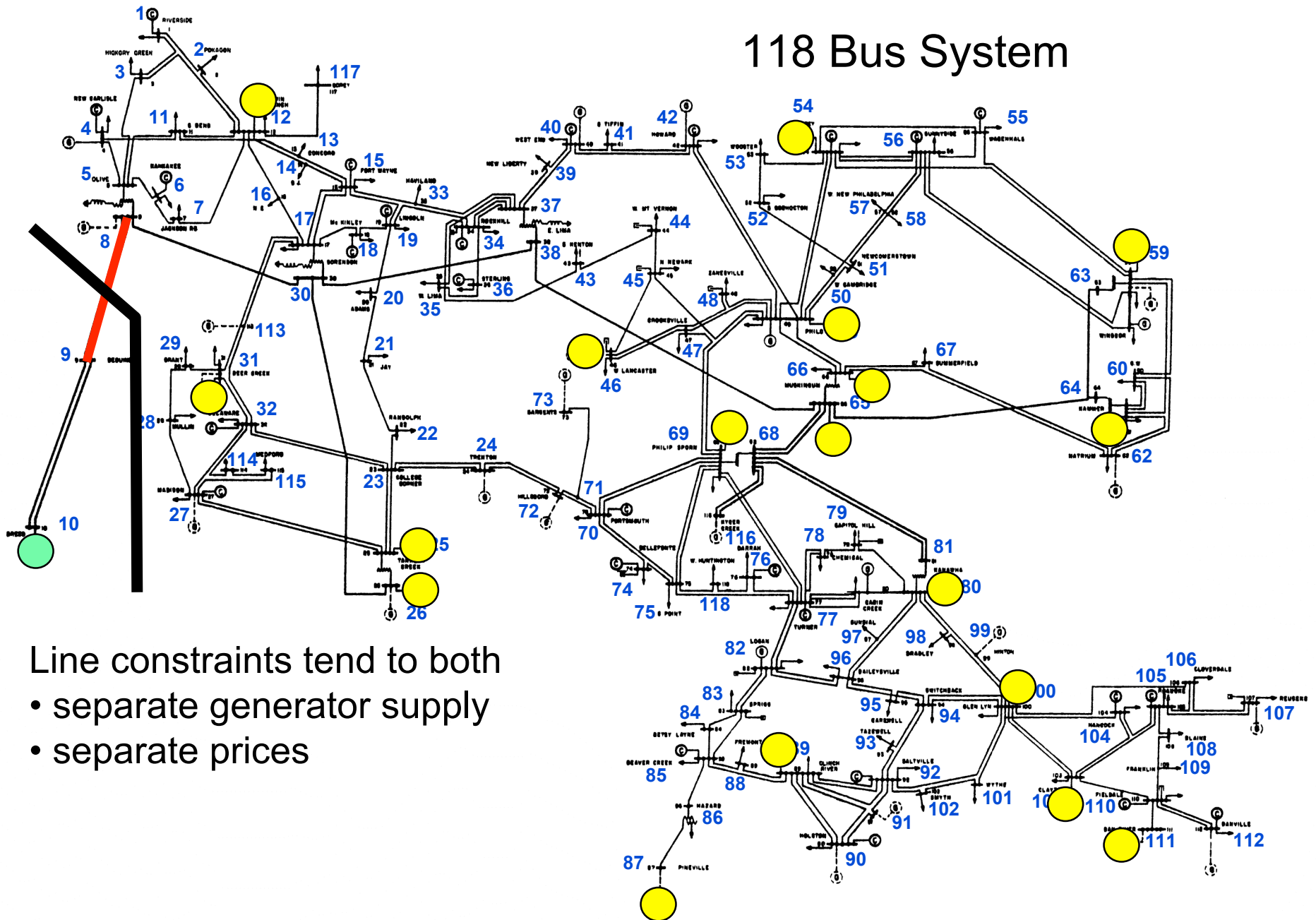
$$\text{basis for } -\Delta\lambda = \begin{bmatrix} I & 0 \end{bmatrix} \Sigma$$

$$\text{Basis} = \begin{bmatrix} + & + & + \\ + & + & + \\ + & + & + \\ + & - & + \\ + & - & - \\ + & - & - \\ + & - & - \end{bmatrix}$$

–Identify suppliers that belong to non-substitutable regions.

–Specific analysis: Identify (small number of) suppliers that can *change prices without changing dispatch*.

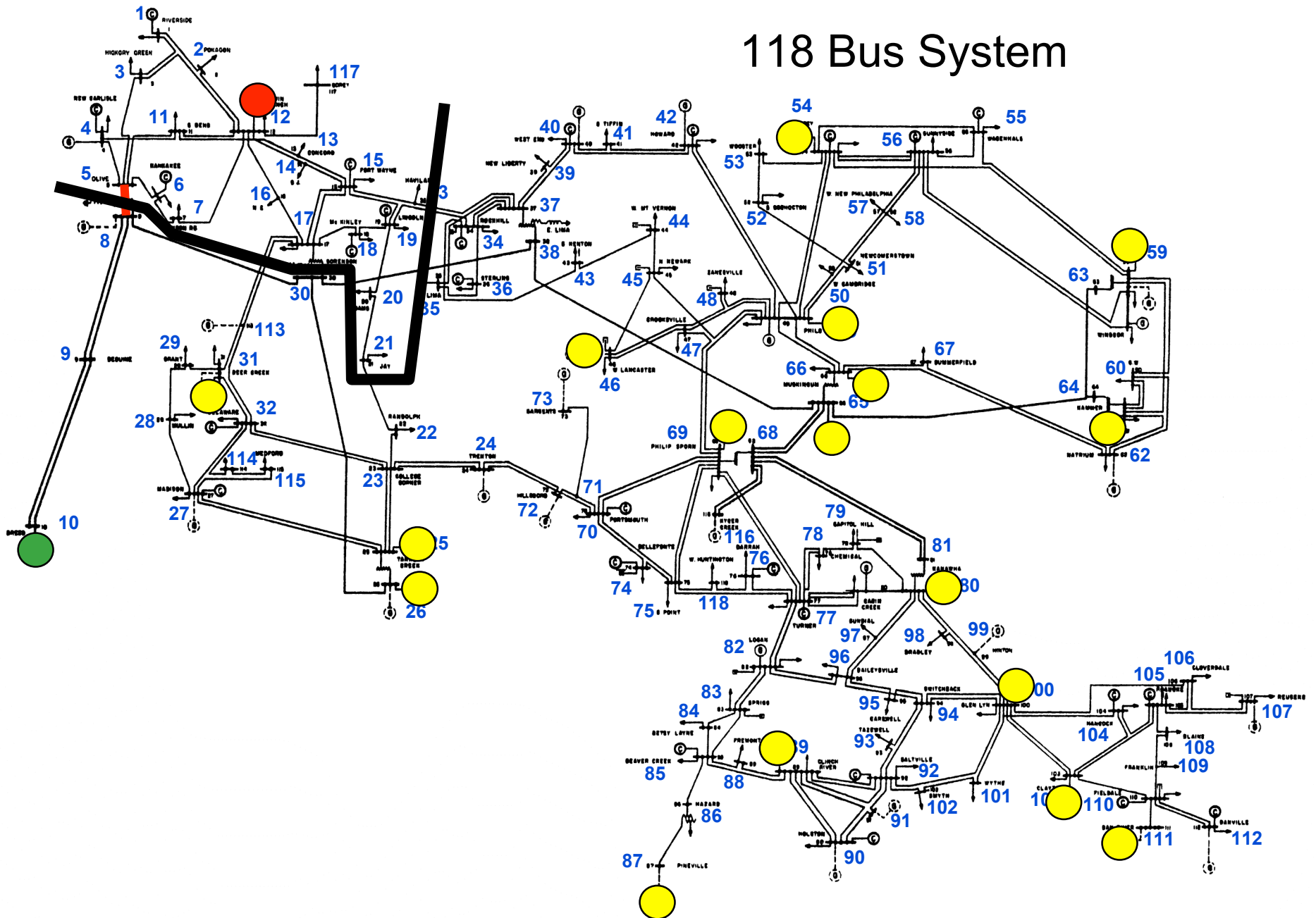
118 Bus System

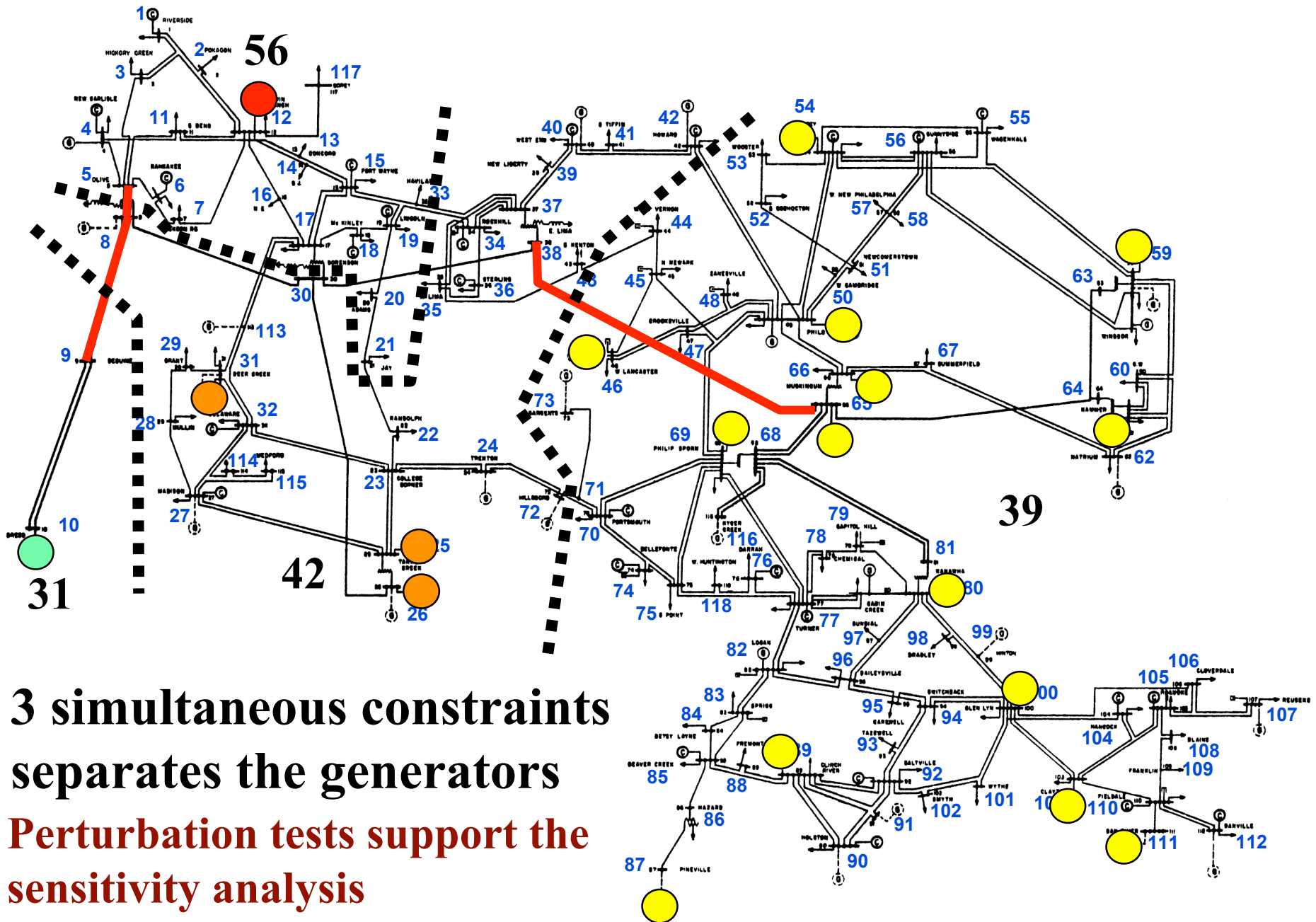


Line constraints tend to both

- separate generator supply
- separate prices

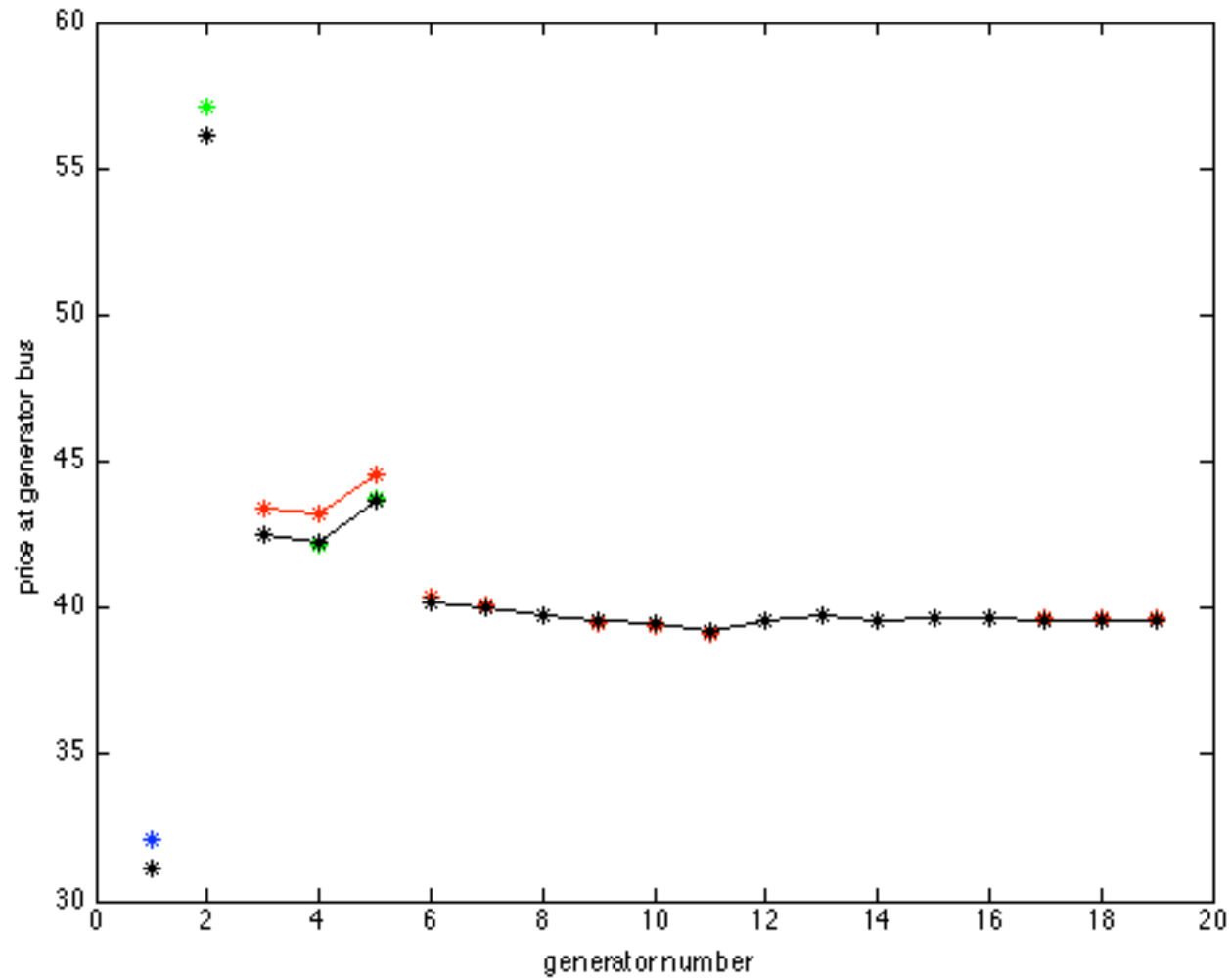
118 Bus System





Perturbation Analysis

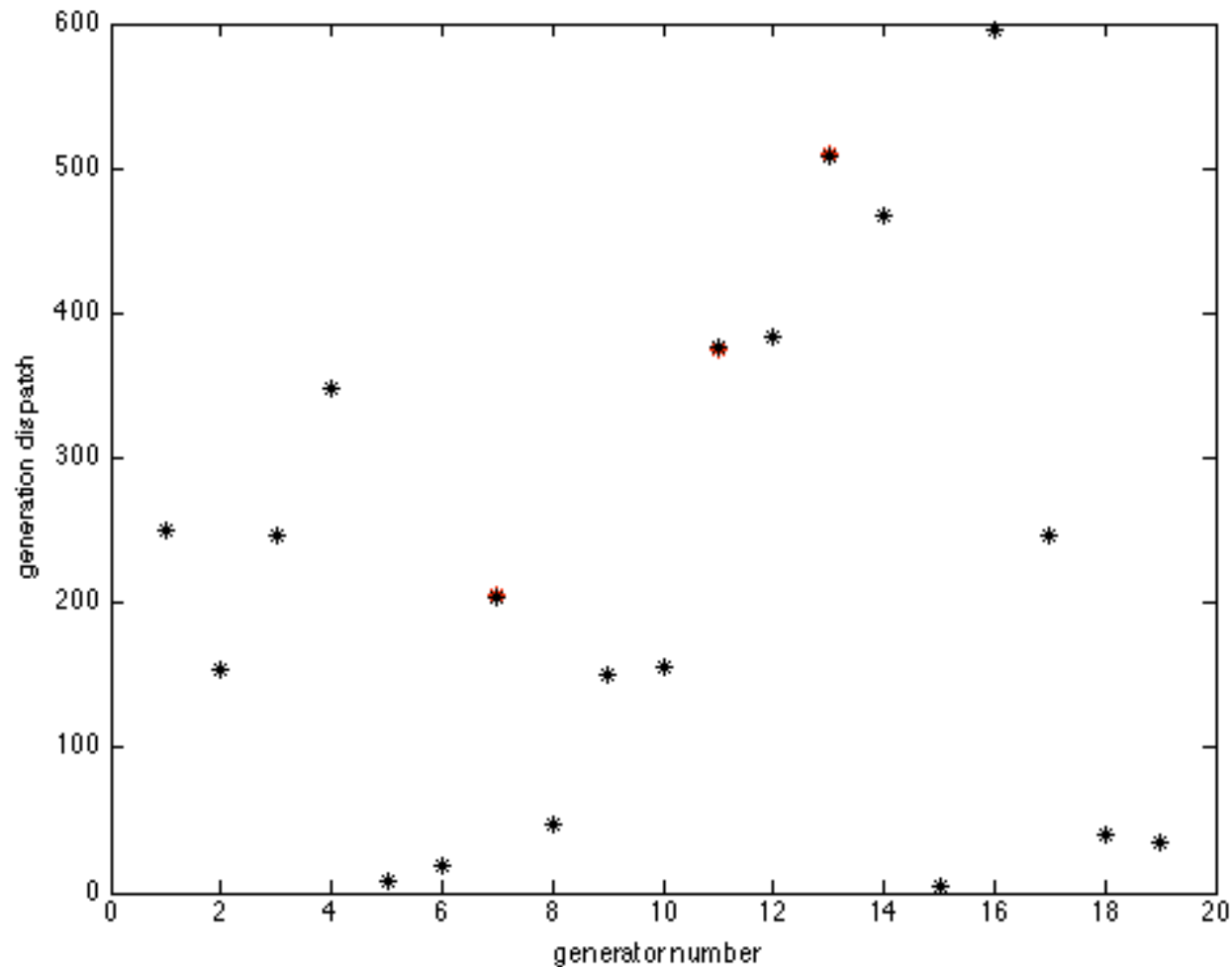
(just checking)



Increase
Prices by
\$1/MWh
(DC OPF)

Perturbation Analysis

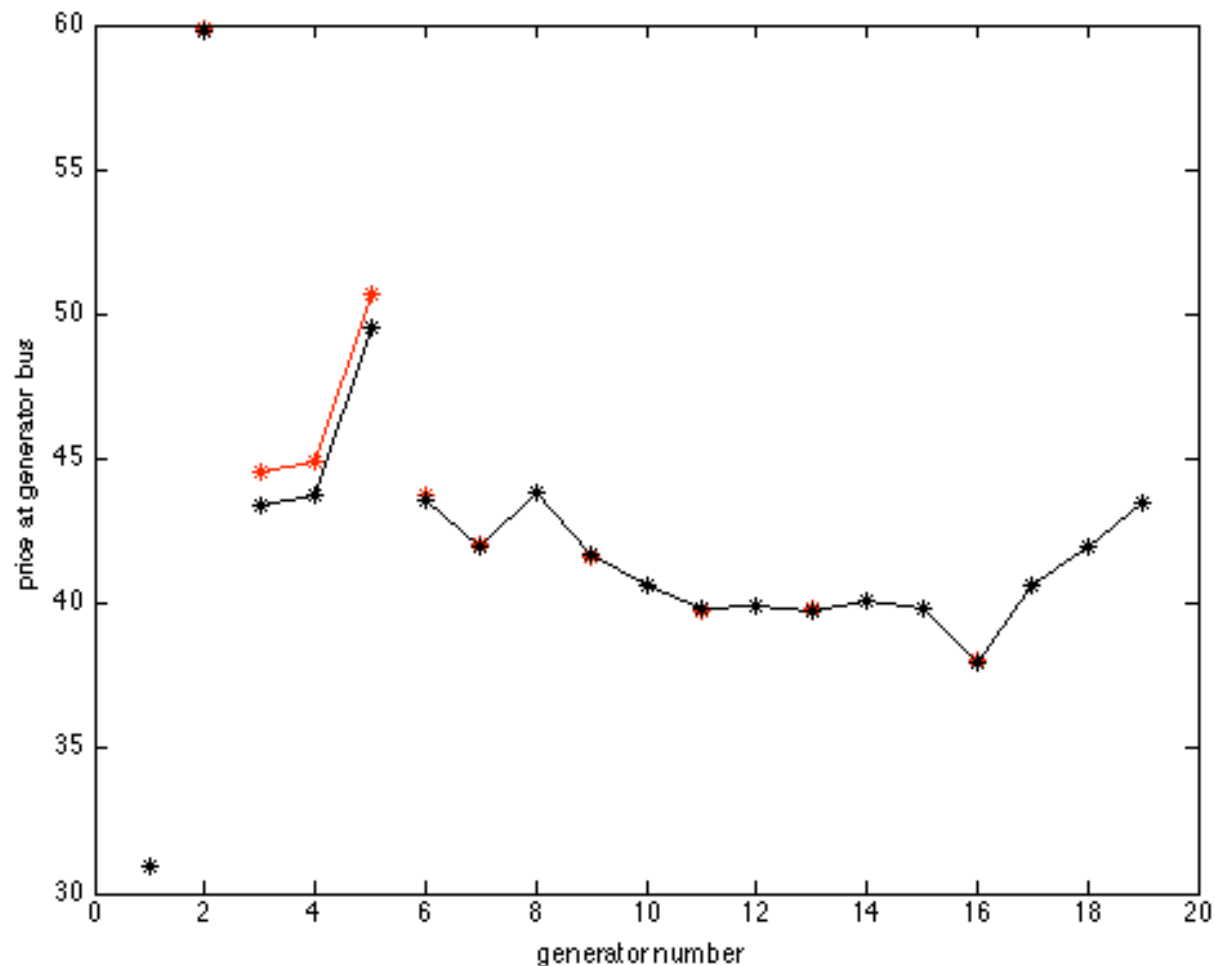
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Dispatches
don't change
(DCOPF)

Perturbation Analysis

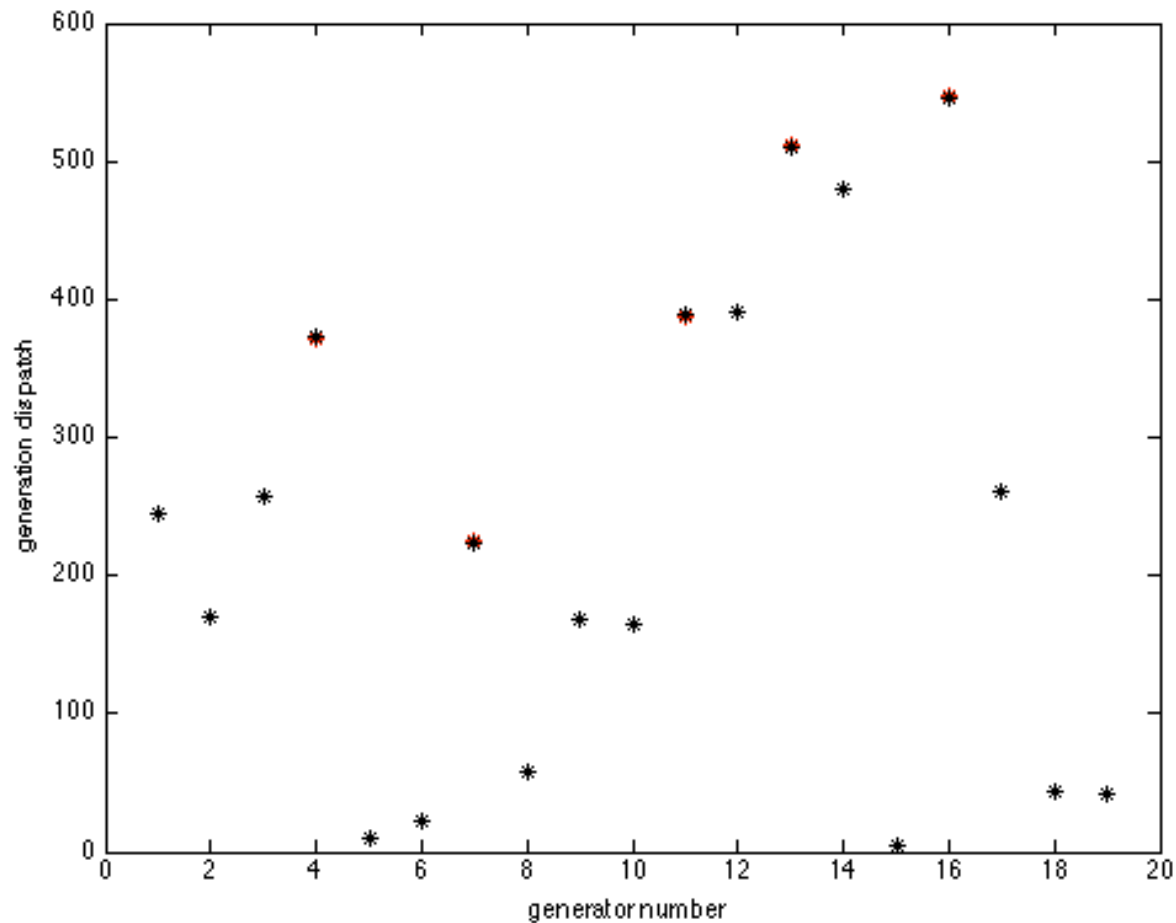
(just checking)



Increase
Prices by
\$1/MWh
In group of
Three
generators
(AC OPF)

Perturbation Analysis

(just checking)



Dispatches
don't change
(ACOPF)

Identify Small Groups

- Vectors of price perturbation for zero dispatch change

-0.99	0.32	0.34	1.00		1.00	-0.00	-0.00	0
0.02	-0.45	0.33	1.00		-0.00	1.00	-0.00	-0.00
0.02	0.06	0.27	1.00		-0.00	-0.02	0.93	0.02
0.02	0.08	0.29	1.00		-0.00	-0.05	1.00	-0.02
0.02	0.02	0.29	1.00		0.00	0.07	0.87	-0.00
0.02	0.06	-0.12	1.00	Regroup	0.00	0.00	0.16	0.74
0.02	0.06	-0.16	1.00		0.00	0.00	0.10	0.80
0.02	0.06	-0.20	1.00		0.00	0.00	0.03	0.87
0.02	0.06	-0.23	1.00		0.00	-0.00	-0.03	0.93
0.02	0.06	-0.24	1.00		-0.00	-0.00	-0.05	0.95
0.02	0.06	-0.27	1.00		-0.00	-0.00	-0.10	1.00
0.02	0.06	-0.22	1.00		-0.00	-0.00	-0.02	0.92
0.02	0.06	-0.20	1.00		0	0.00	0.02	0.88
0.02	0.06	-0.22	1.00		0.00	0.00	-0.01	0.91
0.02	0.06	-0.21	1.00		-0.00	0.00	-0.00	0.90
0.02	0.06	-0.21	1.00		-0.00	0.00	-0.00	0.91
0.02	0.06	-0.22	1.00		0.00	0.00	-0.01	0.91
0.02	0.06	-0.22	1.00		0.00	0.00	-0.01	0.91
0.02	0.06	-0.22	1.00		-0.00	0.00	-0.01	0.91

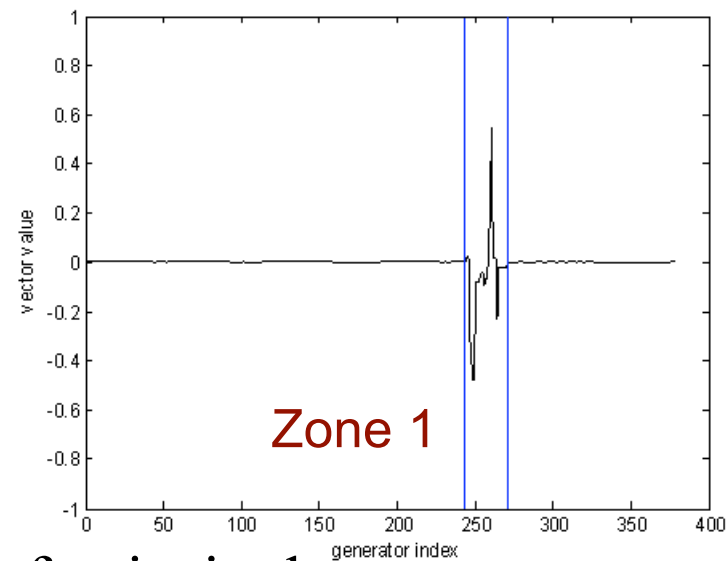
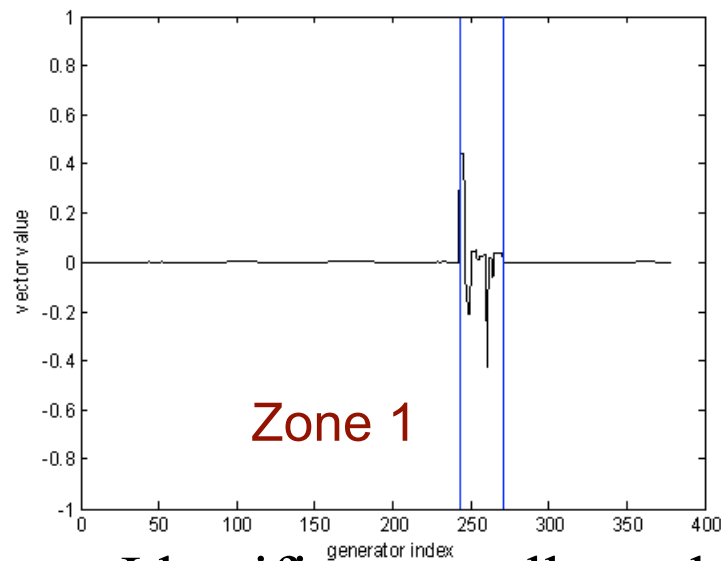
Simultaneous
Effect of
All
constraints

by constraint

Note correlations in rows

Large System Results

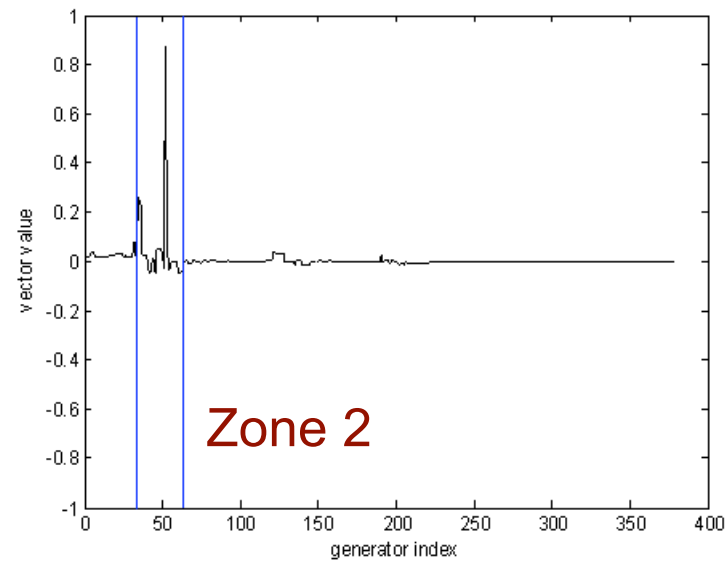
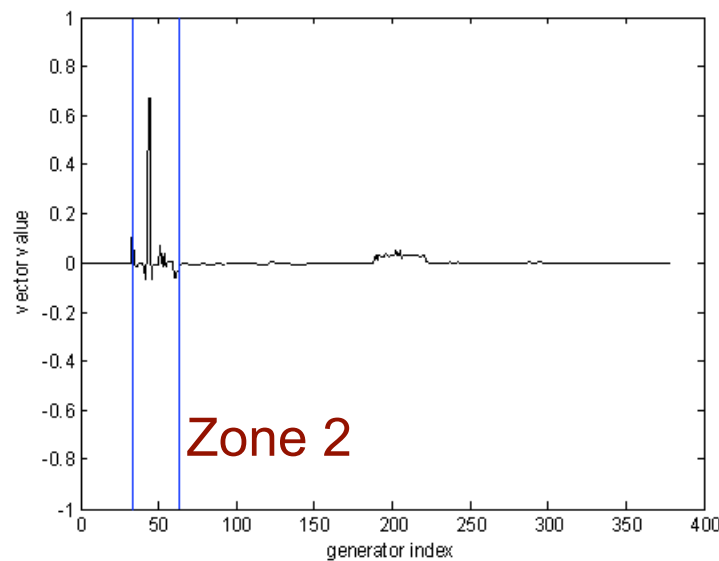
PJM Heavily-Loaded Case, five binding constraints



Identifies a small number of units in the same zone that can change price without changing dispatch.

Large System Results

PJM Heavily-Loaded Case, five binding constraints



Identifies a small number of units in the same zone that can change price without changing dispatch.

Summary

- We've developed a practical system for identifying suppliers with market power potential.
- It uses a sensitivity-based approach that accounts for the network model.
- Demonstrated results on large realistic models.
- Presently developing visualization techniques.